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Topology and shape optimization of continuum structures by genetic algorithm and BEM

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This paper describes the topology and shape optimization scheme of continuum structures by using genetic algorithm (GA) and boundary element method (BEM). The structure profiles are defined by using the spline function surfaces. Then, the genetic algorithm is applied for determining the structure profile satisfying the design objectives and the constraint conditions. The present scheme is applied to minimum weight design of two-dimensional elastic problems in order to confirm the validity.

Keywords: Topology and shape optimization, genetic algorithm (GA), boundary element method, spline function, two-dimensional elastic problem.

1. Introduction

In the shape optimization problem of the continuum structures, one very often encounters the changes of the structural topology such as the appearance, the disappearance and the combination of holes and so on. Since, however, the design sensitivity for the topology change can not be defined exactly, there may be some difficulties when the topology optimization problem is solved by the gradient-type search scheme. In this case, no gradient-type search schemes seem to be attractive. Genetic algorithm (GA) is one of such search schemes and, the computational cost is cheaper than the other no gradient-type schemes such as random search scheme [1, 2]. Therefore, some researches are studying the structural optimization scheme using the genetic algorithm [3–9].

In the existing studies for the structure optimization schemes using genetic algorithm, the profiles of the continuum structures are usually represented by the cells or the function-curves such as the Bezier and spline functions. In the cell representation scheme, the design domain is divided into small square cells. Each cell has binary parameter of which value indicates whether the cell is occupied with material or not. In the function-curve representation scheme, the boundary profile of the structure is represented with the Bezier or spline function curves. They have some difficulties. In the cell representation scheme, the structures have the zigzag-shaped profiles due to the cell and therefore, it is necessary to convert the data of the zigzag-shaped profile to the data of the smooth one because only the data of smooth profiles are necessary at the design and development processes of the structures. Besides, the number of the design variables increases in proportion to the number of the cells. When the object is large-scale structure, the number of the design variables becomes numerous. The function-curve representation scheme has a difficulty related to the topology representation of the structures. Special design variable is necessary for indicating the structural topology.

For overcoming these difficulties, this paper presents the following scheme. The profile of the two-dimensional structure is represented as the cross-section between the three-dimensional spline surface and a cutting plane. The profiles can be represented by smaller number of the variables than the cell representation scheme and, unlike the function-curve representation scheme, the structural topology can be represented without special design variables related to the topology.

In the existing schemes, finite element method is employed for estimating objective functions and constraint conditions. The present scheme, however, employs boundary element method. In the present scheme, the structures with complicated profiles are generated during the optimization process. Since the computational accuracy strongly depends on domain and/or boundary discretization employed for the analysis, the remeshing process for each structure is absolutely necessary at each iteration step. Although finite element method is very powerful tool for the analysis, the computational cost of the remeshing process is very expensive because of the necessity of domain discretization. On the other hand, boundary element method can solve the problem by the boundary discretization alone when the object under consideration is governed with linear and homogeneous differential equation. The computational cost for the boundary discretization is much cheaper than finite element method.

This paper is organized as follows. In the Sec. 2, the representation schemes of the profiles of the continuum structures are compared in order to discuss the features of the present scheme. In the Sec. 3, the algorithm of the present scheme is indicated. In the Sec. 4, the boundary element formulation for the two-dimensional elastic problem is described. In the Sec. 5, the present scheme is applied to two numerical examples. Finally, the Sec. 6 summaries some conclusions.

2. Representation schemes of structural profiles

2.1. Background

2.1.1. Cell representation scheme

The cell representation scheme is very popular for the topology and shape optimization problem of the structure (Fig. 1). The design domain is divided into small square cells which have different binary parameters. The parameter is specified to be zero if the cell is empty and to be one if the cell is full of material.

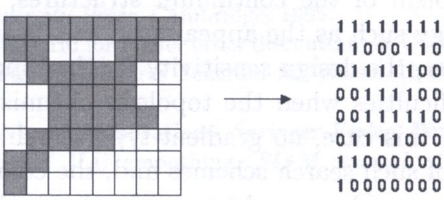

Fig. 1. Cell representation scheme

This scheme has the following advantages:

1. The cell representation scheme is very suitable for finite element analysis and thus, the cells can play as the finite elements.
2. Topology optimization as well as the shape optimization can be performed without special design variables.

