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Knowledge based optimization

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Optimization theory has advanced considerably during the last three decades, as illustrated by a vast number of published books, surveys, and papers concerning this subject. However, for optimization of complex systems that may not be modeled exactly by Ludwig von Bertalanffy's general system theory, we may need a new philosophy based on rather non-conventional logics. It is represented bellow.

Keywords: complex systems, multi-objective optimization, fuzzy logic, modal logic, multi-valued logic

1. Introduction

On the one hand, a complex system is a system that characterizes by a collection of multi-agents, nonlinear interactions, unanticipated global behaviors that result from interaction of agents, between agents and environment. Here, the local interactions among agents may be continuously recombined, revised and the behavior of system tends to arise more from the interactions among its agents than from any constraints imposed onto the structure of the whole system. Moreover, a complex adaptive system, smart structure for example, is a system that adapts itself by self-learning. Its control strategy, by any way, must be as optimum as possible. Hence modeling of these systems requires more than a deductive theory that is based on clear definitions and axioms based on two-valued logic – general basics of Bertalanffy's general systems theory. This is why new modeling paradigms such as cellular automata, multi-agent systems, fuzzy systems and others are generated in modern studies of complexity. This demonstrates the necessity of intelligent decisions based on human knowledge during the optimization process, taking into account both the quantitative, qualitative and changing in time properties of these systems. On the other hand, although an engineer's objectives are usually very general or ambiguous; their decisions are dependent on many factors not known with certainty, the success of the engineering approach is, in fact, evident from the history of technology and the applied science. It reflects correctness of the logical principles of engineers in relation to the real world.

As it is well known, the interpretation of formulae of classical proposition calculus as functions over any Boolean algebra is a generation of the well known truth-table method. It has been known for a long time that the lack of modal connectives: 'possible', necessary' etc., for reasoning would now be hardly recognizable even to engineers who in many cases use other than two-valued logic. Then, without fundamentals of formal logic for reasoning, a decision-maker might, in many cases, use mythical logic. It may result in a set of incomplete decisions, which are sometimes contrary to solutions resulting from mathematical models. To solve this problem, an approach based on the fuzzy set theory was proposed by Bellman and Zadeh (1970) [2], in which the decision-making process is a restriction of objectives and constraints to the values, x, to be chosen by decision-maker. The restriction is given by two fuzzy subsets A and B, their intersection gives the range of the values x satisfying both the objectives and constraints, i.e., it gives the subset C: $\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\}$. An *a priori* justification of union and intersection operators (presented in Bellman's and Zadeh's definition) being suitable for each specific real-world problem is however highly problematic in practice, as pointed out by Sakawa (1993) [6]. In this work, a multi-objective optimization problem is solved by, firstly, using conventional method in order to obtain the Pareto solution set and, secondly, using an expert's experiences to evaluate these solutions, important level of each objective function, and finding the optimization solution based on non-conventional logic.

2. System modeling

Complex system: In the "soil-structure" system, for example, the behavior of a structure is dependant not only a given loading but also on mutual influence of its surrounding media, i.e., any change in the shape of a structure will result in increased or reduced influence of its environment, which cannot be neglected. Besides, the shape of a structure is determined by a series of parameters, which are also mutually dependent on each other. It is represented in the following figure:

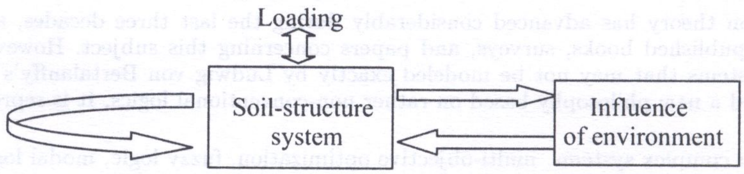

Fig. 1. The complex "soil-structure" system

Then, even the best solution for a single parameter will not suffice to obtain a satisfactory functional effect for the whole system of parameters of the structure i.e., the general property of the structure cannot be explained by reviewing each of its parameters separately one by one. In addition, fuzziness and uncertainties regarding both input data and mutual relations mentioned above lead to a more complex system.

