

Minimum cost design of ring and stringer stiffened cylindrical shells

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In this paper the optimum design of bent cylindrical shells with welded ring or stringer stiffeners are treated. The objective function is the cost of the structure and the constraints are related to overall and local stability. The problem is solved by MathCad 7.plus software and presented also graphically for ring stiffened cylinders and by CFSQP optimization software for stringer stiffened cylinders.

1. INTRODUCTION

Cylindrical shells are widely used in various structures, e.g. belt-conveyor bridges, offshore structures piping systems etc. Belt-conveyor bridges are applied for materials handling purposes. A service walk-way has also to be constructed along the belt-conveyor. Closed belt-conveyor bridges are applied when the handled materials are sensitive to environmental effects (wind, rain etc.). One type of the closed belt-conveyor bridges involves the cylindrical shell bridges which can be provided with ring or stringer stiffeners or even both types of stiffeners. Fabricated cylinders are produced by butt welding together cold or hot formed plate elements. Long fabricated cylinders are generally fabricated by butt welding together a series of short sections. In the case of cylindrical shell bridges there is a great number of welds thus it is necessary to consider also the fabrication cost in the objective function. When considering the loads acting on the structure beyond the live load, the weight of rollers, belt(s) service-walkway as well as the self-weight which can be a considerable part of the total load have to be assumed. The schematic view of a cylindrical shell belt-conveyor bridge can be seen in Fig. 1.

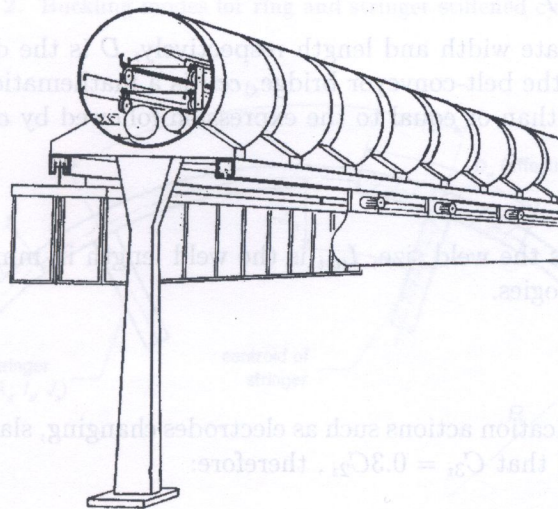


Fig. 1. Schematic view of belt-conveyor bridge

2. OBJECTIVE (COST) FUNCTION

The cost (K) of a structure consists of the material (K_m) and fabrication (K_f) costs. In this case the preparation, assembly, welding and additional (electrodes changing, slag removal, chamfering etc.) costs are considered. Obtaining such cost factors that are valid all over the world is very difficult since cost factors are highly influenced by the technological level of the given country. This problem can be eliminated by the application of fabrication times. Having computed the required time for a fabrication process it can be multiplied by a specific cost factor related to the development level of a country.

$$K = K_m + K_f = k_m \rho V + k_f \sum_i T_i \quad (1)$$

where k_m and k_f are the corresponding cost factors, ρ is the material density, V is the volume of the structure, T_i are the fabrication times.

The variables for the optimisation are the characteristic dimensions of ring stiffened (R or D , h_r , t_r) and stinger stiffened (R or D , h_s , t_s) cylinders, where R is the radius or D is the diameter of shell in both cases, h_r and t_r are the height and thickness of ring stiffeners, h_s and t_s are the height and thickness of stringer stiffeners, Figs. 4 and 3. We have four unknown variables to be optimised for both ring and stringer stiffened cylinders according to the above.

2.1. Fabrication times for weldings

Equation (1) can be written in the following form

$$\frac{K}{k_m} = \rho V + \frac{k_f}{k_m} (T_1 + T_2 + T_3) \quad (2)$$

where

$$T_1 = C_1 \delta_d \sqrt{\kappa \rho V} \quad (3)$$

is the time for preparation, assembly and tacking, δ_d is the difficulty factor expressing the complexity of the structure, κ is the number of structural elements (plates and stiffeners) to be assembled. The required number of plates can be calculated from their dimensions and given by the following equation:

$$n_p = \text{ceil} \left(\frac{D\pi}{B_p} \right) \text{ceil} \left(\frac{L}{L_p} \right) \quad (4)$$

where B_p and L_p are the plate width and length respectively, D is the diameter of the cylindrical shell and L is the length of the belt-conveyor bridge, ceil is a mathematical function which returns the smallest integer greater than or equal to the expression followed by ceil .