However, it has the following disadvantages:

1. Finally obtained structures have zigzag-shaped profiles and therefore, it is necessary to convert the data of the zigzag-shaped profile to the data of the smooth one because only the smooth-shaped profiles are necessary at the design and development processes.

2. A large number of cells are necessary for representing the structures with smooth profiles, which increases the number of design variables and the computational cost.

3. The number of design variables increases in proportion to the number of cells. When the object is a large-scale structure or a structure with complicated profile, huge number of design variables is necessary. Therefore, the computational cost of the optimization process becomes expensive.

2.1.2. Function-curve representation scheme

The function-curve representation scheme has been presented by some researchers (Fig. 2). The structure profile is represented with Bezier or spline function curve [10]. Coordinates of the control points of the function are taken as the design variables in order to define the chromosome.

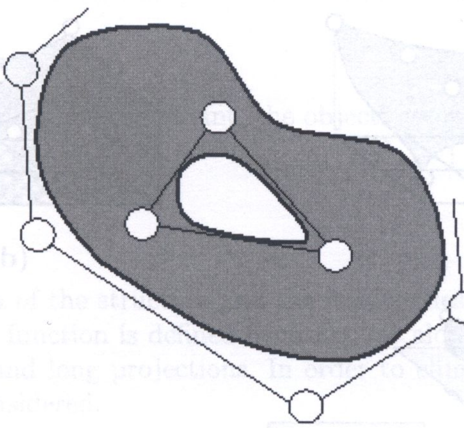

Fig. 2. Function-curve representation scheme

The features of the scheme are summarized as follows:

1. Profiles of the structures can be represented with smooth function-curves. Therefore, unlike the cell representation scheme, the data conversion process is not necessary.

2. The structure profiles are represented by relatively small number of design variables and therefore, the computational cost of the optimization process can be reduced fairly.

3. Additional design variables are necessary for representing the structural topology.

2.1.3. Present scheme

In the present scheme, profile of the two-dimensional structure is represented as the cross-section between three-dimensional spline surface and a plane in parallel with xy-plane. The coordinates of the control points of the curved surface are taken as design variables.

This scheme has the following advantages:

1. A large-scale structure can be represented by much smaller number of design variables than the cell representation scheme.

2. The structure has a smooth-shaped profile and therefore, unlike the cell representation scheme, the data conversion process is not necessary.

3. Unlike the function representation scheme, additional design variable for representing the structural topology is not necessary.

2.2. Profile representation of present scheme

The profile representation scheme of the present method is shown in Fig. 3.

