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Evolutionary computation in optimization and identification

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The aim of the paper is to present the application of the evolutionary algorithms to selected optimization and identification problems of mechanical systems. The coupling of evolutionary algorithms with the finite element method and the boundary element method creates a new artificial intelligence technique that is very suitable in computer aided optimal design and defect detection. Several numerical examples for optimization and identification are presented.

Keywords: evolutionary algorithms, finite element method, boundary element method, optimization, identification

1. Introduction

Many problems of mechanics are formulated as minimization (or maximization) of certain functionals with respect to state fields or design variables.

In the case of the first problem, the functionals are expressed by the total potential energy (for static problems) or by the Hamilton's principal functional (for dynamical problems). Minimization of the total potential energy with respect to state fields leads to equations of equilibrium of mechanical systems. A stationary value of the Hamilton's principal functional describes the real motion of the mechanical system.

From the point of view of optimal design of structures, minimization of the objective functionals, which describe the optimization criteria, with respect to design variables leads to the best structure. The process of looking for the best structure often gives not unique solutions and applications of *methods of artificial intelligence* enable to find the global optimal solutions. Evolutionary algorithms [1, 24] belong to methods which are very promising in computer aided optimal design.

The similar problem like the optimal design is also the identification of certain geometrical or material features (e.g. internal defects) in existing structures having some measurements of state fields or the control of the boundary conditions to secure suitable requirements. Such problems are formulated as minimization of certain functionals. They can be solved by using also artificial intelligence techniques like evolutionary algorithms.

One of the specific features of considered problems is the fact that objective functionals very often do not depend on design variables in the explicit way. In order to find the relationship between changes of design variables and changes of the objective functional one should solve a boundary value or a boundary initial value problem. It can be done by using the *finite element method* (FEM) or the *boundary element method* (BEM)

Numerical techniques as FEM and BEM are routinely used by mechanical and civil engineers in *computer aided design* (CAD) of mechanical components and structures [2, 7, 20].

FEM and BEM although conceptually quite different can be considered as methods that belong to a common class of residual methods. In the last decade FEM and BEM have been often used in *computer aided optimal design* (CAOD) where the design problem is considered as an optimization task.

The aim of the paper is to develop applications of evolutionary computation in the form of coupling of evolutionary algorithms with the finite element method and the boundary element method to CAOD and identification. This work is a review of own original works elaborated by authors.

2. FORMULATION OF THE EVOLUTIONARY OPTIMIZATION

An elastic body, which occupies a domain Ω bounded by a boundary Γ, is considered. Boundary conditions in the form of the displacement and traction fields are prescribed. In the case of dynamical problems initial conditions are also prescribed. One should find an optimal shape of the boundary Γ by minimizing an objective functional

$$\min J_o(\mathbf{x}) \tag{1}$$

with imposed constraints

$$J_\alpha(\mathbf{x}) = 0, \quad \alpha = 1, 2, \ldots, A; \qquad J_\beta(\mathbf{x}) \geq 0, \quad \beta = 1, 2, \ldots, B, \tag{2}$$

where $\mathbf{x}$ is a vector of shape design parameters.

The problem can be solved by using conventional optimization procedures, based on sensitivity information. The boundary element method was applied in several shape optimization problems in which minimization procedures used sensitivity analysis [5, 6, 12]. Unfortunately, approaches based on sensitivity information have some drawbacks: an objective function must be continuous, shape variation of the boundary should be regular, a hessian of the objective function should be positive definite, there is a large probability of convergence to a local optimum, computation starts from a single point narrowing the search domain and a choice of the starting point may exert an influence on the convergence.

Because of these difficulties there is a demand for checking other methods, free from the restrictions mentioned above. The genetic algorithms belong to these methods. Genetic algorithms [1, 24] are stochastic algorithms whose search methods model natural phenomena: the genetic inheritance and the Darvinian strife for survival. Classical genetic algorithms are based only on the fitness value information and coded chromosomes. They work on populations of solutions and use binary operators of crossover and mutation and the probability of operators is constant.

Evolutionary algorithms are considered here as modified and generalized classical genetic algorithms in which populations of chromosomes are not coded binary and ***floating point representation*** is used. They use modified crossover (***simple, arithmetical and heuristic***) and mutation (***uniform, boundary and non-uniform***) operations. The selection is performed in the form of the ***ranking selection*** or the ***tournament selection*** and the probability of operators can be variable [24].

A chromosome, which represents the design vector or matrix, can be expressed in the form

$$\mathbf{x} = \langle x_1, x_2, \ldots, x_i, \ldots x_N \rangle \quad \text{or} \quad \mathbf{x} = [x_{ik}], \quad 1 \leq i \leq N, \quad 1 \leq k \leq K, \tag{3}$$

where there are imposed restrictions on genes x_j and x_{ik} in the form

$$x_{iL} \leq x_i \leq x_{iR}, \qquad x_{ikL} \leq x_{ik} \leq x_{ikR}. \tag{4}$$

Genes x_j and x_{ik} represent geometry or material properties of the structure. Values of genes x_j and x_{ik} belong to the space of real or natural numbers.

The crossover operation swaps some chromosomes of the selected parents in order to create offspring. Simple, arithmetical and heuristic crossover operators are applied.

Simple crossover

This operator needs two parents and produces two children. The simple crossover may produce a child outside the design space. To avoid this, a parameter $\alpha \in [0, 1]$ is applied. For randomly generated crossing parameter i it works as follows (chromosomes x_1, x_2 are parents in the vector form),

$$\mathbf{x}_1 = \langle x_1, x_2, \ldots, x_i, \ldots x_N \rangle \qquad \text{and} \qquad \mathbf{x}_2 = \langle y_1, y_2, \ldots, y_i, \ldots y_N \rangle, \tag{5}$$

$$'\mathbf{x}_1 = \langle x_1, \ldots, x_i, y_{i+1}, \ldots, y_N \rangle, \qquad '\mathbf{x}_2 = \langle x_1, \ldots, x_i, y_{i+1}, \ldots, y_N \rangle. \tag{6}$$

Arithmetical crossover

This operator produces two children that are a linear combination of two parents $\mathbf{x}_1$ and $\mathbf{x}_2$,

$$'\mathbf{x}_1 = \alpha \mathbf{x}_1 + (1 - \alpha) \mathbf{x}_2, \qquad '\mathbf{x}_1 = \alpha \mathbf{x}_2 + (1 - \alpha) \mathbf{x}_1. \tag{7}$$

Heuristic crossover

This operator produces a single offspring from two parents,

$$'\mathbf{x}_3 = r(\mathbf{x}_2 - \mathbf{x}_1) + \mathbf{x}_2, \tag{8}$$

where r is a random value from the range $[0, 1]$ and $J(\mathbf{x}_2) \leq J(\mathbf{x}_1)$.

