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Design of foundation rafts by uncoupled iterative method

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The authors developed a calculation module into a commercial finite element analysis (FEA) program, which is capable of calculating foundation rafts quickly, with correct results for the engineering practice. The calculation method is based on the so-called "uncoupled iterative method", wherein the structure and the soil continuum are analysed separately. The results of one analysis form the boundary conditions for the subsequent analysis as part of an iterative process. The connection between the raft and the soil is considered to be represented by the bedding modulus of the raft and by the soil stresses. The method provides the same displacements for the raft as for the soil surface, provided sufficient convergence can be reached if the raft is not elevated from the soil.

1. Introduction

Every building structure is founded on soil or rock mass. Generally in the static analysis of a structure, especially its foundation, the whole interacting structure-soil system must be taken into account.

In everyday design practice, however, a three-dimensional model of the soil mass cannot be used because of the extreme extent of calculation.

The use of the Winkler model is subject to great discrepancy with the reality, in that for plates resting on soil the Winkler method gives acceptable results only in the vicinity of the load. This is despite the generalisation for inhomogeneous cases where $r(x,y) = k(x,y) \cdot w(x,y)$ (in the formula r is the soil reaction, k is the bedding modulus and w is the deflection). The real r- and w- courses are not affine. In reality, the condition $r = 0$ does not imply the state $w = 0$ at the same place. There may be interaction between two adjacent structures, but the Winkler model cannot express it. From the safety point of view, the bending moments in the foundation plate are most significant. When the load acting on the plate is a uniform pressure, the bending moment values are too small, as compared to reality, and this fact may lead to dangerous conclusions.

The soil-raft interaction can be modelled by analytical methods only in cases under simple condition [7]. These are:

- homogeneous, elastic and isotropic material, infinite half-space or a layer underlain by an absolutely rigid substratum,
- infinitely rigid or infinitely flexible, circular or rectangular foundation slab,
- uniformly or linearly distributed load.

Semi-analytical methods offer solutions also for foundation slabs of finite stiffness [7]. In such cases, the deformation surface of the slab, the bending moments and the bedding pressures are sought in a polynomial form. To these end differential equations describing the states of the slab and of the soil are introduced, constants of the polynomes harmonised, and equilibrium and compatibility equations obtained. This leads to a system of linear equations, the solution of which yields the polynomes for deformations, bending moments and contact pressures. This method can also be extended to the infinite half-space with a soil having an elasticity modulus changing with depth [2, 3]. A precondition for the use of this method is that the load be also specified in polynomial form. Nonetheless, this method still involves essential simplifications as compared to real cases occurring in practice. Furthermore, the application of this method needs a very profound geotechnical knowledge without which its use is limited and may be hazardous.

J.A. Hemsley in [4] discusses some computation methods from the point of view how can be reduced the computational effort. These are:

- the coupled interaction analysis,
- the radius of influence method,
- the coupled iterative method,
- the matrix partitioning method,
- the uncoupled iterative method.

The last four approaches in detail, and recommendations are made with regard to their possible use in design practice. In our process we also applied the last method with some important modifications.

Based on the procedure described hereunder, a programme module was produced for use in conjunction with the static code named FEM Design, developed jointly by Skanska Software (Sweden-Malmö) and PiHun Ltd. (Hungary-Budapest).

2. The Raft Model

In the following we assume a complex model composed of the slab and the connected elastic bedding "under" the slab. The slab is considered to be made up of 6 or 8 node isoparametric Mindlin-Reissner thick plate elements. The degree of freedom of the nodes is three – w, ϕ_x, ϕ_y. The material of the elements can be isotropic or orthotropic. These elements are well known, and their description can be found in the literature [1, 5].

The bedding is also composed of 6 or 8 node isoparametric elastic surface elements. The nodal degree of these elements is one, the vertical displacement is w. Of course the number of nodes and the net pattern are the same for the plate as for the support elements. So we can say one raft element is a made up of a plate and a connected surface support element.