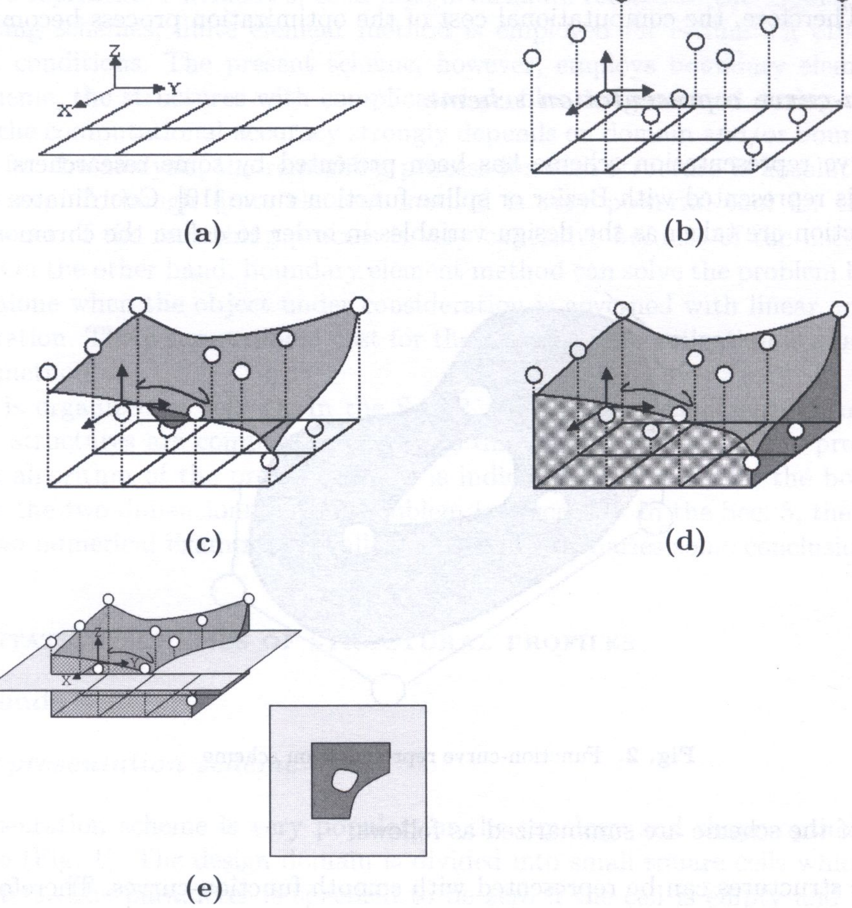

Fig. 3. Shape representation scheme

(a) A system of orthogonal coordinates is taken and lattice points are placed on $x - y$ plane. The coordinates of a lattice point are defined as (X_m, Y_n).

(b) Control points are taken to control the spline surface and the coordinates of a control point are as (X_m, Y_n, Z_{mn}).

(c) A spline surface is formed according to the control points. The coordinates of an arbitrary point on the surface (x, y, z) are given as:

$$\left.\begin{aligned} x(s,t) &= \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} X_m B_{m,k}(s) B_{n,k}(t) \\ y(s,t) &= \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} Y_n B_{m,k}(s) B_{n,k}(t) \\ z(s,t) &= \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} Z_{mn} B_{m,k}(s) B_{n,k}(t) \end{aligned}\right\} \tag{1}$$

where $B_{\ell,k}$ denotes the B-Spline function. Nodes of the spline function are taken so as to satisfy Scoenberg–Whittny condition [10]. s and t denote the parameters taken on the x and y-coordinates. The side constraint conditions of s and t are as follows.

$$\left.\begin{aligned} 0 < s < M-1 \\ 0 < t < N-1 \end{aligned}\right\}. \tag{2}$$

(d) The two-dimensional profile is constructed as the cross-sectional plane between the spline surface and the plane

$$Z = Z_h. \tag{3}$$

3. Optimization algorithm

3.1. Optimization problem

A design requirement is to minimize the total weight of a structure on condition that the maximum stress is kept under the permissible stress of the material. The objective function, the constraint condition and the design variables are defined as follows.

3.1.1. Objective function

If the material of the structure is kept invariant, the objective function for minimizing total weight is defined as

$$\frac{A}{A_0} \to \min \tag{4}$$

where A and A_0 are the area of the structure and the feasible design domain, respectively.

If, however, the objective function is defined from Eq. (4) alone, the final profile may have very strange shape with narrow and long projections. In order to eliminate such profiles, the following objective function is also considered.

$$\frac{L}{L_0} \to \min \tag{5}$$

where L and L_0 denote the circumferences of both structure and the feasible design domain, respectively. By introducing the parameter C_L, the objective function is defined as follows:

$$f = \frac{A}{A_0} + C_L \frac{L}{L_0}. \tag{6}$$

In the following numerical examples, the parameter C_L is specified to 0.1, which is defined from numerical experiments.

3.1.2. Constraint condition

The stress constraint condition is defined as follows:

$$g_0 = \frac{\sigma_{\max}}{\sigma_c} - 1 \geq 0 \tag{7}$$

where $\sigma_{\max}$ and σ_c denote the maximum value of the von Mises equivalent stress at boundary nodes and the permissible stress of the material, respectively.

In the optimization process, the profiles constructed with multi-subdomains may be generated. For excluding such profiles, the following constraint condition is considered.

$$g_1 = N_\Omega - 1 = 0 \tag{8}$$

where N_Ω means the number of the subdomains. The number of subdomains is counted automatically. This constraint condition assures that the final profile is constructed by one domain.

3.1.3. Design variables

Z-coordinate of the control point Z_{mn} is selected as design variable. The side constraint for the variable Z_{mn} is defined as

$$Z_0 \le Z_{mn} \le Z_1 \tag{9}$$

where Z_0, and Z_1 denote the minimum and the maximum values of Z_{mn}, which is defined by a user.