Mutation operators such as the uniform mutation, the boundary mutation,the non-uniform mutation and the special gradient mutation are used,

$$\mathbf{x}_1 = \langle x_1, x_2, \ldots, x_i, \ldots x_N \rangle, \qquad '\mathbf{x}_1 = \langle x_1, x_2, \ldots, 'x_i, \ldots x_N \rangle. \tag{9}$$

Uniform mutation

Children are allowed to move freely within the feasible domain and the gene $'x_i$ takes any arbitrary value from the range $[x_{iL}, x_{iR}]$.

Boundary mutation

The chromosome can take only boundary values of the design space, $'x_i = x_{iL}$ or $'x_i = x_{iR}$. The boundary mutation works very well when the solution lies either on or near the boundary of the feasible search space.

Non-uniform mutation

This mutation operator depends on a generation number t and is used to tune of the system

$$'x_i = \begin{cases} x_i + \Delta(t, x_{iR} - x_i) & \text{if a random digit is 0,} \\ x_i - \Delta(t, x_{iR} - x_i) & \text{if a random digit is 1,} \end{cases} \tag{10}$$

where the function Δ takes value from the range $[0, y]$.

Gradient mutation [26]

This single-argument operator changes any chromosome on the ground of the fitness function gradient,

$$'x_i = x_i + \beta \mathbf{h}(DJ_o/Dx_i), \tag{11}$$

where β is a coefficient determining a step increment in the search direction $\mathbf{h}$ which depends on the fitness function gradient DJ_o/Dx_i.

The objective function given by (1) plays the role of the fitness function. Evolutionary optimization is performed by using an evolutionary algorithm which minimizes the fitness function (1) with respect to design parameters. The constraints (2) are taken into account by the penalty function method.

The flow chart of the proposed approach of the evolutionary optimization using the finite element method and the boundary element method is presented in Fig. 1.

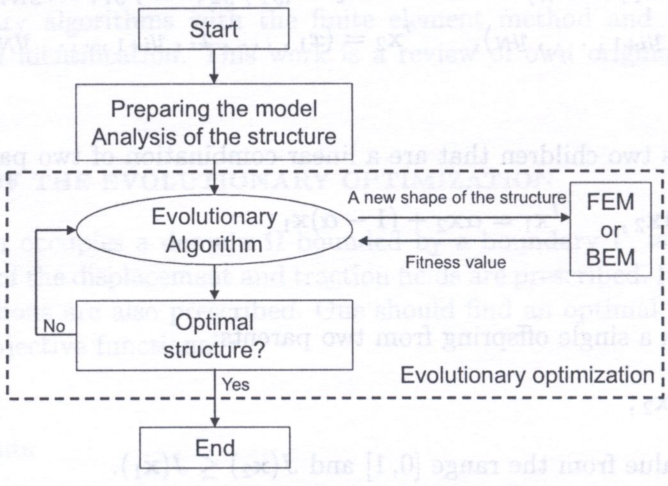

Fig. 1. Block diagram of the evolutionary optimization

Depending on the problem the various FEM and BEM approaches will be applied in evolutionary optimization.

Each of the computer methods is suited to particular problems, and for a given problem one is generally more efficient while the other is less efficient.

Basically the *finite element methods* [2, 20] require the subdivision of the region into domain elements, the response characteristics of each of which are described in a simplified way. The *boundary element methods* [2, 7, 20] on the other hand only require the subdivision of the boundary of the region because the solution of the problem to be solved is taken to be a combination of exact solutions inside the region. The latter method works extremely well for problems with a high ratio of volume to surface area but not so well for those problems with a high ratio of volume to surface area but not so well for those with a low ratio. The opposite is the case for the finite element method. Independently of this cause the finite element method is more effective in nonlinear problems and for framed structures than the boundary element method.

3. APPLICATIONS OF EVOLUTIONARY OPTIMIZATION

3.1. Optimization of truss structures [16]

The criterion of optimization is minimization of the mass of a truss taking into account the following constraints:

- stresses in each truss member are lower than the allowable stress,
- displacements in each truss joint are lower than the admissible value,
- normal forces are lower than the buckling load.

The applied evolutionary algorithm uses operators of the mutation (uniform, non-uniform and boundary), the crossover (simple, arithmetical and heuristic) and the cloning and the ranking selection are implemented.

Design variables are split into three groups containing:

- information about existing truss members,
- information about areas of cross sections of truss members,
- coordinate of free truss joints.

Each group is represented by different kinds of chromosomes. The evolutionary algorithm is used to optimization of:

- number of truss members,
- areas of cross sections of truss members,
- coordinates of free truss joints,
- combinations of above items.

In order to evaluate of constraints the finite element method is used. Displacements of joints are calculated by solving the system of algebraic equations

$$\mathbf{Kq} = \mathbf{F} \tag{12}$$

where **K** is a global stiffness matrix, **q** is a column matrix which contains unknown displacements of joints and **F** is a column matrix which contains acting forces.

Numerical example 1

The truss (Fig. 2a) is optimized for the criterion of minimum mass. The forces are applied in selected joints. Parameters of evolutionary algorithms are: number of generations — 5000; population — 20000. Optimal solution has been found in 2500 generation (Fig. 2b). The mass of the truss before and after optimization is shown in Table 1.

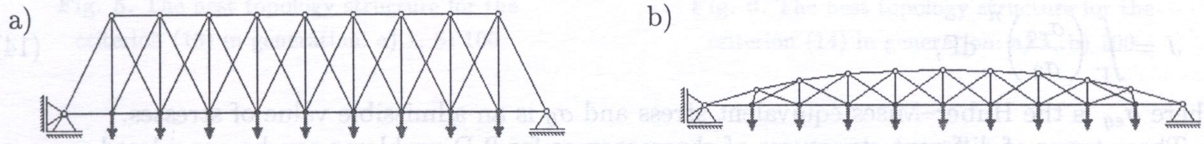

Fig. 2. The truss: a) before and b) after optimization

Table 1. Mass before and after optimization

	Before	After
Mass [kg]	2539.395	2000.352

Numerical example 2

The trussed structure (Fig. 3a) undergoes evolutionary optimization for the criterion of minimum mass. Parameters of evolutionary algorithms are: number of generations — 2000; population — 5000. Optimal solution has been found in 1500 generation (Fig. 3b). The mass of the truss before and after optimization is shown in Table 2.

