Computer Assisted Mechanics and Engineering Sciences, 9: 439–446, 2002.

Oscillations of a hollow piezoceramic cylinder excited by a system of surface electrodes[1]

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(Received Februay 9, 2001)

An antiplane mixed boundary problem of electroelasticity for a hollow piezoelectric cylinder with an arbitrary system of active surface electrodes exciting its oscillations is considered. The solution is carried out on the basis of the approach developed in [4] for investigation of the oscillations of a solid piezoceramic cylinder with a given system of active surface electrodes. Results of numerical realization of the obtained algorithm characterizing amplitude-frequency features of the cylinder and also the behaviour of electroelastic quantities in the cylinder area and on the boundary are given.

1. Introduction

Analysis of interaction of surface electrodes with piezoelectric material in the theory of piezoelectric converters is of great importance, as the edges of the electrodes are sources of concentration of electric and mechanic fields [5]. Criteria of electromechanic destroy of piezoelectrics initiated by the edges of the electrodes are suggested in [1]. Problem of excitation of Rayleigh and Lamb waves by the electrodes in a half-space and strip, respectively, are considered [2, 3]. Some static and dynamic problems for a solid circular cylinder with electrodes are solved by the method of series in [7].

In the given paper the approach [4] based on the method of boundary integral equations for the investigation of electroelastic fields in an infinite along the symmetry axis hollow piezoceramic cylinder, excited by a system of active surface electrodes is presented. It is assumed that the cross-section of the cylinder is restricted by two smooth closed contours, that on the electrodes harmonically

[1]This is an extended version of the article presented at the 8th International Conference on Numerical Methods in Continuum Mechanics, Liptovský Ján, Low Tatras, Slovakia, September 19–24, 2000.

changing with time electric potential differences are given and that the electrodeless areas of the cylinder surfaces contact with vacuum (air). In these conditions in the cylinder the state of antiplane deformation is realized. The resolution equation system of the boundary problem is reduced to two differential Helmholtz and Laplace equations referring to the displacement amplitude and electric potential. To solve the problem integral representations of the solutions, substitution of the limited values of which into the boundary conditions being to a system of integrodifferential equations with discontinuous kernels are built. A method of approximate numerical solution of the given equations based on the method of quadratures [6] is suggested.

2. THE STATEMENT OF THE PROBLEM

Consider a related to the Cartesian coordinate system $Ox_1x_2x_3$ infinite along the axis x_3 hollow piezoceramic cylinder, the cross-section of which is limited by two arbitrary smooth contours C_1 and C_2 (Fig. 1). On the free from mechanical forces outer and inner surfaces of the cylinder there are located $2n_1$ an $2n_2$ infinitely long (in the direction of the axis x_3) thin electrodes, respectively, with preset electric potentials and the electrodeless areas of the cylinder are conjugated with vacuum (air). The boundaries of k electrode located on the contour C_m $(m = 1, 2)$ are determined by the quantities $\alpha_{2k-1}^{(m)}$ and $\alpha_{2k}^{(m)}$ $(k = \overline{1, 2n_1}$ if $m = 1$; $k = \overline{1, 2n_2}$ if $m = 2)$, and the electric potential is given on it by the quantity $\phi_k^{(m)} = \mathrm{Re}(\Phi_k^{(m)} e^{-i\omega t})$. It is assumed that the axis x_3 coincides with the direction of force lines of the electric fields of the preliminary polarization of the piezoceramics.