$$T_2 = \sum_i C_{2i} a_{wi}^n L_{wi} \quad (5)$$

is the time of welding, a_{wi} is the weld size, L_{wi} is the weld length in mm, C_{2i} and n are constants for different welding technologies.

$$T_3 = \sum_i C_{3i} a_{wi}^n L_{wi} \quad (6)$$

is the time of additional fabrication actions such as electrodes changing, slag removal and chamfering. Ott and Hubka [8] proposed that $C_{3i} = 0.3C_{2i}$, therefore:

$$T_2 + T_3 = 1.3 \sum_i C_{2i} a_{wi}^n L_{wi} \quad (7)$$

Values of C_{2i} and n may be given according to the modified data [7] of *COSTCOMP* [4, 2]. It gives welding times and costs for different welding technologies. In this paper welds are considered with GMAW-C process, stiffeners are welded to the cylindrical shell by double fillet welds and shell plates are welded together by V butt welds. The weld size of a fillet weld is a discrete value according to the stiffener thickness and the weld size of a V weld is equal to the shell thickness.

3. DESIGN CONSTRAINTS

The design constraint system has been formulated according to the specifications of API (American Petroleum Institute) [1]. The bulletin provides stability criteria for determining the structural adequacy against buckling of circular members when subjected to bending. The buckling capacities of the cylinders are based on linear bifurcation (classical) analyses reduced by capacity reduction factors which account for the effects of imperfection and nonlinearity in geometry and boundary conditions. The different kind of buckling modes for ring and stringer stiffened cylinders are shown in Fig. 2. An additional deflection constraint has to be also taken into account for both ring and stringer stiffened cylinders because the structure has to be rigid enough against the dynamic effects.

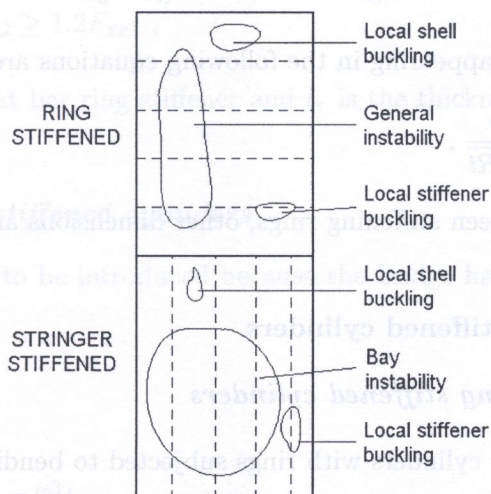


Fig. 2. Buckling modes for ring and stringer stiffened cylinders

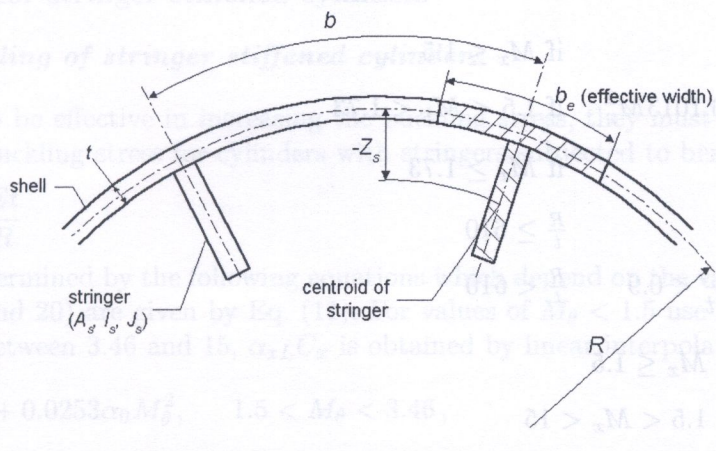


Fig. 3. Section through stringers

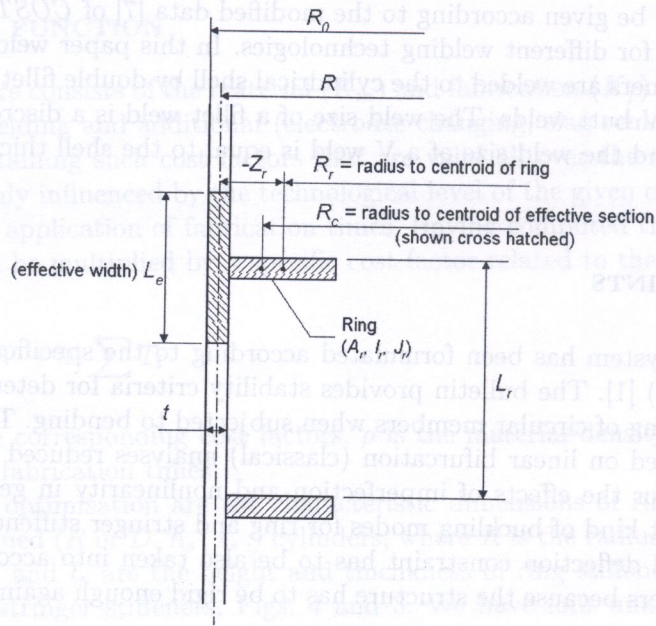


Fig. 4. Section through rings

The values of M_x and M_θ appearing in the following equations are defined as:

$$M_x = \frac{L_r}{\sqrt{Rt}}, \quad M_\theta = \frac{b}{\sqrt{Rt}}, \tag{8}$$

where L_r is the distance between stiffening rings, other dimensions are shown in Figs. 3 and 4.

3.1. Constraints for ring stiffened cylinders

3.1.1. Local buckling of ring stiffened cylinders

The elastic buckling stress for cylinders with rings subjected to bending is defined as:

$$F_{xeL} = \alpha_{xL} \sigma_{xeL} = \alpha_{xL} C_x \frac{Et}{R} \tag{9}$$

where:

$$C_x = \begin{cases} 0.630 & \text{if } M_x \leq 1.5 \\ \frac{0.904}{M_x^2} + 0.1013M_x^2 & \text{if } 1.5 < M_x < 1.73 \\ 0.605 & \text{if } M_x \geq 1.73 \end{cases} \tag{10}$$

$$\alpha_{xL} = \begin{cases} 0.207 & \frac{R}{t} \geq 610 \\ \frac{169\bar{c}}{195 + R/t} < 0.9 & \frac{R}{t} < 610 \end{cases} \tag{11}$$

$$\bar{c} = \begin{cases} 2.64 & M_x \leq 1.5 \\ \frac{3.13}{M_x^{0.42}} & 1.5 < M_x < 15 \\ 1.0 & M_x \geq 15 \end{cases} \tag{12}$$

3.1.2. General instability of ring stiffened cylinders

The elastic buckling stress for cylinders subjected to bending is defined as:

$$F_{xeG} = \alpha_{xG} \sigma_{xeG} = \alpha_{xG} 0.605 E \frac{t}{R} (1 + \bar{A}_r)^{\frac{1}{2}}, \quad \bar{A}_r = \frac{A_r}{L_r t} \quad (13)$$

where A_r is the ring area, L_r is the ring spacing, and the imperfection factor is

$$\alpha_{xG} = \begin{cases} 0.72 & \text{if } \bar{A}_r \geq 0.2 \\ (3.6 + 5.0\alpha_{xL})\bar{A}_r + \alpha_{xL} & \text{if } 0.06 < \bar{A}_r < 0.2 \\ \alpha_{xL} & \text{if } \bar{A}_r \leq 0.06 \end{cases} \quad (14)$$

where α_{xL} is given by Eq. (11) in which $\bar{c} = 1.0$, E is the elastic modulus.

3.1.3. Local stiffener buckling and sizing of stiffeners

To preclude stiffener buckling prior to shell buckling the local stiffener buckling stress must be greater than the shell buckling stress given by the foregoing equations. The local stiffener buckling stress can be assumed to be equal to the yield stress for stiffeners which satisfy the following compact section requirements.

$$\frac{h_r}{t_r} \leq 0.375 \sqrt{\frac{E}{F_y}}, \quad F_{xeG} \geq 1.2 F_{xeL}, \quad (15)$$

where h_r is the height of a flat bar ring stiffener and t_r is the thickness of the bar, F_y is the yield stress.

3.1.4. Deflection of ring stiffened cylinders

An additional constraint has to be introduced because the bridge has to have appropriate rigidity against dynamic loads.

$$w_{\max} \leq w_{adm} = \frac{L}{300} \quad (16)$$

where

$$w_{\max} = \frac{5pL^4}{384EI_x}, \quad I_x = \pi R^3 t. \quad (17)$$

3.2. Constraints for stringer stiffened cylinders

3.2.1. Local buckling of stringer stiffened cylinders

For the stringers to be effective in increasing the buckling stress, they must be spaced sufficiently close. The elastic buckling stress for cylinders with stringers subjected to bending is defined as:

$$F_{xeL} = \alpha_{xL} C_x \frac{Et}{R} \quad (18)$$

where $\alpha_{xL} C_x$ is determined by the following equations which depend on the value of M_θ . The values of α_0 in Eqs. (19 and 20) are given by Eq. (11). For values of $M_\theta < 1.5$ use $M_\theta = 1.5$ in Eq. (19). For values of M_θ between 3.46 and 15, $\alpha_{xL} C_x$ is obtained by linear interpolation.

