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Fuel-efficient attitude maneuvers of flexible spacecraft with residual vibration reduction into an expected level

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A three-dimensional rest-to-rest attitude maneuver of a satellite with flexible solar panels equipped by on-off reaction jets is studied. Results indicate that, under an unshaped input, the maneuver induces an undesirable motion of the satellite as well as vibration of the solar panels. Fuel-efficient input shapers are then applied to reduce the residual oscillation of its attitude. By reducing vibrations at several large-amplitude natural frequencies, the expected pointing precision of the satellite can be satisfied.

1. Introduction

A spacecraft or a satellite in operation needs certain accuracies in its attitude. For example, a satellite required by the KOREASAT must have an antenna beam pointing error not greater than 0.07° in roll and pitch, and not more than 0.2° in yaw [2]. For its precise orientation, the spacecraft during its operation in space requires frequent corrections of its attitude. Attitude maneuver of rigid spacecraft can be done without a lot of vibration problems after reaching its desired attitude. For the flexible spacecraft maneuvering the attitude without regard to system flexibility or without controls on the flexible members, large amplitude transient and steady-state oscillations may occur, especially when the system is equipped with on-off jets. Such a system often needs a rest-to-rest attitude maneuver with small vibration both during and at the end of the maneuver. For example, it may be necessary to generate a torque profile such that the spacecraft is rotated through a desired attitude angle, while the deflections of flexible members remain small throughout the maneuver and go to zero at the end of the maneuver.

To minimize modal vibration in a flexible spacecraft system, which is equipped with on-off reaction jets, the input shaping methods (see Fig. 3) have been developed [3, 4, 6, 7, 9]. Parman and Koguchi [5] demonstrate an application of shaped commands to maneuver attitudes of a flexible satellite with a large number of flexible modes. They show that the resultant vibrations can be reduced drastically when the satellite is subjected to shaped inputs suppressing the vibration at the frequency with largest vibration amplitude. However, the residual roll oscillations of the satellite are still greater than the permissible attitude error of the precise-oriented satellite, such as the satellite required by KOREASAT, because the vibrations at other frequencies are still large enough.

This paper presents computer simulations of rest-to-rest attitude maneuvers of a satellite with flexible solar panels under shaped inputs. The model developed by Parman and Koguchi [5] is used. The offset angle of the solar panels is set to non-zero value. For this setting angle, there is a coupling motion between roll and yaw displacements of the satellite. The fuel-efficient shaped input is selected to produce zero vibration at the frequency with the largest amplitude of residual attitude oscillation and small vibration at several other frequencies in order to result the permissible residual attitude oscillation. The maximum deflections of solar panels during the attitude maneuver will also be discussed in order to find the strategy for choosing an input shaper with small deflection of flexible members.

2. A SATELLITE WITH SYMMETRICAL FLEXIBLE SOLAR PANELS

The flexible spacecraft studied in this paper is a satellite containing a rigid main body and two symmetrical flexible solar panels. The finite element model of the satellite can be seen in Fig. 1. For the application of finite element method to discretize elastic deformations of the solar panels, each solar panel is divided into 16 rectangular plate elements as shown in Fig. 1. By using such a division, each solar panel has 27 nodal points. The elements are numbered from 1 through 16 on the right-hand-side and from 17 through 32 on the left-hand-side, while their nodal points are numbered from 1 through 27 on the right-hand-side and from 28 through 54 on the left-hand-side. The solar panels are oriented towards the sun, and the declination with respect the X_b-axis of rigid main body-fixed frame is identified by the offset angle δ. Only out-of-plane deformations of the solar panels are considered.

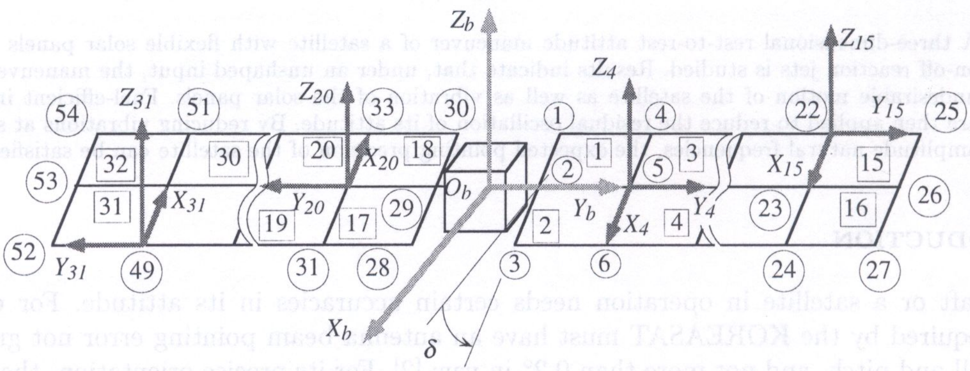

Fig. 1. The model of satellite being investigated: element numbering and nodal point numbering

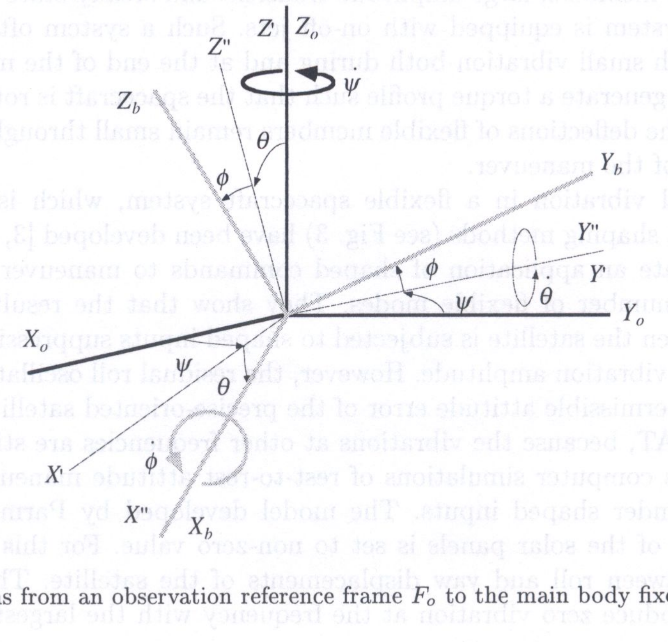

Fig. 2. The rotations from an observation reference frame F_o to the main body fixed reference frame F_b