3.2. Genetic algorithm

3.2.1. Fitness function

The fitness function *fitness* is defined from the objective function as follows:

$$fitness = 1 - f. \tag{10}$$

The constraint conditions are usually added into the fitness function by introducing the penalty parameter. In the present scheme, however, the constraint conditions are not included into the fitness function because the population is organized by the individuals satisfying the constraint conditions alone.

3.2.2. Genetic coding

Length of chromosome is equal to total number of the control points ($M \times N$). Each gene of the chromosome is related to the z-coordinate of the control point Z_{mn} as follows:

$$Z_{mn} = Z_0 + (Z_1 - Z_0)\frac{G_{m+M\times n}}{G_{\max} - 1} \tag{11}$$

where $G_{\max}$ is the maximum value of the genes.

3.2.3. Genetic operations

(1) Selection:

As the selection operator, this paper employs ranking selection, in which parents are selected from the population according to the ranking of the fitness function of the individuals, instead of the value of the fitness function itself. Moreover, the elitist scheme is employed so that the best individual at each population survives at the next generation.

(2) Crossover:

One-point crossover is employed in this study.

(3) Mutation:

Mutation operation changes the value of a gene into different value which is selected randomly from arbitrary values.

Algorithm

Figure 4 shows the algorithm of the present scheme. Firstly, population is organized by individuals satisfying the constraint conditions alone. Fitness function of each individual is estimated by using boundary element method and then, the genetic operators such as the selection, the crossover and the mutation are applied to the population to create new individuals. New population is organized with the individuals satisfying the constraint conditions. Number of the individuals is kept invariant during the process.

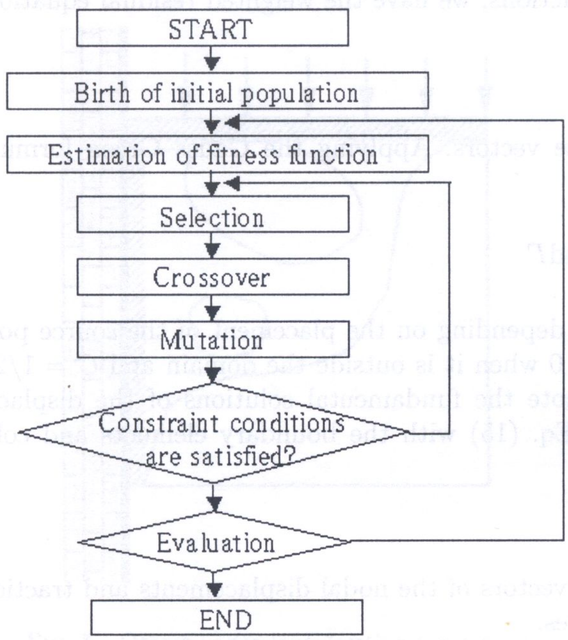

Fig. 4. Flowchart of present scheme

4. Boundary element method

4.1. Numerical analysis scheme

Finite element method is the most popular tool for the analysis and employed for evaluating the objective functions and the constraint conditions in the structural optimization problem. On the other hand, the present scheme employs the boundary element method for the purpose. The computational accuracy of the numerical analysis scheme such as finite and boundary element methods strongly depends on the finite and boundary element meshes employed for the analysis. Since, in the present scheme, several structures with complicated profiles are generated during the optimization process, remeshing process is absolutely necessary at each iteration step. Although finite element method is very powerful tool for numerical analysis, domain discretization by finite elements is necessary. The computational cost for the domain discretization is very expensive. On the other hand, boundary element method can solve the problem by the boundary discretization alone when the object under consideration is governed by the linear and homogeneous differential equation. Therefore, the computational cost for the boundary element mesh generation is much cheaper than that for the finite element mesh.

4.2. Boundary element formulation

We shall explain the BEM analysis in two-dimensional elastic problem [11, 12].

