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Estimating the error of coordinate measuring machines

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This paper presents a strategy to analyze the accuracy of measurements obtained in the calibration process of a Coordinate Measuring Machine (CMM). Thereafter, the operator will be able to find the most appropriate placement region for a part to be analyzed in order to reduce the error and take into account the shape of each part. Such strategy was developed through the visualization of error contours in the measurement space. Five different error indexes were developed for this analysis, associating different values to each point of the measurement space, allowing the user to choose the aspect he wants to take into consideration in the measurement process: centering of samples, dispersion, average errors, and others. Finally, the results are transformed into images, analysed and compared.

1. Introduction

The quality of products described through precise dimensional characteristics can be verified through careful checking of measurements, taking into account the desired, exact value and the value obtained from the part.

Measures obtained from a CMM (Coordinate Measuring Machine) approximate numerical features of the object. This instrument, the CMM, as described in [1], makes possible the pointwise metrological recognition by grouping the constituent surfaces of the object.

The measurement of objects by coordinate sampling is achieved considering a particular measurement region defined by the displacement system of the mobile parts of the CMM. This measurement is submitted to, at least, three kinds of error: systematical, random and rounding error.

With a relatively simple strategy to analyze the accuracy of measurements obtained in the CMM calibration process, the operator may find the most appropriate placement region to place a part in order to minimize error taking into account the shape of each part. This strategy will be developed in the following sections.

2. Structure and operation of a CMM

A Coordinate Measuring Machine is an electronic and mechanic system built to obtain cartesian coordinates of points on solid surfaces. CMMs differ but they present common fundamental characteristics. In Fig. 1 below we can see the parts composing a CMM.

The grouping system (that will touch the object) is a very delicate part of the CMM and measures coordinates of points placed on a rigid surface when these touch a contact sphere. The function of the grouping system is to transmit a touch event to the processor so that coordinates are noted down and mobile parts are locked. With this system we can distinguish between points less than 10^{-3} mm apart of each other, being sufficient for most applications.

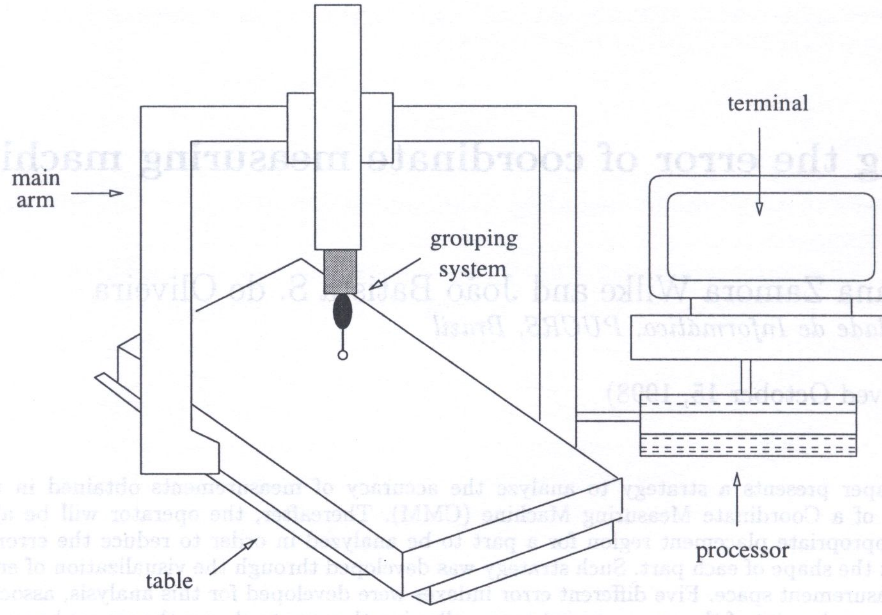

Fig. 1. Coordinate measuring machine (CMM)

3. Sources of error in a CMM

Errors in operations with the CMM come basically from the acquisition of the coordinates. The error factors are classified as internal – originating from the machine itself; and external – dependent from environmental conditions.

Internal factors

- Coordinate system: the most significant internal factor consists in the deviations of the real coordinate system with respect to the ideal coordinate system. These deviations are caused by distortions in the shape and orientation of the guidances and relationed parts.
- Algorithms: CMMs have a processor to handle the readings and to introduce corrections, calculate distances, centers, and so on. The algorithms that carry out these functions can have unexpected behavior if used near their validity range.