$$\alpha_{xL} C_x = \frac{3.254}{M_\theta^2} + 0.0253\alpha_0 M_\theta^2, \quad 1.5 < M_\theta < 3.46, \quad (19)$$

$$\alpha_{xL} C_x = 0.605\alpha_0, \quad M_\theta > 15. \quad (20)$$

3.2.2. Bay instability of stringer stiffened cylinders

The theoretical elastic buckling load for bay instability is given by the following orthotropic shell equation (21). When the stringers are not sufficiently close together so that the shell plating is fully effective, the rigidity parameters (E_x , E_θ , D_x , D_θ , $D_{x\theta}$, $G_{x\theta}$) of Eq. (22) are modified by the ratio of effective width to stiffener spacing. Equation is given for b_e which is the effective width of plate in the x direction. When $b_e < b$, set $\nu = 0$; otherwise $\nu = 0.3$. m and n are half waves and waves into which the shell will buckle in the longitudinal and circumferential direction, respectively. For the following equation to be valid, the bay instability stress should be less than 1.5 times the local shell buckling stress.

$$N_{xeB} = \frac{A_{33} + \frac{A_{12}A_{23} - A_{13}A_{22}}{A_{11}A_{22} - A_{12}^2}A_{13} + \frac{A_{12}A_{13} - A_{11}A_{23}}{A_{11}A_{22} - A_{12}^2}A_{23}}{Y} \quad (21)$$

where

$$\begin{aligned} A_{11} &= E_x \left(\frac{m\pi}{L}\right)^2 + G_{x\theta} \left(\frac{n}{R}\right)^2 \\ A_{22} &= E_\theta \left(\frac{n}{R}\right)^2 + G_{x\theta} \left(\frac{m\pi}{L}\right)^2 \\ A_{33} &= D_x \left(\frac{m\pi}{L}\right)^4 + D_{x\theta} \left(\frac{m\pi}{L}\right)^2 \left(\frac{n}{R}\right)^2 + D_\theta \left(\frac{n}{R}\right)^4 + \frac{E_\theta}{R^2} \\ A_{12} &= (E_{x\theta} + G_{x\theta}) \left(\frac{m\pi}{L}\right) \left(\frac{n}{R}\right) \\ A_{23} &= \frac{E_\theta}{R} \left(\frac{n}{R}\right) \\ A_{13} &= \frac{E_{x\theta}}{R} \left(\frac{m\pi}{L}\right) + C_x \left(\frac{m\pi}{L}\right)^3 \\ E_x &= \frac{Et}{1-\nu^2} \left(\frac{b_e}{b}\right) + \frac{EA_s}{b} \\ E_{x\theta} &= \frac{\nu Et}{1-\nu^2} \\ E_\theta &= \frac{Et}{1-\nu^2} \\ G_{x\theta} &= \frac{Gt b_e}{2 b} \\ D_x &= \frac{Et^3}{12(1-\nu^2)} \frac{b_e}{b} + \frac{EI_s}{b} + \frac{EA_s Z_s^2}{b} \\ D_\theta &= \frac{Et^3}{12(1-\nu^2)} \\ D_{x\theta} &= \frac{\nu Et^3}{6(1-\nu^2)} + \frac{Gt^3 b_e}{6 b} + \frac{GJ_s}{b} \\ C_x &= \frac{EA_s Z_s}{b} \\ Y &= \left(\frac{m\pi}{L}\right)^2. \end{aligned} \quad (22)$$

The elastic buckling stress in the longitudinal direction for the bay instability mode of cylinders subjected to bending is given by Eq. (23) with N_{xeB} determined from Eq. (21). When $b_e < b$,

the values for F_{xeB} must be determined by iteration since the effective width is a function of the buckling stress.

$$F_{xeB} = \alpha_{xB} \frac{N_{xeB}}{t_x} \quad (23)$$

where

$$t_x = \frac{A_s + b_e t}{b} \quad (24)$$

$$\alpha_{xB} = \begin{cases} 0.65, & \text{if } \bar{A}_s \geq 0.06 \\ \alpha_{xL}, & \text{if } \bar{A}_s < 0.06 \end{cases} \quad (25)$$

$$b_e = 1.9t \sqrt{\frac{E}{F_{xeB}}} \leq b \quad (26)$$

$\bar{A}_s = \frac{A_s}{bt}$ and α_{xL} is given by Eq. (11) with $\bar{c} = 1.0$.