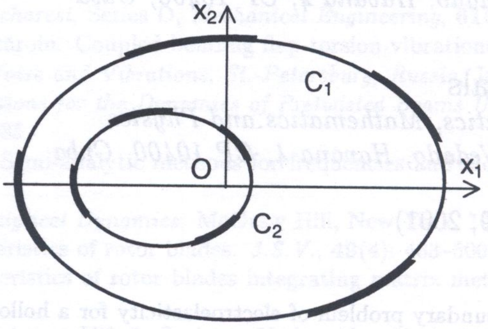

Fig. 1

In the given conditions in the hollow cylinder with electrodes the state of anti-plane deformation [7] is realized. In quasistatic approximation the system of equations of antiplane boundary problem of electroelasticity is reduced to two differential equations related to the displacement $u_3 = \mathrm{Re}(U_3 e^{-i\omega t})$ and electric potential $\phi = \mathrm{Re}(\Phi e^{-i\omega t})$, [4],

$$c_{44}^E \nabla^2 u_3 + e_{15} \nabla^2 \phi = \rho \frac{\partial^2 u_3}{\partial t^2}, \qquad e_{15} \nabla^2 u_3 - \varepsilon_{11}^S \nabla^2 \phi = 0 \,. \tag{1}$$

Here c_{44}^E, ε_{11}^S, e_{15} and ρ are the shift modulus measured at a constant electric field, dielectric permittivity, piezoceramic constant measured at constant deformation and material density, respectively.

From Eq. (1) there follow the relationships

$$\nabla^2 u_3 - c^{-2} \frac{\partial^2 u_3}{\partial t^2} = 0 \,, \qquad \nabla^2 F = 0 \,,$$

$$\phi = \frac{e_{15}}{\varepsilon_{11}^S} u_3 + F \,, \qquad c = \sqrt{\frac{c_{44}^E (1 + k_{15}^2)}{\rho}} \,, \qquad k_{15} = \frac{e_{15}}{\sqrt{c_{44}^E \varepsilon_{11}^S}} \,. \tag{2}$$

The mechanical and electric quantities may be expressed by the functions u_3 and F by the formulas

$$\tau_{13} - i\tau_{23} = 2\frac{\partial}{\partial z}\left[c_{44}^E(1+k_{15}^2)u_3 + e_{15}F\right],$$
$$D_1 - iD_2 = -2\varepsilon_{11}^S\frac{\partial F}{\partial z}, \tag{3}$$
$$E_1 - iE_2 = -2\frac{\partial}{\partial z}\left(F + \frac{e_{15}}{\varepsilon_{11}^S}u_3\right), \qquad z = x_1 + ix_2 .$$

In Eq. (3) τ_{ij} is the stress of a longitudinal shear, D_j and E_j are the components of induction vectors and electric field intensity.

The mechanical and electric boundary conditions on the surface of the cylinder allowing for Eqs. (2), (3) will be written in the following form,

$$\frac{\partial}{\partial n}\left\{c_{44}^E(1+k_{15}^2)u_3 + e_{15}F\right\} = 0 \quad \text{on } C = C_1 \cup C_2,$$
$$\phi = F + \frac{e_{15}}{\varepsilon_{11}^S}u_3 = \phi^*(\zeta, t), \qquad \zeta \in C_\phi, \tag{4}$$
$$D_n = -\varepsilon_{11}^S\frac{\partial F}{\partial n} = 0 \quad \text{on } C\backslash C_\phi .$$

Here C_ϕ is the part of the united contour C corresponding to the electroded surface of the cylinder; the derivative along the normal to the contour C is designated by the differential operator $\partial/\partial n$.

The previously written equalities (2) related to the amplitude quantities take following form,

$$\nabla^2 U_3 + \gamma^2 U_3 = 0, \qquad \nabla^2 F^* = 0, \qquad \Phi = \frac{e_{15}}{\varepsilon_{11}^S}U_3 + F^*, \qquad \gamma = \frac{\omega}{c}. \tag{5}$$

Thus, the stated problem is reduced to the determination of the function U_3 and F^* from the differential equations (5) and boundary conditions (4).

