Computer Assisted Mechanics and Engineering Sciences, 6: 39–62, 1999.

Interactive methodology for reliability-based structural design and optimization

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(Received March 3, 1998)

Recent advances in reliability methods, optimization as well as design sensitivity analysis have resulted in development of computational systems supporting RBDO processes for medium/large structures. For RBDO the efficiency problems are critical and in order to get the optimum design a number of fast approximate methods have been recently proposed. These methods, tested for rather small problems, show acceptable accuracy and speed up computations considerably. However, when applied in the automated way to medium/large scale problems they may cause severe convergence problems or lead to a poor local minimum after expensive computations.

Instead of an automated optimization procedure, an interactive approach is proposed. Implemented in the POLSAP-RBO system it allows to combine effective interactive design methods with visual capabilities to efficiently generate optimum design. Benchmark studies of an offshore jacket structure show efficiency of the interactive approach which employs integration, approximation and reduction techniques for maximizing efficiency of RBDO.

1. Introduction

Design optimization has undergone recently a substantial progress. The techniques currently available have matured to the point that optimization methods are being added to many existing commercial finite element codes. However, most of these developments deal only with deterministic parameters. For the rational design it is crucial to account for uncertain properties of material, loading and geometry as well as the mathematical model of the system. Moreover, reliability performances should be introduced as the most rational safety measures. Deterministic optimization enhanced by reliability performances and formulated within the probabilistic framework is called Reliability-Based structural Design and Optimization (RBDO).

RBDO should be considered as an important ingredient in the design of advanced structural systems [17, 21, 23, 10]. However, wider applications still exhibit severe limitations related mostly to low computational efficiency. Sources of these problems may mainly be attributed to deeply nested architecture of RBDO procedures involving structural analysis (FEA), design sensitivity analysis (DSA), reliability, reliability sensitivity analysis and optimization. Practically, the RBDO process can be considered as two-level optimization process with underlying computational-intensive tasks of DSA and FEA. For large structural problems the computational time can be prohibitively high.

Efficient RBDO system supporting design of large structural systems must adopt the fastest and most accurate algorithms for the above mentioned procedures. In optimization, algorithms based upon sequential quadratic programming (SQP) are considered the most efficient and reliable [20]. In the reliability analysis, the first or second order reliability methods (FORM/SORM) should be used provided that sensitivity information is available [2]. Otherwise, the advanced simulation methods combined with the so called response surface method are considered as the most efficient [16]. Structural optimization and the FORM/SORM algorithms require gradients of structural performances with respect to design variables. Since purely analytical methods cannot be generally employed, computer-based design sensitivity methods must be adopted. To assure the generality and efficiency there are two basic sensitivity methodologies for implementation in a RBDO code, referred to as the continuum and discrete methods of design sensitivity analysis [9, 11].

Combining the most efficient methods, the first integrated computational systems for RBDO have emerged [18, 4]. However, applications of RBDO to large systems, characterized by a large number of stochastic variables and failure functions are still not straightforward, the typical problems being those of a relatively low computational efficiency and weak convergence of the optimization process. Carrying out RBDO in a fully automated way quite often leads to a failure due to errors in the problem definition, or to a poor local minimum after expensive computation. In a fully automated optimization procedure a significant part of information may be wasted since at each optimization step the reliability, design sensitivity and structural analyses must be repeated. The existing information, computed at the earlier steps, is not used any more at the next steps. Therefore, instead of automating the optimization process, an interactive approach appears potentially more useful for effective large-scale RBDO. Interactive design environment should allow to combine effective interactive design methods with both visual capabilities to efficiently lead to optimum design, and approximation and reduction optimization techniques to maximize computational efficiency.

According to these requirements, an integrated computational system POLSAP-RBO is being developed to allow carrying out interactive RBDO processes. In the present version only 3D truss structures are considered.

In this paper, the efficiency, robustness, and reliability of the interactive methodology is compared with automated strategies for solving RBDO of large structural systems using accurate and approximate methods. A benchmark example of a reliability-based sizing and configuration optimization of an offshore jacket is presented.