3.2.3. Local stiffener buckling and sizing of stiffeners

The stiffener buckling for stringers is the same as it is for rings but instead of the general instability the bay instability stress should be used.

$$\frac{h_s}{t_s} \leq 0.375 \sqrt{\frac{E}{F_y}}, \quad F_{xeB} \geq 1.2F_{xeL}, \quad (27)$$

where h_s is the height of a flat bar stringer stiffener and t_s is the thickness of the bar.

3.2.4. Deflection of stringer stiffened cylinders

The deflection constraint is the same as for ring stiffeners in Eqs. (16) and (17), but in the second moment of inertia I_x the effect of the stringers are involved.

$$I_x = \pi R^3 t + 2h_s t_s R^2 \left(1 + 2 \sum_{i=1}^{\frac{n_s}{4}-1} \sin^2 \left(i \frac{2\pi}{n_s} \right) \right) \quad (28)$$

if n_s divisible by 4 without remainder,

$$I_x = \pi R^3 t + 2h_s t_s R^2 \left(1 + 2 \sum_{i=1}^{\text{int}(\frac{n_s}{4})} \sin^2 \left(i \frac{2\pi}{n_s} - \frac{\pi}{n_s} \right) \right) \quad (29)$$

if n_s not divisible by 4 without remainder.

4. STRESS CALCULATION

The computed stresses [1] in stiffened cylindrical shells may differ from the membrane stress of unstiffened shells. Equations are given below for correction factors, which account for the load sharing effects of the various elements. In Eqs. (31) and (32), M is the bending moment at the cross section under consideration which is defined by Eq. (30):

$$M = \frac{pL^2}{8}, \quad p = p_l + p_b + p_r + p_w + p_0, \quad (30)$$

where p is factored uniformly distributed load containing the live load (p_l), loads of the weight of the belt (p_b), rollers (p_r), service-walkway (p_w) and self-weight (p_0).

4.1. Bending stress for ring stiffened cylinders

The bending stress for ring stiffened cylinders is defined as follows:

$$f_b = \frac{M}{\pi R^2 t} K_b, \quad K_b = \frac{1 + 0.5 \frac{t}{R}}{1 + 0.25 \left(\frac{t}{R}\right)^2}. \quad (31)$$

4.2. Bending stress for stringer stiffened cylinders

The bending stress for stringer stiffened cylinders is defined as follows:

$$f_b = \frac{M}{Q_a \pi R^2 t_e} \quad (32)$$

where

$$t_e = t + \frac{A_s}{b}, \quad Q_a = \frac{A_s + b_e t}{A_s + b t}. \quad (33)$$

The factor Q_a is a ratio of the effective area to the actual area, $Q_a = 1.0$ for the local shell buckling mode.

4.3. Allowable stresses

The allowable stress F_b for bending is to be taken as the lowest value given for all modes of failure. In our case the stiffeners are sized in accordance with the methods given in section 3.1.3 and 3.2.3, therefore only the shell buckling mode should be considered in the equations which follow. The allowable stresses must be greater than the applied stresses.

$$f_b \leq F_b = F_{xcL} \quad (34)$$

where F_{xcL} is the smaller of the values given by the following equations:

$$F_{xcL} = F_{xeL}, \quad (35)$$

$$F_{xcL} = \begin{cases} \frac{233F_y}{166 + R/t} \leq F_y & R/t < 300, \\ 0.5F_y & R/t \geq 300. \end{cases} \quad (36)$$

5. OPTIMUM SEARCH FOR RING AND STRINGER STIFFENED CYLINDERS

5.1. Grapho-analytical optimum design for ring stiffened cylinders

By the appropriate reduction (some of the constraints are considered to be active) of the number of unknown variables (R, t, h_r, t_r) the problem leads to an optimization task having two unknown variables (R, t) or for the better graphical representation we use the square diameter of the shell (D^2) and the slenderness ($\delta_c = D/t$). In this case the permissible region defined by the constraints can be plotted in a coordinate system of the two unknown variables. The basic principle of this solution is that the optimum point can be found at the point where the level line of the cost function touches the permissible region. The flowchart of the process can be seen in Fig. 5. A suitable program has been developed implementing the algorithm for MathCad 7.0 software.