3. Integrodifferential equations of the boundary problem of electroelasticity

To reduce the stated problem to integral equations let us write down the representations of the sought-for functions in the following form

$$U_3(x_1, x_2) = \int_{C_1} p_1(\zeta)H_0^{(1)}(\gamma r)\,ds + \int_{C_2} p_2(\zeta^*)H_0^{(1)}(\gamma r^*)\,ds,$$
$$F^*(x_1, x_2) = \int_{C_1} f_1(\zeta)\frac{\partial}{\partial n}\ell n r\,ds + \int_{C_2} f_2(\zeta^*)\frac{\partial}{\partial n}\ell n r^*\,ds, \tag{6}$$
$$r = |\zeta - z|, \qquad r^* = |\zeta^* - z|, \qquad \zeta \in C_1, \qquad \zeta^* \in C_2 .$$

Here $H_\nu^{(1)}(x)$ is the Hankel function of the first type of order ν, ds is an element of the arc length of the contour C.

Substituting the limited values of the functions (7) at $z \to \zeta_0 \in C_1$ and $z \to \zeta_0^* \in C_2$ calculated by using Sohotsky–Plemelj formulas [4] for singular integrals, in the boundary conditions (4) we come to the system of integrodifferential equations of the second type,

$$
\begin{aligned}
&-2ip_1(\zeta_0) + \int_{C_1} \{p_1(\zeta)\, g_1(\zeta,\zeta_0) + f_1'(\zeta)\, g_2(\zeta,\zeta_0)\}\, ds \\
&\qquad + \int_{C_2} \{p_2(\zeta^*)\, g_1(\zeta^*,\zeta_0) + f_2'(\zeta^*)\, g_2(\zeta^*,\zeta_0)\}\, ds = 0, \qquad \zeta_0 \in C_1, \\
&2ip_2(\zeta_0^*) + \int_{C_1} \{p_1(\zeta)\, g_1(\zeta,\zeta_0^*) + f_1'(\zeta)\, g_2(\zeta,\zeta_0^*)\}\, ds \\
&\qquad + \int_{C_2} \{p_2(\zeta^*)\, g_1(\zeta^*,\zeta_0^*) + f_2'(\zeta^*)\, g_2(\zeta^*,\zeta_0^*)\}\, ds = 0, \qquad \zeta_0^* \in C_2, \\
&\pi f_1(\zeta_0) + \int_{C_1} \{p_1(\zeta)\, g_3(\zeta,\zeta_0) + f_1(\zeta)\, g_4(\zeta,\zeta_0)\}\, ds \\
&\qquad + \int_{C_2} \{p_2(\zeta^*)\, g_3(\zeta^*,\zeta_0) + f_2(\zeta^*)\, g_4(\zeta^*,\zeta_0)\}\, ds = \Phi_1^*(\zeta_0), \qquad \zeta_0 \in C_{1\phi}, \\
&-\pi f_2(\zeta_0^*) + \int_{C_1} \{p_1(\zeta)\, g_3(\zeta,\zeta_0^*) + f_1(\zeta)\, g_4(\zeta,\zeta_0^*)\}\, ds \\
&\qquad + \int_{C_2} \{p_2(\zeta^*)\, g_3(\zeta^*,\zeta_0^*) + f_2(\zeta^*)\, g_4(\zeta^*,\zeta_0^*)\}\, ds = \Phi_2^*(\zeta_0^*), \qquad \zeta_0^* \in C_{2\phi}, \\
&\int_{C_1} f_1'(\zeta)\, g_5(\zeta,\zeta_0)\, ds + \int_{C_2} f_2'(\zeta^*)\, g_5(\zeta^*,\zeta_0)\, ds = 0, \qquad \zeta_0 \in C_1 \backslash C_{1\phi}, \\
&\int_{C_1} f_1'(\zeta)\, g_5(\zeta,\zeta_0^*)\, ds + \int_{C_2} f_2'(\zeta^*)\, g_5(\zeta^*,\zeta_0^*)\, ds = 0, \qquad \zeta_0^* \in C_2 \backslash C_{2\phi},
\end{aligned}
\tag{7}
$$

where

$$
g_1(\xi,\eta) = \frac{2}{\pi i}\,\mathrm{Re}\frac{e^{i\psi_0}}{\xi-\eta} + \gamma H_1(\gamma r_0)\cos(\psi_0-\alpha_0),
$$