Such support element model of the bedding is better than the Winkler type springs, because their element stiffness matrix is not diagonal, and the reactions can be obtained as distributed surface loading. The elastic response of the support elements can be characterised by the bedding modulus k. The k modulus is constant for one element, but it can be different from element to element. The bedding modulus can be calculated by taking into account the physical properties of the layered elastic soil.

With this model it is possible and easy to take discontinuities like holes in the plate or in the soil into calculation. Support elements can be located outside the plate, too, but in this case they have importance only in the calculation of the soil surface deflections.

Considering the described model the following system of linear equations must be satisfied:

$$[\mathbf{K}] \cdot \left\{\mathbf{w}^f\right\} = \{\mathbf{f}\} \tag{1}$$

where $[\mathbf{K}]$ is the global stiffness matrix of the plate and surface support elements, $\left\{\mathbf{w}^f\right\}$ is the nodal displacement vector, $\{\mathbf{f}\}$ is the nodal load vector, which is calculated from the structural loads acting on the slab.

3. THE SOIL MODEL

3.1. Basic assumptions

The soil mass considered in this method is a pseudo-elastic, generally layered halfspace. In any layer, the physical constant – the $M(z)$ compression modulus – can vary linearly with depth (Fig. 1). In the special case of a single layer with constant modulus, the classical Boussinesq halfspace applies. The compression modulus of the k-layer is calculated from the following formula:

$$M_k(z) = M_{k0} + m_k \cdot (z - \sum_{i=1}^{k-1} h_i), \qquad 0 \leq z \leq H, \tag{2}$$

where M_{k0} is the compression modulus at the top of the k-layer, m_k is a factor of modulus change in the k-layer, z is the depth under the foundation plate, h_i is the thickness of the i-layer, H is the limit depth (the soil is considered only down to this limit depth H). The elastic properties of the soil have the following relation:

$$M = E \cdot \frac{1-\mu}{(1+\mu)\cdot(1-2\cdot\mu)}, \tag{3}$$

where E is the elasticity modulus, μ is the Poisson ratio and M is the compression (oedometric) modulus. For each raster point on the raft base, the data of the nearest borehole profile is applied (Fig. 1). The vertical stresses in the soil are obtained by Boussinesq theory [7] (Fig. 2)

$$\sigma = P \cdot \frac{3 \cdot z^3}{2\cdot\pi\cdot R^5} \tag{4}$$

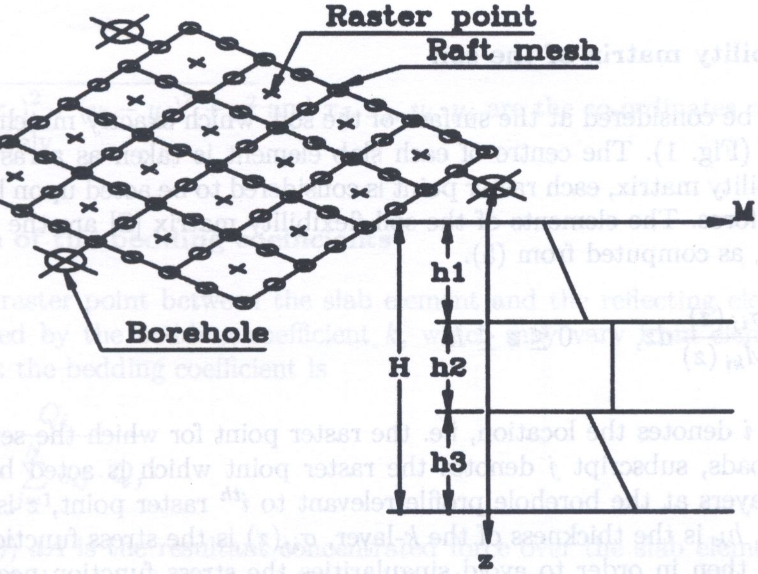

Fig. 1. FE-grid of the raft, raster points on the soil surface, location of boreholes and borehole profile

