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Identification of an equivalent model for granular soils by FEM/NMM/p-EMP hybrid system

Ewa Pabisek
Institute for Computational Civil Engineering
Cracow University of Technology
Warszawska 24, 31-155 Cracow, Poland
e-mail: E.Pabisek@L5.pk.edu.pl

The application of FEM/NMM/p-EMP computational hybrid system in formulation of the Neural Material Model (NMM) for granular soils is presented. NMM is a Multi Layer Preceptron formulated 'on-line'. The cumulative algorithm of the autoprogressive method was implemented into the FEM program. The patterns for NMM training were generated in the rigid strip footing analysis. Pseudo-empirical equilibrium paths p-EMP for verification of the NMM were computed by a FEM program for the elastic-plastic Drucker-Prager material model. The discussed inverse problem of NMM identification is illustrated by two study cases of footing: 1) rigid strip footing on plane semispace, 2) inclined slope analysis. It was numerically proved that the NMM identified in the first study case can be successfully applied to the analysis of the latter one.

Keywords: Artificial Neural Network (ANN), Neural Material Model (NMM), hybrid computational system, constitutive modelling.

1. Introduction

The basic goal of material modelling is an appropriate formulation of constitutive equations in order to predict the physical behaviour of the investigated material. Formulation of general constitutive relations is a complex problem when material parameters are unknown 'a priori' and they have to be estimated by data from tests on real solids or engineering structures.

Material models can also be formulated in an implicit way, without the phenomenological approach, by means of Artificial Neural Network (ANN), adopted as aproximators of nonlinear functions. In such an approach the form of the material stiffness matrix can be applied as a Neural Material Model (NMM), that is a projection done by ANN. NMM describes the stress-strain relations from which the constitutive matrix can be computed. In such an approach, the consistent (algorithmic) tangent operator (matrix) component E_{ij}^{NMM} is derived explicitly from the parameters and input/output current vectors of NMM. The algorithmic tangent operator is needed in each step of a generalised Newton-Raphson method. The approach, proposed in [2], can be adopted in any NMMs modelling different nonlinear material relations.

Computation of NMM network weights can be carried out by means of: 1) autoprogressive Ghaboussi approach [1, 2] called in [4] algorithm A. In this algorithm the NMM network is retrained on each load increment, cf. Fig. 1; 2) cumulative algorithm B proposed by Shin and Pande in [7], in which the NMM is retrained at the end of each loading program cycle. In these algorithms, the material model NMM is prepared by means of FEM/NMM/p-EMP hybrid computational program. The system is composed of three components, i.e. Finite Element Method (FEM), Neural Material Model (NMM) and pseudo-Empirical data p-EMP.

The FEM/NMM/p-EMP hybrid system is characterised by a very high fusion level of the components. This means that the applied components interact with each other during 'on line' formulation of NMM. The real structure responses, measured at structure level, are related to the external load