$$
g_2(\xi,\eta) = \frac{e_{15}}{c_{44}^E(1+k_{15}^2)}\, g_5(\xi,\eta), \qquad g_3(\xi,\eta) = \frac{e_{15}}{\varepsilon_{11}}\, H_0^{(1)}(\gamma r_0),
$$

$$
g_4(\xi,\eta) = \mathrm{Re}\frac{e^{i\psi}}{\xi-\eta}, \qquad g_5(\xi,\eta) = \mathrm{Im}\frac{e^{i\psi_0}}{\xi-\eta},
$$

$$
H_1(x) = \frac{2i}{\pi x} + H_1^{(1)}(x), \qquad r_0 = |\xi-\eta|, \qquad f_m'(\zeta) = \frac{df_m}{ds},
$$

$$
\alpha_0 = \arg(\xi-\eta), \qquad \psi_0 = \psi(\eta), \qquad \psi = \psi(\xi).
$$

Here ψ is the angle between the normal to the contour C and the axis x_1, $\Phi_m^*(\zeta_0)$ are piecewise-constant functions defining the values of the electric potential on the electrodes located on the outer ($m=1$) and inner ($m=2$) surfaces of the cylinder.

Defining the functions $p_m(\zeta)$ and $f_m(\zeta)$ from the system (7) by formulas (3) using the representations (6) it is possible to calculate all the components of the electrostatic field in the cylinder area.

Let us determine the expression for the density amplitude of distribution of the electric charges $q_k^{(m)}(\beta)$ on k-th electrode located on the contour C_m ($m=1,2$). Introducing the parametrization of the contour C_m with the help of the equality $\zeta = \zeta(\beta)$, $\zeta^* = \zeta^*(\beta)$ ($0 \le \beta \le 2\pi$) and taking into account the fact that the cylinder is conjugated with vacuum we will write

$$
q_k^{(m)}(\beta) = D_n^{(m,k)}(\beta), \qquad \alpha_{2k-1}^{(m)} < \beta < \alpha_{2k}^{(m)} \qquad (m=1,2). \tag{8}
$$

Here $D_n^{(m,k)}(\beta)$ is an amplitude of the normal component of the vector of electric induction on the corresponding electroded area of the contour C. Using the integral representations (6) for the functions $F^*(x_1, x_2)$ and allowing for (4) and (8), we find

$$q_k^{(m)}(\beta_0) = -\varepsilon_{11}^S \left\{ \int_{C_1} f_1'(\zeta)\, \mathrm{Im} \frac{e^{i\psi_0}}{\zeta - \eta}\, \mathrm{d}s + \int_{C_2} f_2'(\zeta^*)\, \mathrm{Im} \frac{e^{i\psi_0}}{\zeta^* - \eta}\, \mathrm{d}s \right\}, \qquad \eta \in C_{\phi_k}, \tag{9}$$

where $C_{m\phi_k}$ is a part of the contour C_m on which k-th electrode is located.

Integrating the expression (9) on the variable β_0 in the limits from $\alpha_{2k-1}^{(m)}$ to $\alpha_{2k}^{(m)}$, we obtain the amplitude value of the summarized charge of k-th electrode located on the contour C_m related to its length. The current flowing through the given electrode may be determined by the formula

$$I_k^{(m)}(t) = \mathrm{Re}\left\{ i\omega e^{-i\omega t} \int_{\alpha_{2k-1}^{(m)}}^{\alpha_{2k}^{(m)}} q_k^{(m)}(\beta_0)\, s'(\beta_0)\, \mathrm{d}\beta_0 \right\}, \qquad s'(\beta_0) = \frac{\mathrm{d}s}{\mathrm{d}\beta_0}. \tag{10}$$

Equation (10) allows to find anti-resonance frequencies at which $I_k^{(m)}(t) = 0$.