Although this theory strictly applies to the homogeneous, elastic infinite half-space, it can be reliably used in practice for settlement calculations. In the calculation of settlements, the surface load is assumed to be transferred to the soil via the plate elements resting on the soil with no tensile stresses between slab and soil. In order to obtain the stress (σ) at a point at depth z, the cumulative effect of all plate elements is considered. The settlement is calculated in the usual way from the stresses and from the compression modulus, as:

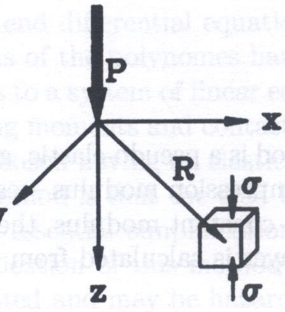

Fig. 2

$$w = \int_H \frac{\sigma(z)}{M(z)} dz \tag{5}$$

where w is the settlement. The value of H is normally determined by national standards. In Hungary, it is taken equal to the depth where the excess soil stress σ due to loading reduces to 20% of the effective overburden pressure, σ_0:

$$\sigma(H) = 0.2 \cdot \sigma_0(H). \tag{6}$$

Alternatively, H can be obtained from the dimensions of the raft, for example, with large plates the limit depth

$$H = 0.5 \cdot B \tag{7}$$

where B is the smaller dimension (width) of the slab. Further formulae for the calculation of the limit depth can be found in [6].

3.2. The flexibility matrix of the soil

An FE-grid will be considered at the surface of the soil, which exactly matches the FE-grid for the foundation slab (Fig. 1). The centre of each slab element is taken as a raster point. In order to obtain the flexibility matrix, each raster point is considered to be acted upon by a unit concentrated (point) vertical force. The elements of the soil flexibility matrix [S] are the deflections pertaining to the unit load, as computed from (5).

$$s_{ij} = \sum_{k=1}^{L} \int_{h_k} \frac{\sigma_{ij}(z)}{M_{ki}(z)} dz, \qquad 0 \le z \le H \tag{8}$$

where subscript i denotes the location, i.e. the raster point for which the settlement is calculated from the unit loads, subscript j denotes the raster point which is acted by the unit load, L is the number of layers at the borehole profile relevant to i^{th} raster point, z is the depth below the foundation level, h_k is the thickness of the k-layer, $\sigma_{ij}(z)$ is the stress function.

When $i = j$, then in order to avoid singularities the stress function needs to be adjusted by dividing the area of the slab element pertaining to the raster point considered into triangles and calculating the stress by integrating the partial effects obtained from (4), (Fig. 3).

In polar co-ordinate system, the partial stress obtained by integration with respect to r over the area of a triangular subelement is

$$\sigma_\Delta(z) = \frac{1}{2\pi A_\Delta} \int_{-\vartheta_1}^{\vartheta_2} 1 - \left(\frac{z \cdot \cos\vartheta}{\sqrt{m^2 + (z \cdot \cos\vartheta)^2}} \right)^3 d\vartheta,$$

$$x_p = t \cdot x_2 + (1-t) \cdot x_1; \quad y_p = t \cdot y_2 + (1-t) \cdot y_1, \tag{9}$$
$$t = \frac{(y_c - y_1) \cdot (y_2 - y_1) + (x_c - x_1) \cdot (x_2 - x_1)}{(y_2 - y_1)^2 + (x_2 - x_1)^2}.$$