External factors

- Temperature: this is the most difficult factor to control. The temperature modifies the shape of the parts of CMM and alters its configuration. Basically, temperature changes in a determined place or has different values according to each place. The reference temperature accepted is 20°C.
- Environmental humidity: environmental humidity also influences the results of the measurements. It can expand the volume of the granite table or favour corrosion.
- Dust: it can rest on the object or the grouping sphere and change the measurement.
- Vibrations: the most important vibrations are transmitted to the machine through its foundations. Vibrations can be also transmitted by the atmosphere (propagation of sound waves or draught).
- Operator: the operator can affect the measurement as the exaggerated contact of the hands with the object causes differences in temperature.

4. Errors originating from a CMM

The errors originating from a CMM are classified in, at least, three kinds of error: systematical, random and rounding error.

1. Systematical error: this error is due to permanent defects of construction of the instrument and has always the same value to each measurement – thus, it is predictable.

2. Random error: the random error is caused by alterations (not perceptible) of the instrument, of the object to be measured, of the environment and of the others. These errors cannot be estimated separately but, nevertheless, can be measured quantitatively through calibration.

3. Rounding error: rounding error (see [2]) is inherent to hardware and its control depends on numerical methods and techniques of scientific computation. In this way, we can say that the rounding error is related to processing in finite precision. However, in most cases the rounding error is seldom taken into consideration for all practical purposes because the introduced error will be irrelevant if it is compared with the mechanic errors of the CMM.

5. Formulation of the problem

The measurement of objects by coordinates is achieved considering a particular measurement space, defined by the displacement system of the mobile parts of the machine.

Thus, let there be a tridimensional measurement region $S_p = \{(x_i, y_i, z_i) : x_i, y_i, z_i \in R, 1 \leq i \leq n\}$, where n is the number of positions in the measurement region. So, S_p is a set of points that can be reached by the mechanism, and we then define error sets originating from m_i samples $x_1^i, x_2^i, \ldots x_{m_i}^i$ associated to each point x_i in S_p and obtained through calibration. So, these error sets can be grouped according to each axis:

$$S_x^i = \{x_j^i - x_i : 1 \leq i \leq n, \quad 1 \leq j \leq m_i\} \tag{1}$$

$$S_y^i = \{y_j^i - y_i : 1 \leq i \leq n, \quad 1 \leq j \leq m_i\} \tag{2}$$

$$S_z^i = \{z_j^i - z_i : 1 \leq i \leq n, \quad 1 \leq j \leq m_i\} \tag{3}$$

That is, for each position $(x_i, y_i, z_i) \in S_p$ are associated three sets of errors obtained from the sampled coordinates: S_x^i, S_y^i and S_z^i as shown in Fig. 2 below, for some i and $m_i = 3$.

Moreover, we may introduce a generic error function

$$\varepsilon_k^i : R^3 \times S_x^i \times S_y^i \times S_z^i \longrightarrow R \tag{4}$$

to provide values of error indexes at position (x_i, y_i, z_i) taking into account the kind of error we want to measure, the original point and the errors of the measurements obtained.

6. Proposed solution

Starting from a data file originated from a calibration containing the original n positions and the m_i measurements associated to each position, a set of routines was implemented to calculate the error index associated to each point of the measurement region. This error function associates each point of the measurement space to an error calculated from the m_i measurements obtained for the point. So, with these error values we will be able to plot a surface and find the measurement region with smallest error. In this way, errors are associated to a color scale and, finally, a color surface is visualized.

Six different types of error indexes will be developed for this analysis, working differently as they allow the user to choose special aspects he wants to take into consideration in the measurement process: sample centering, sample spreading, and others.