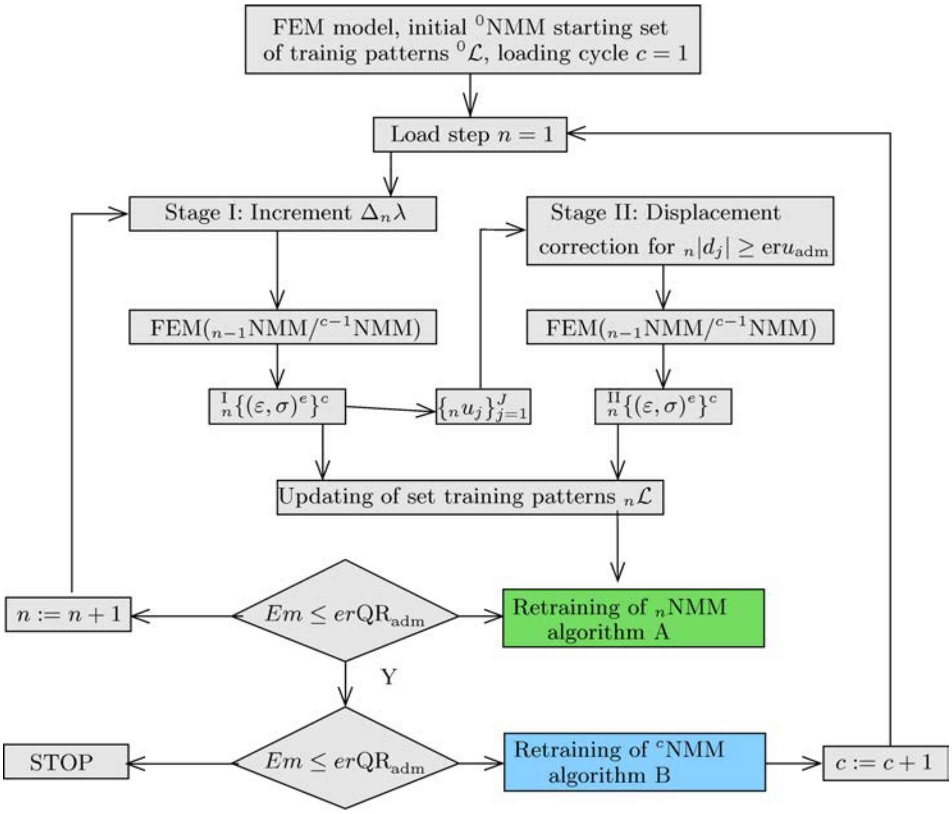

Fig. 1. Flow chart.

parameter p-EMP $\rightarrow \{P_j \rightarrow {}_{\mathrm{n}}u_j^{\mathrm{m}}\}_{j=1}^J$, where ${}_{\mathrm{n}}u_j^{\mathrm{m}}$ – displacements as structure responses measured at $j = 1,\ldots,J$ points at n load level P_j. The formulation of NMM is carried out iteratively, following the measured loading process (P_j, u_j^{m}). This process serves fulfilling the response (e.g. displacements) compatibility relations ${}_{\mathrm{n}}u_j ={}_{\mathrm{n}} u_j^{\mathrm{m}}$, where ${}_{\mathrm{n}}u_j$ – computed structural responses.

In paper [5], a comparison of the numerical efficiency of algorithms A and B was done. This issue was also discussed in [4] on an example of a simple plane truss analysed in [1]. In this example it was assumed that the truss was made of the Osgood-Ramberg material. It was numerically demonstrated that algorithm A is comparable to B from the view point of the number of total load cycles c, see Fig. 1.

The hybrid system FEM/NMM/p-EMP was also applied to identification of equivalent material (in the sense of homogenization theory) in the analysis of plane stress boundary value problems, cf. [4, 6]. Pseudo-empirical loading paths $P({}_{\mathrm{n}}u_j^{\mathrm{m}})$ were computed by means of a standard FEM program, assuming an elasto-plastic material model with the Huber-Mises-Hencky (HMH) yield function. The quality of the $\mathrm{NMM_{HMH}}$ material model built was verified by means of stress maps obtained by FEM/HMH and FEM/$\mathrm{NMM_{HMH}}$ programs. On the basis of the obtained results, it can be concluded that each neural model material defined in the hybrid FEM/NMM, implements correctly the identified constitutive relationships. Additionally, by the use of combined techniques of 'on-line' and 'off-line', the formulated network can have improved properties. It can allow an analysis of various more complicated 2D boundary value problems.

In the presented paper the identification of NMM for the soil continua is discussed. An equivalent NMM was formulated during the analysis of the strip footing problem. The pseudo-empirical equilibrium path was determined by FEM program, which assumed the elastic-plastic model of Drucker-Prager material, well related to granular soils. This model is based on the generalization of the HMH criterion for materials which depends on the hydrostatic pressure, i.e. the material models take into account the first stress invariant I_1 in the yield function. The corresponding hybrid FEM/NMM/p-EMP program was based on the cumulative algorithm B, see Fig. 1.