4. NUMERICAL SOLUTION OF THE SYSTEM (7)

Let us consider one of the methods of numerical realization of the system (7). Let us build the interpolating Lagrange polynomial for the sought-for functions $p(\zeta)$ and $f'(\zeta)$ in the nodes $\beta_j = 2\pi(j-1)/N$ $(j = \overline{1, N})$. Such polynomial has the form [6]

$$\begin{aligned} L_N[p_*(\beta); \beta] &= \frac{1}{N} \sum_{j=1}^{N} p_j^0 \sin \frac{N(\beta_j - \beta)}{2} \operatorname{cosec} \frac{\beta_j - \beta}{2}, \\ L_N[f_*'(\beta); \beta] &= \frac{1}{N} \sum_{j=1}^{N} f_j^0 \sin \frac{N(\beta_j - \beta)}{2} \operatorname{cosec} \frac{\beta_j - \beta}{2}, \\ p(\zeta) = p_*(\beta), \quad p_j^0 &= p_*(\beta_j), \quad f(\zeta) = f_*(\beta), \quad f_j^0 = f_*'(\beta_j). \end{aligned} \tag{11}$$

It must be mentioned here that the formulas (11) are valid for odd numbers of the node division of the contour C.

Integration of the second formula (11) using the equation cited in [8],

$$\int \frac{\sin(2m+1)x}{\sin x}\, \mathrm{d}x = 2 \sum_{k=1}^{m} \frac{\sin 2kx}{2k} + x,$$

brings to the following expression for the function $f_*(\beta)$,

$$\begin{aligned} M_N[f_*(\beta); \beta] &= \frac{1}{N} \sum_{j=1}^{N} f_j^0\, \Omega_j(\beta) + A, \\ \Omega_j(\beta) &= -2 \sum_{k=1}^{\frac{N-1}{2}} \frac{\sin k(\beta_j - \beta) - \sin k\beta_j}{k} + \beta. \end{aligned} \tag{12}$$

The constant A appearing here must be determined from the conditions of the periodicity of the function $f_*(\beta)$ which due to Eq. (12) has the following form,

$$\sum_{j=1}^{N} f_j^0 = 0. \tag{13}$$

Applying (12) we also find the quadrature formula

$$\int_0^{2\pi} f_*(\beta)\, G(\beta,\beta^*)\, \mathrm{d}\beta = \frac{2\pi}{N^2}\sum_{j=1}^{N} f_j^0 \sum_{m=1}^{n} \Omega_{jm}\, G(\beta_m\,,\beta^*) + A\frac{2\pi}{N}\sum_{m=1}^{N} G(\beta_m\,,\beta^*)\,, \tag{14}$$

where $\Omega_{jm} = \Omega(\beta_m)$. In the node collocations $\beta_\ell^* = \pi(2\ell-1)/N$ $(\ell = \overline{1,N})$, the polynomial (11) has the following value at odd value of N,

$$L_n[p_*(\beta);\beta_\ell^*] = \frac{1}{N}\sum_{j=1}^{N} p_j^0(-1)^{\ell+j}\operatorname{cosec}\frac{\beta_\ell^*-\beta_j}{2} \qquad (\ell=\overline{1,N})\,. \tag{15}$$

For singular integral in (2.20) the formula analogous to the formula of calculating regular integrals [6] appears,

$$\int_0^{2\pi} f_*'(\beta_j)\operatorname{Im}\frac{e^{i\psi_0}}{\zeta(\beta)-\zeta_0(\beta_\ell^*)}\, s'(\beta)\,\mathrm{d}\beta = \frac{2\pi}{N}\sum_{j=1}^{N} f_j^0 \operatorname{Im}\frac{e^{i\psi_0(\beta_\ell^*)}}{\zeta(\beta_j)-\zeta_0(\beta_\ell^*)}\, s'(\beta_j)\,. \tag{16}$$

Now, substituting the integrals in Eq. (7) by finite sums of the formulas (14), (16) and using the equalities (12), (13) and (15), we come to the system $4N+2$ of algebraic equations related to the values of the functions $p_m(\zeta)$ and $f_m'(\zeta)$ in the nodes of the interpolation β_j $(j=\overline{1,N})$ and the constant $A_m (m=1,2)$.