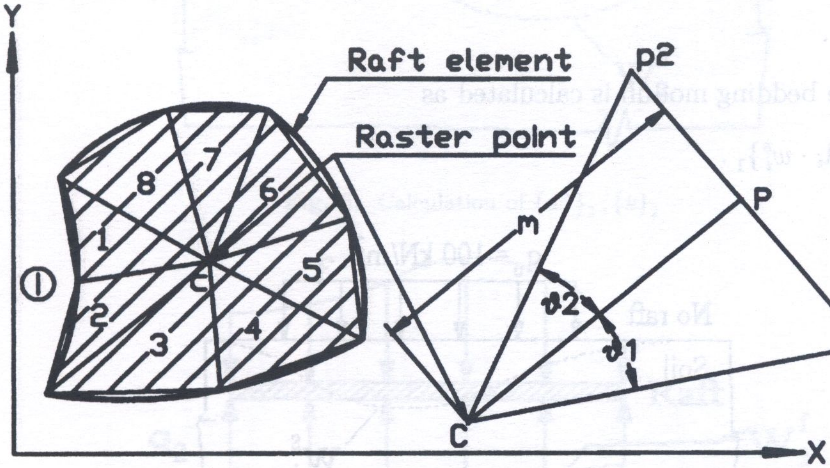

Fig. 3

The angles ϑ_1 and ϑ_2 are to be interpreted according to the sign convention assumed (i.e. if $t < 0$, then $\vartheta_1 < 0$, and for $t > 1$, $\vartheta_2 < 0$), A_Δ is the area of a triangular. The integration with respect to ϑ is carried out numerically, and the overall value of the stress σ_{ii} is calculated by summing up the partial values obtained for the triangles.

When $i \neq j$ then the stress function is

$$\sigma(z)_{ij} = \frac{3 \cdot z^3}{2 \cdot \pi \cdot R_{ij}^5} \tag{10}$$

where $R_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + z^2}$ and x_i, x_j, y_i, y_j are the co-ordinates of the i^{th} and j^{th} raster points, respectively.

3.3. Determination of the bedding coefficients

The interaction at a raster point between the slab element and the reflecting element of the soil surface is characterised by the bedding coefficient k, which may vary from element to element. Under the i^{th} element the bedding coefficient is

$$k_i = \frac{Q_i}{A_i \cdot w_i^s} = \frac{Q_i}{A_i \cdot \sum_{j=1}^{N} s_{ij} \cdot Q_j} \tag{11}$$

where $Q_i = \int_{A_i} q(x,y)\, dA$ is the resultant concentrated force over the slab element at the raster point; A_i is the area of the slab element, w_i^s is the deflection in the raster point, N is the number of raster points.

4. THE ITERATION ALGORITHM

The algorithm for the soil-raft interaction can be described in four steps. The theoretical basis of the method, which is very simple, is shown by the next four figures. The figures represent a vertical cross-section of a raft on a soil continuum.

Step 1.
This step is made only once, and the purpose of it is to define the initial values of the bedding moduli for the raft elements. The area covered by the raft has to load by $q_0 = 100\ \mathrm{kN/m^2}$ as a uniformly distributed surface load (Fig. 4). So, the nodal load vector at the raster points on the soil surface is $\{q_i\}_0 = \{100 \cdot A_i\}$ and the surface settlements $\{w^s\}_1$ can be obtained from

$$\{w_i^s\}_1 = [S]\,\{q_i\}_0\,. \tag{12}$$

The initial set of the bedding moduli is calculated as

$$\{k_i\}_1 = \{q_i\}_0 \,/\, \{A_i \cdot w_i^s\}_1\,. \tag{13}$$

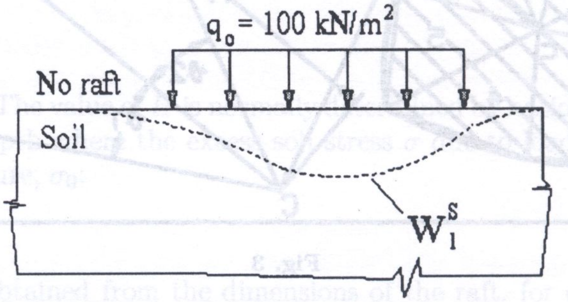

Fig. 4. Define the initial values of $\{w^s\}_1, \{k\}_1$

Step 2.
First, using the $\{k_i\}_1$ bedding moduli, the $[\mathbf{K}]_1$ stiffness matrix of the structure has to be determined. Then the entire structural loading $\{f_i\}$ is applied to the raft (Fig. 5) and by solving the equation system