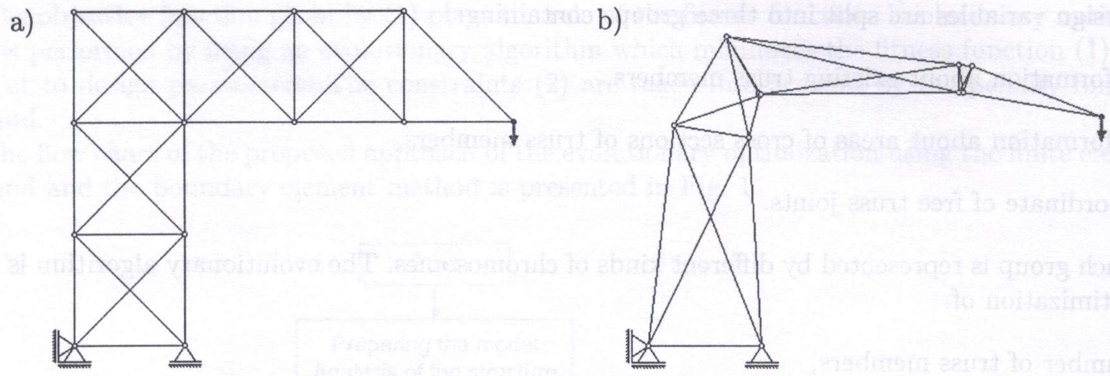

Fig. 3. The trussed structure: a) before and b) after optimization

Table 2. Mass before and after optimization

	Before	After
Mass [kg]	5947.43944	3406.069

3.2. Topology optimization

An application of classical genetic algorithms to shape optimization of 2-D elastic structures was proposed in [19]. This concept was extended to the generalized shape optimization (topology optimization + shape optimization) using evolutionary algorithms and BEM [13, 21, 22]. The generalized shape optimization based on the evolutionary algorithm and FEM is considered in [27].

A new approach for topology optimization based on the evolutionary algorithm is proposed [17]. The number of voids and their shape and position are entirely controlled by the evolutionary algorithm.

Two different fitness functions are considered,

$$J = \int_\Omega d\Omega \tag{13}$$

and

$$J = \int_\Gamma \left(\frac{\sigma_{eq}}{\sigma_0}\right)^n d\Gamma, \tag{14}$$

where σ_{eq} is the Huber–Mises equivalent stress and σ_0 is an admissible value of stresses.

Three types of different structures of chromosomes for 2-D problems can be considered,

$$\mathbf{X} = \langle x_1\,, y_1\,, \mathbf{r}_1\,, x_2\,, y_2\,, \mathbf{r}_2\,, \ldots, x_{n_{\max}}\,, y_{n_{\max}}\,, \mathbf{r}_{n_{\max}}\rangle, \tag{15}$$

$$\mathbf{X} = \langle n, x_1\,, y_1\,, \mathbf{r}_1\,, x_2\,, y_2\,, \mathbf{r}_2\,, \ldots, x_{n_{\max}}\,, y_{n_{\max}}\,, \mathbf{r}_{n_{\max}}\rangle, \tag{16}$$

$$\mathbf{X} = \langle w_1\,, w_1\,, \ldots, w_{n_{\max}}\,, x_1\,, y_1\,, \mathbf{r}_1\,, x_2\,, y_2\,, \mathbf{r}_2\,, \ldots, x_{n_{\max}}\,, y_{n_{\max}}\,, \mathbf{r}_{n_{\max}}\rangle, \tag{17}$$

where: $n_{\max}$ is maximum number of voids, genes x_i and y_i are co-ordinates of the centre of a void, the gene $\mathbf{r}_i$ is a vector of shape parameters. For a circular void $\mathbf{r}_i$ contains only one element which is a radius, for an elliptical void $\mathbf{r}_i$ represents two radii, for a more shape-complicated void $\mathbf{r}_i$ is a set of radii associated with control points (NURBS).

In the case of the chromosome (15) the number of voids is governed by the condition: *if* $r_i < r_{\min}$ *then the i-th void does not exist.* For the chromosome (16) the number of voids is controlled by the gene n. In the case of the chromosome (17) the number of voids is governed by controlling parameters $w_i = \{true; false\}$.

In the evolutionary optimization of topology the following operators have been applied: (i) uniform, non-uniform, boundary and gradient mutations, (ii) simple, arithmetical and heuristic crossovers and (iii) the tournament selection.

Numerical example 3

2-D elastic structure has been undergone topology optimization (Fig. 4).

Circular voids have been introduced into the domain Ω of the structure. The maximum number of voids was $n_{\max} = 4$ and the size of population was 10000. The geometry of the structure for the fitness functions (13) is presented in Fig. 5. Stress constrains were imposed. Figure 6 shows the geometry of the structure for the fitness function (14) and a constraint for the volume was applied. The structures were modelled by the boundary element method.

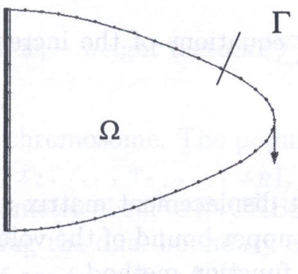

Fig. 4. **The structure before optimization**

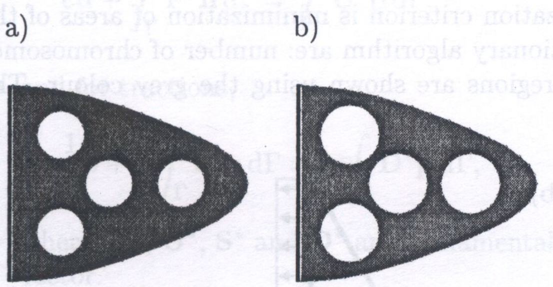

Fig. 5. **The best topology structure for the criterion (13) in generation: a) 1, b) 100**

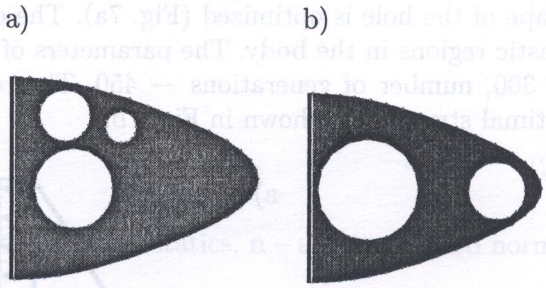

Fig. 6. **The best topology structure for the criterion (14) in generation: a) 1, b) 100**

3.3. Shape optimization of elastoplastic structures

The application of an evolutionary algorithm and the finite element method in shape optimization of elastoplastic solids is presented [14, 23]. The optimization criterion is minimization of areas of the plastic regions in the body. The fitness function is formulated in the form of the domain integral,

$$J = \int_{\Omega} \left[\frac{\sigma_{eq}}{\sigma_o} \right]^n \mathrm{d}\Omega. \tag{18}$$

Minimization of this functional reduces values of stresses and areas of plastic regions. Constraints in the form of upper bound of volume are imposed.

In order to minimize the number of design parameters, the boundary of the solid is described by means of the NURBS (Non-Uniform B-Splines) curves. The FEM mesh is generated by the Delaunay method using the TRIANGLE program.

One assumes that the solids are made from elastoplastic material with hardening. The fitness function is evaluated for each chromosome in terms of the incremental finite element method [20].