The attitude angles of the satellite are expressed in Bryant's angles: roll angle ϕ, pitch angle θ, and yaw angle ψ; where the definition of these angles can be seen in Fig. 2. The equations of motion of the satellite have been developed by Parman and Koguchi [5]. For small attitude angle displacements, they can be written in the following matrix form:

$$\begin{bmatrix} m\mathbf{U}_3 & \mathbf{Q} & \mathbf{W} \\ \mathbf{Q}^T & \mathbf{I} & \mathbf{A} \\ \mathbf{W}^T & \mathbf{A}^T & \mathbf{M} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{r}} \\ \ddot{\mathbf{\Theta}} \\ \ddot{\mathbf{d}} \end{Bmatrix} + \begin{bmatrix} \mathbf{0}_{3\times 3} & \mathbf{Q}\tilde{\boldsymbol{\omega}}_0 & \mathbf{0}_{3\times f} \\ \mathbf{0}_{3\times 3} & \mathbf{I}\tilde{\boldsymbol{\omega}}_0 & \mathbf{0}_{3\times f} \\ \mathbf{0}_{f\times 3} & \mathbf{A}^T\tilde{\boldsymbol{\omega}}_0 & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{\Theta}} \\ \dot{\mathbf{d}} \end{Bmatrix} + \begin{Bmatrix} \mathbf{0}_3 \\ \mathbf{0}_3 \\ \mathbf{K}\mathbf{d} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_b \\ \boldsymbol{T}_b \\ \boldsymbol{F}_a \end{Bmatrix} \tag{1}$$

where m is the mass of the satellite, $\mathbf{Q}$ is the coupling matrix for the translational and rotational displacements of the rigid main body, $\mathbf{W}$ is the coupling matrix for the translational displacements of the rigid main body and the displacements of flexible solar panels, $\mathbf{I}$ is the inertia matrix, $\mathbf{A}$ is the coupling matrix for the rotational displacements of the rigid main body and the displacements of flexible solar panels; $\mathbf{M}$, $\mathbf{D}$, and $\mathbf{K}$ are the mass matrix, the damping matrix, and the stiffness matrix of flexible solar panels respectively; $\tilde{\boldsymbol{\omega}}_0$ is the skew symmetric matrix of angular velocity of the satellite orbit $\boldsymbol{\omega}_0$, $\mathbf{r}$ is the translational displacement of the rigid main body, $\mathbf{\Theta}$ is the rotational displacement of the rigid main body which is expressed in a vector of Bryant's angles, and $\mathbf{d}$ is the displacements of the flexible solar panels. $\boldsymbol{F}_b$ and $\boldsymbol{T}_b$ are external forces and torques vectors acting on the rigid main body, $\boldsymbol{F}_a$ is the vector of external forces and torques acting on the solar panels, while f is the total number of degrees of freedom of the solar panels. $\mathbf{0}_{3\times 3}$, $\mathbf{0}_{3\times f}$, and $\mathbf{0}_{f\times 3}$ are zero matrices, and $\mathbf{0}_3$ is a zero vector. In this paper, the flexible structural subsystems are supposed to have no dissipation properties, so that $\mathbf{D} = \mathbf{0}$.

3. Fuel-efficient input shaper to reduce residual vibration in rest-to-rest slew maneuvers

The flexible satellite studied here is equipped with on-off reaction jets, so it cannot produce variable amplitude actuation forces; the satellite must be moved with constant amplitude force pulses. For this kind of satellite, when the bang-bang inputs were used for rest-to-rest slew maneuvers, the residual vibrations of flexible members and the attitude oscillation of the main body can be very large [5] and greater than the permissible values. By shaping the inputs, the residual vibrations can be reduced.

The inputs to the system under consideration must consist only of positive or negative constant-amplitude force pulses. For rest-to-rest slews, the commands must contain both positive and negative pulses so that the system can be accelerated and then decelerated back to zero velocity. Singhose, Bohlke, and Seering [8] demonstrated that fuel-efficient shaped command profiles spend smaller amounts of fuel compared with time-optimal profiles, while time lengths needed for maneuvers are almost the same. So, it is very useful to use the fuel-efficient shaped commands to control the spacecraft. A series of alternating-sign pulses for a rest-to-rest fuel-efficient slew maneuver can be generated arbitrarily by convolving a step with an input shaper in the form of

$$\begin{cases} \begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} A_1 & A_2 & \cdots & A_n \\ t_1 & t_2 & \cdots & t_n \end{bmatrix} \\ A_i = 1 \qquad (i = 1, 3, \ldots, \frac{n}{2} - 1, \frac{n}{2} + 2, \frac{n}{2} + 4, \ldots, n) \\ A_i = -1 \qquad (i = 2, 4, \ldots, \frac{n}{2}, \frac{n}{2} + 1, \frac{n}{2} + 3, \ldots, n - 1) \end{cases} \tag{2}$$

where $i = 1, \cdots, n$, while A_i is an amplitude of the ith impulse, t_i is a time location of the ith impulse, and n is an even integer. Eqs. (2) lead to *constraint equations of commands* for rest-to-rest fuel-efficient maneuvers. An example of Eqs. (2) is demonstrated by Fig. 3, where a step convolved with this type of input shaper results in a series of alternating sign, variable-width pulses.