2. Reliability-based structural design and optimization

2.1. Parameters in RBDO

Size, shape, material and loading parameters defining a structural model can be considered either deterministic or stochastic. They are denoted by x and X, respectively. The lowercase x will also be used to denote a certain realization of the stochastic parameter X.

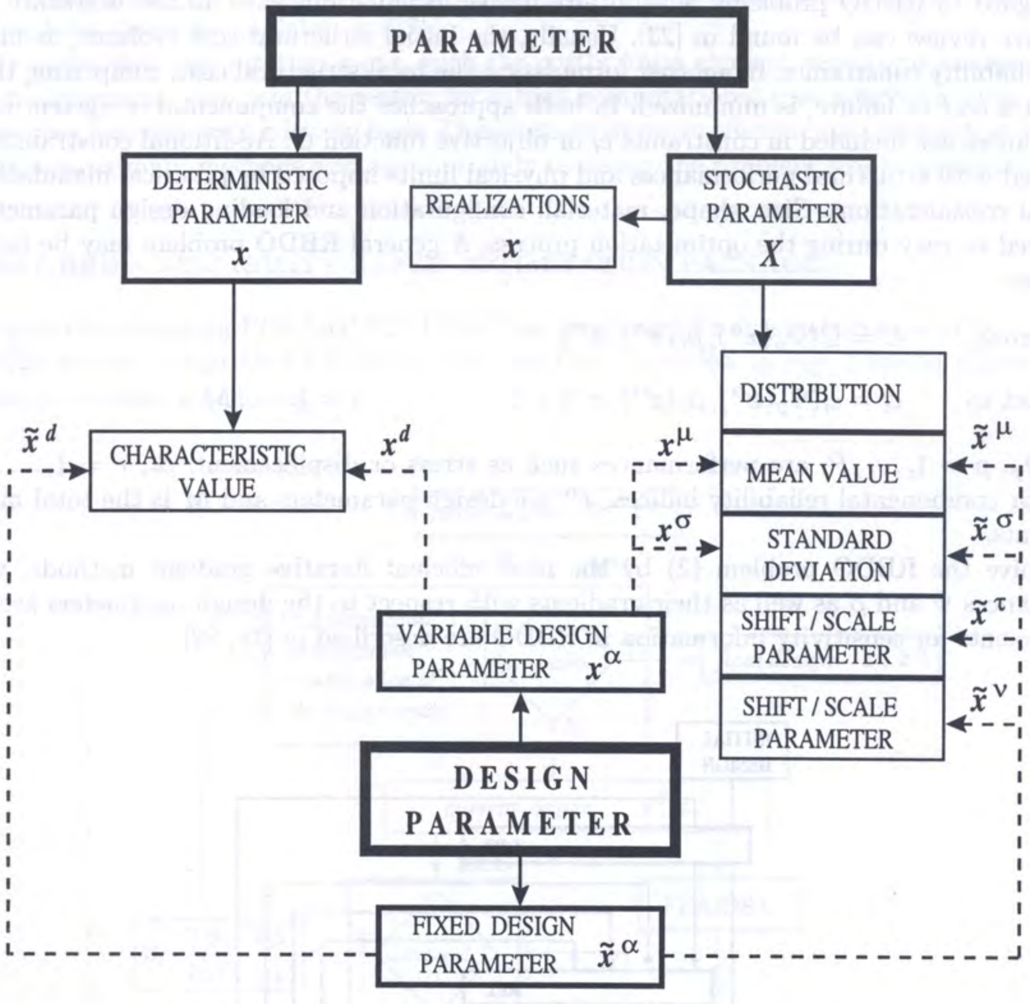
Design parameters, i.e. parameters which are used in the optimization process can be associated with deterministic parameters x^d or descriptors (moments) of distributions of stochastic parameters such as mean value and standard deviation, x^μ, x^σ, respectively. If they are allowed to vary during the optimization process they are called the variable design parameters. If they are kept constant during the optimization process they are called the fixed design parameters and are marked by a tilde sign, $\tilde{x}^\alpha$, see Fig. 1.