Adaptive systems: Adding sensory to the material or structure leads to what can be called a sensory material or structure. If material or a structure includes also actuators, they will be called an active material or structure (see Boller Chr. [3]). Consider a cantilever beam, for example, of length L, which is shown in the following figure:

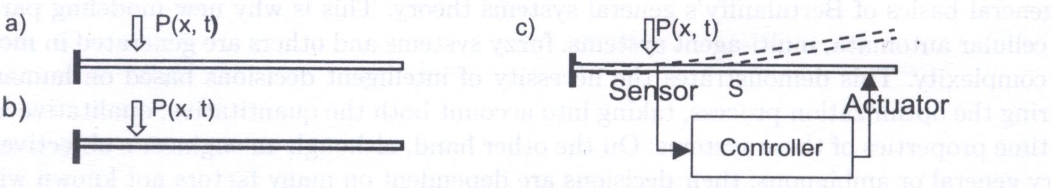

Fig. 2. Vibration control structure (a), passive control (b), active control (c)

with a force $P(x,t)$ applied at a distance x from its fixed end at time t, resulting in a deflection $y(x,t)$ of the beam from its stationary position at the point where the force has been applied. The motion of the beam in transverse vibration is governed by the fourth-order partial differential equation:

$$\frac{(EI)}{(\rho A)}\frac{\partial^4 y(x,t)}{\partial x^4} + \frac{\partial^2 y(x,t)}{\partial t^2} = \frac{1}{m} P(x,t) \tag{1}$$

where ρ, A, I and E denote the mass density, cross-sectional area, moment of inertia of the beam and the Young modulus respectively. The corresponding boundary conditions at the fixed and free ends of the beam are given by:

$$[y(0,t) = 0] \wedge \left[\frac{\partial y(0,t)}{\partial x} = 0\right]; \quad \left[\frac{\partial^2 y(L,t)}{\partial x^2} = 0\right] \wedge \left[\frac{\partial^3 y(L,t)}{\partial x^3} = 0\right]. \tag{2}$$

Traditional passive control of vibration suppression consist of mounting passive material on the beam (Fig. 2b) i.e. by increasing lateral stiffness through bracing. Active vibration control (Fig. 2c), in the current trend towards lightweight structures. It consists of rather artificially generating canceling source, processed by the self-turning control algorithm and fed to the actuator in order to destructively interfere with the unwanted source detected by the sensor when an earthquake or other shock, wind for example, occurs. It is clear, in this case, that although the conventional theory of automata:

$$S_{t+1} = f(S_t, U_t); \qquad Y_t = g(S_t, U_t) \tag{3}$$

in which, S_t, U_t, Y_t denote state, the input and the output at time t, respectively; or differential equations:

$$\frac{\mathrm{d}Y_i}{\mathrm{d}t} = f_i\left(Y_1, Y_2, ..., Y_n\right), \qquad i = 1, 2, ..., m \tag{4}$$

have made a significant contribution to the Bertalanffy's general system theory, they are not sufficient for dealing with complex and complex adaptive systems. Modern studies in complexity have generated new kinds of modeling paradigms such as cellular automata, Wolfram (1984, 1988) [8, 9] using the production rule:

$$\begin{gathered}\{\mathrm{state}(x, y, t) = 1\} \wedge \{\mathrm{state}(x,\ y - 1,\ t) = 1\} \wedge \{\mathrm{state}(x,\ y + 1,\ t) = 1\} \\ \wedge\{\mathrm{state}(x', y', t) = 0 \quad \text{for the other cells} \quad (x', y') \quad \text{in vicinity}\}\end{gathered} \tag{5}$$

or 'if – then' rule for fuzzy systems, (Wang L. and Mendel JM. 1991) [7]:

$$R^j : \text{if } \{(x_1 = A_1^j) \wedge (x_2 = A_2^j) \wedge ... \wedge (x_n = A_n^j)\} \text{ then } (y = B^j) \tag{6}$$

and others, in which, human behavior and knowledge are incorporated. It is indicated that modeling of the complex systems may require logical truth values that lie wholly beyond the framework of two-valued logic.

3. Knowledge based optimization

Truth: From a logical point of view, we recognize the real world through discrete values of the truth. In human history, primitive man described discrete values of truth, 'the being' and 'the non-being', by a flame. In Aristotle's syllogism it appeared as distinguishing any things existing and non-existing in respect to the real world. The first value of the truth was stated by true (T) and then $\{\neg T = F$ (false)$\}$. Although Aristotle himself, who developed the theory of the syllogism in a nearly perfect form based on two-valued logic, worked also on a theory of modal syllogism, in which premises and conclusions may contain the terms "necessary, $\Box$" and "possible, $\Diamond$", i.e., he used modal logic. Multi-valued logic in which, the truth-values are labeled by rational numbers in the unit interval [0,1] was created by Łukasiewicz in 1920. Other logic, in which linguistic variables are introduced – fuzzy logic, based on multi-valued logic was created by Zadeh in 1962 [10]. Changing of the symbols of the truth is represented in the Fig. 3.