The next constraint equations are *rigid body motion constraints*. These constraints must ensure that the system's mass centre will move to the desired amount. For the system under only torque

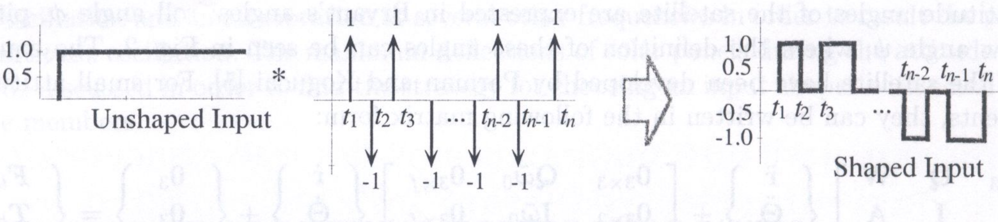

Fig. 3. Input shaping to generate a series of alternating-sign pulses

inputs as studied in this paper, these equations are governed by the equations of desired angle velocity:

$$\begin{Bmatrix} \dot{\phi}_d \\ \dot{\theta}_d \\ \dot{\psi}_d \end{Bmatrix} = \int \begin{bmatrix} I_x & I_{xy} & I_{xz} \\ I_{xy} & I_y & I_{yz} \\ I_{xz} & I_{yz} & I_z \end{bmatrix}^{-1} \begin{Bmatrix} T_{b_x}(t) \\ T_{b_y}(t) \\ T_{b_z}(t) \end{Bmatrix} \mathrm{d}t \tag{3}$$

and angle displacement:

$$\begin{Bmatrix} \phi_d \\ \theta_d \\ \psi_d \end{Bmatrix} = \int\int \begin{bmatrix} I_x & I_{xy} & I_{xz} \\ I_{xy} & I_y & I_{yz} \\ I_{xz} & I_{yz} & I_z \end{bmatrix}^{-1} \begin{Bmatrix} T_{b_x}(t) \\ T_{b_y}(t) \\ T_{b_z}(t) \end{Bmatrix} \mathrm{d}t\,\mathrm{d}t \tag{4}$$

where I_x, I_y, I_z, I_{xy}, I_{xz}, and I_{yz} are components of the inertia matrix of the satellite, $\mathbf{I}$, T_{b_x}, T_{b_y}, and T_{b_z} are components of the torque $\boldsymbol{T}_b$ along X_b, Y_b, and Z_b-axes respectively, depending on time t.

The main purpose of input shaping is to limit the amount of residual vibration that occurs when the system reaches its desired setting point. The constraint equation limiting vibration amplitude can be formulated as a ratio of residual vibration amplitude with input shaping divided by residual vibration without shaping. When the techniques are designed for use with on-off actuators, it would be appropriate to use a baseline input. Whenever possible, the bang-bang input is used as the standard for commands that maneuver from rest to rest.

A bang-bang input can be viewed as a step input convolved with a shaper of the form

$$\begin{bmatrix} A_{bb_j} \\ t_{bb_j} \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & t_{bb_2} & t_{bb_3} \end{bmatrix} \tag{5}$$

where t_{bb_2} and t_{bb_3} in this equation are a switching time and a length of the bang-bang respectively. The bang-bang is a time-optimal input for rest-to-rest maneuvers with three impulses.

The amplitude of residual vibration of an undamped second-order system when subjected to a sequence of impulses can be expressed as a summation of the responses to individual impulses that is given in reference [1]. For the system with a lot number of flexible modes, the expression giving the amplitude for the jth mode ($j = 1, 2, \ldots$) is

$$R_{amp}(\omega_j) = C_j \sqrt{\left[\sum A_i \sin(\omega_j t_i)\right]^2 + \left[\sum A_i \cos(\omega_j t_i)\right]^2} \tag{6}$$

where ω_j is the jth undamped natural frequency and C_j is the constant for its response. We can construct the ratio of shaped to unshaped vibration by dividing Eq. (6) for the shaped input by the equivalent equation for the impulse sequence corresponding to the unshaped input. The percentage vibration relative to a bang-bang is

$$V(\omega_j) = \sqrt{\frac{\left[\sum A_i \sin(\omega_j t_i)\right]^2 + \left[\sum A_i \cos(\omega_j t_i)\right]^2}{\left[\sum A_{bb_j} \sin(\omega_j t_{bb_j})\right]^2 + \left[\sum A_{bb_j} \cos(\omega_j t_{bb_j})\right]^2}} \tag{7}$$

where A_{bb_j} and t_{bb_j} describe the input shaper corresponding to the bang-bang and are given by Eq. (5).

By using Eqs. (7) of the desired modes as *constraint equations of residual vibration*, we can set the level of vibration for a certain system frequency ω_j to the quantity V. The determination of V depends on the vibration amplitude resulting from the bang-bang and the expected residual vibration amplitude resulting from the shaped input. If the vibration of the bang-bang is too large, while the expected vibration level is small enough, then V must be set to a very small value. The known zero vibration (ZV) and zero vibration and derivative (ZVD) shapers use the criteria of $V = 0$, i.e. the shaped vibration amplitude is 0% of the baseline vibration amplitude.

