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Unsteady flow and heat transfer due to a suddenly stopped continuous moving surface

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An approximate solution for the problem of unsteady flow and heat transfer caused by a suddenly stopped continuous moving surface and its gradual cooling has been obtained by solving the non-linear governing equations with the implicit finite difference scheme. Stability and convergence of the scheme are first verified. Then, the influence of the plate velocity, Prandtl number, time of stopping of the plate t_1 and the cooling constant on the flow pattern, temperature, wall shear stress and heat flux is analyzed. It is found that velocity, temperature, wall shear stress and heat flux decrease in time. When the plate moves faster, fluid velocity, wall shear stress and heat transfer intensity are augmented whereas temperature goes down. The Prandtl number increases and the cooling constant reduces temperature and heat flux.

NOMENCLATURE

t – time
t_1 – stopping time of the plate
T – the temperature
T_w – uniform temperature of the plate
T_∞ – uniform temperature of the surrounding fluid
θ – non-dimensional temperature
x, y – non-dimensional co-ordinate variables in two-dimension
u, v – non-dimensional velocity components in x and y directions
u_w – plate velocity
U – characteristic velocity
L – characteristic length
a_1 – constant
C – cooling constant
τ – wall shear stress
q – heat flux
Pr – Prandtl number
Re – Reynolds number

1. INTRODUCTION

Unsteady boundary layer flow and heat transfer situation resulting from a continuously moving surface finds application in a number of manufacturing, technological and engineering processes. For example, materials manufactured by extrusion processes and heat-treated materials travelling between a feed roll and a wind-up roll or on a conveyor belt possess the characteristics of a continuously moving surface [4] which may be suddenly stopped. Other examples of these processes include glass blowing, continuous casting, cooling of metallic sheets, cooling of electronic chips, crystal growing, melt spinning and many others.

In 1961, Sakiadis was the first person investigating the flow due to a moving surface issuing from a slit into a fluid at rest. Since then, several investigators have considered various aspects of this problem including steady state and unsteady flows. Buhler and Zierep [1] have provided interesting analytical solutions for problems of motion of a viscous fluid past an infinite porous flat plate which is suddenly set into motion at a constant velocity.

Recently Ingham and Pop [3] have analyzed the flow and heat transfer due to a suddenly stopped and cooled plate. As they have pointed out, cooling down a continuous moving heated plate to that of the temperature of the surrounding fluid instantaneously at time $t = 0$ is highly impractical. So a realistic situation has to be assumed and this motivated for the present study.

More realistic unsteady flow and heat transfer is discussed when the plate is suddenly stopped at time $t = t_1$ and then cooled gradually, say in a decreasing exponential order. The boundary layer equations are solved by implicit finite difference scheme which is unconditionally stable and convergent. This study provides the answer to the question of impact of the stopping time, Prandtl number, cooling constant and the plate velocity on temperature and flow field in the boundary layer over the plate.

2. FORMULATION OF THE PROBLEM

Let us consider a heated flat plate moving at a constant velocity u'_w that enters through a slit in a large mass of viscous, incompressible fluid. We assume that the motion of the fluid is laminar and of the boundary layer type. Uniform temperature of the surrounding fluid is T_∞ and the plate is maintained at a uniform temperature T_w, greater than T_∞. The plate is suddenly stopped at time $t' = t'_1$ and gradually cooled.

Stationary frame of axis (x', y') where x' and y' are parallel and normal to the plate respectively are taken in the analysis. The origin of these cartesian coordinates is on the surface of the plate $y' = 0$ and the slit is at $x' = 0$. The physical model of the problem is shown in Fig. 1.