2. General remarks on the formulation of NMM

In [6] two algorithms were discussed, i.e. either the autoprogressive algorithm A or the cumulative algorithm B, see Fig. 1, for NMM formulation by 'on-line' technique. The processing carried out by these algorithms and comparison of their numerical efficiency was discussed in [5].

The NMM parameter identification was performed adopting the hybrid program during the analysis of various boundary value problems. According to the algorithm shown in Fig. 1 formulation of an NMM model always starts from the introduction of an initial NMM^0 model. Such a model might be commonly formulated as a three layer Multi-Layer Perceptron (MLP), [3]. This MLP is used to regression mapping, written in short as:

$$\mathbf{y} = \text{NMM}(\mathbf{x}, \mathbf{w}), \tag{1}$$

where $\mathbf{x}$, $\mathbf{y}$ – input/output and $\mathbf{w}$ – weights vector of NMM. In the case of plane stress BV problems the input/output vectors were assumed in [2, 8] in 9D and 3D vector spaces:

$$\underset{(9\times1)}{\mathbf{x}} = \{{}_{n+1}\boldsymbol{\varepsilon},\ {}_{n}\boldsymbol{\varepsilon},\ {}_{n}\boldsymbol{\sigma}\}, \qquad \underset{(3\times1)}{\mathbf{y}} = \{{}_{n+1}\boldsymbol{\sigma}\}, \tag{2}$$

with components

$${}_{k}\boldsymbol{\varepsilon} = {}_{k}\{\varepsilon_x,\ \varepsilon_y,\ \gamma_{xy}\}, \qquad {}_{k}\boldsymbol{\sigma} = {}_{k}\{\sigma_x,\ \sigma_y,\ \tau_{xy}\} \tag{3}$$

at two $k = n,\ n+1$ load levels. In paper [4] both $\mathbf{x}$ and $\mathbf{y}$ were formulated for only one $k = n+1$ load level:

$$\underset{(3\times1)}{\mathbf{x}} = \{{}_{n+1}\boldsymbol{\varepsilon}\}, \qquad \underset{(3\times1)}{\mathbf{y}} = \{{}_{n+1}\boldsymbol{\sigma}\}. \tag{4}$$

Such an approach leads to smaller network NMM than proposed in [1]. It was proved in [5] that due to 'on line' technique computation, vectors (4) can be adopted without losing the accuracy of neural approximation. The reduction of $\mathbf{x} \in \mathcal{R}^{\text{D}}$ to D = 3 simplified the analysis because of decreasing dimension of the weights space $\mathbf{w}$, corresponding to the number of generalised weights (NMM synaptic weights and biases). Additionally, due to application of the 'on-line' and 'off-line' techniques combined it was possible to formulate the NMM network with improved approximation properties. It was proved on examples of plane stress BV problems, discussed in [4].

Beside the prediction of stresses $\boldsymbol{\sigma}(\boldsymbol{\varepsilon})$ at the FE Gauss integration points, the NMM network also allows analytical computation of the algorithmic tangent operator components ${}_{n_1}E_{ij}^{\text{NMM}}$. These components are functions of the input/output values and values of the synaptic weights of the formulated NMM network, cf. [2]. In such a sense, this network cannot be called a "black box". This property of NMM is explored in the hybrid computational programs formulated on the base of algorithms shown in Fig. 1.

3. Numerical examples

3.1. Rigid strip footing on plane semi-space

The strip footing problem is modelled as a rigid strip applied at the upper edge of the plane subspace shown in Fig. 2. The investigated problem corresponds to the plane strain BV problem shown in Fig. 2a. The original infinite semi-space is replaced by two subdomains BVs O-C-G-F and O-B-D-E. In Figs. 2b, c the geometry, FE meshes and boundary conditions are shown.