The constitutive equation is given as follows,

$$\mathrm{d}\boldsymbol{\sigma} = \mathbf{D}_T(\boldsymbol{\sigma})\,\mathrm{d}\boldsymbol{\varepsilon}, \tag{19}$$

where $\mathbf{D}_T(\boldsymbol{\sigma})$ is the tangent constitutive matrix.

The tangent stiffness matrix is formulated by

$$\mathbf{K}_T = \int_\Omega \mathbf{B}^T \mathbf{D}_T(\boldsymbol{\sigma}) \mathbf{B}\,\mathrm{d}\Omega \tag{20}$$

where $\mathbf{B}$ is the geometrical matrix.

The resulting system of algebraic equations of the incremental finite element method may be presented as

$$\mathbf{K}_T\,\mathbf{dq} = \mathbf{dF} \tag{21}$$

in which $\mathbf{dq}$ is an unknown increment displacement matrix and $\mathbf{dF}$ is an increment force matrix.

The constraints in the form of the upper bound of the volume and geometrical restrictions of are taken into account using the penalty function method.

Numerical example 4

The plate with NURBS modelled of the external boundary shape and NURBS modelled of the shape of the hole is optimized (Fig. 7a). The optimization criterion is minimization of areas of the plastic regions in the body. The parameters of evolutionary algorithm are: number of chromosomes — 300, number of generations — 450. The plastic regions are shown using the grey colour. The optimal structure is shown in Fig. 7b.

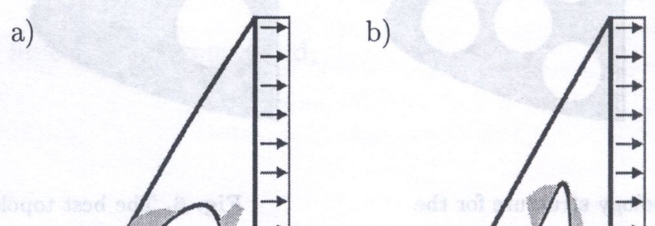

Fig. 7. The elasto-lastic plate: a) before and b) after optimization

3.4. Shape optimization of cracked structures

In the shape optimization of cracked structures three kinds of the criteria can be proposed [3, 8]:

- minimization of the maximum crack opening (*MCO*),

$$MCO = \max(u) = \max(u^+ - u^-), \tag{22}$$

where: $u = \sqrt{u_i u_i}$ and u^+, u^- are the displacement values of the coincident nodes laying on the opposite sides of the crack,

- minimization of the reduced J-integral in the form,

$$J_{\text{red}} = \left(J_1^2 + J_2^2\right)^{1/2}, \tag{23}$$

where: J_1 and J_2 are Rice J-integrals for 1 and 2 tips of the crack,

- minimization of the reduced stress intensity factor in the form

$$K_{\text{red}} = \sum_{i=1}^{n} w_i K_i \tag{24}$$

where: K_i – stress intensity factors, w_i – weight factors ($\sum w_i = 1$), $n = 4$ for one internal crack (2-D problems).

The design vector $\mathbf{x}$ is treated as a chromosome. The population consists of randomly generated set of feasible chromosomes $\mathbf{x} = (x_1, x_2, \ldots, x_r, \ldots, x_R)$, where x_r is a gene which represents coordinate of a control point of the parametrical curve (NURBS or B-spline) modelling the boundary.

The direct problem is solved by using the dual boundary element method (DBEM). Two vector boundary integral equations of DBEM are formulated [7]:

- for displacements,

$$\mathbf{cu} + \int_\Gamma \mathbf{P}^*\mathbf{u}\,d\Gamma = \int_\Gamma \mathbf{U}^*\mathbf{p}\,d\Gamma, \tag{25}$$

- and for tractions,

$$\frac{1}{2}\mathbf{p} + \mathbf{n}\int_\Gamma \mathbf{S}^*\mathbf{u}\,d\Gamma = \mathbf{n}\int_\Gamma \mathbf{D}^*\mathbf{p}\,d\Gamma, \tag{26}$$

where: $\mathbf{P}^*$, $\mathbf{U}^*$, $\mathbf{S}^*$ and $\mathbf{D}^*$ are fundamental solutions of elastostatics, $\mathbf{n}$ – a unit outward normal vector.

Numerical example 5

The boundary of a plate contained two cracks (Fig. 8a) is optimized. The criterion of minimum K_{red} is applied. Parameters of evolutionary algorithms are: number of generations— 1500; number of chromosomes — 100. Constraints are imposed on on the boundary equivalent Huber–Mises stresses. Optimal solution has been found in 1280 generation (Fig. 8b, Table 3).

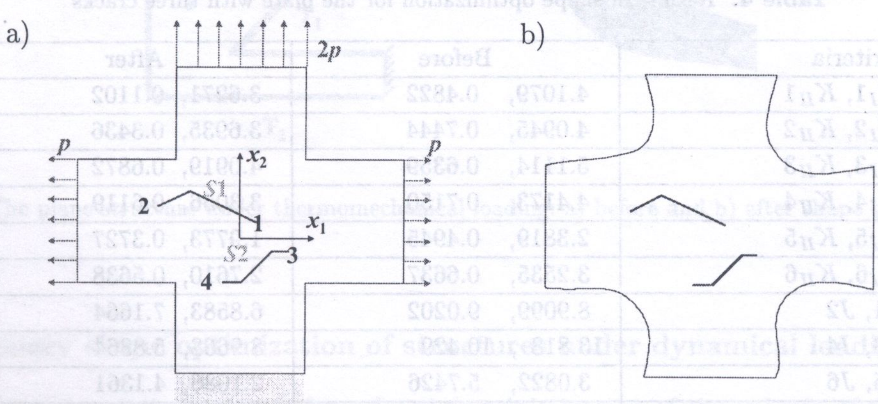

Fig. 8. Plate with two cracks: a) before and b) after optimization

Table 3. Results of shape optimization for the plate with two cracks

Criteria	Before	After
K_I1, $K_{II}1$	5.2456, 1.5965	4.6088, 1.2586
K_I2, $K_{II}2$	4.3746, 3.1095	3.4366, 2.5185
K_I3, $K_{II}3$	6.0637, 0.4374	5.4322, 0.6571
K_I4, $K_{II}4$	4.4595, 0.1934	4.1093, 0.5211
K_{red}	4.2679	3.9072
$J1$, $J2$	15.6422, 15.0033	11.8879, 9.4548
$J3$, $J4$	19.2496, 10.3772	15.5943, 8.936
$MCO1$, $MCO2$	1.3821, 0.8829	1.1099, 0.8001
$\sigma_{eq_{\max}}$	75.6664	51.2198

Numerical example 6

A plate contains three cracks (Fig. 9a) and the criterion of minimum MCO_{red} is applied. Parameters of evolutionary algorithms are: number of generations — 2000; number of chromosomes — 100 chromosomes. Optimal solution has been found in 1075 generation. Results are presented in Fig. 9b and Table 4.