5. Examples

As an example let us consider a hollow cylinder (material – ceramics PZT-4 [7]), the cross-selection of which is restricted by two circular contours ($\zeta = R_1 e^{i\beta}$, $\zeta^* = R_2 e^{i\beta} + a$, $\beta \in [0, 2\pi]$). The cylinder is excited by four electrodes located in pairs on its outer and inner surfaces. Solution of the system of integrodifferential equations (7) for this case was fulfilled numerically according to the above indicated scheme.

In Fig. 2a amplitude-frequent characteristics for the quantities $Q^* = \left|Q/\left(\varepsilon_{11}^S \Phi_1^{(1)}\right)\right|$ characterizing the summarized charge Q on the electrode $(m=k=1)$ are presented. The curves 1 and 2 are built for values $a=0$, $R_2/R_1 = 0.5$ and 0.8, respectively. The potentials on the electrodes are

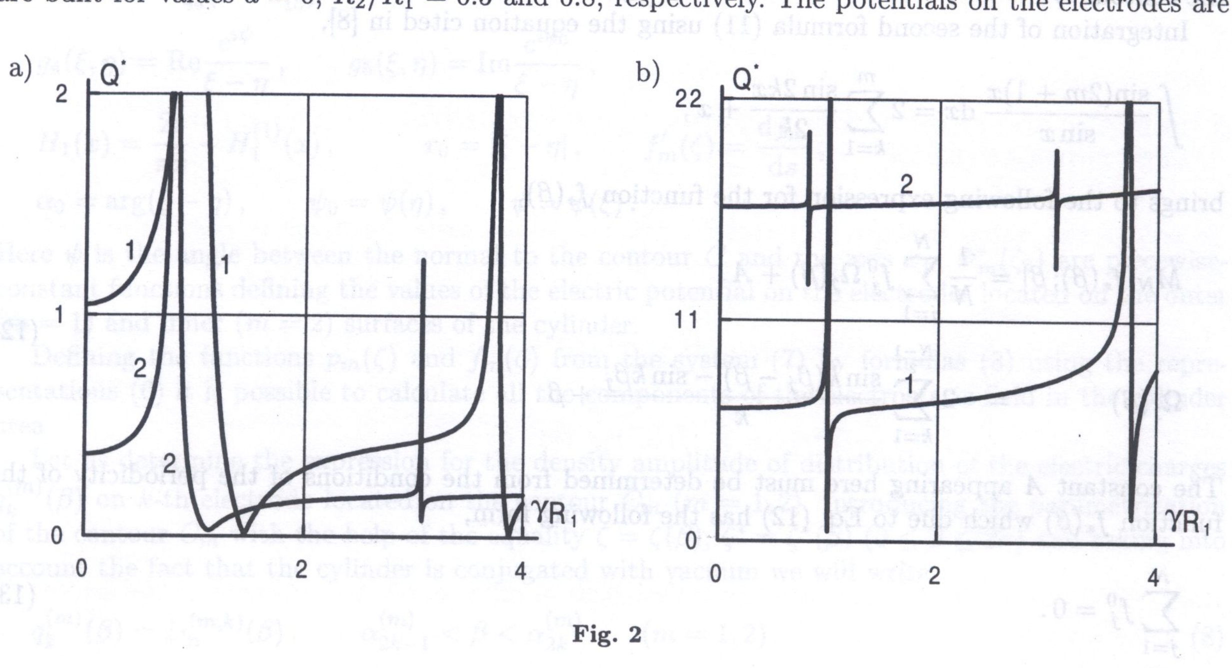

Fig. 2

written in the following form: $\Phi_1^{(1)} = \Phi_2^{(2)} = 1\,\mathrm{V}$, $\Phi_2^{(1)} = \Phi_1^{(2)} = -1\,\mathrm{V}$. Location of the electrodes was fixed by the quantities $\alpha_1^{(m)} = 5\pi/14$, $\alpha_2^{(m)} = 9\pi/14$, $\alpha_3^{(m)} = 19\pi/14$, $\alpha_4^{(m)} = 23\pi/14$ $(m = 1, 2)$. The analogous results for the cases $\Phi_1^{(1)} = \Phi_1^{(2)} = 1\,\mathrm{V}$, $\Phi_2^{(1)} = \Phi_2^{(2)} = -1\,\mathrm{V}$ are presented in Fig. 2b.