The NMM was formulated for a small domain O-B-D-E by means of p-EMP data. The equilibrium path $P(v_A^{\text{m}})$ was computed using the FEM program.

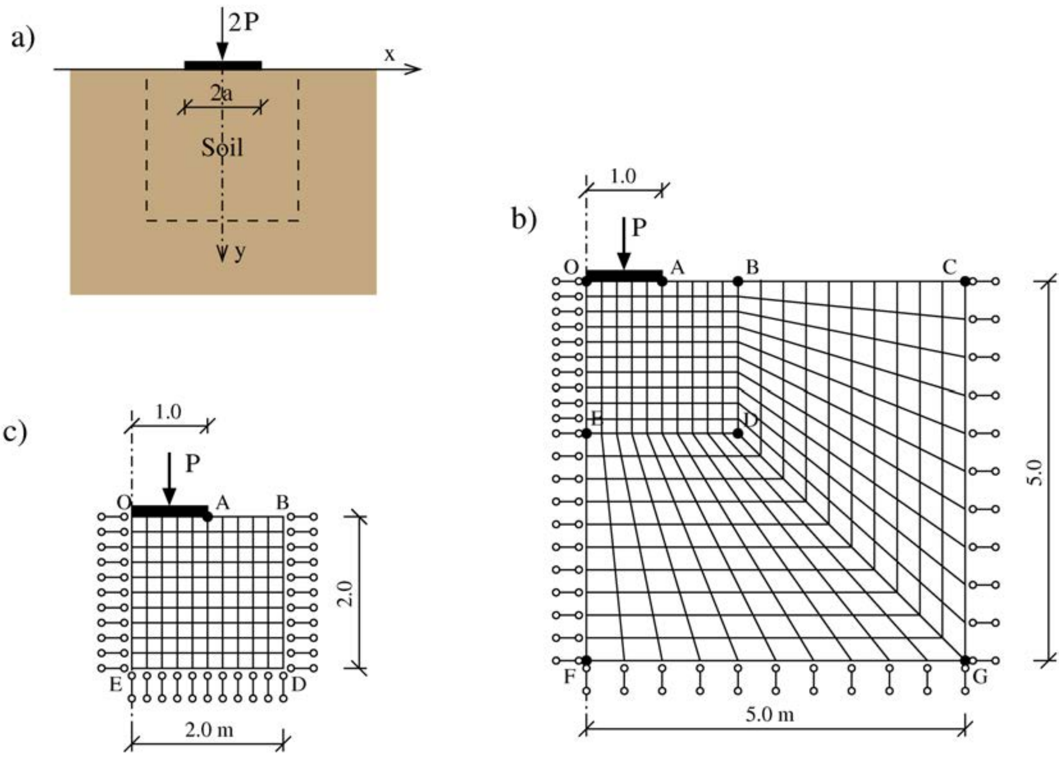

Fig. 2. Plane strain BV problems discussed: a) infinite semi-space, b) finite domain O-C-G-F, c) finite domain O-B-D-E.

3.1.1. Drucker-Prager model of soil

From among many models of soils the Drucker-Prager model is worth emphasising as a continuous version of the Coulomb-Mohr model, see [10]. These models can be applied to the analysis of granural soils.

The DP model is applied in soil plasticity analysis adopting the yield function, which is generalisation of the HMH yield function. It depends not only on the stress deviator second invariant J_2 but also on the first invariant of stress tensor I_1. The DP yield function can be written as follows:

$$f \equiv \sqrt{3J_2} + \alpha I_1 - \beta c(\kappa) = 0, \tag{5}$$

where: the material constants α and β are used

$$\alpha = \frac{6 \sin \varphi}{3 - \sin \varphi}, \qquad \beta = \frac{6 \cos \varphi}{3 - \sin \varphi}, \tag{6}$$

where φ – internal friction angle. The cohesion function was adopted in linear form:

$$c(\kappa) = \sigma_Y + E_p \kappa, \tag{7}$$

where σ_Y – yield stress, E_p – linear strain hardening modulus, κ – invariant plastic strain measure:

$$\int \sqrt{2/3 \dot{\varepsilon}^p_{ij} \cdot \dot{\varepsilon}^p_{ij}} \, dt = \int \dot{\lambda}(1 + 2/9\widetilde{\alpha}^2) \, dt \tag{8}$$