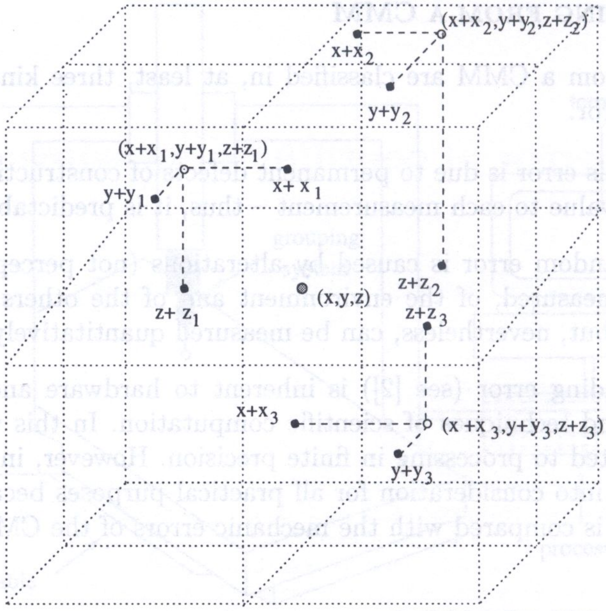

Fig. 2. S_x^i, S_y^i, S_z^i, **for a given point** i

7. Measurement indexes (ε_K)

The error value relative to each point of the measurement region will be calculated through six different forms, allowing the user to take into account any of several desired aspects. Next we present the measurement indexes and thereafter some examples of visualization will be given.

7.1. Largest absolute error (ε_1)

Let there be the set S_i and the following associated values

$$\nabla x_i = \min\{x_j^i : 1 \leq j \leq m_i\} \qquad \Delta x_i = \max\{x_j^i : 1 \leq j \leq m_i\}$$
$$\nabla y_i = \min\{y_j^i : 1 \leq j \leq m_i\} \qquad \Delta y_i = \max\{y_j^i : 1 \leq j \leq m_i\}$$
$$\nabla z_i = \min\{z_j^i : 1 \leq j \leq m_i\} \qquad \Delta z_i = \max\{z_j^i : 1 \leq j \leq m_i\}$$

This index determines the largest error obtained in the calibration of each point independently of the axes. The measurement index associated to point (x_i, y_i, z_i) and denoted by ε_1^i is evaluated by

$$\varepsilon_1^i = \max(|\nabla x_i|, |\Delta x_i|, |\nabla y_i|, |\Delta y_i|, |\nabla z_i|, |\Delta z_i|) \tag{5}$$

7.2. Largest absolute error + displacement from the origin (ε_2)

This index is similar to the previous but it considers the distance to the origin as well as the absolute error. It tries to include the displacement of the mechanism as the CMM will move longer if the object is far from the origin, and the probability of mechanical misalignments increase. The measurement index associated to (x_i, y_i, z_i) and denoted by ε_2^i is calculated as

$$\varepsilon_2^i = \max(|\nabla x_i|, |\Delta x_i|, |\nabla y_i|, |\Delta y_i|, |\nabla z_i|, |\Delta z_i|) \cdot \|(x_i, y_i, z_i)\| \tag{6}$$

7.3. Average error (ε_3)

This index takes the average error of the calibration on each axis. We can take just one of the axes (or two) into consideration and create a new index, for example, considering some of the other axes as irrelevant, as this will be useful for placing one-dimensional and two-dimensional parts. The measurement index associated to (x_i, y_i, z_i) denoted by ε_3^i is calculated through

$$\varepsilon_3^i = \frac{1}{m_i}\left(\sum_{j=1}^{m_i} x_j^i + \sum_{j=1}^{m_i} y_j^i + \sum_{j=1}^{m_i} z_j^i\right) \tag{7}$$

7.4. Centering error (ε_4)

This index shows the degree of centering of the samples with respect to (x_i, y_i, z_i) (see Fig. 3). The measurement index is denoted by ε_4^i and calculated as

$$\varepsilon_4^i = \frac{1}{m_i}\left\|\sum_{m_i}(S_x^i\,,\,S_y^i\,,\,S_z^i) - S_i\right\| \tag{8}$$

The more distant from the point are the samples, the largest will be the value of this index. If the centering is good, the center of the samples is near to (x_i, y_i, z_i)