$$[\mathbf{K}]_1 \cdot \left\{w_i^f\right\}_1 = \{f_i\} \tag{14}$$

the soil surface stresses (or distributed reactions) $\{q_i\}_1$ are obtained by the conventional bedded plate analysis.

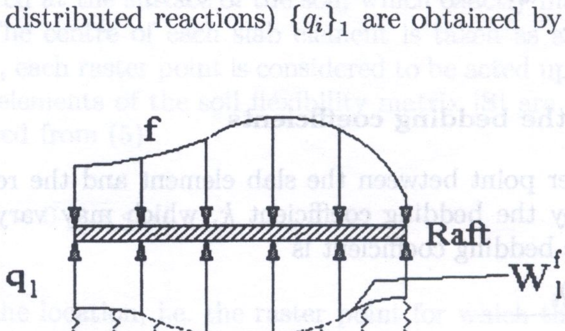

Fig. 5. Calculate the $\left\{w^f\right\}_1, \{q\}_1$

Step 3.
In the third step the soil is loaded by the soil stresses calculated directly from the real raft loads $-\{q_i\}_1$. The result is a new set of $\{w_i^s\}_2, \{k_i\}_2$ values (Fig. 6).
Step 4.
New $[K]_2$ structural stiffness matrix is determined taking into account the latest set of bedding moduli $\{k_i\}_2$, then with the original loads $\{f_i\}$ a new $\left\{w_i^f\right\}_2, \{q_i\}_2$ can be calculated (Fig. 7).

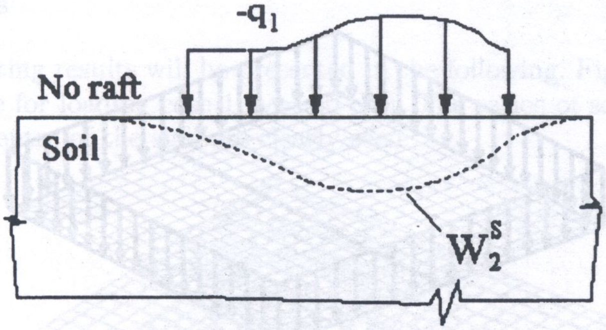

Fig. 6. Calculation of $\{w^s\}_2, \{k\}_2$

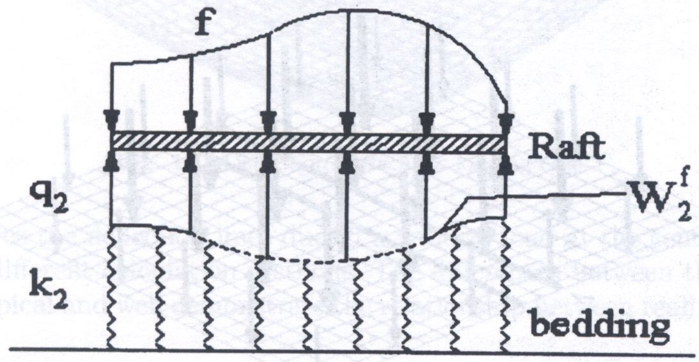

Fig. 7. Calculation of $\{w^f\}_2, \{q\}_2$

The iteration process is continued until either the prescribed level of convergence or the prescribed maximum number of the iteration steps is attained. If the greatest difference between bedding moduli in step r and step $r+1$ is less than the prescribed value, the convergence criteria is satisfied. The error of the r^{th} step is calculated from the following formula:

$$\varepsilon_r = \frac{|\{k_i\}_r - \{k_i\}_{r-1}|_{\max}}{(\{k_i\}_r)_{\max}}, \qquad (r \geq 2) \tag{15}$$

5. Numerical examples

In addition to the testing of the new program module, a comparison with the results obtained by the Winkler bedding principle was made for 4 loading cases which correspond to actual problems.