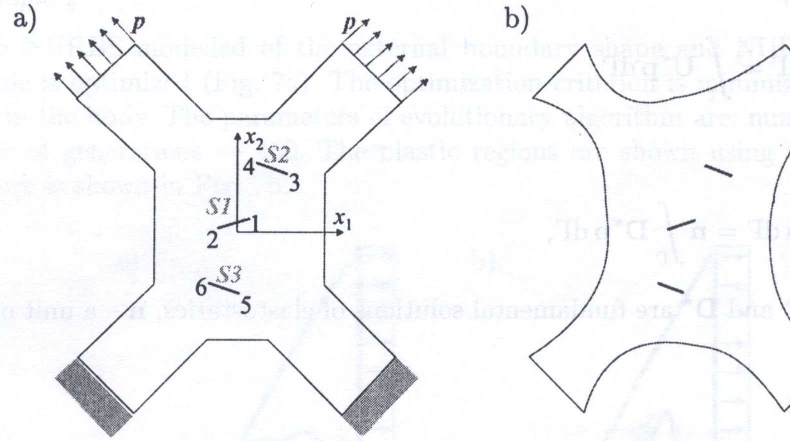

Fig. 9. Plate with three cracks: a) before and b) after optimization

Table 4. Results of shape optimization for the plate with three cracks

Criteria	Before	After
K_I1, $K_{II}1$	4.1079, 0.4822	3.6271, 0.1102
K_I2, $K_{II}2$	4.0945, 0.7444	3.6935, 0.3436
K_I3, $K_{II}3$	5.1114, 0.6359	4.0919, 0.6872
K_I4, $K_{II}4$	4.4173, 0.7150	3.3056, 0.6119
K_I5, $K_{II}5$	2.3819, 0.4945	1.9773, 0.3727
K_I6, $K_{II}6$	3.2535, 0.6637	2.7610, 0.5638
$J1$, $J2$	8.9099, 9.0202	6.8583, 7.1664
$J3$, $J4$	13.818 , 10.429	8.9668, 5.8864
$J5$, $J6$	3.0822, 5.7426	2.1086, 4.1361
$MCO1$, $MCO2$, $MCO3$	1.871 , 2.185 , 1.298	1.654 , 1.703 , 1.085
MCO_{red}	1.8389	1.53386
$\sigma_{eq_{\max}}$	27.6659	27.6009

3.5. Shape optimization of structures under thermomechanical loading

In the shape optimization of thermoelastic structures the criterion of minimum global compliance of structure is proposed [10, 11, 18],

$$J = \int_\Gamma \left(\frac{1}{2}\mathbf{p}\cdot\mathbf{u}\right) \mathrm{d}\Gamma. \tag{27}$$

In order to evaluate the fitness functions (27) the direct boundary-value problem of linear, homogeneous, isotropic, steady-state thermoelasticity theory is solved. The boundary only integral equation for thermoelastic problem is obtained [7],

$$\mathbf{cu} + \int_\Gamma \mathbf{P}^*\mathbf{u}\,\mathrm{d}\Gamma = \int_\Gamma \mathbf{U}^*\mathbf{p}\,\mathrm{d}\Gamma + \int_\Gamma (\mathbf{P}T - \mathbf{Q}q)\,\mathrm{d}\Gamma, \tag{28}$$

in which needed boundary temperature and its normal derivative are calculated by solving the boundary integral equation for the heat conduction problem,

$$cT + \int_\Gamma Q^* T\,\mathrm{d}\Gamma = \int_\Gamma T^* q\,\mathrm{d}\Gamma, \tag{29}$$

where: Q^* and T^* are fundamental solutions for the Laplace equation, $\mathbf{P}$ and $\mathbf{Q}$ are known coordinate functions.

Numerical example 7

The plane structure (Fig. 10a) is optimized for the criterion of minimum global compliance. Only the parts of the boundary where temperatures T_1 and T_2 are assumed undergo the shape modification with preservation of constant volume. The Bézier curves are used to model the optimized boundary. The traction field $\overline{p}$ is applied. Parameters of evolutionary algorithms are: number of chromosomes — 100; number of generations — 1000. The optimal shape of the structure with conditions: $T_1 = 0°$C, $T_2 = 200°$C, $p = 50$ kN/m is presented in Fig. 10b.

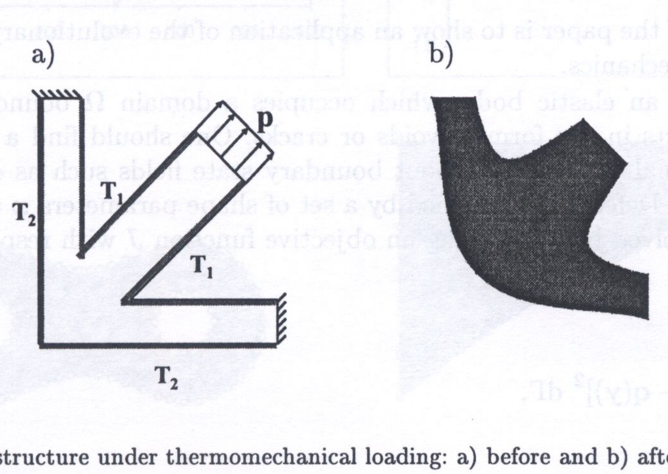

Fig. 10. The plane structure under thermomechanical loading: a) before and b) after shape optimization

3.6. Evolutionary shape optimization of structures under dynamical loading

The problem of shape optimization is formulated as minimization of the volume of the body,

$$\min J = \int_\Omega \mathrm{d}\Omega, \tag{30}$$

with constraints imposed on boundary equivalent stresses and boundary displacements,

$$\begin{aligned} &\sigma_{\mathrm{eq}}(\mathbf{y},\mathbf{t}) - \sigma_{\mathrm{o}} \leq 0, && \forall \mathbf{y} \in \Gamma \quad \text{and} \quad \forall \mathrm{t} \in [0,\mathrm{T}], \\ &u(\mathbf{y},\mathbf{t}) - u_{\mathrm{o}} \leq 0, && \forall \mathbf{y} \in \Gamma \quad \text{and} \quad \forall \mathrm{t} \in [0,\mathrm{T}], \end{aligned} \tag{31}$$

where: $u = \sqrt{\mathbf{u}\cdot\mathbf{u}}$ and u_o is an admissible value of displacements.