It is reasonable to consider that only the mean value and/or the standard deviation of a stochastic parameter may be defined as variable design parameters, other distribution descriptors being assumed fixed, as illustrated in Fig. 1.

2.2. Formulation of RBDO

It is fairly typical in civil and mechanical engineering that some quantities which describe a structural system and applied loads should be modelled as stochastic parameters. They define a space $\mathcal{X}$ of stochastic parameters. Then, a potential element failure mode is described by a failure function $g(\boldsymbol{x}) = 0$, dividing the space $\mathcal{X}$ into a failure domain, where $g(\boldsymbol{x}) \leq 0$, and a safe domain, where $g(\boldsymbol{x}) > 0$. In FORM/SORM, the basic uncertain variables X are transformed to a space of standard independent and normally distributed variables U (with realizations denoted by $\boldsymbol{u}$) [13, 6].

In the FORM approach the failure function $g(\boldsymbol{x})$, transformed to the standard space, is expanded up to the first order at a point $\boldsymbol{u}^*$ defined as the point on $g(\boldsymbol{u}) = 0$ at the minimum distance from



Rys. 1. Deterministic, stochastic and design parameters in RBDO

x^d, $\tilde{x}^d$ – variable and fixed design parameter, respectively, associated with a deterministic parameter

x^μ, $\tilde{x}^\mu$ – variable and fixed design parameter, respectively, associated with the mean value of a stochastic parameter

x^σ, $\tilde{x}^\sigma$ – variable and fixed design parameter, respectively, associated with the standard deviation of a stochastic parameter

$\tilde{x}^\tau$, $\tilde{x}^\nu$ – fixed design parameters, associated with the shift/scale parameters of distribution of stochastic variable

the origin of the coordinate system. The reliability index β is equal to this distance and, therefore, is evaluated by solving the minimization problem [13]

$$\beta \equiv \|\boldsymbol{u}^*\| = \min\|\boldsymbol{u}\| \qquad \text{subject to} \quad g(\boldsymbol{u}) = 0 \tag{1}$$

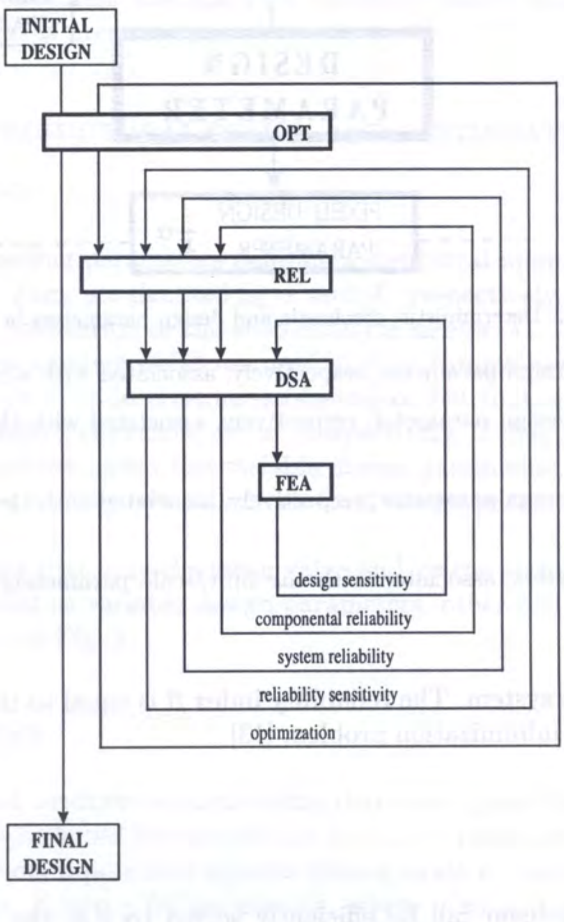
The above optimization problem can be efficiently solved by e.g. the Rackwitz-Fiessler method combined with SQP [3]. Resulting iterative algorithm is applicable to both small and large scale reliability problems. Another possibility is offered by Fast Probability Integration procedure incorporated in the probabilistic structural analysis program NESSUS [15]. It computes distribution functions of structural response using advanced mean-value first-order methods.