4. Rest-to-rest 3-dimensional attitude maneuvers of the flexible satellite

In this section, rest-to-rest attitude maneuvers of the flexible satellite are simulated. The observed satellite is supposed to has no control and no damping properties on the flexible solar panels. The control inputs only work on the main body, at the center of mass of the satellite, as on-off reaction jets with constant amplitude force or torque pulses. It is assumed that the amplitudes of torque pulses produced by the system, both T_{b_x}, T_{b_y}, and T_{b_z}, are 20 Nm, and they cannot be produced simultaneously. The main body of satellite has the values of $I_{b_x} = 2,406$ kgm^2, $I_{b_y} = 2,284$ kgm^2, $I_{b_z} = 378$ kgm^2, and $I_{b_{xy}} = I_{b_{xz}} = I_{b_{yz}} = 0$, where I_{b_x}, I_{b_y}, I_{b_z}, $I_{b_{xy}}$, $I_{b_{xz}}$, and $I_{b_{yz}}$ are components of the inertia matrix of the main body. The parameters of flexible solar panels can be seen in Table 1. The offset angle of solar panels is $\delta = 30°$. For this configuration, the whole satellite has the values of $I_x = 17,535$ kgm^2, $I_y = 2,384$ kgm^2, $I_z = 15,557$ kgm^2, $I_{xy} = I_{yz} = 0$, and $I_{xz} = 43$ kgm^2. An initial condition of the satellite is given to be undeformed state in the rest condition (i.e. $\dot{\phi} = \dot{\theta} = \dot{\psi} = 0$). The initial values of ϕ, θ, and ψ are also zero. The desired attitude displacements are 10° (i.e. 0.1745 rad) in roll, 5° in pitch, and $-10°$ in yaw.

Table 1. Parameters of the solar panels of satellite.

Description	Values
Number of solar panels	2
Dimension of each solar panel (m^3)	$12 \times 2.4 \times 0.03$
Young's modulus (N/m^2)	0.6×10^8
Poisson ratio	0.3
Mass density (kg/m^3)	120
Number of elements in each solar panel	16
Dimension of each element (m^3)	$1.5 \times 1.2 \times 0.03$
Offset angle, δ (degrees)	30
Distance between panel's root and O_b (m)	1.80

4.1. Attitude maneuvers of the satellite under bang-bang torque commands

The shortest duration time constant-amplitude commands for rest-to-rest slew maneuvers are bang-bang inputs. Constraint equations that must be satisfied for this rest-to-rest slew maneuver are $\left\{\dot{\phi}_d, \dot{\theta}_d, \dot{\psi}_d\right\}^T = \{0, 0, 0\}^T$ and $\{\phi_d, \theta_d, \psi_d\}^T = \{0.1745, 0.0873, -0.1745\}^T$. The profiles of bang-bang torques needed consist of 24.70 seconds long of T_{b_x}, 6.45 seconds long of T_{b_y}, and 23.28 seconds long of T_{b_z} bang-bangs. The satellite is subjected firstly to the roll torque command, then the pitch torque command, and finally the yaw torque command. Under these inputs, the attitude angles move to the desired values. After the maneuvers, the roll and yaw angles oscillate in large amplitudes dominantly at the period of 17.49 seconds (i.e. $\omega = 0.3593$ rad/sec). The amplitude of the residual roll angle oscillations is 5.15° , while the amplitude of the yaw oscillation is 3.37° as shown in Fig. 4(a). Of course such residual oscillations are unfavorable and may disturb the satellite missions. The residual pitch angle oscillation of the satellite studied here is very small. The amplitude is not greater than 0.0065°, and it looks like a straight line when it was plotted in Fig. 4(a). The largest residual vibrations of solar panels happen at nodes 25 and 52. The node 25, shown in Fig. 4(b), experiences a very large local vertical vibration of 2.186 m amplitude. Compared with the solar panel's length of 12 m, this deflection is about 18.2%.

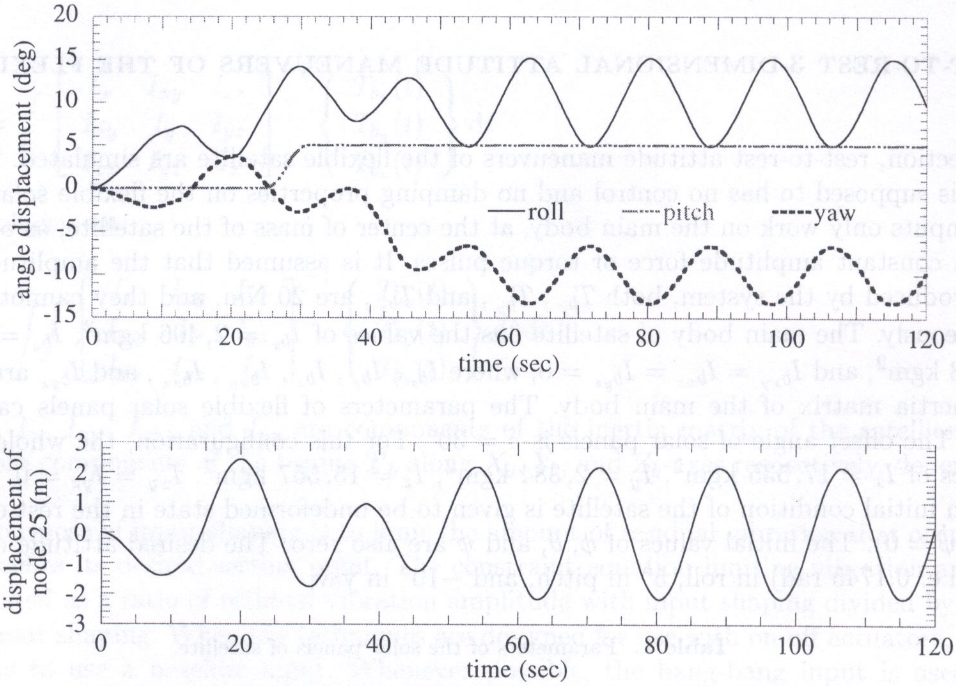

Fig. 4. Time responses under the bang-bang torque inputs: (a) attitude angles of the main body, and (b) vertical displacement of node 25 of solar panel