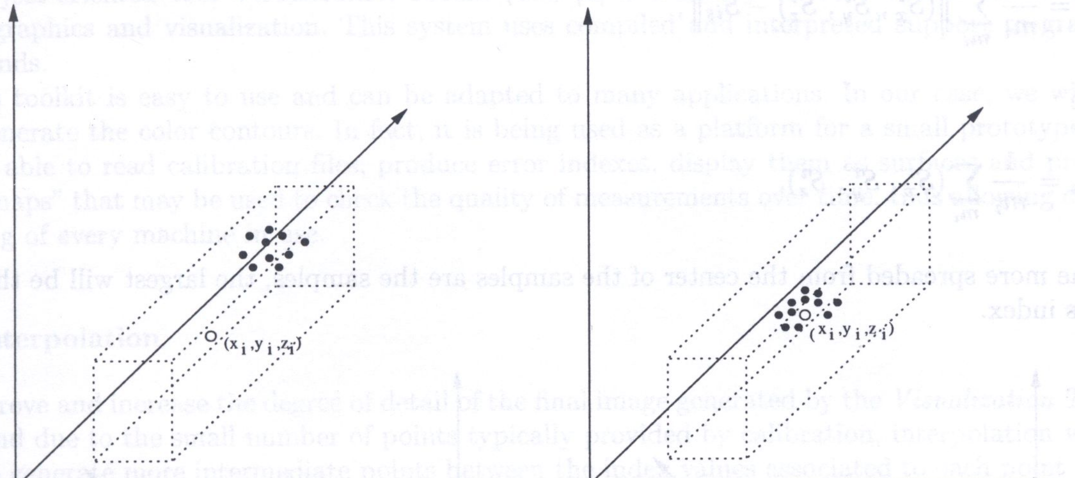

Fig. 3. Centering index

7.5. Spreading of the ideal point (ε_5)

This index shows the degree of spreading of the samples regarding the ideal point (see Fig. 4), so that the more spreaded from the ideal value are the samples, the largest will be the error index. This index is calculated through

$$\varepsilon_5^i = \frac{1}{m_i}\sum_{m_i}\left\|(S_x^i\,,\,S_y^i\,,\,S_z^i) - S_i\right\| \tag{9}$$

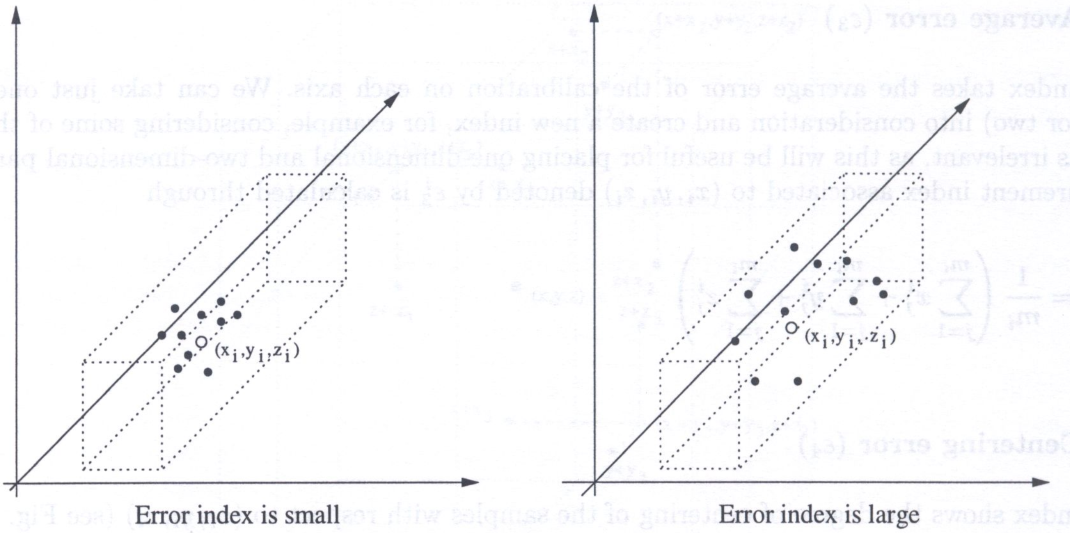

Fig. 4. Spreading of the ideal point

7.6. Spreading of the center of the samples (ε_6)

This index shows the degree of spreading of the errors regarding the center of the samples (see Fig. 5), being denoted by ε_6^i and calculated through

$$\varepsilon_6^i = \frac{1}{m_i} \sum_{m_i} \left\| (S_x^i\,,\, S_y^i\,,\, S_z^i) - S_{is} \right\|, \tag{10}$$

where

$$S_{is} = \frac{1}{m_i} \sum_{m_i} (S_x^i\,,\, S_y^i\,,\, S_z^i). \tag{11}$$

The more spreaded from the center of the samples are the samples, the largest will be the value of this index.