Besides the yield function also plastic potential g has to be introduced:

$$g = \sqrt{3J_2} + \widetilde{\alpha} I_1 - \widetilde{\beta} c(\kappa) \tag{9}$$

in order to calculate the plastic strain rate vector $\boldsymbol{\varepsilon}^p$ for the non-associated plasticity:

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\frac{\partial g}{\partial \boldsymbol{\sigma}}. \tag{10}$$

In (9) $\widetilde{\alpha}$, $\widetilde{\beta}$ are a function of dilatancy angle ψ similarly to the definition of α and β:

$$\widetilde{\alpha} = \frac{6\sin\psi}{3-\sin\psi}, \qquad \widetilde{\beta} = \frac{6\cos\psi}{3-\sin\psi}. \tag{11}$$

3.1.2. Analysis of a strip rigid footing problem

The strip load $2P$ applied to the foundation which affects vertical displacement $v_{\mathrm{A}}^{\mathrm{m}}$ at the edge point A, cf. Fig. 2. The following set of material properties was assumed: $E = 50$ MPa, $E_p = 1$ kN/m^2, $\sigma_Y = 1$ kN/m^2, $\nu = 0.25$, $\varphi = 30°$, $\psi = 3°$. The displacements $v = $ const. along the contact boundary of foundation and ground were also assumed.

In the plane strain state discussed, the input and output vectors have the following form:

$$\underset{(3\times1)}{\mathbf{x}} = \{\varepsilon_x, \varepsilon_y, \gamma_{xy}\}, \qquad \underset{(4\times1)}{\mathbf{y}} = \{\sigma_x, \sigma_y, \sigma_z, \tau_{xy}\}. \tag{12}$$

The initial neural material model ^{0}NMM corresponds to the linear elastic material. After preliminary computation the following neural network architecture was accepted ^{0}NMM: 3-20-20-4. Bipolar sigmoid activation functions were used in the hidden layers and output neurons. The network was trained by means of Levenberg-Marquardt method. This initial ^{0}NMM network was then retrained after each load cycle, according to algorithm B, implemented in the FEM/NMM/p-EMP hybrid program.

The process of the model NMM$_{\text{O-B-D-E}}$ formulation was ended after $c = 6$ load cycles. The stopping criterion corresponded to fulfilling the compatibility of displacements

$$_n d_j \equiv {}_n\left|\frac{v_j^{\mathrm{m}} - v_j}{v_j^{\mathrm{m}}}\right| \cdot 100\% < \mathrm{er}u_{\mathrm{adm}},$$

for all the load steps n and each measured point j along the contact zone of the foundation and upper edge of the domain $0 - A$, see Fig. 2b.

In Fig. 3a the equilibrium paths $P(v_{\mathrm{A}})$ are shown for subsequent six load cycles, computed by the FEM/NMM$_{\text{O-B-D-E}}$ program.

The identified neural model NMM$_{\text{O-B-D-E}}$ can be applied in the analysis of BV problem for the original domain O-C-G-F shown in Fig. 2. The equilibrium path $P(v_{\mathrm{A}})$, computed by the