Since the civilization began, number has been regarded as a symbol of truth. Here, it is important to note that "truth" of primitive man – flame characterizes the real existence of any agent being in the real world, while "truth" of civilizable man characterizes rather our logical reason only. For example, in Levis's modal logic – "truth" is considered as the ultimate status symbol of possibility. Let us return to the early "black or white" reasoning, for example, beginning with Aristotle's syllogistic, i.e. using the two symbols: T (true) and F (false), we represent "black" by two possibilities $(B, \neg B)$ and "white" by, $(W, \neg W)$. Then, we have 4 possible cases:

$$\text{"black or white"} = \{BW,\ B\neg W,\ \neg BW,\ \neg B\neg W\}. \tag{7}$$

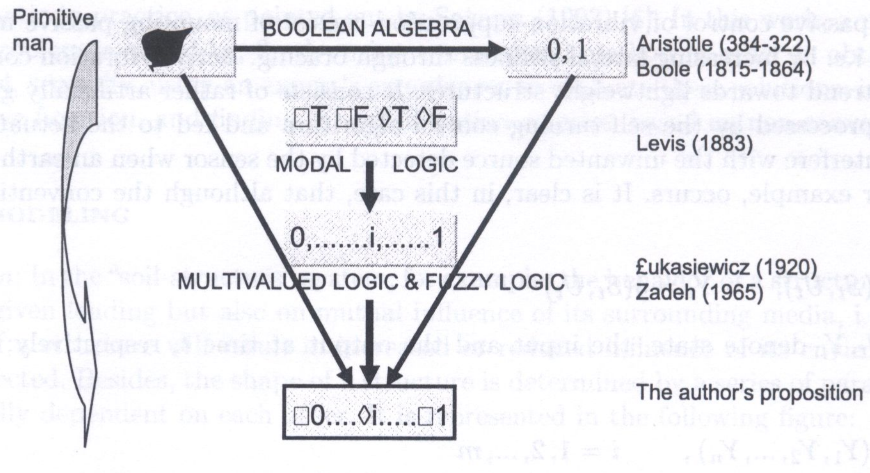

Fig. 3. The evolution of the truth

Now, from modal logic using box connective, □, and diamond connective, ◊, we can represent "black" by 4 symbols, ($\Box B$, $\Diamond B$, $\neg\Box B$, $\neg\Diamond B$), where, $\Box B$ and $\Diamond B$ are read: "B is necessary" and "B is possible" respectively; "white" by ($\Box W$, $\Diamond W$, $\neg\Box W$, $\neg\Diamond W$). We can obtain 16 possible cases:

$$\text{"black or white"} = \{\Box B\Box W, \Box B\Diamond W, \Box B\neg\Box W, \Box B\neg\Diamond W, \Diamond B\Box W, \Diamond B\Diamond W, \Diamond B\neg\Box W, \Diamond B\neg\Diamond W,$$
$$\neg\Box B\Box W, \neg\Box B\Diamond W, \neg\Box B\neg\Box W, \neg\Box\neg\Diamond W, \neg\Diamond B\Box W, \neg\Diamond B\Diamond W, \neg\Diamond B\neg\Box W, \neg\Diamond B\neg\Diamond W\}. \quad (8)$$

It is clear that many possible cases, which may be more practical, may be lost in reasoning based on two-valued logic. It expresses an effect of modal logic. Semantics of this logic was represented by Saul Kripke (1956) (see Boolos [4]), in which, Leibniz's fantasy of the actual world seems as one "possible world" among other. It can be represented by the following triple K:

$$K = \{W, R, V\}; \qquad R \subset W^2$$

$$\bigvee_{w_i \subset W} \bigwedge_{w_j \subset W} (w_i R w_j) : \{V(\Box A, w_i) = \text{true iff } [V(A, w_j) = \text{true}]\}, \quad i, j \in \{1, 2, ..., n\}, \quad i \neq j \quad (9)$$

in which, W denotes the set of possible worlds, R is binary relation (between possible worlds), V is relation between sentence letters and world, and A denotes a sentence. Algebraic investigations concerning modal logic lead to interpretations the algebra of modal-fuzzy truth-values, τ, (see Chi 2000 [5]) as follows.