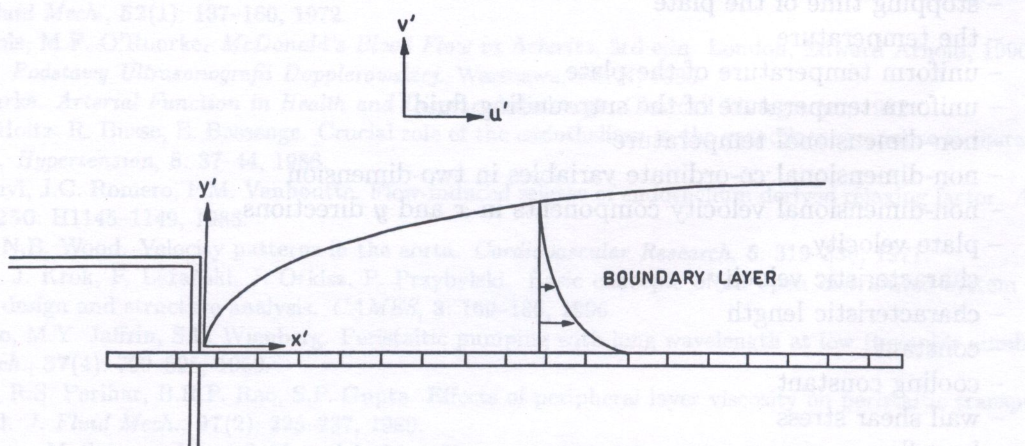

Fig. 1. Physical model of the problem

Governing boundary layer equations and energy equation (after neglecting viscous dissipation) are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2}, \tag{3}$$

where Pr is the Prandtl number.

In the above equations, the following non-dimensional variables are used,

$$\begin{aligned} &x=\frac{x'}{L}; \quad y=\frac{\sqrt{Re}}{L}y'; \quad t=\frac{U}{L}t'; \quad u_w=\frac{u'_w}{U}; \\ &u=\frac{1}{U}u', \quad v=\frac{\sqrt{Re}}{U}v'; \quad \theta=\frac{T-T_\infty}{T_w-T_\infty}, \end{aligned} \tag{4}$$

where Re is the Reynolds number, U and L are characteristic velocity and length respectively.

The corresponding initial and boundary conditions become

$$\begin{aligned} &\text{at } t=0, \ \ 0<y<\infty: && u(x,y)=v(x,y)=\theta(x,y)=0, \\ &\text{for } 0\le t<t_1 \text{ at } y=0: && u(x,y)=u_w, \quad v(x,y)=0, \quad \theta(x,y)=1, \\ &\text{for } t\ge t_1 \text{ at } y=0: && u(x,y)=0, \quad v(x,y)=0, \quad \theta(x,y)=e^{C(t_1-t)}, \\ &\text{for } t\ge 0 \text{ as } y\to\infty: && u(x,y)=\theta(x,y)=0, \\ &\text{for } t\ge 0 \text{ at } x=0: && u(x,y)=\theta(x,y)=0, \end{aligned} \tag{5}$$

where $C=\frac{a_1 L}{U}$ is a constant, called the cooling constant.

Further, the wall shear stress and heat flux in non-dimensional form are given by

$$\tau=\left[\frac{\partial u}{\partial y}\right]_{y=0}, \tag{6}$$

$$q=-\left[\frac{\partial \theta}{\partial y}\right]_{y=0}. \tag{7}$$

3. SOLUTION OF THE PROBLEM

In order to solve these unsteady non-linear, coupled equations (1)–(3) under the conditions given by Eq. (5), an implicit finite difference scheme of Crank–Nicolson is used. The finite difference equations corresponding to Eqs. (1)–(3) are given by

$$\begin{aligned} \frac{1}{4\Delta x}\left[u_{i,j}^{n+1}-u_{i-1,j}^{n+1}+u_{i,j}^{n}-u_{i-1,j}^{n}+u_{i,j-1}^{n+1}-u_{i-1,j-1}^{n+1}+u_{i,j-1}^{n}-u_{i-1,j-1}^{n}\right] \\ +\frac{1}{2\Delta y}\left[v_{i,j}^{n+1}-v_{i,j-1}^{n+1}+v_{i,j}^{n}-v_{i,j-1}^{n}\right]=0, \end{aligned} \tag{8}$$