The governing equation without body force and the boundary conditions are given as:

$$\sigma_{ij,j} = 0 \quad (\text{in } \Omega) \tag{12}$$

and

$$\left.\begin{aligned} u_i &= \overline{u}_i \quad (\text{on } \Gamma_u) \\ t_i &= \overline{t}_i \quad (\text{on } \Gamma_t) \end{aligned}\right\} \tag{13}$$

where u_i, t_i and σ_{ij} denote the displacement, the traction and the stress components in the two-dimensional coordinates, respectively, and $(\ )_{,j}$ the derivative in the j direction. Taking the Kelvin solutions as the weight functions, we have the weighted residual equation;

$$\int_{\Omega} \sigma_{ij,j} u^*_{ki} e_k \mathrm{d}\Omega = 0 \tag{14}$$

where e_k denotes the base vectors. Applying the Gauss–Green formula, we have the boundary integral equation;

$$Cu_i = \int_{\Gamma} (u^*_{ij} t_j - t^*_{ij} u_j) \mathrm{d}\Gamma \tag{15}$$

where C is the parameter depending on the placement of the source point; $C = 1$ when the point is inside the domain, $C = 0$ when it is outside the domain and $C = 1/2$ when it is on the smooth boundary. u^*_{ij} and t^*_{ij} denote the fundamental solutions of the displacements and the tractions, respectively. Discretizing Eq. (15) with the boundary elements and collocating the nodes on the elements, we have

$$\mathbf{Hu} = \mathbf{Gt} \tag{16}$$

where $\mathbf{u}$ and $\mathbf{t}$ denote the vectors of the nodal displacements and tractions, respectively. $\mathbf{H}$ and $\mathbf{G}$ are their coefficient matrices.

Applying the boundary conditions to the above system of equations, we have

$$[\mathbf{H}_u \ \ \mathbf{H}_t] \left\{ \begin{array}{c} \bar{\mathbf{u}}_u \\ \mathbf{u}_t \end{array} \right\} = [\mathbf{G}_u \ \ \mathbf{G}_t] \left\{ \begin{array}{c} \mathbf{t}_u \\ \bar{\mathbf{t}}_t \end{array} \right\} \tag{17}$$

where the subscripts u and t denote Γ_u and Γ_t, respectively. Assembling the system of equations, we have

$$[-\mathbf{G}_u \ \ \mathbf{H}_t] \left\{ \begin{array}{c} \mathbf{t}_u \\ \mathbf{u}_t \end{array} \right\} = [-\mathbf{H}_u \ \ \mathbf{G}_t] \left\{ \begin{array}{c} \bar{\mathbf{u}}_u \\ \bar{\mathbf{t}}_t \end{array} \right\} \tag{18}$$

and

$$\mathbf{Ax} = \mathbf{b}. \tag{19}$$

This system of equations is solved for $\mathbf{x}$ to determine the unknown nodal values.

5. Numerical examples

5.1. Example 1

The object domain under consideration is shown in Fig. 5. The left edge of the object is fixed at the wall and the load of 50 kg/mm^2 is uniformly given on the upper edge. The plane stress state is considered in the domain.

Slash-marked part of the object domain is unchanged during the optimization process because they are the loaded part and the part attached to the wall. The design objective is to minimize the area of the object on the condition that the maximum equivalent stress does not exceed the permissible stress of material. The analysis is carried out according to the parameters shown in Table 1.

Figure 6 shows the convergence properties of the fitness values of best individuals at five runs starting from the different initial population. The abscissa and the ordinate denote the fitness value and the generation, respectively. The fitness values increase as the generation goes and converge to any values at 50th generation. Figure 7 shows the profiles of the best individuals at each generation of a run. The best individual at the initial steps has two holes. Finally, the best individual converges to the profile with one hole as the generation goes.

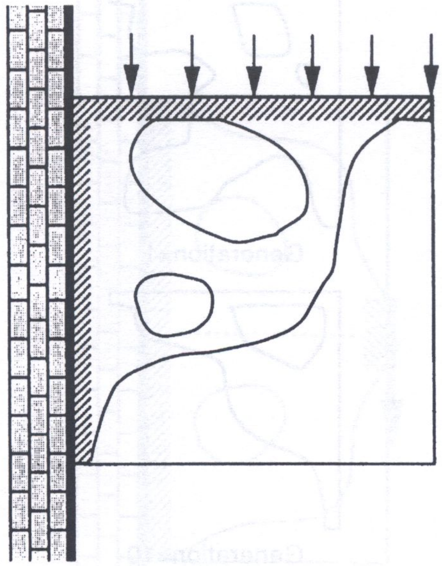

Fig. 5. Object under consideration (example 1)