From Fig. 2 it follows that with decrease of the parameter R_2/R_1 there is observed the displacement of the resonance frequencies to the right and in this case the displacement is increased with increase of the number of resonance frequencies.

Investigation of the influence of electric loading on the distribution of mechanical fields in the area of a hollow cylinder is of great interest. In Figs. 3a,b there are shown module level lines of the amplitude of displacement correspondingly for the values $\gamma R_1 = 0.5$, $\alpha_1^{(m)} = 5\pi/14$, $\alpha_2^{(m)} = 9\pi/14$, $\alpha_3^{(m)} = 19\pi/14$, $\alpha_4^{(m)} = 23\pi/14$ and $\alpha_1^{(m)} = \pi/6$, $\alpha_2^{(m)} = 5\pi/6$, $\alpha_3^{(m)} = 7\pi/6$, $\alpha_4^{(m)} = 11\pi/6$ $(m = 1, 2)$. The values of the potentials were presented equal to $\Phi_1^{(1)} = \Phi_1^{(2)} = 1\,\mathrm{V}$, $\Phi_2^{(1)} = \Phi_2^{(2)} = -1\,\mathrm{V}$.

a) b)

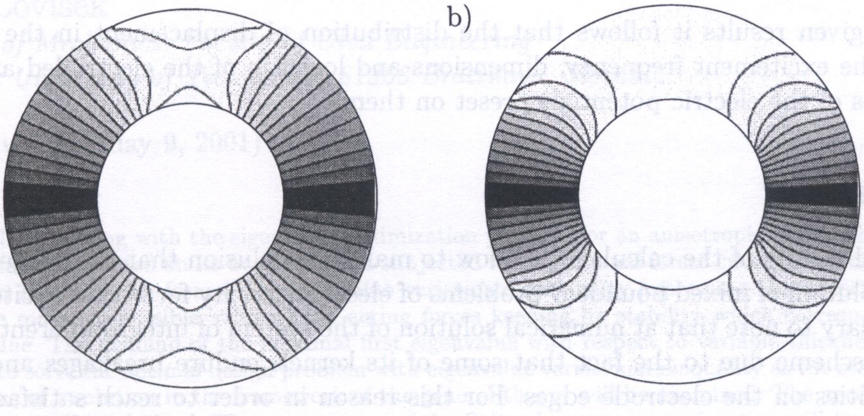

Fig. 3

Level lines of the quantity $|U_3|$ for the cases $\Phi_1^{(1)} = -1\,\mathrm{V}$, $\Phi_1^{(2)} = 5\,\mathrm{V}$, $\Phi_2^{(1)} = 1\,\mathrm{V}$, $\Phi_2^{(2)} = -5\,\mathrm{V}$ and $\Phi_1^{(1)} = -5\,\mathrm{V}$, $\Phi_1^{(2)} = 1\,\mathrm{V}$, $\Phi_2^{(1)} = 5\,\mathrm{V}$, $\Phi_2^{(2)} = -1\,\mathrm{V}$ are represented in Figs. 4a,b. The design variables were assumed to be equal to $\alpha_1^{(m)} = \pi/6$, $\alpha_2^{(m)} = 5\pi/6$, $\alpha_3^{(m)} = 7\pi/6$, $\alpha_4^{(m)} = 11\pi/6$ $(m = 1, 2)$, $\gamma R_1 = 3$.

a) b)