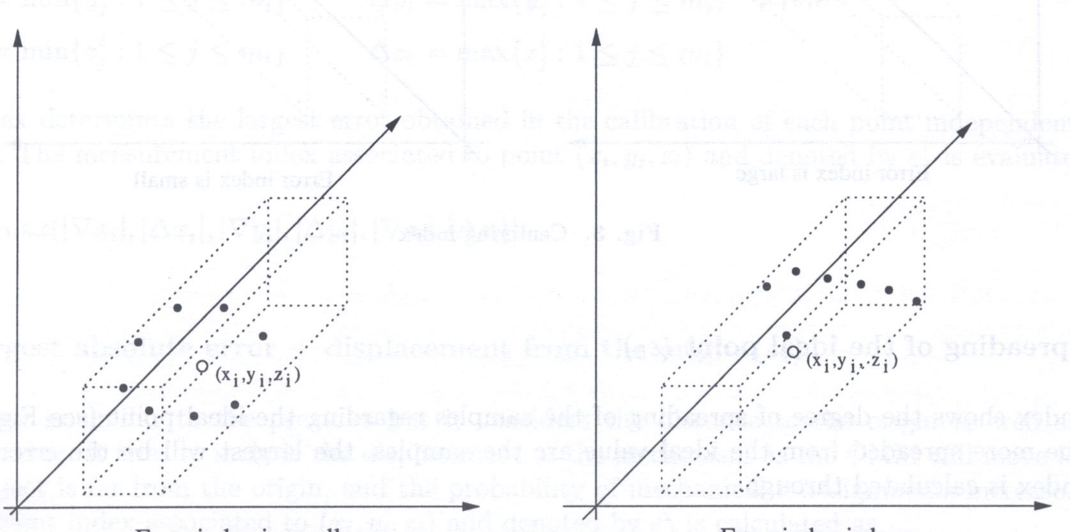

Fig. 5. Spreading of the center of the samples

7.7. Normalization and units

The data describing the measurement error are obtained from the calibration of a CMM. The data file contains the coordinates of each point of the measurement region and the errors originating from m_i measurements associated to these coordinates. Usually, these data are given in μm, that is 10^{-6} m. So, after these computations, all error indexes will have μm as their units. After values are computed for each samples point, is is customary to normalize the error index by dividing them by the largest error obtained, thus keeping all values in the range $[0, 1]$.

8. VISUALIZATION PROCESS

From the file containing the coordinates and calibration data, the routine that calculates the measurement index corresponding to each point of the measurement region is executed. Thus, a file with the error indexes is generated.

To generated more points and to improve the final image the interpolation routine is needed. This routine generates the index value between consecutive points of the measurement region, and a larger data file in the format used by the *Visualization Toolkit* is generated.

Using the visualization tool the indexes are associated to a color scale and color contours of surfaces are shown.

8.1. *Visualization Toolkit (vtk)*

The object-oriented tool *Visualization Toolkit (vtk)* [3] is a library of C++ classes for tridimensional graphics and visualization. This system uses compiled and interpreted support to graphics commands.

This toolkit is easy to use and can be adapted to many applications. In our case, we will use it to generate the color contours. In fact, it is being used as a platform for a small prototype that will be able to read calibration files, produce error indexes, display them as surfaces and produce "error maps" that may be used to check the quality of measurements over time, thus allowing careful checking of every machine in use.

8.2. Interpolation

To improve and increase the degree of detail of the final image generated by the *Visualization Toolkit (vtk)* and due to the small number of points typically provided by calibration, interpolation will be used to generate more intermediate points between the index values associated to each point of the measurement region.

Consider the measurement grid $5 \times 5 \times 5$ (Fig. 6) and the index value (ε_k^i) associated to each one of the 125 points.