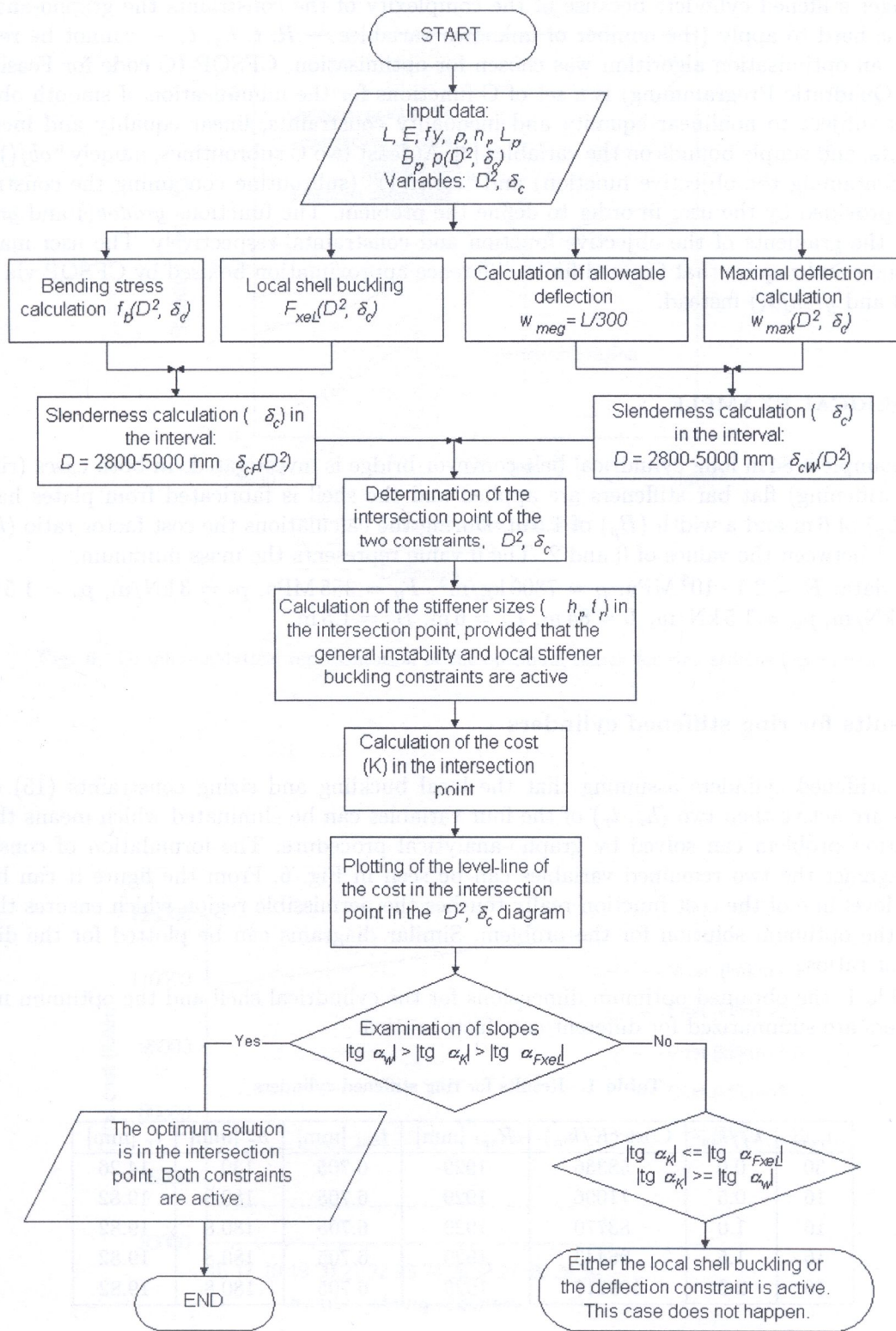


Fig. 5. Flowchart of the grapho-analytical solution for ring stiffened cylinders

5.2. Adoption of FSQP algorithm for stringer stiffened cylinders

For stringer stiffened cylinders because of the complexity of the constraints the grapho-analytical solution is hard to apply (the number of unknown variables — R, t, h_s, t_s — cannot be reduced) therefore an optimisation algorithm was chosen for optimisation. CFSQP (C code for Feasible Sequential Quadratic Programming) is a set of C functions for the minimization of smooth objective functions subject to nonlinear equality and inequality constraints, linear equality and inequality constraints, and simple bounds on the variables [3]. At least two C subroutines, namely “*obj()*” (subroutine containing the objective function) and “*constr()*” (subroutine containing the constraints), must be provided by the user in order to define the problem. The functions *gradob()* and *gradcn()* compute the gradients of the objective function and constraints, respectively. The user may omit this routines and require that forward finite difference approximation be used by CFSQP via calling *grobfd()* and *grcnfd()* instead.