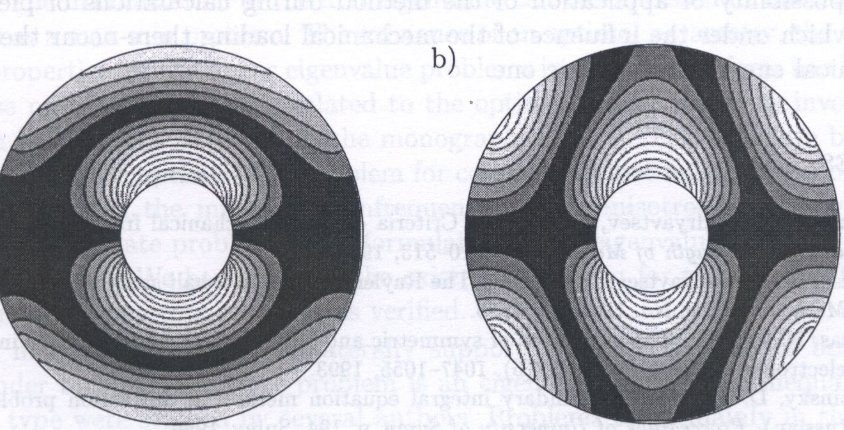

Fig. 4

Figures 5a,b illustrate the lines of the level $|U_3|$ in the area of an eccentric cylinder, respectively, at $\gamma R_1 = 1$ and 6 for the values of the potential $\Phi_1^{(1)} = \Phi_2^{(2)} = 1\,\mathrm{V}$, $\Phi_2^{(1)} = \Phi_1^{(2)} = -1\,\mathrm{V}$ (location of the electrodes corresponds to Fig. 4).

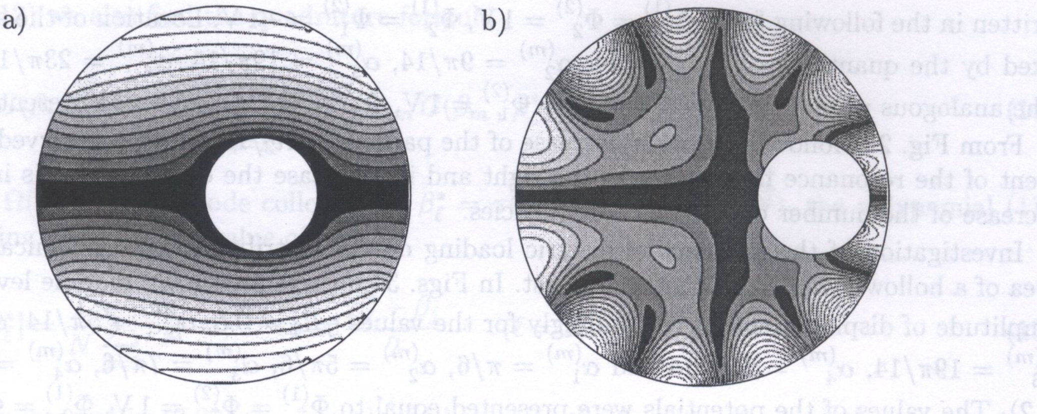

Fig. 5

From the given results it follows that the distribution of displacement in the cylinder mostly depends on the excitement frequency, dimensions and location of the electroded areas and also on the differences of the electric potentials preset on them.

6. Conclusion

The presented results of the calculations allow to make a conclusion that the suggested approach is efficient for solution of mixed boundary problems of electroelasticity for bodies excited by electrodes.

It is necessary to note that at numerical solution of the system of integrodifferential equations (7) by the given scheme due to the fact that some of its kernels endure breakages and the "densities" have singularities on the electrode edges. For this reason in order to reach satisfactory accuracy a substantial number of interpolation nodes should be taken on the boundary cylinder contours, which increases the processing time. Despite this fact the investigated approach attracts by its universality permitting to study different variants of loading the cylinder without any principle complications of the calculation algorithms.

The considered approach to the solution of mixed boundary problems of electroelasticity may be applied for calculations of the characteristics of piezoelectric transforms of different geometric forms under the electric loading with the help of multielectric systems. We must also note the fundamental possibility of application of the method during calculations of piezoelectric voltage generator in which under the influence of the mechanical loading there occur the direct conversion of the mechanical energy into electric one.

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