4.2. Attitude maneuvers of the satellite under shaped torque commands

The attitude maneuvers of the flexible satellite under shaped torque inputs are observed in this subsection. The attitude angles have to be changed to the desired values as explained in the early of this section, while their residual oscillations must be smaller than the permissible pointing errors of the satellite. For the system having a lot of flexible modes such as the finite element model of satellite studied here, when the vibration at a natural frequency with the strongest residual vibration is suppressed, for example to be zero, other frequencies can amplify the resulted residual vibration so that it is still larger than the expected level relating to its mission. In such case, the vibration at several natural frequencies needs to be reduced until the resulting residual vibration is within the acceptable level.

The desired attitude angle displacements of the satellite observed in this study are 10° in roll, 5° in pitch, and −10° in yaw, while the residual attitude angle oscillations are not greater than 0.07° in roll and pitch, and 0.2° in yaw. These values of residual angle oscillations are as same as the permissible maximum antenna beam pointing errors of the satellite required by the KOREASAT [2]. When bang-bang torque inputs are used for a maneuver as described in Sec. 4.1, the very strong residual roll and yaw oscillations happen at $\omega = 0.3593$ rad/sec. There is a coupling motion between roll and yaw, while the pitch motion is independent. So, in order to make the residual roll oscillation smaller than 0.07°, the input shapers are applied to the roll and yaw commands. Remembering that the amplitude of roll oscillation resulting from the bang-bang commands is 5.15°, the sum of V expressed in Eq. (7) for the roll and yaw command shapers must be smaller than 1.36%. In this paper, the input shapers are applied to the roll and yaw torque commands by setting the percentage of vibration V at $\omega = 0.3593$ rad/sec to be zero. The number of impulses of input shapers is selected to be 8 in the following configuration:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 0 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 \end{bmatrix} \tag{8}$$

where fuelling periods happen at $t_1 \rightarrow t_2$, $t_3 \rightarrow t_4$, $t_5 \rightarrow t_6$, and $t_7 \rightarrow t_8$; and the non-fuelling at $t_2 \rightarrow t_3$, $t_4 \rightarrow t_5$, and $t_6 \rightarrow t_7$. The value of t_1 is zero in Eq. (8). A time duration of command is selected to be shorter than 40 seconds.

Elegantly, to achieve a precise orientation of the flexible satellite, the residual vibration at all natural frequencies of the system must be damped. Such a vibration reduction is almost impossible to be done in the system with a lot number of flexible modes. So, to achieve the residual attitude oscillation within the permissible level, the vibrations at large-amplitude natural frequencies have to be set to small values. In this paper, besides at $\omega = 0.3593$ rad/sec, the vibration reductions at other two higher natural frequencies, i.e. 0.9563 and 1.1166 rad/sec, are considered.

As the yaw input, a configuration of shaped command is applied by setting $V(1.1166) \leq 12.5\%$ and minimizing t_8. As the roll inputs, three configurations of shaped commands are chosen, as listed in Table 2. The last two columns of this table are the percentages of vibration at $\omega = 0.9563$ rad/sec and $\omega = 1.1166$ rad/sec. Case 1 has a minimum value of $V(1.1166)$ for $100\% \leq V(0.9563) \leq 150\%$; Case 2 has a minimum value of t_8 for $450\% \leq V(0.9563) \leq 500\%$ and $50\% \leq V(1.1166) \leq 100\%$; and Case 3 has a minimum value of $V(0.9563)$ for $100\% \leq V(1.1166) \leq 150\%$.

Table 2. Time location of impulses for shaping the torque inputs.

Roll cases	t_1 (sec)	t_2 (sec)	t_3 (sec)	t_4 (sec)	t_5 (sec)	t_6 (sec)	t_7 (sec)	t_8 (sec)	$V(0.9563)$ (%)	$V(1.1166)$ (%)
Case 1	0	3.87	8.34	12.44	19.26	23.31	27.76	31.68	136.7	6.1
Case 2	0	0.20	3.43	11.83	20.83	29.23	32.46	32.66	490.3	64.1
Case 3	0	1.10	4.60	11.55	21.50	28.46	32.02	33.11	12.4	141.4
Yaw	0	0.41	5.11	12.28	21.76	23.94	24.70	30.10	140.8	11.4

The bang-bang pitch command is still applied to the satellite since the residual pitch oscillation resulting is only 0.0065° in amplitude, as described in Sec. 4.1, where it is smaller than the permissible maximum pointing error of 0.07° in pitch. In simulations done in this paper, the commands are applied separately. The satellite is subjected firstly to the shaped roll command, then to the bang-bang pitch command, and finally to the shaped yaw command. It means that for the roll command, $t_j = (t_j)_{\text{roll}}$; for the pitch command, $t_{bb_j} = (t_8)_{\text{roll}} + (t_{bb_j})_{\text{pitch}}$; and for the yaw command, $t_j = (t_8)_{\text{roll}} + (t_3)_{\text{pitch}} + (t_j)_{\text{yaw}}$; where $(t_j)_{\text{roll}}$ and $(t_j)_{\text{yaw}}$ are t_j for roll and yaw commands based on Eq. (8) separately, and $(t_{bb_j})_{\text{pitch}}$ is t_{bb_j} based on Eq. (5).

Residual roll oscillation.