6. NUMERICAL EXAMPLE

In this example a 84 m long cylindrical belt-conveyor bridge is investigated. In both cases (ring and stringer stiffening) flat bar stiffeners are applied and the shell is fabricated from plates having a length (L_p) of 6 m and a width (B_p) of 1.5 m. During the calculations the cost factor ratio (k_f/k_m) is changed between the values of 0 and 2. The 0 value represents the mass minimum.

Basic data: $E = 2.1 \cdot 10^5$ MPa, $\rho = 7800$ kg/m³, $F_y = 355$ MPa, $p_l = 3$ kN/m, $p_b = 1.5$ kN/m, $p_r = 1.5$ kN/m, $p_w = 1.5$ kN/m, $L = 84$ m, $L_p = 6$ m, $B_p = 1.5$ m.

6.1. Results for ring stiffened cylinders

For ring stiffened cylinders assuming that the local buckling and sizing constraints (15) of ring stiffeners are active then two (h_r, t_r) of the four variables can be eliminated which means that the optimization problem can be solved by grapho-analytical procedure. The formulation of constraints plotted against the two remained variables can be seen in Fig. 6. From the figure it can be seen that the level line of the cost function really touches the permissible region which ensures that the point is the optimum solution for the problem. Similar diagrams can be plotted for the different cost factor ratios.

In Table 1, the obtained optimum dimensions for the cylindrical shell and the optimum number of stiffeners are summarized for different cost factor ratios.

Table 1. Results for ring stiffened cylinders

$n_{r_{opt}}$	k_f/k_m	Cost (K/k_m)	R_{opt} [mm]	t_{opt} [mm]	h_r [mm]	t_r [mm]
30	0.0	58336	1929	6.705	130	14.26
16	0.5	71096	1929	6.705	180.8	19.82
16	1.0	83770	1929	6.705	180.8	19.82
16	1.5	96443	1929	6.705	180.8	19.82
16	2.0	109117	1929	6.705	180.8	19.82

Figure 7 shows the total cost of a cylindrical ring stiffened shell against the number of stiffeners for different cost factor ratios. It can be seen that the costs are rising by the increase in the number of stiffeners while for mass minimum a slight decrease can be observed (however the relevant line seems to be horizontal) if the number of stiffeners are increased.

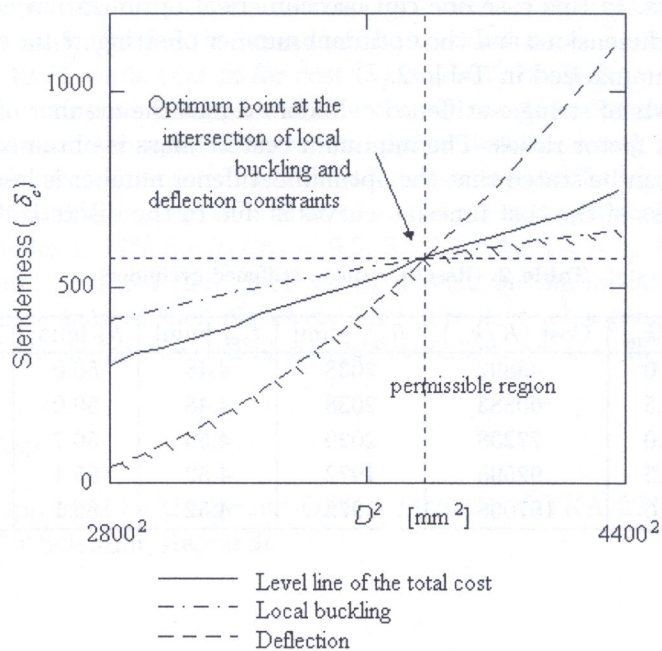


Fig. 6. Grapho-analytical representation of the optimum search for ring stiffened cylinders