Table 1. Parameters for simulation

Number of control points	$M = N = 8$
Range of Z-coordinates	$Z_0 = 0,\ Z_1 = 1$
Height of a cutting plane	$Z_h = 0.5$
Maximum value of genes	$G_{\max} = 3$
Number of the individuals	100
Crossover rate	0.8
Mutation rate	0.016
Permissible stress	$\sigma_c = 500\,\mathrm{kg/mm}^2$
Order of Spline function	3

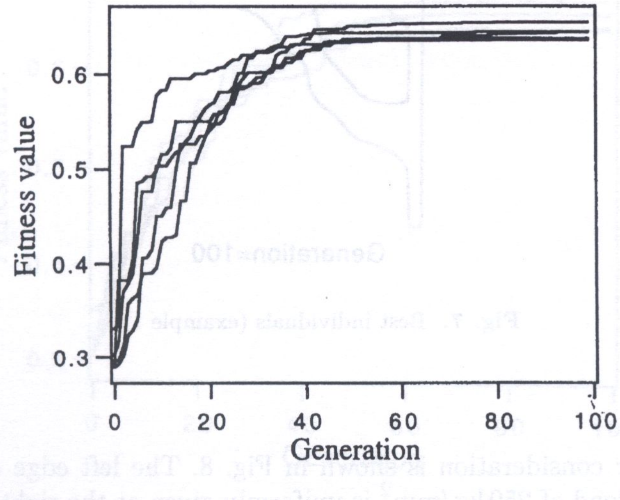

Fig. 6. Fitness value of best individuals (example 1)

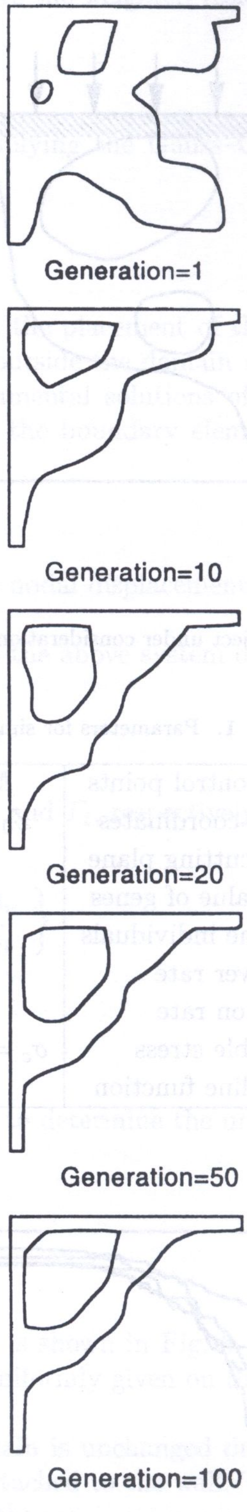

Fig. 7. Best individuals (example 1)

5.2. Example 2

The object domain under consideration is shown in Fig. 8. The left edge of the object domain is fixed at the wall and the load of 250 kg/mm^2 is uniformly given at the right wall. Plane stress state is considered in the domain.

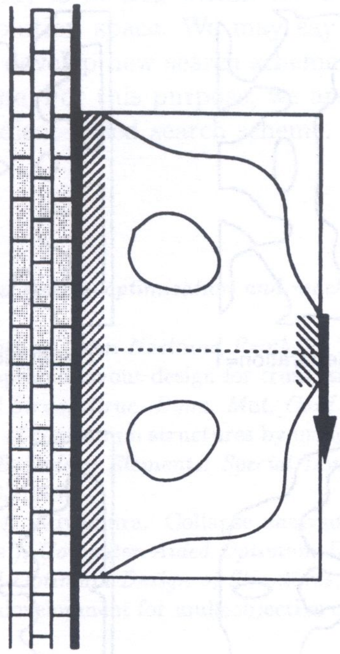

Fig. 8. Object under consideration (example 2)

Slash-marked part of the object domain is unchanged during the optimization process because they are the loaded part and the part attached to the wall. The objective of the optimization is to minimize the area of the object under the condition that the maximum equivalent stress does not exceed the permissible stress of material. The same parameters as the example 1 are used for analysis.

Figure 9 shows the convergence properties of the objective functions of five runs starting from the different initial population. The objective function at each case decreases gradually and converges to any values at 50th iteration step. Figure 10 shows the profiles of the best individuals at each time step. At the beginning of the simulation, the best individual has two holes. The profile finally goes to the profile with one hole.