In regard to RBDO problems, several alternative formulations exist in the literature. A comprehensive review can be found in [23]. Usually, the initial structural cost (volume) is minimized under reliability constraints. In another formulation the total structural cost, comprising the initial cost and a cost of failure, is minimized. In both approaches the componental or system reliability performances are included in constraints c_i or objective function C. Additional constraints may be associated with structural performances and physical limits imposed by practical manufacturing or technical considerations. Size, shape, material, configuration and loading design parameters may be allowed to vary during the optimization process. A general RBDO problem may be formulated as follows:

$$\begin{aligned} &\text{minimize} && C = C(\Psi_p(\boldsymbol{x}^\alpha), \beta_r(\boldsymbol{x}^\alpha), \boldsymbol{x}^\alpha) \\ &\text{subject to} && c_i = c_i(\Psi_p(\boldsymbol{x}^\alpha), \beta_r(\boldsymbol{x}^\alpha), \boldsymbol{x}^\alpha) \geq 0 \qquad i = 1, \ldots, M \end{aligned} \tag{2}$$

where Ψ_p, $p = 1, \ldots, P$, are performances such as stress or displacement, β_r, $r = 1, \ldots, m_r$, are system or componental reliability indices, $\boldsymbol{x}^\alpha$ are design parameters and M is the total number of constraints.

To solve the RBDO problem (2) by the most efficient iterative gradient methods, values of performances Ψ and β as well as their gradients with respect to the design parameters are needed. Requirements for sensitivity information in RBDO are described in [19, 22].



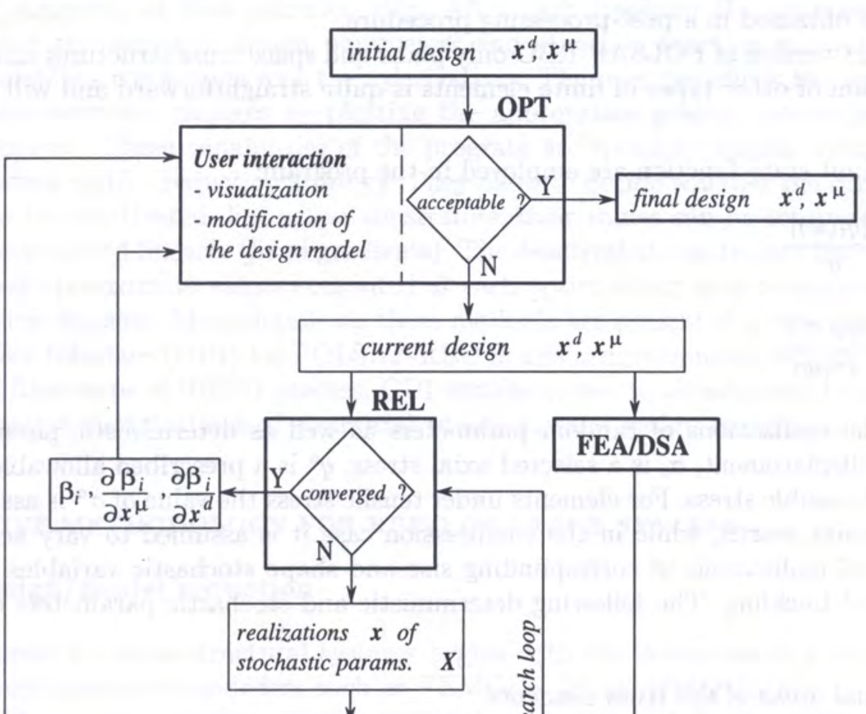
Rys. 2. RBDO flow chart

A characteristic feature of RBDO are nested iteration loops as shown in Fig. 2 presenting a typical flow chart. The outer loop is associated with optimization, the intermediate one with reliability analysis, and the inner loop with design sensitivity analysis. Data flow for this complex system has been discussed in detail in [19].

Deeply embedded computation loops, with the costly finite element structural analysis performed in the innermost loop, are the reason for a high computational cost which for large realistic problems often becomes prohibitively high. Therefore, to improve efficiency, an interactive approach exploiting approximate methods and appropriately reducing the problem size is necessary.