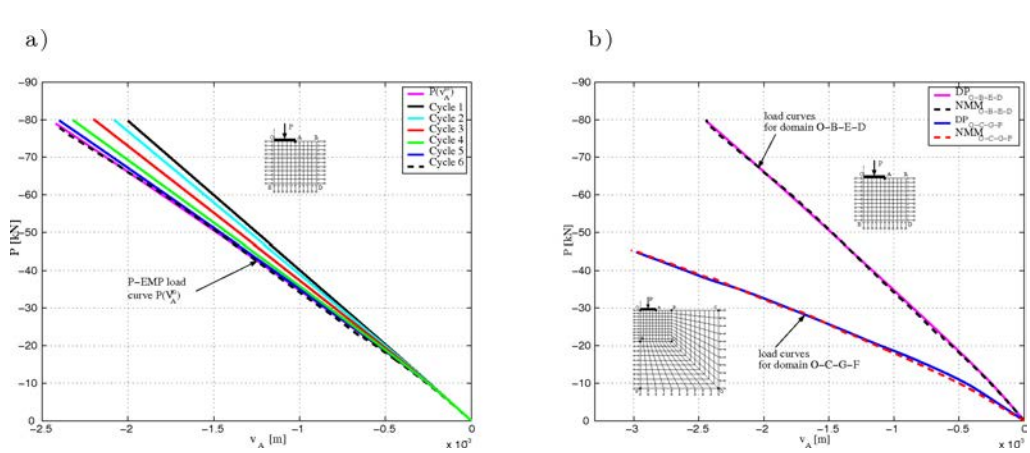

Fig. 3. a) Equilibrium paths $P(v_A)$ computed for O-B-D-E domain during training NMM$_{\text{O-B-D-E}}$ process; b) pseudo-empirical equilibrium paths $P(v_A)$ computed by FEM/DP and FEM/NMM$_{\text{O-B-D-E}}$ program.

hybrid program FEM/NMM$_{\text{O-B-D-E}}$ for the original domain, coincides with the pseudo-empirical path computed by means of FEM/DP program. Then, the DP material model was adopted to the analysis of the BV problem for the extended domain O-C-G-F, see Fig. 3b.

In subsequent Fig. 4a, b the maps of pressure-depended stresses

$$\sigma_{\text{pde}} = \sqrt{3J_2} + \frac{6\sin\varphi}{3-\sin\varphi} I_1 \tag{13}$$

are shown. The maps in Figs. 4, 5 were computed by means of FEM/DP, vs. the maps obtained after application of the hybrid system FEM/NMM$_{\text{O-B-D-E}}$, to the analysis of subdomain O-B-D-E and O-C-G-F.

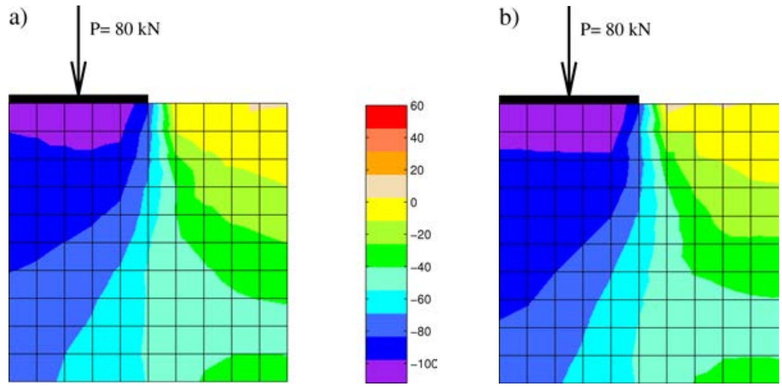

Fig. 4. Distribution of $\sigma_{\text{pde}}(x,y)$ for O-B-D-E domain for a) equivalent NMM$_{\text{O-B-D-E}}$ model, b) DP model.