More points are defined in this region generating a new grid $(9 \times 9 \times 9)$. The evaluation of the index value associated to the new points generated is made in Fig. 7.

9. RESULTS AND IMAGES

The input data file is described considering a measurement region of $5 \times 5 \times 5$ samples, that is, 125 points. To each one of these points are associated 3 coordinates (x, y, z) and six samples are taken from calibration. A file with the index values is generated and the interpolation routine is used. After that, a data file in the format used by the *Visualization Toolkit* is generated and the image is produced (see for example Fig. 8). The axes x, y and z are represented in violet, green and

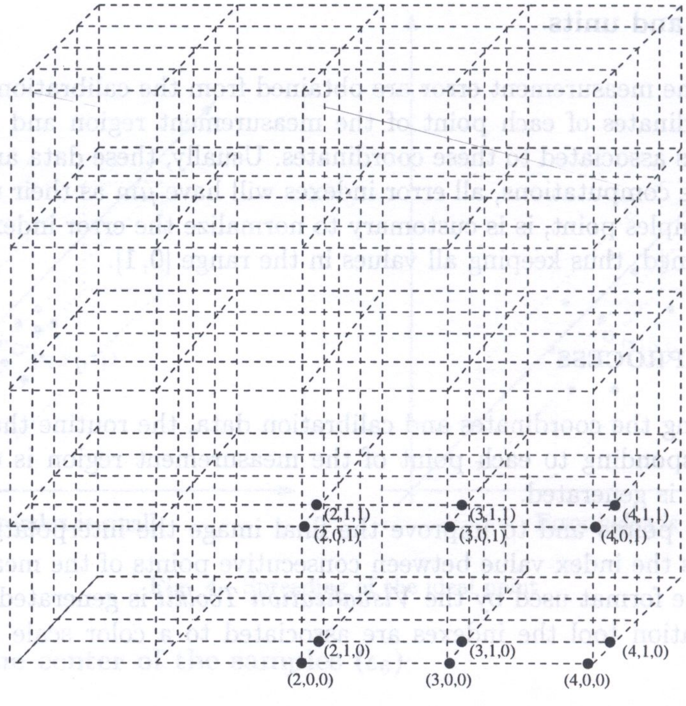

Fig. 6. Measurement grid considered $(5 \times 5 \times 5)$

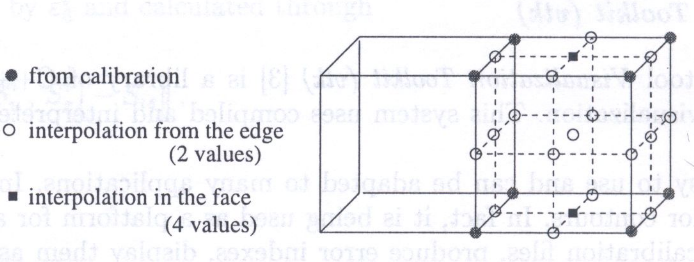

Fig. 7. Points generated through interpolation

blue, respectively. The small cube shows the system origin and the initial position of the grouping system.

The number of contours and their color can be controlled by the user, and in this example we have that the green region represents a region of small error and the violet region a region of large error. Thus, it is advisable to place the object to be measure in the green region.

10. EXAMPLES

1. Measurement index ε_2

   Considering two input data files containing test data, the following images were obtained (Fig. 9).

   This index considers the distance to origin. Thus, we can observe that the regions of small and large error (data files 1 and 2) are similar, but far from origin the error is largest.

2. Measurement index ε_5

   The following images were obtained considering the same data files (Fig. 10).

   This index measures the spreading of the errors. Analyzing this index, we can observe that the spreading of the errors with respect to the original point is largest in the region x with small y (data file 1 and 2).