$$\tau : w \to [0, 1]. \quad (10)$$

It enables us to define arithmetic operations on modal-fuzzy truth-values in terms of arithmetic operations on fuzzy numbers. From that, truth dimension based on modal-fuzzy truth-values, like tense logic, is intended as kind of universal quantifiers of uncertainties over time. In this dimension, the logical link – an implication we deal with, is not a causal link based on two-valued logic:

$$p \to q = 0 \text{ or } 1. \quad (11)$$

As it is not certain, we may rather use modal operator "possible", which leads to implication in the framework of modal logic:

$$p \Rightarrow q = \neg\Diamond(p \wedge \neg q) \quad (12)$$

which represents exactly what Levis had in mind by saying that "implicate" is uncertain. Łukasiewicz's logic implication (multi-valued logic) enables us to quantify this uncertainty:

$$p \to q = \min[1, 1 - \tau(p) + \tau(q)]. \quad (13)$$

Moreover, in truth dimension we can use a special linguistic term called linguistic hedges (or simply hedges) generated in fuzzy logic, by which other linguistic terms are modified, to modify fuzzy estimates. For example, the proposition "ϕ is big", which is assumed to mean "ϕ is big is true", may be modified by the hedge "very" or "fairly" in the following ways:

$$\text{“}\phi \text{ is very big is true”}, \quad \text{“}\phi \text{ is big is very true”}, \quad \text{“}\phi \text{ is big is fairly true”}. \tag{14}$$

The hedges "very" and "fairly" are interpreted as the unary operations:

$$\text{very}(a) = a^2, \qquad \text{fairly}(a) = a^{1/2}. \tag{15}$$

It indicates that the first-order Boolean logic restricts itself to the study of true and false logic expressions. For complex and adaptive systems, however, we need logical truths that lie wholly beyond the limit of first-order logic. A broader analysis of the inner structure of logic expressions is achieved in predicate logic and non-conventional logics (modal logic, multi-valued logic and fuzzy logic), which we call higher-order logics. Then, to obtain intelligent decision during optimization process, important changes in philosophy of optimization are required.

4. Multi-objective optimization based on expert's experiences

Generally, structural optimization problem has been restricted to the multi-objective optimization model:

$$\underset{\forall \mathbf{X}\in E, \mathbf{Y}\in R}{\text{maximum(minimum)}}\{\mathbf{F} = (f_1(\mathbf{X},\mathbf{Y}), f_2(\mathbf{X},\mathbf{Y}), .., f_k(\mathbf{X},\mathbf{Y}))\} \tag{16}$$

where: $\mathbf{X} = (x_1, x_2, ..., x_n)$ is a vector of design variables, $\mathbf{Y} = (y_1, y_2, ..., y_r)$ is a vector of random variables, $f_i(\mathbf{X},\mathbf{Y})$, $i = 1, 2, ..., k$ are objective functions, k denotes number of objective functions, $R = \{y|u(y)\}$, where $u(y)$ is any probability distribution of random variables, E denotes the permissible space which is expressed as:

$$E = \{\mathbf{X}|g_j(\mathbf{X},\mathbf{Y}) \geq 0, j = 1, 2, ..., m\} \tag{17}$$

where: $g(\mathbf{X}, \mathbf{Y})$ denotes the design constraints, m is number of the design constraints. Let a set Q be in the domain of the objective function, f, such that:

$$f : Q \to RL \tag{18}$$

where RL is an order structure of solution space used to define alternative solution. When the preferences between alternatives are described by one objective function, some logical operators can be used to observe two directions characterized by two relations: greater or lower one. Here, "true" and "false" are sufficient to define either existence or non-existence of the optimal solution. When, in contrast, preferences of the alternatives described in multi-objective optimization are defined using the Pareto-optimal concept, in which general system of objective functions becomes:

$$\min_{\mathbf{X}\in E, \mathbf{Y}\in R} \{\mathbf{Z}(\mathbf{X},\mathbf{Y}) = [z_1(\mathbf{X},\mathbf{Y}), z_2(\mathbf{X},\mathbf{Y}), ..., z_k(\mathbf{X},\mathbf{Y})]\} \tag{19}$$

i.e., evaluation of each objective function, $f(X) \in P$, where P denotes the Pareto solution set obtained from conventional method, is based on its relative deviation $Z(\mathbf{X})$ from the extreme solution $f^0(\mathbf{X})$:

$$\bigwedge_{f_r(\mathbf{X})\in P} Z_r(\mathbf{X}) = \frac{|f_r(\mathbf{X}) - f_r^0(\mathbf{X})|}{f_r^0(\mathbf{X})} \tag{20}$$