The roll angle motions of the main body of satellite for all cases are plotted in Fig. 5. We can see in Fig. 5(a), that the rest of roll angle has been changed successfully to the desired new value of 10° after attitude maneuvers. Case 1 with $V(0.9563) = 136.7\%$ and $V(1.1166) = 6.1\%$ for the roll command gives a favorable residual roll angle oscillation with a total amplitude of about 0.031° as shown in Fig. 5(b), a reduction of about 0.6% compared with the result of the bang-bang input, which is smaller than the permissible maximum pointing error in roll. For Case 2, where the roll command has $V(0.9563) = 490.3\%$ and $V(1.1166) = 64.1\%$, the total amplitude of residual roll oscillation is about 0.101°, at the period of 5.63 sec (or $\omega = 1.1166$ rad/sec). It means that the use of input of Case 2 reduces the residual roll angle oscillation to about 2.0% compared with the result of the bang-bang input, but it is still greater, about 1.44 times, than the permissible maximum error. The residual roll angle oscillation after maneuver resulting in Case 3, where $V(0.9563) = 12.4\%$ and $V(1.1166) = 141.4\%$ for the roll command, has a total amplitude of about 0.185°, or about 3.6% of the result of the bang-bang input, and this value does not satisfy the precision requirement of the satellite pointing.

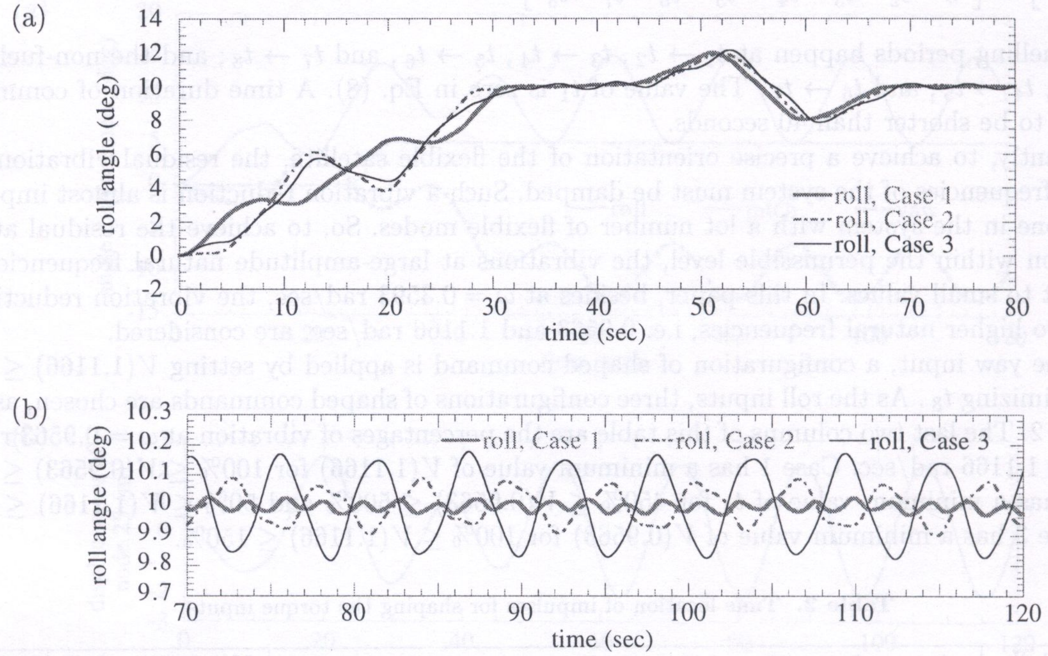

Fig. 5. Time responses of roll angle displacement of the main body: (a) during maneuver, and (b) residual oscillation

Remembering the percentages of vibration at $\omega = 0.9563$ and 1.1166 rad/sec done by each case in connection with their residual roll oscillations resulting, the model of satellite observed in this study has a tendency to give a larger response amplitude at 1.1166 rad/sec compared with the 0.9563 rad/sec one. Case 2 shows sharply that the amplitude of the residual roll oscillation at $\omega = 0.9563$ rad/sec is not large. Although the values of $V(0.9563) = 490.3\%$ and $V(1.1166) = 64.1\%$, the resultant residual roll oscillation is smaller than the one of Case 3, where the values of $V(0.9563) = 12.4\%$ and $V(1.1166) = 141.4\%$. So, besides at its 0.3593 rad/sec natural frequency, we must give attention in the value of percentage vibration at 1.1166 rad/sec in selecting shaped inputs in order to obtain residual roll oscillation at the expected level.

Residual pitch and yaw oscillations.

For the all cases simulated here, the pitch angles move to 5° and have a favorable residual oscillation in 0.0065° total amplitude as shown in Fig. 6. The remaining of yaw angle after maneuvers is −10° as plotted in Fig. 7(a). Case 3 gives the largest residual yaw oscillation as shown in Fig. 7(b), about 0.120°, which is within the permissible pointing error in yaw. The total amplitudes of Cases 1 and 2 are 0.020° and 0.065° respectively, and these values satisfy the requirement of the satellite attitude accuracy in yaw described previously. Since the yaw motion couples with the roll one, the response shape and periods of oscillation appearing in the both motions are same for each case. We can see this phenomenon by comparing Fig. 5(b) and 7(b).