5.1. Comparative test cases

In all four load cases the same raft with an area of 32 m×32 m was considered. The slab thickness was assumed to vary from 20 to 40 to 60 cm in each case. For the concrete, the modulus of elasticity $E_c = 30000$ MPa, and the Poisson ratio $\mu_c = 0.2$ was taken into consideration. One soil layer with a constant compression modulus of $M = 200$ MPa, and with a limit depth of $H = 16$ m was assumed. By a comparison of average settlements, an equivalent bedding coefficient of $k = 21.4$ MPa/m was obtained. In case 1 (Fig. 8) the uniform surcharge load on the raft was 150 kN/m^2. In case 2 (Fig. 9), the same total load was transferred to the raft via columns arranged in a 6 m×6 m raster. In case 3 (Fig. 10) one part of the raft area was loaded by 50 kN/m^2. This case corresponds to a high-rise building with a larger protruding underground garage area. In case 4 (Fig. 11) again a set of point loads was considered.

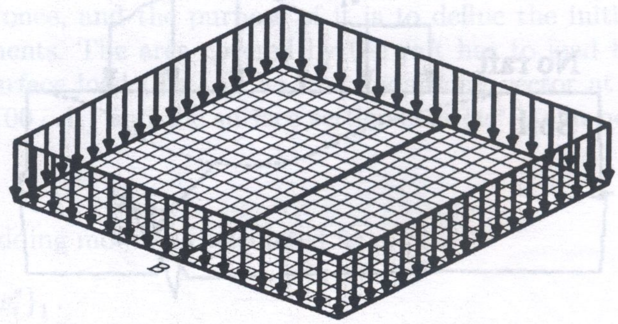

Fig. 8. Loading case 1

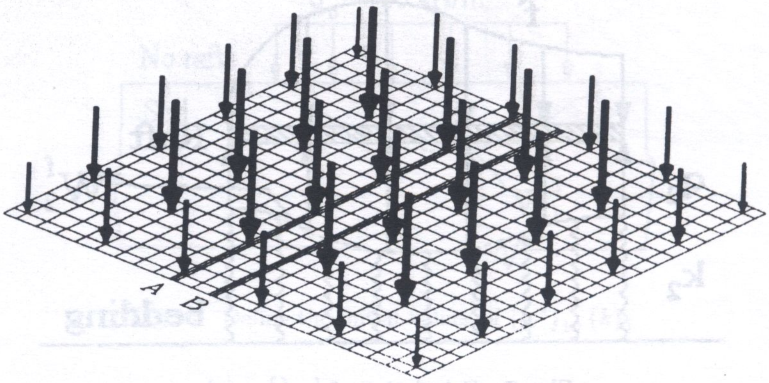

Fig. 9. Loading case 2

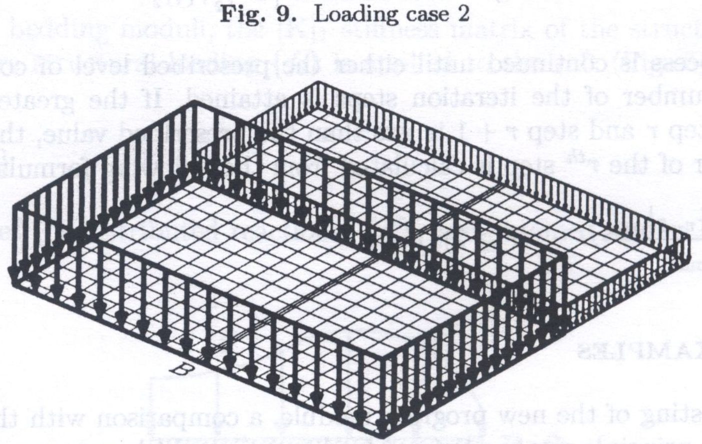

Fig. 10. Loading case 3

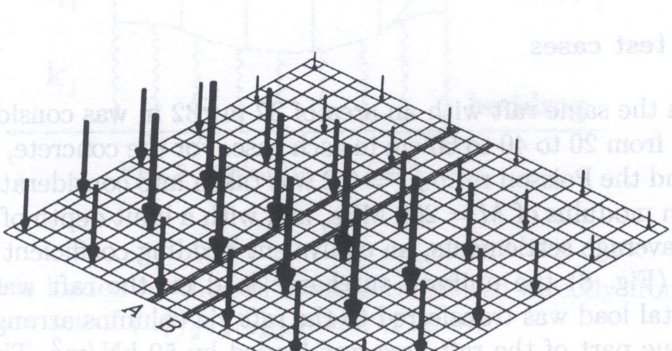

Fig. 11. Loading case 4

5.2. Important results

Only some of the interesting results will be presented in the following. Figure 12 shows the characteristic deflection surface for loading case 1 ($v = 60$ cm). The ratios of settlements at the corners, midpoints of sides and centre of the slab are clearly seen.

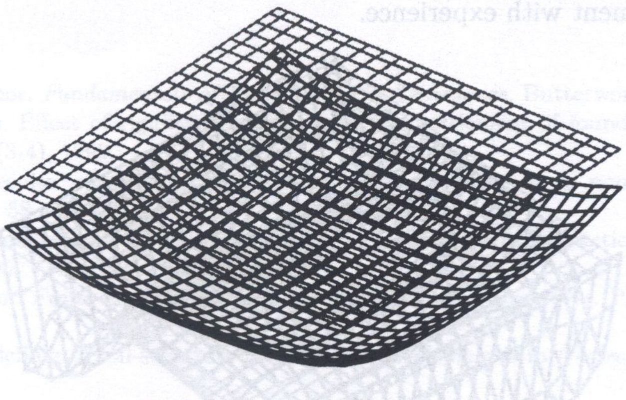

Fig. 12. Deflection surface for loading case 1

Figure 13 compares the deflection lines due to a uniform load at the central section B (Figs. 8, 10) as obtained by different calculation methods. The differences between the Winkler-model and the soil-model are typical and well demonstrate the relationship between reality and the two models.