We suppose:

$$Z_r(\mathbf{X}^i) < Z_r(\mathbf{X}^j) \Leftrightarrow f_r(\mathbf{X}^i) \succ f_r(\mathbf{X}^j), \quad r = 1, 2, ..., k \tag{21}$$

where "$\succ$" denotes the preferred relation, is read as "better" or "predominates". It is seen that "$\succ$" is a partial ordering over Q. Each value z_i, $i = 1, 2, ..., k$ of the attribute $\mathbf{Z}$ may be described in the real line as a vector:

$$\mathrm{z}_i \overset{RL}{=} (\mathrm{z}_i^1, \mathrm{z}_i^2, ..., \mathrm{z}_i^n). \tag{22}$$

Evaluation by an expert, usually through truth measures, maps a concrete value z_i^n from a semantic scale (measuring in various traditional units) onto a universal scale (measuring in truth measures). On this scale, the linguistic estimates of the expert may be formalized as fuzzy subset independent on the semantic of the objective functions.

$$\tau : \mathbf{Z} \to [0,1]. \tag{23}$$

It represents the degree of the expert's aspiration according to each objective function. Let ${}^0z_i$ be the totally required level for f_i while ${}_0z_i$ is an unacceptable level for f_i. The truth function $\tau(\mathbf{X},\mathbf{Y})$, which is a strictly monotone and continuous function with respect to $\mathrm{z}_i(\mathbf{X}, \mathbf{Y})$, will be determined as truth measure as follows:

$$\tau(\mathbf{X},\mathbf{Y}) = \begin{cases} 0 & \text{for} : z_i(\mathbf{X},\mathbf{Y}) = {}_0z_i(\mathbf{X},\mathbf{Y}), \\ 1 - z_i(\mathbf{X},\mathbf{Y}) & \text{for} : z_i(\mathbf{X},\mathbf{Y}) \in ({}^0z_i(\mathbf{X},\mathbf{Y}), {}_0z_i(\mathbf{X},\mathbf{Y})), \\ 1 & \text{for} : z_i(\mathbf{X},\mathbf{Y}) = {}^0z_i(\mathbf{X},\mathbf{Y}). \end{cases} \tag{24}$$

It is then possible to transfer, through truth measures, the values of the objective functions from the semantic scale to linguistic estimates on the universal scale. Then, for each value z_i we have correspondingly a fuzzy truth value (in the sense of multi-valued logic). Then an ideal solution $\mathbf{X}^*$ is perceived as the preferred solution in respect of truth τ so that z_i should be in the left vicinity of ${}^0\mathrm{z}_i$. To express the experiences extracted from the expert, three hedges are used: T, FT, VT (true, fairly true, very true). We can construct a modified proposition, H, using a linguistic hedge representing the expert's judgment to restrict $\tau(\mathbf{X}, \mathbf{Y})$ to which any hedge will be assigned. The preferred degree of the expert's estimates is presented by $\tau^*(\mathrm{z})$:

$$\tau^*(\mathrm{z}) = TFM\{H(\tau(X, Y)) = s, \qquad s \in (T, FT, VT)\} \tag{25}$$

where, $TFM\{.\}$ is Zadeh–Baldwin's logic operation called the truth function modification (see Baldwin 1979 [1]). In truth dimension, by verbalization procedure, the expert can change any knowledge in respect of truth as well as integration with another. In multi-optimization problems, let $\tau^{\text{imp}}(f_i(\mathbf{X}, \mathbf{Y})) \in [0,1]$ determined by designer, be an important degree of the objective function f_i. Then, each Pareto-optimal solution must be subjected to the condition: "All of the preferred degrees of the expert $\tau^*(\mathrm{z}_i)$, $i = 1, 2, ..., k$ are satisfied with important degrees of all the corresponding objective function $\tau^{\text{imp}}(f_i)$". That is to say:

"It is not possible that both important degrees of all the corresponding objective functions are true and decision-maker's estimates are false"