The displacement field $\mathbf{u}(\mathbf{y},\mathbf{t})$ is computed by solving a boundary-initial value problem of elastodynamics by the dual reciprocity boundary element method (DRBEM). The vector boundary integral equation of DRBEM used in the paper has the form [7]

$$\mathbf{cu} + \int_\Gamma \mathbf{P}^*\mathbf{u}\,\mathrm{d}\Gamma - \int_\Gamma \mathbf{U}^*\mathbf{p}\,\mathrm{d}\Gamma = \rho\left(-\mathbf{c}\Psi^k + \int_\Gamma \mathbf{U}^*\Sigma^k\,\mathrm{d}\Gamma - \int_\Gamma \mathbf{P}^*\Psi^k\,\mathrm{d}\Gamma\right)\ddot{\alpha}^k, \tag{32}$$

where: ρ is a mass density, $\mathbf{U}^*$ and $\mathbf{P}^*$ are fundamental solutions of elastostatics, Ψ^k and Σ^k are pseudo-displacement and -traction fields, respectively, generated by f^k, where $\mathbf{u}(\mathbf{z}) = \alpha^k f^k(\mathbf{z})$, $\mathbf{z} \in \Omega$.

Numerical examples 8 and 9

Shape optimization problem of dynamical loaded structures (Fig. 11a) is optimized for the criterion of minimum volume with imposed stress and displacement constraints. The gradient mutation operator, based on the steepest descent mutation, was controlled by a neural network. The shape of the external boundary was specified by NURBS curves and coordinates of control points played the role of genes. The dot lines denote boundaries that undergo shape optimization (Fig. 11a). Dynamical loads acting on the structures are presented in Fig. 11b. The final optimal shapes of the structures are shown in Fig. 11c.

4. Formulation of the evolutionary identification

The aim of this part of the paper is to show an application of the evolutionary algorithms to inverse problems of applied mechanics.

It is assumed that an elastic body, which occupies a domain Ω bounded by a boundary Γ, contains internal defects in the form of voids or cracks. One should find a position and shape of defects having additional information about boundary state fields such as displacements, stresses or natural frequencies. Defects are specified by a set of shape parameters $\boldsymbol{x} = (x_i)$, $i = 1, 2, \ldots, N$. The problem can be solved by minimizing an objective function J with respect to $\boldsymbol{x} = (x_i)$:

- for static problems,

$$J = \frac{1}{2}\int_\Gamma [\hat{\mathbf{q}}(\mathbf{y}) - \mathbf{q}(\mathbf{y})]^2\,\mathrm{d}\Gamma, \tag{33}$$

- for dynamical problems:

$$J = \frac{1}{2}\int_\Gamma [\hat{\mathbf{q}}(\mathbf{y},t) - \mathbf{q}(\mathbf{y},t)]^2\,\mathrm{d}\Gamma\mathrm{d}t, \tag{34}$$

where $\hat{\mathbf{q}}$ are measured values of state fields such as e.g. displacements, stresses or natural frequencies, $\mathbf{q}$ are computed values of the same state fields.

The fitness function (33) was applied in crack identification [3, 4, 9].

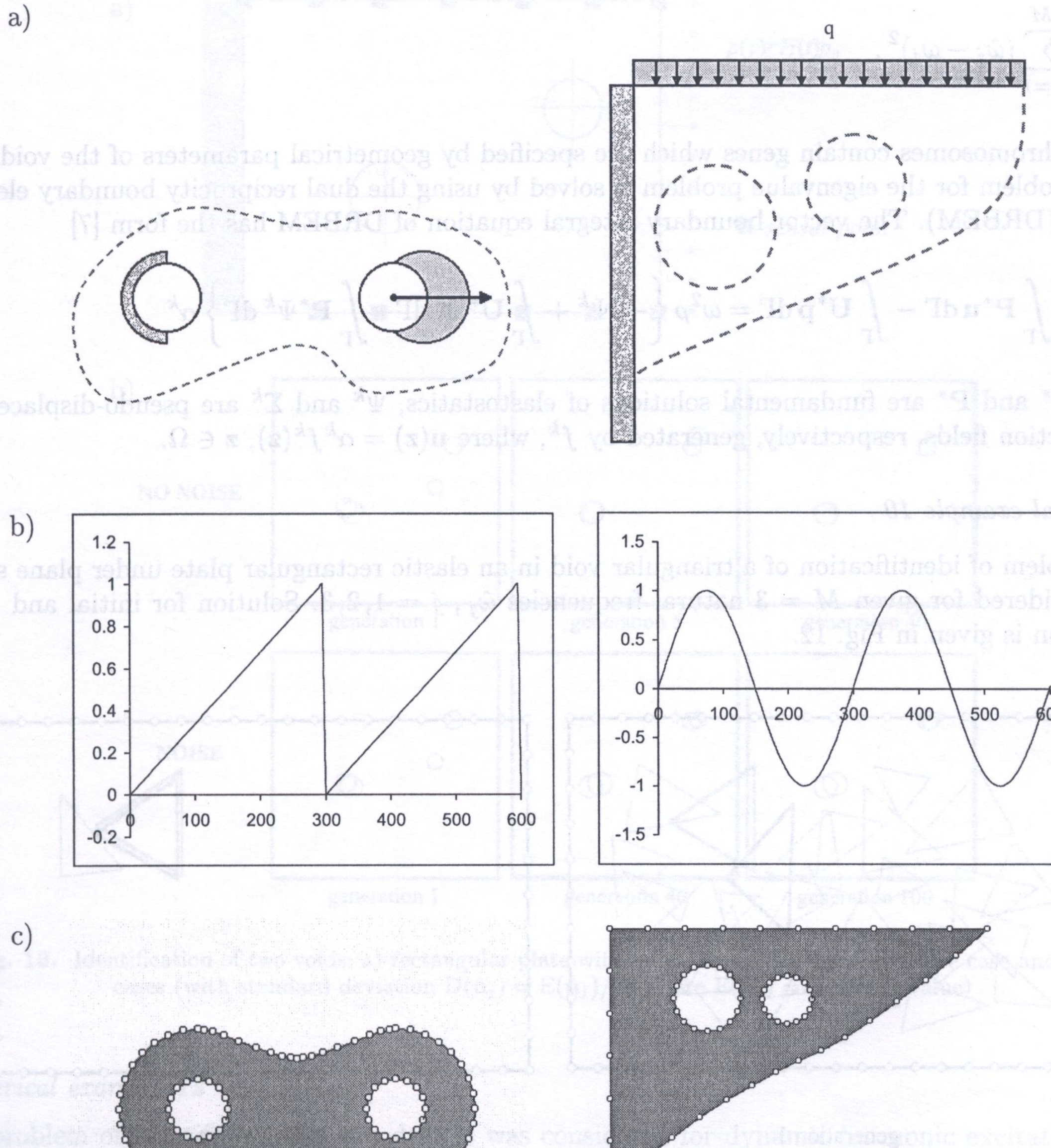

Fig. 11. Shape optimization of two structures: (a) structures with boundaries undergoing shape optimization, (b) dynamical load acting upon structures, (c) optimal shape of structures