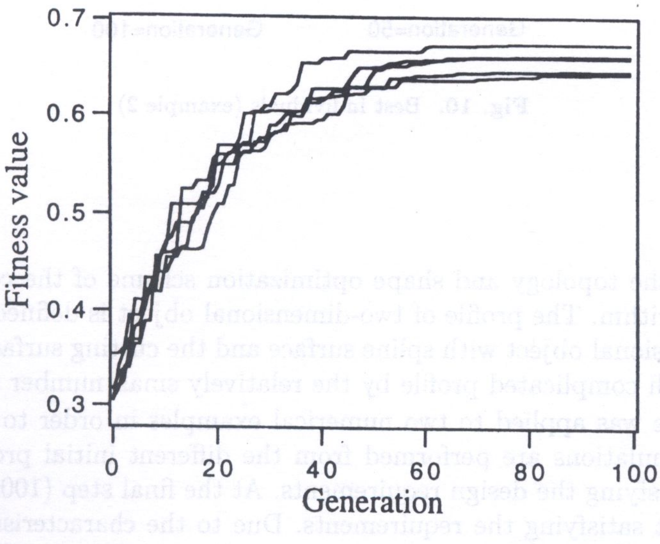

Fig. 9. Fitness value of best individuals (example 2)

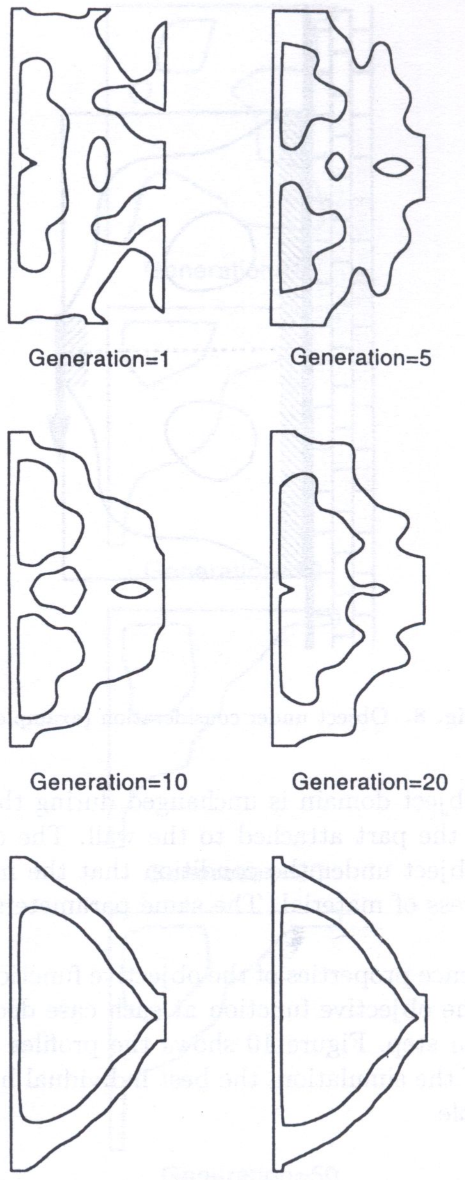

Fig. 10. Best individuals (example 2)

6. Conclusions

This paper describes the topology and shape optimization scheme of the continuum structures by using the genetic algorithm. The profile of two-dimensional object is defined as the intersection between the three-dimensional object with spline surface and the cutting surface in order to effectively express the object with complicated profile by the relatively small number of design variables.

The present scheme was applied to two numerical examples in order to discuss its features. At each example, five simulations are performed from the different initial profiles. Finally, we could obtain the profiles satisfying the design requirements. At the final step (100th iteration step), there exist some profiles not satisfying the requirements. Due to the characteristic of genetic algorithm depending on design sensitivity, it is somewhat time-consuming to reach the optimal solution from quasi-optimal solution although the quasi-optimal solution can be found within relatively short computing time. The aim of our study is to find within the real time the quasi-optimal solution in the discrete and multi-variable solution space. We may say that the aim of our study can be satisfied. It is necessary, however, to develop new search scheme such that the optimal solution can be found within short computing time. For this purpose, we are studying the combination scheme of the genetic algorithm and the gradient-based search scheme.

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