$$\frac{1}{\Delta t}\Big[u_{i,j}^{n+1}-u_{i,j}^{n}\Big]+u_{i,j}^{n}\frac{1}{2\Delta x}\Big[u_{i,j}^{n+1}-u_{i-1,j}^{n+1}+u_{i,j}^{n}-u_{i-1,j}^{n}\Big]$$
$$+v_{i,j}^{n}\frac{1}{4\Delta y}\Big[u_{i,j+1}^{n+1}-u_{i,j-1}^{n+1}+u_{i,j+1}^{n}-u_{i,j-1}^{n}\Big]$$
$$=\frac{1}{2(\Delta y)^2}\Big[u_{i,j-1}^{n+1}-2u_{i,j}^{n+1}+u_{i,j+1}^{n+1}+u_{i,j-1}^{n}-2u_{i,j}^{n}+u_{i,j+1}^{n}\Big],\quad (9)$$

$$\frac{1}{\Delta t}\Big[\theta_{i,j}^{n+1}-\theta_{i,j}^{n}\Big]+u_{i,j}^{n}\frac{1}{2\Delta x}\Big[\theta_{i,j}^{n+1}-\theta_{i-1,j}^{n+1}+\theta_{i,j}^{n}-\theta_{i-1,j}^{n}\Big]$$
$$+v_{i,j}^{n}\frac{1}{4\Delta y}\Big[\theta_{i,j+1}^{n+1}-\theta_{i,j-1}^{n+1}+\theta_{i,j+1}^{n}-\theta_{i,j-1}^{n}\Big]$$
$$=\frac{1}{Pr}\frac{1}{2(\Delta y)^2}\Big[\theta_{i,j-1}^{n+1}-2\theta_{i,j}^{n+1}+\theta_{i,j+1}^{n+1}+\theta_{i,j-1}^{n}-2\theta_{i,j}^{n}+\theta_{i,j+1}^{n}\Big]\quad (10)$$

The region of integration is considered as a rectangle with sides $x=2$ and $y=14$. Here $y=14$ corresponds to $y=\infty$ for it lies far outside the momentum and energy boundary layers. This maximum value of y was chosen after some preliminary investigations. Here i, j, n designate x, y, t respectively. Some preliminary calculations have been performed and the following mesh sides are selected,

$$\Delta x = 0.04, \qquad \Delta y = 0.2,$$

and the time step $\Delta t = 0.01$.

The finite difference equation (10), at a particular time level n, at each internal nodal point on a particular i-level, constitutes a tri-diagonal system of equations, which are solved by Thomas algorithm [2]. Thus, the values of θ are known at every nodal point on a particular i-level at $(n+1)^{\text{th}}$ time level. Similarly values of u are calculated from Eq. (9), and then v is calculated explicitly from (8) at every nodal point. Computations are carried out by moving along the i-direction.

4. Stability analysis

The stability criterion of the finite difference scheme used is established using the von Neumann method [2] for regular space and time discretization.

By assuming that a general term of the Fourier expansion for u and θ at an initial time $(t=0)$ is $e^{i\alpha x}e^{i\beta y}$, one can define these modes at time t as

$$u = F(t)\,e^{i\alpha x}e^{i\beta y}, \quad (11)$$
$$\theta = G(t)\,e^{i\alpha x}e^{i\beta y}. \quad (12)$$

Let $F(t)$ and $G(t)$ be denoted as F_1 and G_1 at the next time $t+\Delta t$. Substituting Eqs. (11) and (12) into (9) and (10), assuming u and v are constants, one gets

$$\frac{1}{\Delta t}[F_1-F]+\frac{u}{2\Delta x}\big[(F_1+F)(1-e^{-i\alpha\Delta x})\big]+\frac{v}{2\Delta y}[i\sin(\beta\Delta y)(F_1+F)]$$
$$=\frac{1}{(\Delta y)^2}\,[(F_1+F)(\cos(\beta\Delta y)-1)],$$

$$\frac{1}{\Delta t}[G_1-G]+\frac{u}{2\Delta x}\big[(G_1+G)(1-e^{-i\alpha\Delta x})\big]+\frac{v}{2\Delta y}[i\sin(\beta\Delta y)(G_1+G)]$$
$$=\frac{1}{Pr(\Delta y)^2}\,[(G_1+G)(\cos(\beta\Delta y)-1)]\,.$$