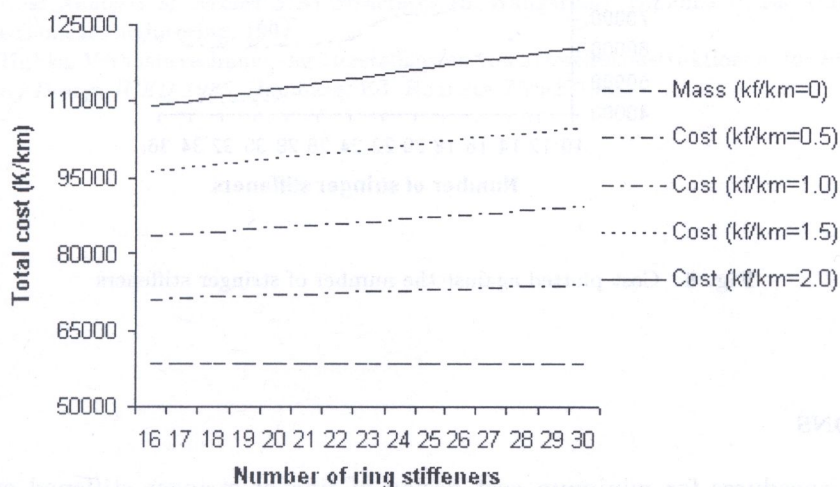


Fig. 7. Cost plotted against the number of ring stiffeners

6.2. Results for stringer stiffened cylinders

For stringer stiffened cylinders the grapho-analytical solution is not applicable because of the complexity of the constraints. In this case one can use numerical optimization algorithms to solve the problem. The optimum dimensions and the optimum number of stringers for cylindrical shells found by FSQP method are summarized in Table 2.

In Fig. 8 the total costs of stringer stiffened cylinders versus the number of stringer stiffeners are plotted for different cost factor ratios. The minimum cost or mass is obtained at different numbers of stringer stiffeners. It can be stated that the optimum stiffener number is less for higher cost factor ratios. The unsmoothness of the cost function curves is due to the discretization of weld sizes.

Table 2. Results stringer stiffened cylinders

n_{sopt}	k_f/k_m	Cost (K/k_m)	R_{opt} [mm]	t_{opt} [mm]	h_s [mm]	t_s [mm]
32	0.0	43892	2038	4.48	50.0	6.00
32	0.5	60883	2038	4.48	50.0	6.00
30	1.0	77238	2029	4.50	56.7	6.22
24	1.5	92595	1972	4.52	85.1	9.33
24	2.0	107066	1972	4.52	85.1	9.33

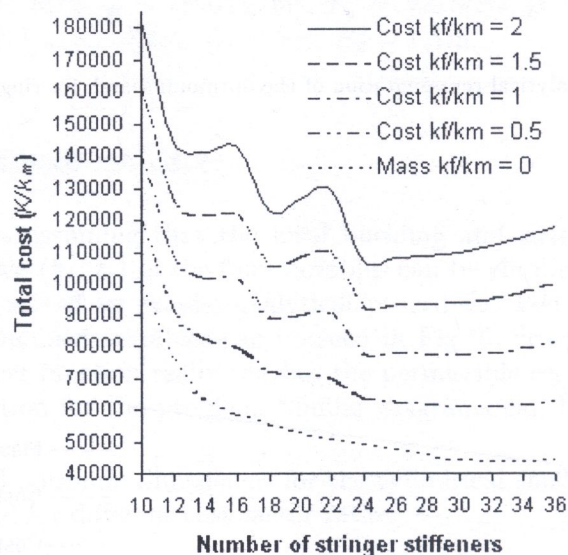


Fig. 8. Cost plotted against the number of stringer stiffeners

7. CONCLUSIONS

The developed procedures for minimum cost design of ring or stringer stiffened cylinders can be efficiently applied for the given task and are capable of finding the global optimum. In further researches investigations should be extended for finding the additional cost influencing factors by which the cost function should be modified.

The obtained results can be summarized as follows. In both cases (ring or stringer stiffening) it can be stated that the optimal number of stiffeners is always higher for mass minimum than for

cost minimum, which is also a general rule for stiffened structures. The selection of the number of stiffeners has particular importance considering that, in case of ring stiffeners, if one chooses the number of stiffeners for 30 instead of the optimum 16 (assuming cost minimum $k_f/k_m = 2$) than the total cost increases by 10%. This observation is more valid for stringer stiffening as if one selects the number of stringer stiffeners for 10 instead of the relevant optimum number, the mass of the structure increases by 125% while that of for cost ($k_f/k_m = 2$) by 70%.

On the basis of the obtained results it is also turned out that the mass or/and cost of the structure can be decreased if stringer stiffeners are applied instead of ring stiffeners. If we compare the mass and cost minimum results for ring stiffened and stringer stiffened cylinders, than the mass of a ring stiffened shell is 33% greater than that of a stringer stiffened shell, while these values for cost minimum are as follows: 17% for $k_f/k_m = 0.5$, 8.5% for $k_f/k_m = 1$, 4% for $k_f/k_m = 1.5$ and 2% $k_f/k_m = 2$. It should be noted that these values depend on the initial parameters such as the length of the bridge, dimension of shell plates etc. therefore it is not a general rule that the cost or mass can be decreased by the application of stringer stiffeners, however in most cases this is valid.

ACKNOWLEDGEMENT

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