5. APPLICATIONS OF EVOLUTIONARY IDENTIFICATION

5.1. Detection of voids based on measured natural frequencies

The identification problem is formulated as a minimization of the sum of differences between calculated natural frequencies ω_j (for a given position of the void) and measured natural frequencies $\hat{\omega}_j$,

$$J = \sum_{j=1}^{M} (\hat{\omega}_j - \omega_j)^2 . \tag{35}$$

The chromosomes contain genes which are specified by geometrical parameters of the void. The direct problem for the eigenvalue problem is solved by using the dual reciprocity boundary element method (DRBEM). The vector boundary integral equation of DRBEM has the form [7]

$$\mathbf{cu} + \int_\Gamma \mathbf{P}^* \mathbf{u} \, d\Gamma - \int_\Gamma \mathbf{U}^* \mathbf{p} \, d\Gamma = \omega^2 \rho \left\{ -\mathbf{c} \Psi^k + \int_\Gamma \mathbf{U}^* \Sigma^k \, d\Gamma - \int_\Gamma \mathbf{P}^* \Psi^k \, d\Gamma \right\} \alpha^k \tag{36}$$

where $\mathbf{U}^*$ and $\mathbf{P}^*$ are fundamental solutions of elastostatics, Ψ^k and Σ^k are pseudo-displacement and -traction fields, respectively, generated by f^k, where $\mathbf{u}(\mathbf{z}) = \alpha^k f^k(\mathbf{z})$, $\mathbf{z} \in \Omega$.

Numerical example 10

The problem of identification of a triangular void in an elastic rectangular plate under plane strain was considered for given $M = 3$ natural frequencies $\hat{\omega}_j$, $j = 1, 2, 3$. Solution for initial and 18-th generation is given in Fig. 12.

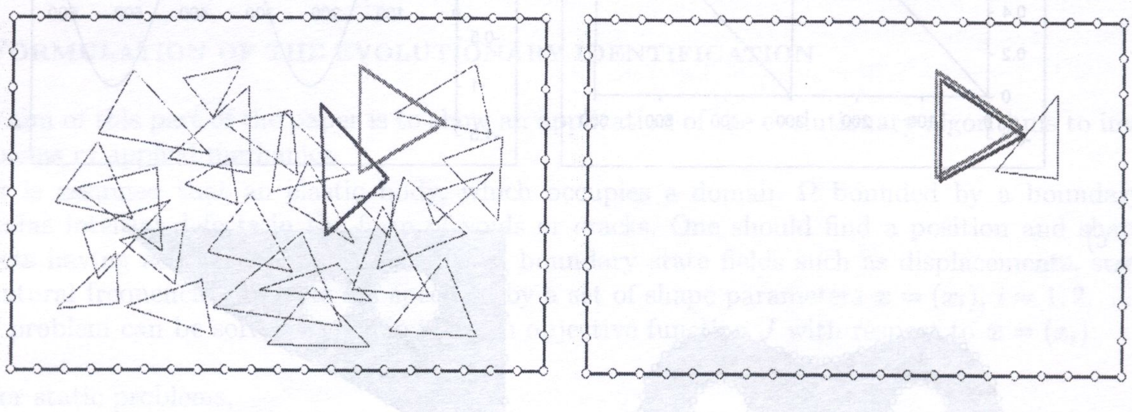

Fig. 12. Identification of a triangular void; population of voids in a) 1st generation, (b) 18th generations

5.2. Detection of defects based on measured dynamical boundary displacements

The problem of a void identification in elastic structures being under dynamical loads is considered [25].

The fitness function has the form (34). The displacement field $\mathbf{u} = \mathbf{q}(\mathbf{y}, t)$ is computed by solving a boundary-initial value problem of elastodynamics by DRBEM.

Numerical example 11

The plate with two circular voids was loaded dynamically (Fig. 13a). Heaving measured displacements at sensor points one should determine positions and circular radii. The problem was solved for deterministic experimental data of displacements (*no noise* case) and for stochastic variation with normal distribution (*noise* case) (Fig. 13b)

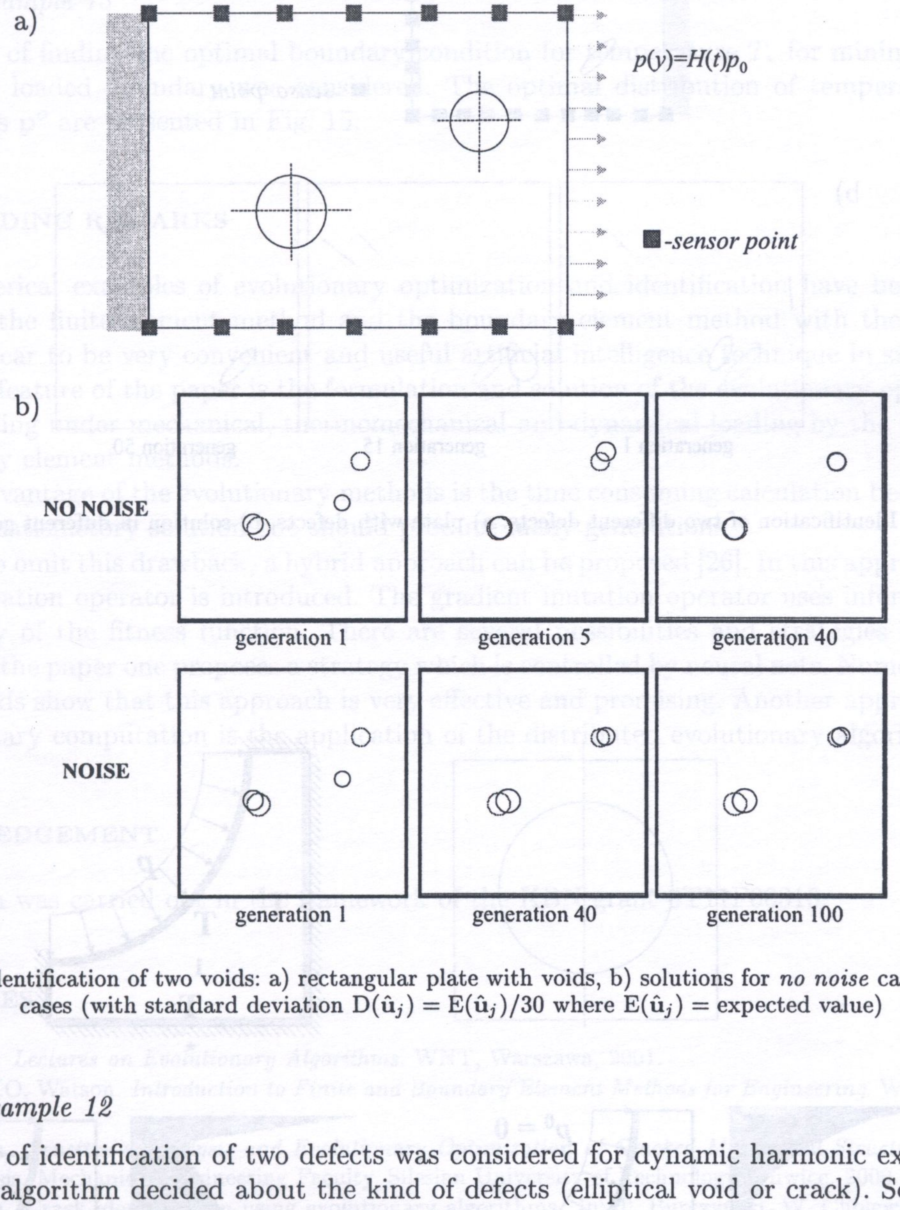

Fig. 13. Identification of two voids: a) rectangular plate with voids, b) solutions for *no noise* case and *noise* cases (with standard deviation $D(\hat{u}_j) = E(\hat{u}_j)/30$ where $E(\hat{u}_j)$ = expected value)

Numerical example 12

The problem of identification of two defects was considered for dynamic harmonic excitation. The evolutionary algorithm decided about the kind of defects (elliptical void or crack). Solution of the problem was achieved in 50 generations (Fig. 14).