Upon simplifying the above equations, they read

$$(1+A)F_1 = (1-A)F, \tag{13}$$

$$(1+B)G_1 = (1-B)G, \tag{14}$$

where

$$A = \frac{u\Delta t}{2\Delta x}\left[1-e^{-i\alpha\Delta x}\right] + i\,\frac{v\Delta t}{2\Delta y}\sin(\beta\Delta y) - \frac{\Delta t}{(\Delta y)^2}[\cos(\beta\Delta y)-1], \tag{15}$$

$$B = \frac{u\Delta t}{2\Delta x}\left[1-e^{-i\alpha\Delta x}\right] + i\,\frac{v\Delta t}{2\Delta y}\sin(\beta\Delta y)\ \frac{\Delta t}{Pr(\Delta y)^2}[\cos(\beta\Delta y)-1]. \tag{16}$$

Thus, Eqs. (13) and (14) become

$$F_1 = \left[\frac{1-A}{1+A}\right]F, \qquad G_1 = \left[\frac{1-B}{1+B}\right]G,$$

or in a matrix form

$$\begin{bmatrix} F_1 \\ G_1 \end{bmatrix} = \begin{bmatrix} \frac{1-A}{1+A} & 0 \\ 0 & \frac{1-B}{1+B} \end{bmatrix} \begin{bmatrix} F \\ G \end{bmatrix} \tag{17}$$

where

$$\begin{bmatrix} \frac{1-A}{1+A} & 0 \\ 0 & \frac{1-B}{1+B} \end{bmatrix}$$

is referred to as the amplification matrix.

Finite difference system is stable if the modulus of each eigenvalue of the amplification matrix do not exceed unity.

Clearly, the eigenvalues of the amplification matrix (17) are $\frac{1-A}{1+A}$ and $\frac{1-B}{1+B}$. Here u is everywhere non-negative and v is everywhere non-positive.

Let

$$a = \frac{u\Delta t}{2\Delta x}, \qquad b = \frac{|v|\Delta t}{2\Delta y}, \qquad c = \frac{\Delta t}{(\Delta y)^2}.$$

Therefore $\frac{v\Delta t}{2\Delta y} = -b$. Then Eq. (15) becomes

$$\begin{aligned} A &= a\left[1-e^{-i\alpha\Delta x}\right] - bi\sin(\beta\Delta y) - c[\cos(\beta\Delta y)-1] \\ &= 2a\sin^2\left[\frac{\alpha}{2}\Delta x\right] + 2c\sin^2\left[\frac{\beta}{2}\Delta y\right] + i[a\sin(\alpha\Delta x) - b\sin(\beta\Delta y)]. \end{aligned} \tag{18}$$

Since the real part of A is positive, $|\frac{1-A}{1+A}| \le 1$ always. Similarly one can prove that $|\frac{1-B}{1+B}| \le 1$.

Therefore, the scheme is unconditionally stable. The local truncation error is $O(\Delta t^2 + \Delta y^2 + \Delta x)$ and it tends to zero as Δt, Δx and Δy go towards zero. Hence, the scheme is compatible and in the virtue of the Lax theorem, it is convergent.

5. Results and Discussion

Results of performed calculations are depicted graphically by means of figures for x component of velocity u, heat transfer θ, wall shear stress τ and heat flux q at different time stopping of t_1, different time t, plate velocity u_w, Prandtl number Pr, the cooling constant C.

Throughout the study, calculations are carried out at $x = 2$, where the boundary layer is already fully devolped. Moreover, calculations show that the v-component of the velocity is non-positive and negligibly small and its absolute value only slightly increases in time and with the higher plate velocity. Therefore, only the velocity u is further discussed in detail.

Since the transient flow is studied, calculations have been carried out for different times, say, $t = 2.0, 2.5, 3.0, 4.0, 6.0$ and various t_1.

Figure 2 elucidates the effect of time on the velocity profiles when $t_1 = 5.0$. The similar effect of the velocity decrease is observed when $t_1 = 2.0$, 10.0 and 15.0. Further it is noticed that the velocity profiles have similar trend for $t_1 = 10.0$ and $t_1 = 15.0$. Significance of the plate velocity u_w is analyzed in Fig. 3. It is seen that its increase accelerates the flow.