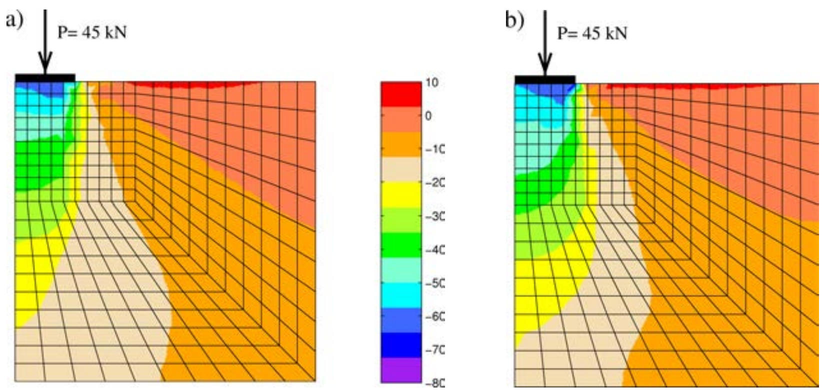

Fig. 5. Distribution of $\sigma_{\text{pde}}(x,y)$ for O-C-G-F domain for a) equivalent NMM$_{\text{O-B-D-E}}$ model, b) DP model.

Looking at Figs. 4a and 4b we can state that distribution stresses σ_{pde} computed by means of both programs are very similar to each other. Such a conclusion can be treated as a numerical proof.

The maps of stresses shown in Fig. 5a, b were computed in the analysis of subdomain O-C-G-F by the FEM/DP and FEM/NMM$_{\text{O-B-D-E}}$. The stress distributions are also very similar in both figures. The conclusion is that the equivalent material modelled on subdomain O-B-D-E is really a homogenisation material.

3.2. Inclined slope analysis

The NMM$_{\text{O-B-D-E}}$ formulated above for the BV footing problem, was applied to the inclined slope analysis shown in Fig. 6. It was assumed that material characteristics are the same as in Example 3.1, i.e. the model of equivalent material could be adopted as a neural model NMM$_{\text{O-B-D-E}}$.

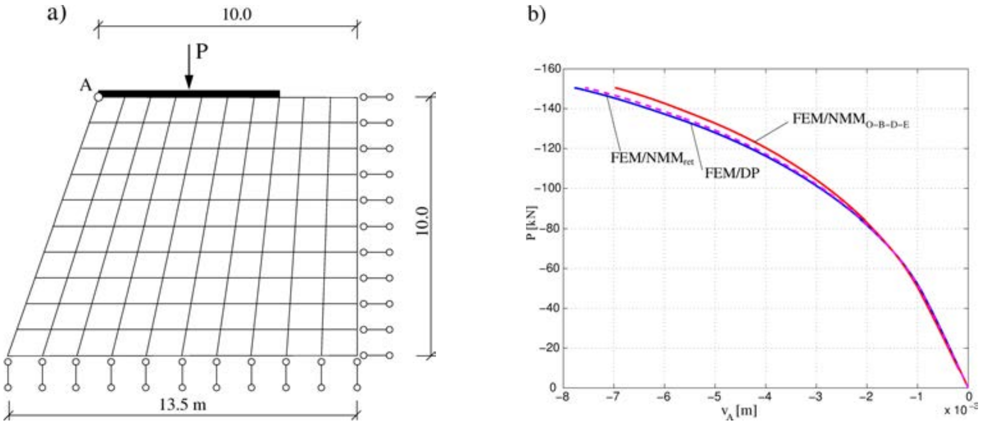

Fig. 6. a) Geometry of slope; b) equilibrium paths $P(v_A)$ computed by FEM/DP, FEM/NMM$_{\text{O-B-D-E}}$ and FEM/NMM$_{\text{ret}}$ hybrid program.

The BV problem was analysed for the geometrical and loading data shown in Fig. 6a. The equilibrium path was computed for the vertical displacement at point A of the slope applying the standard FEM/DP program. In Fig. 6b the equilibrium path is shown, obtained by the FEM/NMM$_{\text{O-B-D-E}}$ hybrid program. The two paths are similar to each other.