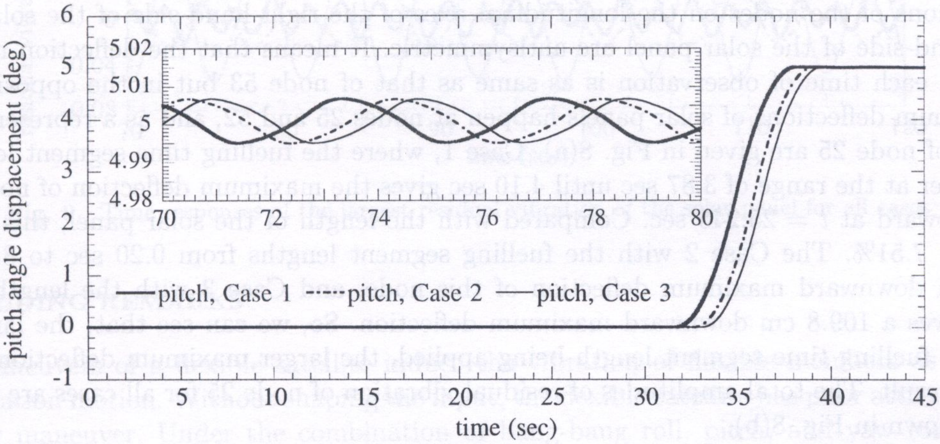

Fig. 6. Time responses of pitch angle displacement of the main body

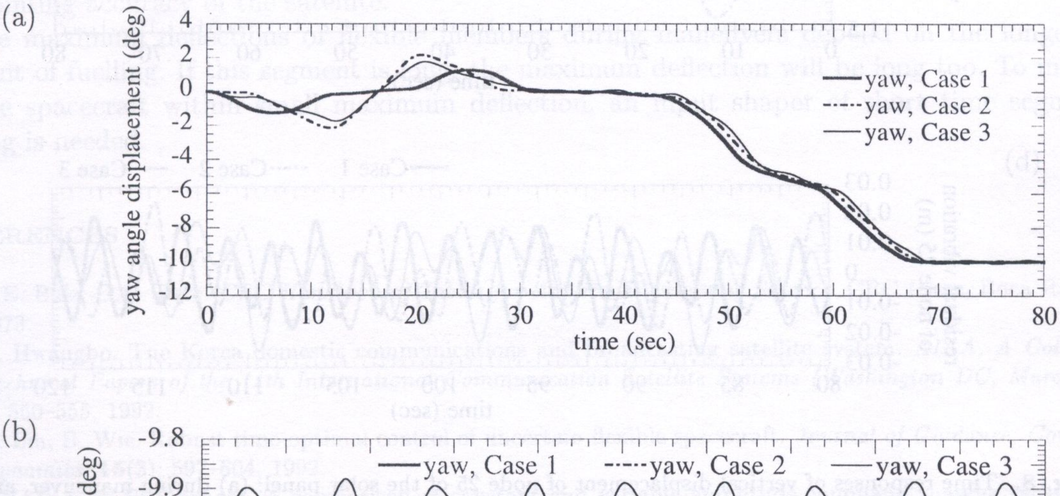

Fig. 7. Time responses of yaw angle displacement of the main body: (a) during maneuver, and (b) residual oscillation

Maximum deflections of solar panels.

Since the solar panels of the satellite are supposed to have only out-of-plane elastic deformations, the deflections of the nodes on the longitudinal axes of the right-hand-side of the solar panel and the left-hand-side of the solar panel are antisymmetric. It means that the deflection magnitude of node 26 at each time of observation is as same as that of node 53 but in the opposite direction. The maximum deflections of solar panels happen at nodes 25 and 52, and as a representation, time responses of node 25 are given in Fig. 8(a). Case 1, where the fuelling time segment lengths of the input shaper at the range of 3.87 sec until 4.10 sec gives the maximum deflection of node 25, about 90.2 cm upward at $t = 25.245$ sec. Compared with the length of the solar panel, this deflection is only about 7.51%. The Case 2 with the fuelling segment lengths from 0.20 sec to 8.40 sec gives a 136.3 cm downward maximum deflection of this node, and Case 3 with the lengths of 1.09 to 6.96 sec gives a 109.8 cm downward maximum deflection. So, we can see that, the larger value of the longest fuelling time segment length being applied, the larger maximum deflection of the solar panel will result. The total amplitudes of residual vibration of node 25 for all cases are smaller than 3 cm, as shown in Fig. 8(b).