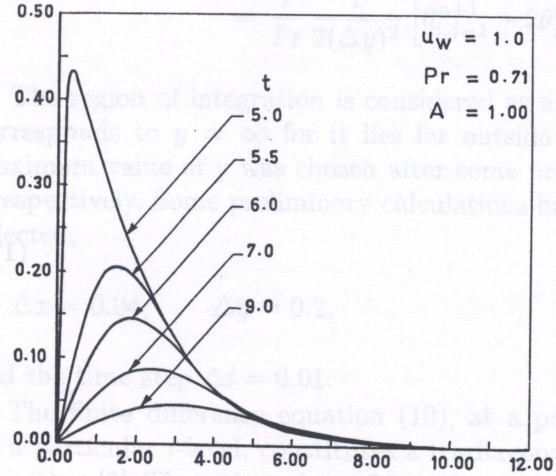

Fig. 2. Velocity profiles at different times for $t_1 = 5$

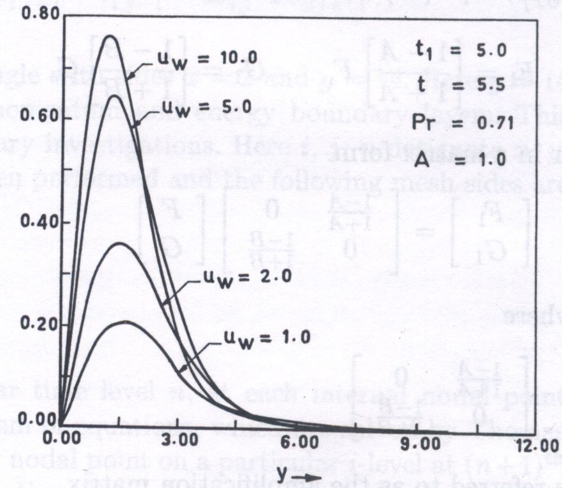

Fig. 3. Influence of u_w on velocity profiles

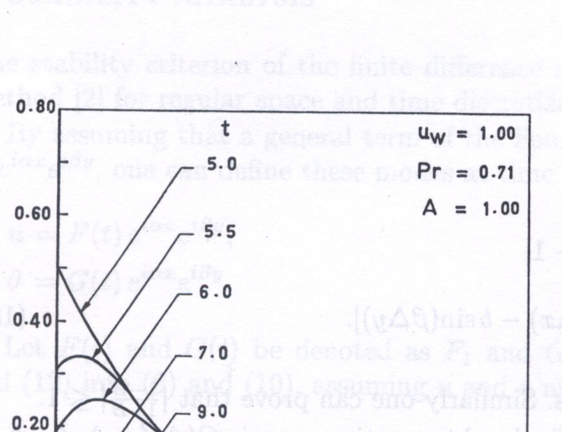

Fig. 4. Temperature profiles at different times for $t_1 = 5.0$

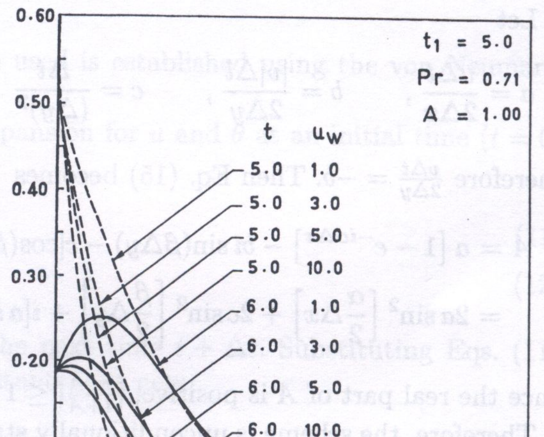

Fig. 5. Influence of u_w on temperature profiles

Figure 4 displays temperature profiles at various time for $t_1 = 5.0$. Expected temperature decrease is visible there. Figure 5 illustrates the influence of the plate velocity u_w on the temperature profiles within the boundary layer. The temperature decreases for increasing u_w.

Figure 6 shows the impact of Prandtl number on temperature distribution. Increase of Pr reduces both the temperature and the thermal boundary thickness. Figure 7 confirms that the higher value of the cooling constant C results in more severe temperature decrease.