5.3. Inverse problems in thermoelasticity

The problem of finding an optimal distribution of temperature on the one part of the boundary Γ_* is solved for minimum of displacements on another part of the boundary Γ_p [11, 18],

$$J = \int_{\Gamma_p} \left(\frac{u}{u_o}\right)^n d\Gamma_p , \tag{37}$$

where u is total displacements at the boundary Γ_p, u_o is an admissible displacement.

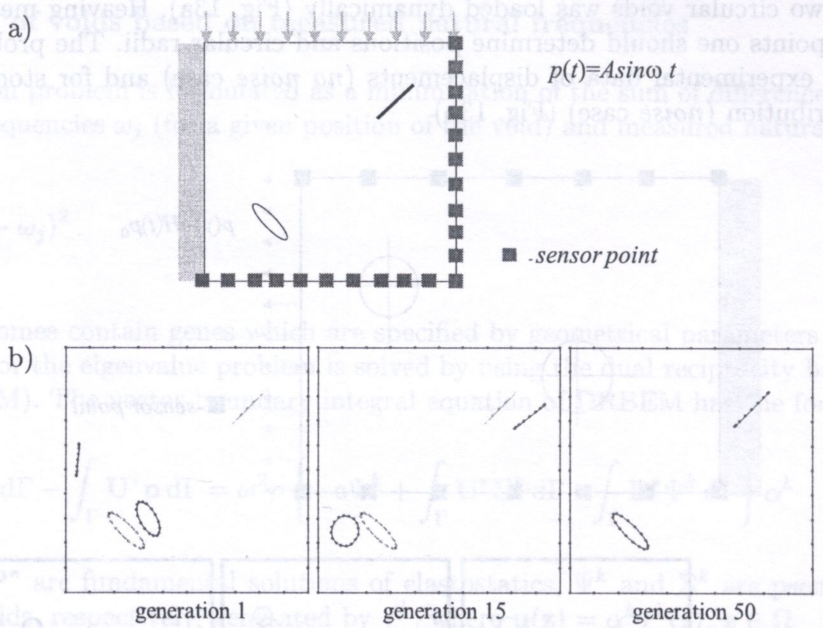

Fig. 14. Identification of two different defects: a) plate with defects, b) solution in different generations

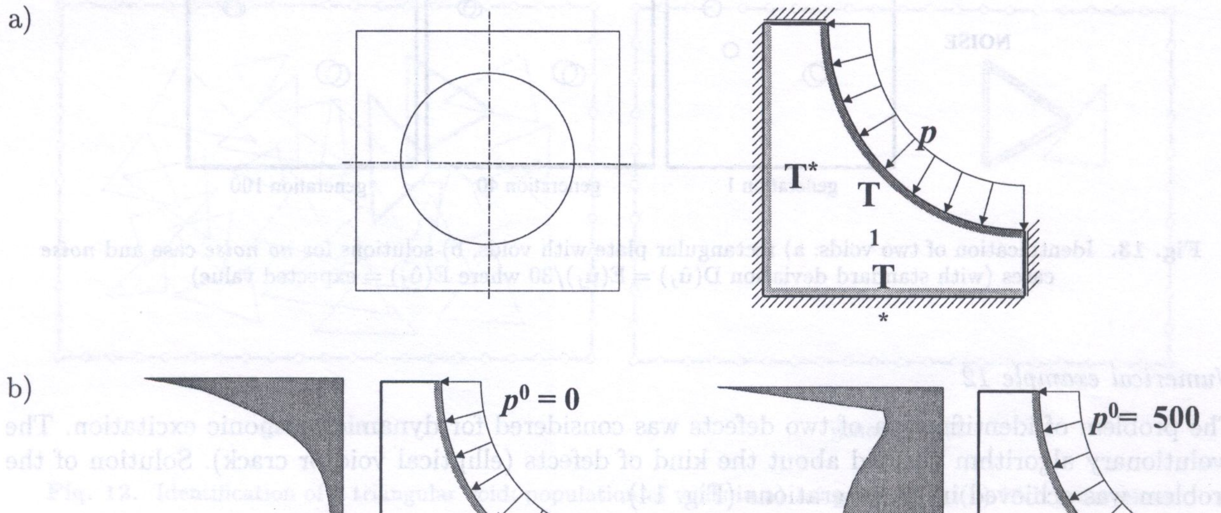

Fig. 15. Solution for thermoelastic solid, a) a plate with boundary conditions, b) optimal distribution of temperature

In order to evaluate the fitness function (37) the direct boundary-value problem of linear, homogeneous, isotropic, steady-state thermoelasticity theory is solved. The boundary only integral equations for thermoelastic problem have the form (28) and (29).

In this case the distribution of temperature is represented by genes, which are described by control points of Bézier curves.

Numerical example 13

The problem of finding the optimal boundary condition for temperature T_* for minimum displacements of the loaded boundary was considered. The optimal distribution of temperature for two traction fields $\mathbf{p}^o$ are presented in Fig. 15.

6. Concluding remarks

Several numerical examples of evolutionary optimization and identification have been presented. Coupling of the finite element method and the boundary element method with the evolutionary methods appear to be very convenient and useful artificial intelligence technique in such problems. The original feature of the paper is the formulation and solution of the evolutionary optimization of structures being under mechanical, thermomechanical and dynamical loading by the finite element and boundary element methods.

One disadvantage of the evolutionary methods is the time consuming calculation because in order to achieve a satisfactory solution one should produce many generations.

In order to omit this drawback, a hybrid approach can be proposed [26]. In this approach a special gradient mutation operator is introduced. The gradient mutation operator uses information based on sensitivity of the fitness function. There are several possibilities and strategies of using such mutation. In the paper one proposes a strategy which is controlled by neural nets. Numerical tests for detecting voids show that this approach is very effective and promising. Another approach to speed the evolutionary computation is the application of the distributed evolutionary algorithms [15].

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