In order to improve the fitting of the equilibrium paths mentioned above, the retraining of the material model NMM$_{\text{O-B-D-E}}$ was carried out. In the same Fig. 6b a new equilibrium path FEM/NMM$_{\text{ret}}$ is shown. This path fits much better the FEM/DP path than non-retrained curve FEM/NMM$_{\text{O-B-D-E}}$.

The maps with distribution of pressure depended stress $\sigma_{\text{pde}}(x, y)$ are shown in Fig. 7. The distribution of stresses is similar for the applied material models.

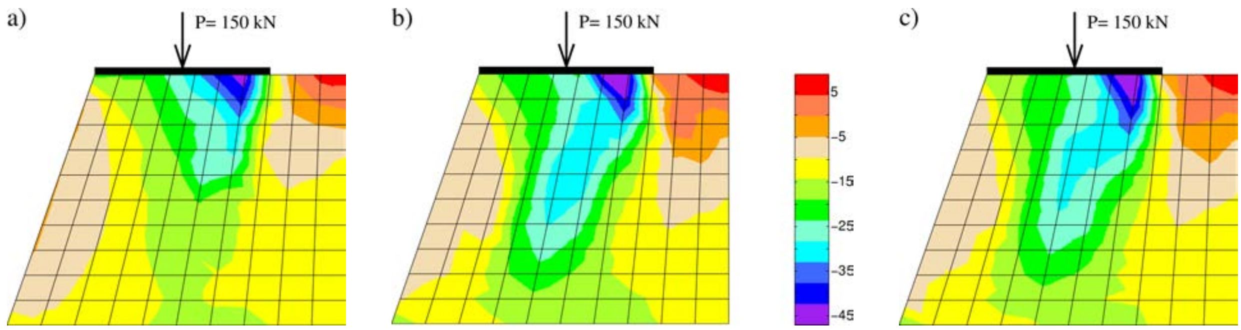

Fig. 7. Distribution of $\sigma_{\text{pde}}(x, y)$ for: a) equivalent NMM$_{\text{O-B-D-E}}$ model, b) equivalent NMM$_{\text{ret}}$ model, c) DP model.

The retrained model NMM$_{\text{ret}}$ gives the stress σ_{pde} shown in Fig.7b. The obtained stress distribution fits well the map obtained for the Drucker-Prager material model DP, see Fig.7c. Comparing Figs 7a and b, we can conclude that the retrained model NMM$_{\text{ret}}$ approximates model DP better than model NMM$_{\text{O-B-D-E}}$.

3.3. Application of Bayesian Neural Networks

In both study cases 3.1 and 3.2 the application of Bayesian approach have been investigated. According to earlier recognition cf. [9] the True Bayesian Neural Network and Semi-Bayesian NN were eliminated because of a great numerical complexity of the considered problems. Now the investigations have been directed to the application of Gaussian Processes model and the corresponding implementation is under development.

4. Final conclusion

1. The main goal of the paper was to present a new non-destructive method for identification of an unknown material model in real soil. This model, called Neural Material Model (NMM) was formulated using the computational hybrid system FEM/NMM/p-EMP, integrating the FE method with ANN. The NMM parameters were calibrated 'on line' by means of pseudo-empirical component p-EMP, which was the equilibrium curve, computed at selected points j by the FEM program using elasto-plastic Drucker-Prager material model.

2. The numerical tests performed point to the fact that identification of pressure depended material model is much more complicated than in the case of HMH material, dependent only on the stress deviator invariant J_2. This was the main reason why the cumulative algorithm B was used instead of the original autoprogressive algorithm A.

3. The two study cases analysed prove that a material can be efficiently identified on a small sub-domain O-B-D-E in order to diminish the numerical effort of identification. Then the identified model of soil NMM can be applied to the analysis of the BV problem with changed geometry and boundary conditions.

4. In case of inclined slope the equivalent material model needs minor retraining.

5. The Bayesian appoaches are difficult to be applied in the analysis of two considered study cases because of their numerical complexity. The Gaussian Processes model is considered as a candidate to be efficient tool but it needs further investigation.

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