Figure 8 confirms that the increase of u_w causes higher wall shear stresses at early times of the analyzed process. They, however, go down rapidly, quickly approaching zero.

The impact of plate velocity u_w on the heat flux is exhibited through Fig. 9. It is generally

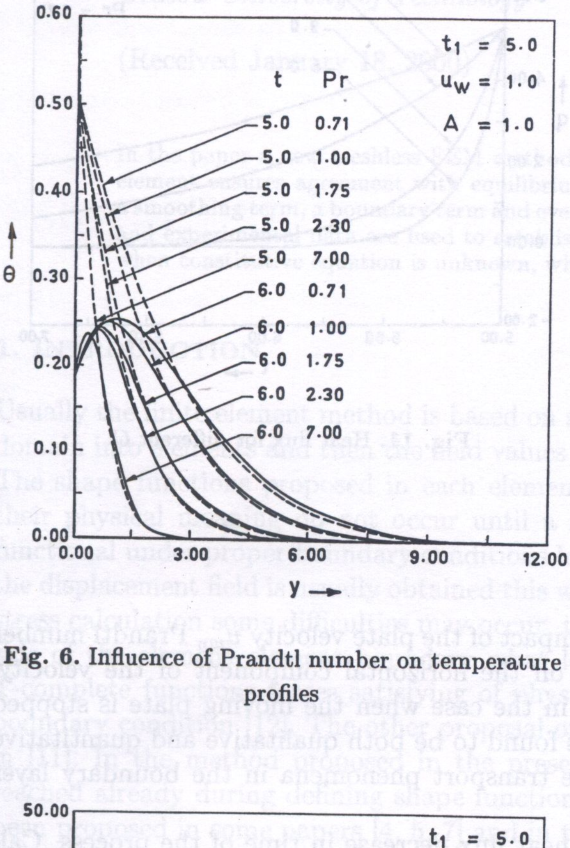

Fig. 6. Influence of Prandtl number on temperature profiles

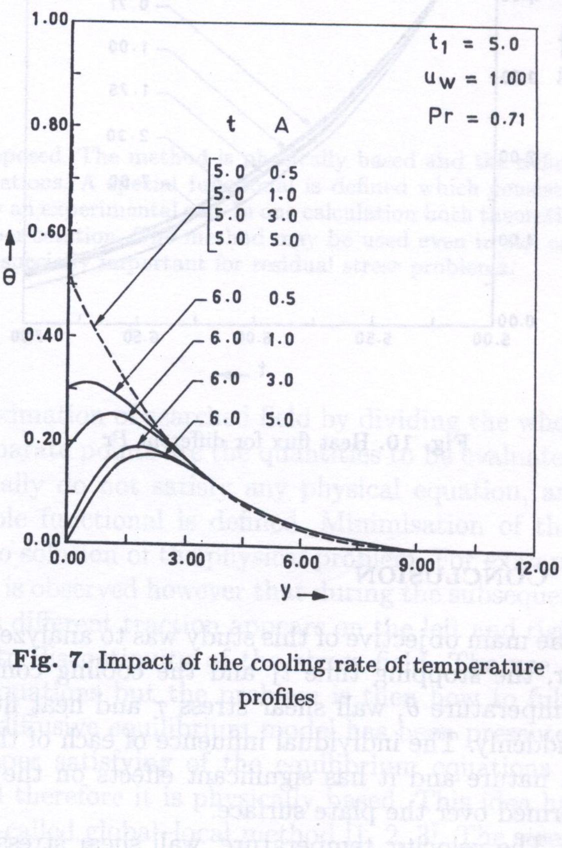

Fig. 7. Impact of the cooling rate of temperature profiles

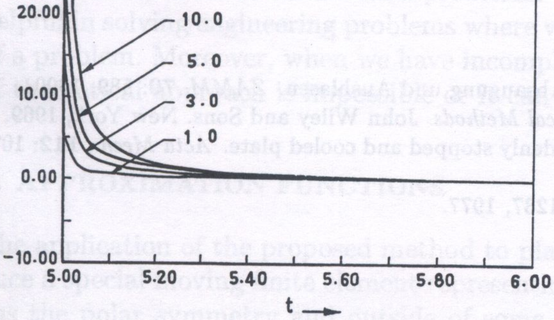

Fig. 8. Skin friction for different u_w

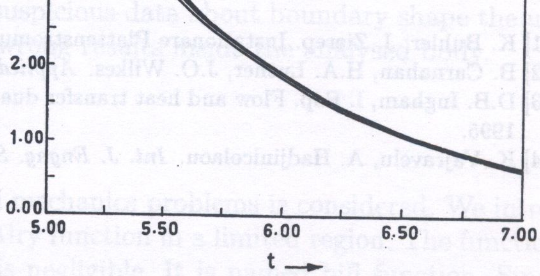

Fig. 9. Heat flux for different u_w

observed that the heat flux falls down gradually with respect to time, irrespective of t_1. Figure 10 shows the influence of Prandtl number on q. Initially, the higher Pr increases the heat flux till $t \simeq 5.2$ and this trend is just reversed for $t > 5.2$. Figure 11 shows how the constant C influences the intensity of heat transfer. The increase of C decreases the heat flux.

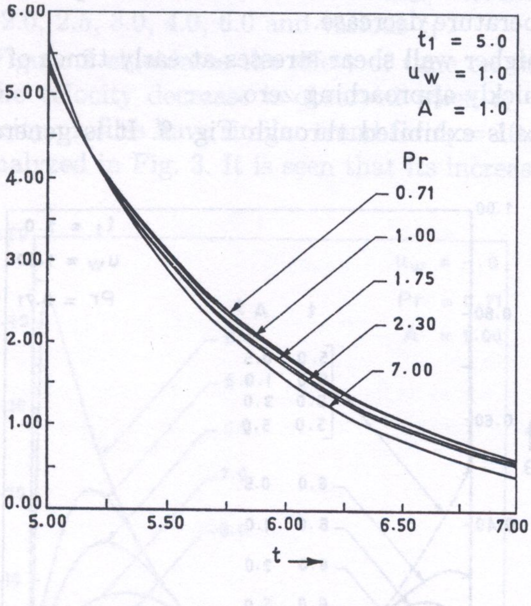

Fig. 10. Heat flux for different Pr

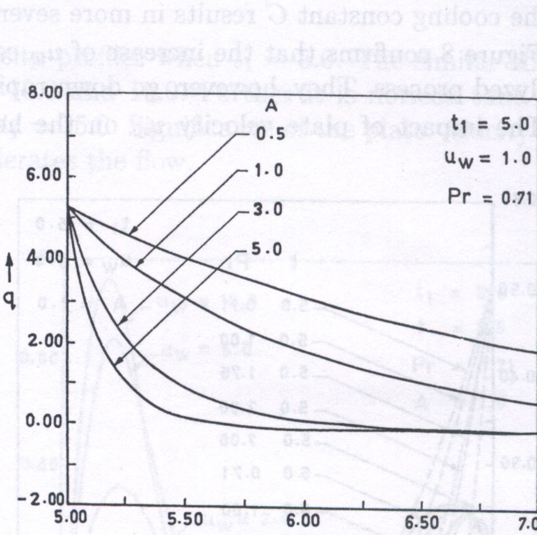

Fig. 11. Heat flux for different C

6. Conclusion

The main objective of this study was to analyze the impact of the plate velocity u_w, Prandtl number Pr, the stopping time t_1 and the cooling constant on the horizontal component of the velocity, temperature θ, wall shear stress τ and heat flux q, in the case when the moving plate is stopped suddenly. The individual influence of each of them is found to be both qualitative and quantitative in nature and it has significant effects on the entire transport phenomena in the boundary layer formed over the plate surface.

The velocity, temperature, wall shear stress and heat flux decrease in time of the process. Calculations carried out for different stopping times reveal, that the velocity, temperature, wall shear stress and heat flux have a similar trend for various t_1. The plate velocity increases the fluid velocity, wall shear stress and heat flux, whereas it reduces the temperature. Higher Pr lowers the temperature and heat flux after sometime of the process reversing the beginning trend.

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