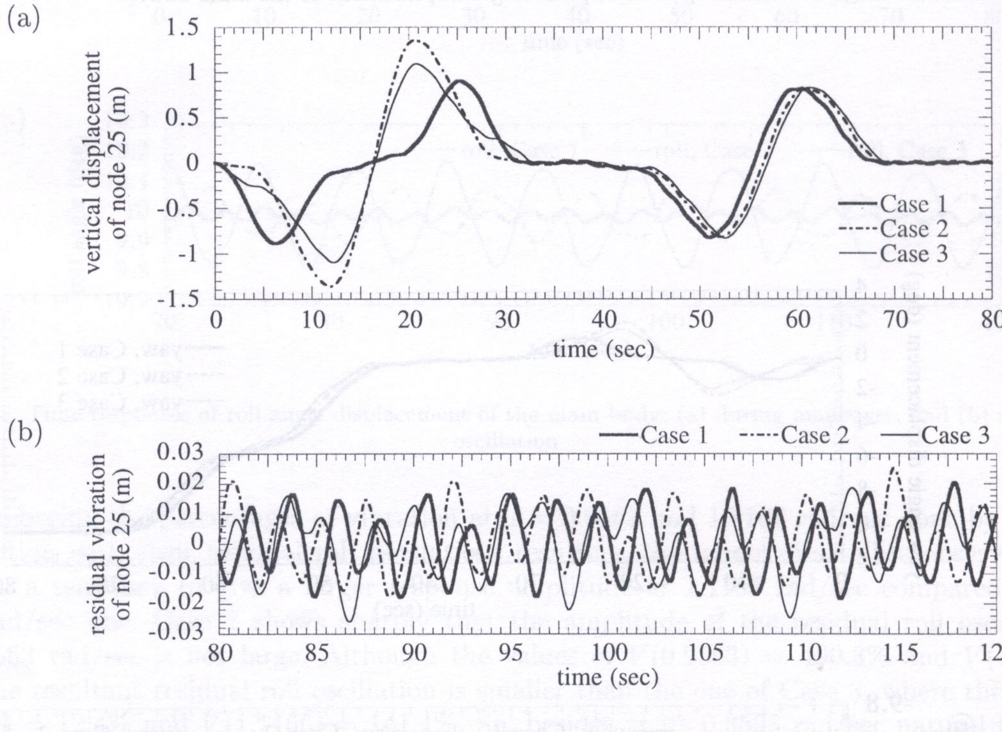

Fig. 8. Time responses of vertical displacement of node 25 of the solar panel: (a) during maneuver, and (b) residual vibration

Maximum residual vibration of solar panels.

The use of shaped inputs has reduced successfully the residual vibrations of the flexible modes. The maximum residual vibrations of solar panels happen at nodes 25 and 52 for Case 1, at nodes 13 and 40 for Case 2, and at nodes 16 and 43 for Case 3. The total amplitude of the largest residual vibration of panels for Case 1 is 2.2 cm, Case 2 is 3.2 cm, and Case 3 is 5.9 cm (about 0.49% of solar panel length) as shown in Fig. 9. Such as the roll and yaw motions, Case 1 also gives the smallest residual vibration of the solar panels.

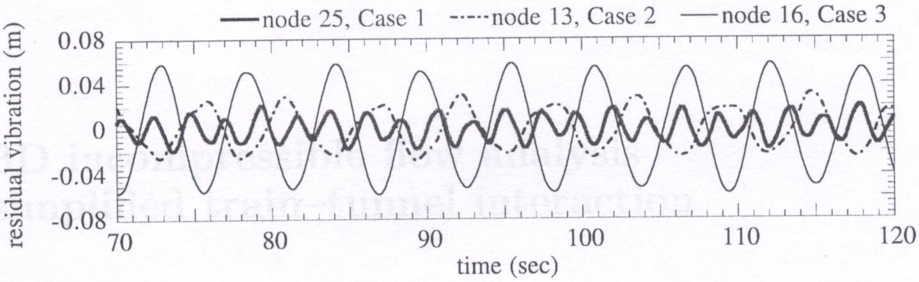

Fig. 9. Time responses of the largest residual vibration of the solar panel for all cases

5. Concluding remarks

Attitude maneuvers of a flexible satellite induce the vibration of flexible members as well as the satellite libration motion. Without shaping the input, the flexible satellite has poor attitude accuracy after a slew maneuver. Under the combination of bang-bang roll, pitch, and yaw torque inputs, for 10° desired roll angle displacement, the satellite studied in this paper has residual roll angle oscillation with an amplitude greater than 5°. Shaped inputs show their capability to reduce residual oscillation and vibration slightly. For the shaped inputs with eight impulses compared here, the maximum total amplitudes of residual roll oscillation become smaller than 0.185°, the largest total amplitudes of residual vibration of 12 meters solar panel's length are not greater than 5.9 cm, and the total amplitudes of tip solar panel's residual vibration are not greater than 3 cm.

To maneuver a flexible satellite with residual attitude angle oscillations at a low level by using a shaped input, it is necessary to reduce residual vibrations to small values, especially the vibrations at ω associated with large amplitudes. The permissible level of residual vibration itself depends on the pointing accuracy of the satellite.

The maximum deflections of flexible members during maneuvers depend on the longest time segment of fuelling. If this segment is long, the maximum deflection will be long too. To maneuver flexible spacecraft within small maximum deflection, an input shaper of short time segments of fuelling is needed.

References

[1] R.E. Bolz, G.L. Tuve. *CRC Handbook of Tables for Applied Engineering Science*. CRC Press, Boca Raton, FL, 1973.

[2] H. Hwangbo. The Korea domestic communications and broadcasting satellite system. *AIAA, A Collection of Technical Papers of the 14th International Communication Satellite Systems (Washington DC, March 22-24)*, **1**: 550–555, 1992.

[3] Q. Liu, B. Wie. Robust time-optimal control of uncertain flexible spacecraft. *Journal of Guidance, Control, and Dynamics*, **15**(3): 597–604, 1992.

[4] L.Y. Pao, W.E. Singhose. A comparison of constant and variable amplitude command shaping techniques for vibration reduction. *IEEE Conference on Control Applications (Albany, NY), Institute of Electrical and Electronics Engineers*, New York, 875–881, 1995.

[5] S. Parman, H. Koguchi. Rest-to-rest attitude maneuvers of a satellite with flexible solar panels by using input shapers. *Computer Assisted Mechanics and Engineering Sciences*, **5**(4): 421–441, 1998.

[6] K. Rogers, W.P. Seering. Input shaping for limiting loads and vibration in systems with on-off actuators. *AIAA Guidance, Navigation, and Control Conf.*, San Diego, CA, 1996.

[7] T. Singh, S.R. Vadali. Robust time-optimal control: a frequency domain approach. *Journal of Guidance, Control, and Dynamics*, **17**(2): 346–353, 1994.

[8] W. Singhose, K. Bohlke, W. Seering. Fuel-efficient shaped command profiles for flexible spacecraft. *Proceedings of the 1995 AIAA Guidance, Navigator, and Control Conference*, 1995.

[9] B. Wie, R. Sinha, Q. Liu. Robust time-optimal control of uncertain structural dynamic systems. *Journal of Guidance, Control, and Dynamics*, **15**(5): 980–983, 1993.