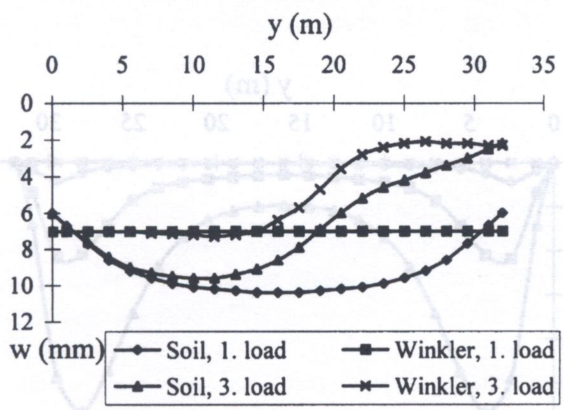

Fig. 13. Deflections at section B, under uniform load

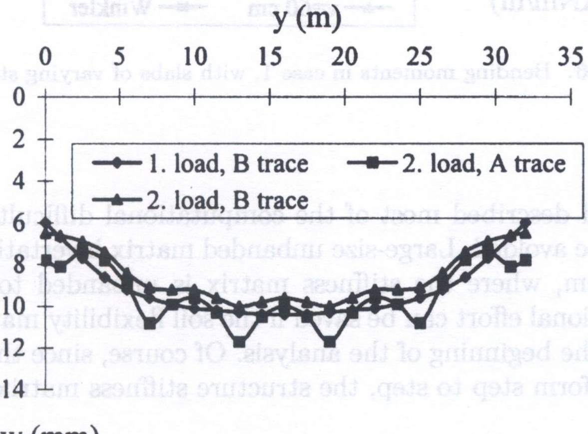

Fig. 14. Typical deflection curves for loading cases 1 and 2

Figure 14 shows a comparison of deflections under a uniform load (Fig. 8) with those due to point loads acting on the slab (Fig. 9). The settlements due to uniform load appear to be the mean value of the settlements at the "column strip" (section A) and the "mid-field strip" (section B).

Figure 15 shows a typical distribution diagram of bending moments in the longitudinal direction, under a uniform load. Finally, Fig. 16 shows bending moments in slabs of different stiffness, their ratios being good agreement with experience.

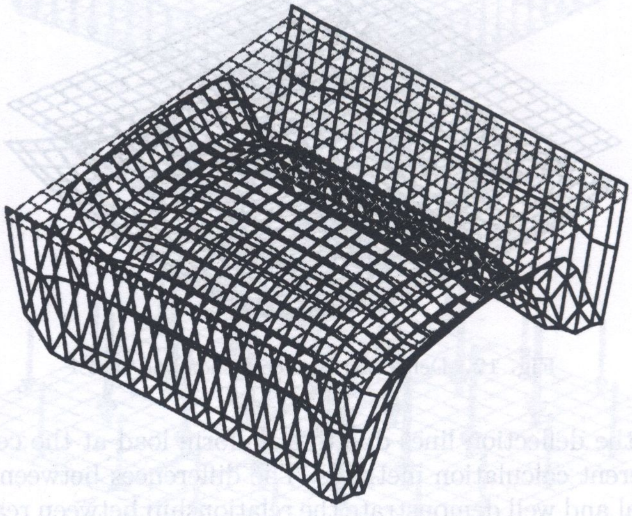

Fig. 15. Bending moments diagram for case 1, $v = 60$ cm

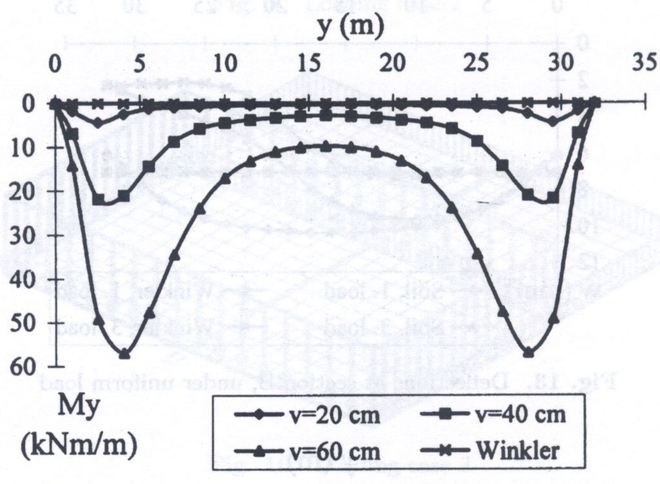

Fig. 16. Bending moments in case 1, with slabs of varying stiffness

6. Conclusions

By the use of the method described most of the computational difficulties that occur with other interactive methods can be avoided. Large-size unbanded matrix invertation, as well as the solution of a large equation system, where the stiffness matrix is unbanded too is not necessary. More generally, much computational effort can be saved if the soil flexibility matrix needs to be computed and stored only once, at the beginning of the analysis. Of course, since the bedding modulus of the support elements change form step to step, the structure stiffness matrix has to be recomputed at every step.

The modified program now enables raft foundation design to achieve the same level of reliability from geotechnical point of view as it was attained in the design of structures long time ago.

ACKNOWLEDGEMENTS

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