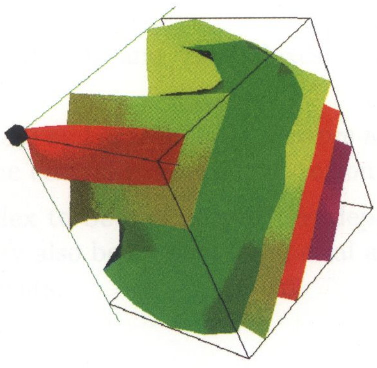

Fig. 8. Image produced

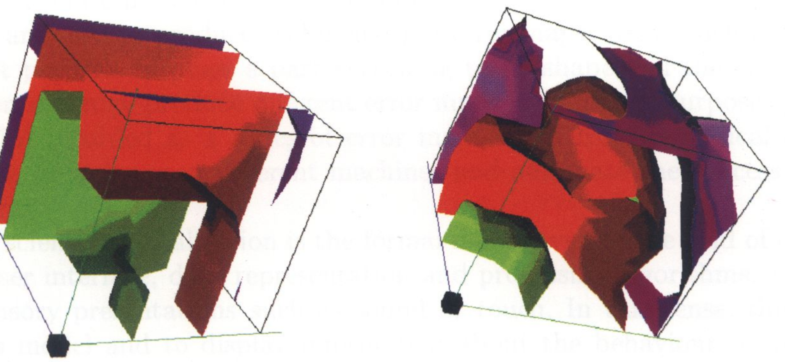

Fig. 9. ε_2(data file 1) and ε_2(data file 2)

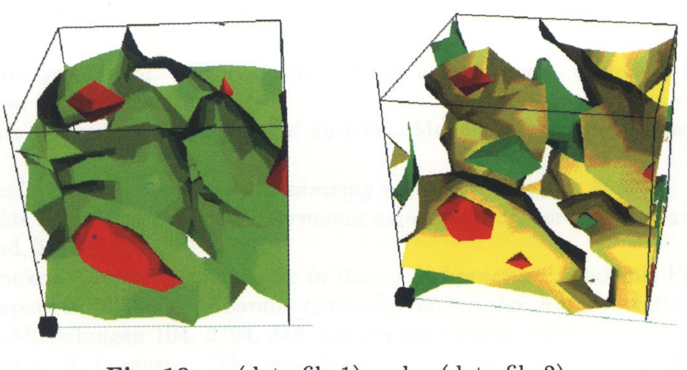

Fig. 10. ε_5(data file 1) and ε_5(data file 2)

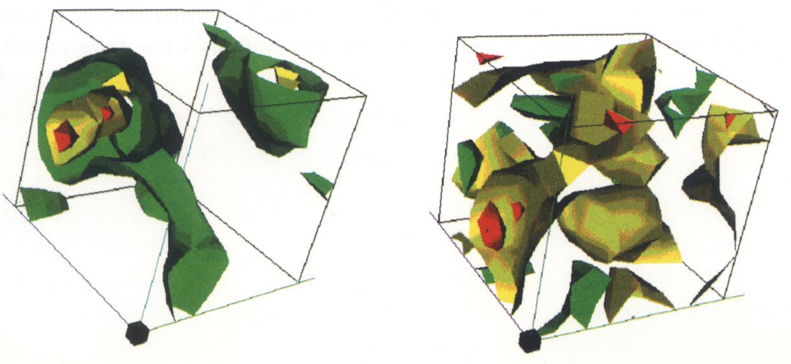

Fig. 11. ε_4(data file 2) and ε_5(data file 2)

3. $\varepsilon_4 \times \varepsilon_5$ (data file 2)

Considering the second input data file and the indexes ε_4 and ε_5 the following images were obtained (Fig. 11).

We can observe that centering (ε_4) and spreading (ε_5) errors are large for small y in both images. Besides, the value is small in the central measurement region.

Of course, the choice of one index to be used in practice depends on what we want to measure or analyse. Different indexes may also be created for special applications or to describe different characteristics of the measurements.

11. Conclusions

This paper deals with the practical issues of using computers in the process of placing objects to be measured by a coordinate measuring machine and to produce better results.

Calibration data are used to produce tridimensional error maps. Using such maps, the operator may choose the best position to place a part according to its shape and the error that has to be minimized. Moreover, one may produce different error maps for different purposes and mantain the "history" of the CMM recorded as a series of error maps taken from periodical calibrations. We could perform comparisons between different machines and automate the process of positioning a part to minimize error.

According to [3], scientific visualization is the formal name given to the field of computer science that encompasses user interface, data representation and processing algorithms, visual representations, and other sensory presentations such as sound or touch. In this sense, this paper presents possibilities both to model and to display information about the behaviour of measurement machines, making them easier to use effectively.

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