In logic term of modal logic, we can describe this condition as follows:

$$\neg\Diamond\{(\tau^{\text{imp}}(f_i) = \text{true}) \wedge \neg(\tau^*(\mathrm{z}_i) = \text{true})\}. \tag{26}$$

It is the interpretation of logic implication "$\Rightarrow$" of modal logic, i.e.,

$$\{\tau^{\text{imp}}(f_i) = \text{true}\} \Rightarrow \{\tau^*(\mathrm{z}_i) = \text{true}\} \quad \equiv \quad I\{[\tau^{\text{imp}}(f_i) = \text{true}], [\tau^*(\mathrm{z}_i) = \text{true}]\} \tag{27}$$

where, I denotes a fuzzy implication, which is a function expressed in the form:

$$I : [0,1] \times [0,1] \to [0,1]. \tag{28}$$

We can perform the quantification of this fuzzy implication using multi-valued logic.

$$I\{\tau^{\text{imp}}(f_i), \tau^*(z_i)\} = \min\{1, 1 - \tau^{\text{imp}}(f_i) + \tau^*(z_i)\}. \tag{29}$$

Thus, we obtain a new condition, for which an "optimistic" possibility of the decision maker is formulated as:

$$\underset{i}{\text{minimum}} \quad I\{\tau^{\text{imp}}(f_i(\mathbf{X}, \mathbf{Y})), \tau^*(z_i)\}, \quad i = 1, 2, 3, ..., k. \tag{30}$$

The optimal solution, SF, of the fuzzy optimization problem can be defined by maximizing "optimistic" possibility of the decision-maker i.e., using the max-min strategy for a game:

$$SF = \max \underset{i}{\text{minimum}} \; I\{\tau^{\text{imp}}(f_i(\mathbf{X}, \mathbf{Y})_j), \tau^*(z_i)_j\}, \quad i = 1, 2, 3, ..., k \tag{31}$$

where, $j = 1, 2, ..., m$, which denotes j-th solution being in the Pareto-solution set.

5. Calculation example

Figure 4 submits the calculation scheme:

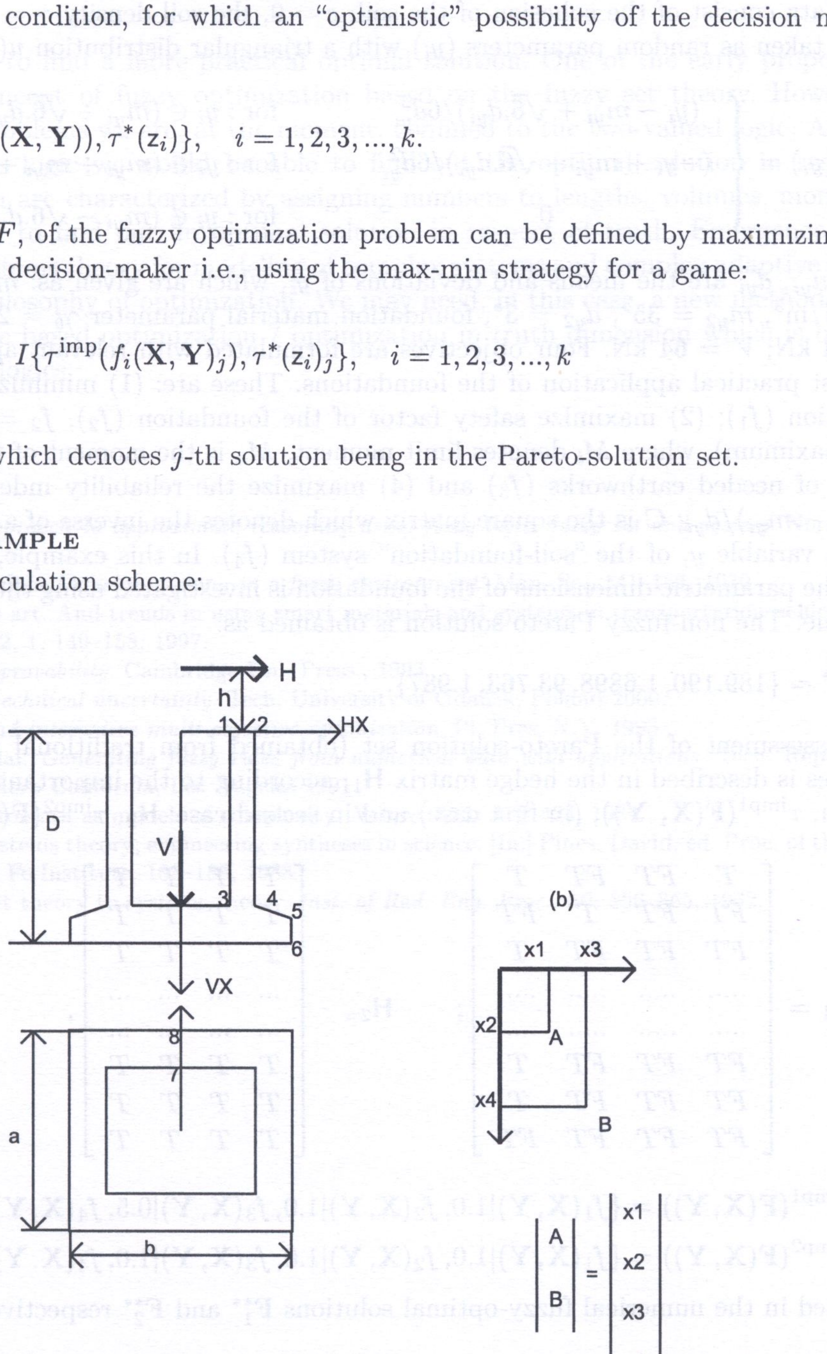

Fig. 4. Computation scheme of foundation: (a) – the foundation scheme, (b) – determination of coordination of characteristic points, where $x1 = xA_1$, $x2 = xA_2$, $x3 = xB_1$, $x4 = xB_2$

in which the shape of the foundation is captured on the fixed mesh determined by the following shape conditions defined by the expert:

$$0.45 <= x1_1 <= 1.0; \quad 0.85 <= x2_1 <= 1.25; \quad x1_2 = x2_2 = 0.0;$$
$$1.5 <= x3_2 <= 3.0; \quad 0.85 <= x4_1 <= 1.25; \quad 1.0 <= x5_1 <= 2.5; \quad 1.3 <= x5_2 <= 4.8;$$
$$2.0 <= x6_2 <= 4.0; \quad x1_1 = x3_1 = x7_1; \quad x3_2 = x4_2; \quad x5_1 = x6_1 = x8_1;$$
$$0.2 <= x2_1 - x1_1 <= 0.6; \quad 0.3 <= x4_1 - x3_1 <= 0.6; \quad 0.3 <= x5_2 - x4_2 <= 0.8;$$
$$0.5 <= x6_2 - x4_2 <= 1.0; \quad x2_1 - x1_1 <= x4_1 - x3_1; \quad x5_2 - x4_2 <= x6_2 - x3_2.$$

Input data consist of the cohesion of the soil: $c = 0$, the soil density $\gamma = y_1$ and the friction angle $\varphi = y_2$ taken as random parameters (y_i) with a triangular distribution $u(y)$:

$$u(y_i) = \begin{cases} (y_i - m_{yi} + \sqrt{6}.d_{yi})/6d_{yi}^2 & \text{for} : y_i \in (m_{yi} \div \sqrt{6}.d_{yi} - m_{yi}), \\ (-y_i + m_{yi} + \sqrt{6}.d_{yi})/6d_{yi}^2 & \text{for} : y_i \in (m_{yi} \div m_{yi} + \sqrt{6}.d_{yi}), \\ 0 & \text{for} : y_i \notin (m_{yi} - \sqrt{6}.d_{yi} \div m_{yi} + \sqrt{6}.d_{yi}) \end{cases}$$

where m_{yi}, d_{yi} are the means and deviations of y_i, which are given as: $m_{y1} = 16.5$ kN/m^3, $d_{y1} = 1.92$ kN/m^3, $m_{y2} = 35°$, $d_{y2} = 3°$, foundation material parameter $\gamma_b = 22$ kN/m^3; External load: $H = 44$ kN; $V = 64$ kN. Four objectives are formulated with relevant applicable results to cover the most practical application of the foundations. These are: (1) minimize material volume of the foundation (f_1); (2) maximize safety factor of the foundation (f_2), $f_2 = M_l \ / \ M_h$ (f_2 >1.2 and $f_2 \rightarrow$ maximum), where M_l denotes limit moment, M_h is the moment of the force H; (3) minimize volume of needed earthworks (f_3) and (4) maximize the reliability index, $\beta = \min(\mathbf{y}'^T \ \mathbf{C}\mathbf{y}')^{0.5}$; $y_i' = (y_i - m_{yi})/d_{yi}$; $\mathbf{C}$ is the square matrix which denotes the inverse of a correlation matrix of the random variable y, of the "soil-foundation" system (f_4). In this example, effect of interaction between the parametric dimensions of the foundation is investigated using the simulation-optimization technique. The non-fuzzy Pareto solution is obtained as:

$$\mathbf{F}^* = \{189.190, 1.6898, 93.763, 1.987\}.$$

Next, assessment of the Pareto-solution set (obtained from traditional method) using linguistic estimates is described in the hedge matrix $\mathbf{H}_1$, according to the important degree of each objective function, $\tau^{\text{imp1}}(\mathbf{F}(\mathbf{X}, \mathbf{Y}))$, (in first case) and in second case: $\mathbf{H}_2$, $\tau^{\text{imp2}}(\mathbf{F}(\mathbf{X}, \mathbf{Y}))$,

$$\mathbf{H}_1 = \begin{bmatrix} T & FT & FT & T \\ FT & FT & T & FT \\ FT & FT & FT & T \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ FT & FT & FT & T \\ FT & FT & FT & T \\ FT & FT & FT & FT \end{bmatrix}; \quad \mathbf{H}_2 = \begin{bmatrix} T & T & T & T \\ T & T & T & T \\ T & T & T & T \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ T & T & T & T \\ T & T & T & T \\ T & T & T & T \end{bmatrix},$$

$$\tau^{\text{imp1}}(\mathbf{F}(\mathbf{X}, \mathbf{Y})) = \{f_1(\mathbf{X}, \mathbf{Y})|1.0, f_2(\mathbf{X}, \mathbf{Y})|1.0, f_3(\mathbf{X}, \mathbf{Y})|0.5, f_4(\mathbf{X}, \mathbf{Y})|0.8\},$$
$$\tau^{\text{imp2}}(\mathbf{F}(\mathbf{X}, \mathbf{Y})) = \{f_1(\mathbf{X}, \mathbf{Y})|1.0, f_2(\mathbf{X}, \mathbf{Y})|1.0, f_3(\mathbf{X}, \mathbf{Y})|1.0, f_4(\mathbf{X}, \mathbf{Y})|1.0\};$$

it resulted in the numerical fuzzy-optimal solutions $\mathbf{F}_1^{**}$ and $\mathbf{F}_2^{**}$ respectively.

$$\mathbf{F}_1^{**} = \{231.694, 1.8978, 107.486, 2.398\} \quad \text{with} \quad SF_1 = 0.835,$$
$$\mathbf{F}_2^{**} = \{189.190, 1.6898, 93.763, 1.987\} \quad \text{with} \quad SF_2 = 0.859.$$

It is indicated that the changing of the hedge matrix, $\mathbf{H}$, (assessment of the Pareto-solution set) and the important degree of each objective function, $\tau^{\text{imp}}(\mathbf{F}(\mathbf{X},\ \mathbf{Y})$ allows us to obtain different foundation shapes from the Pareto-solution set. It is interesting to note that when linguistic estimates, "true", of the expert are used for all of the Pareto-solution set (second case), we obtain $\mathbf{F}_2^{**} = \mathbf{F}^*$. It indicates that the result $\mathbf{F}_2^{**}$ obtained, based on fuzzy logic (in the second case), is equivalent with the result $\mathbf{F}^*$, obtained from traditional optimization problem based on two-valued logic. It indicates, in this case, that binary logic is from the quantitative point of view a particular reduction of multi-valued logic. Besides, this result reflects the consistency of the proposed method.

6. Conclusions

Many efforts have made to find a more practical optimal solution. One of the early proposals for such studies was the concept of fuzzy optimization based on the fuzzy set theory. However, to solve the optimization problems we are, at the moment, confined to the two-valued logic. Although in the framework of this logic we would be able to find out the optimal solution in respect to different measures, which are characterized by assigning numbers to lengths, volumes, money etc., we would lose an ability to find out an optimal solution in respect of truth. Emergence of new approaches to general systems theory for modeling of complex systems and complex adaptive systems requires changing the philosophy of optimization. We may need, in this case, a new methodology of optimization – knowledge based optimization / optimization in truth dimension which is based on rather non-conventional logics.

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