3. POLSAP-RBO – RELIABILITY-BASED OPTIMIZATION PACKAGE

The computational system POLSAP-RBO has been developed for interactive as well as automatic RBDO. The system comprises FEA/DSA, REL and OPT modules. In Fig. 3 the data flow and the main system modules are shown.



Rys. 3. Data flow chart in POLSAP-RBO

The OPT module uses sequential quadratic programming algorithms. Values of reliability indices and their gradients are calculated by COMREL – program for time-invariant component reliability analysis [5], using FORM, SORM or Mean Value First Order (MVFO) methods.

To calculate the structural response and its sensitivities with respect to all design parameters and also to calculate the value of objective function and its gradient, the FEM/DSA package – POLSAP-DSA is used. System sensitivity information is computed by the DSA module using the highly efficient discrete (analytical) method of the design sensitivity analysis [11]. The design sensitivity expressions are obtained by taking analytical derivatives of the finite element matrix equation with respect to design variables. This method yields the exact design sensitivity of an approximate finite element model.

Using the discrete design sensitivity method two approaches are available: direct and adjoint variable approach. To get the design sensitivity information for K performances, L load cases and N design variables by the adjoint variable method, L equilibrium systems, and additional K matrix equations for adjoint loads must be solved. In the direct differentiation approach, L equilibrium systems and $N \times L$ equations for displacement derivatives with respect to each design parameter must be solved. For $N \times L < K$ the direct differentiation method and for $N \times L > K$ the adjoint variable method is preferred. Therefore, before starting the RBDO procedure, the user decides on whether the adjoint or direct approach is more appropriate for the problem on hand. Both the approaches are quite efficient since the additional equations are solved with the same stiffness matrix as used for the original equilibrium solution. Therefore, having inverted the stiffness matrix, sensitivities are obtained in a post-processing procedure.

In the current version of POLSAP-RBO only plane and space truss structures can be optimized, but incorporation of other types of finite elements is quite straightforward and will be undertaken in the future.

Two types of limit state function are employed in the program:

$$g(\boldsymbol{x}) = 1 - \frac{|q_i(\boldsymbol{x})|}{q^a} \tag{3}$$

$$g(\boldsymbol{x}) = 1 - \frac{|\sigma_i(\boldsymbol{x})|}{\sigma^a(\boldsymbol{x})} \tag{4}$$

where $\boldsymbol{x}$ are the realizations of random parameters as well as deterministic parameters, q_i is a selected nodal displacement, σ_i is a selected axial stress, q^a is a prescribed allowable displacement and σ^a is an admissible stress. For elements under tensile stress the value of σ^a is assumed constant during the β-point search, while in the compression case it is assumed to vary according to the current values of realizations of corresponding size and shape stochastic variables; this allows to account for local buckling. The following deterministic and stochastic parameters can be used in the analysis:

- cross-sectional areas of the truss elements
- Young modulus of the individual elements
- nodal coordinates
- load case multipliers

Design parameters are defined as the mean values of the stochastic parameters $\boldsymbol{x}^\mu$ and deterministic parameters $\boldsymbol{x}^d$. A typical RBDO problem is formulated as follows:

$$\begin{array}{lll} \text{minimize} & C(\boldsymbol{x}^\alpha) & \alpha = \{d, \mu\} \\ \text{subject to} & \beta_i(\boldsymbol{x}^\alpha) \geq \beta_i^{\min} & i = 1, \ldots, m_r \\ & c_i(\boldsymbol{x}^\alpha) \geq 0 & i = 1, \ldots, m_d \\ & {}^l x_i^\alpha \leq x_i^\alpha \leq {}^u x_i^\alpha & i = 1, \ldots, N \end{array} \tag{5}$$

where c_i, $i = 1, \ldots, m_d$ are deterministic constraints, m_r is the number of reliability constraints, N is the number of design variables and ${}^l x_i^\alpha$, ${}^u x_i^\alpha$ are the lower and upper bounds, respectively, imposed on design variables.

Design variables of the same type can be linked into groups; a linear relationship between them may be established as well.

POLSAP-RBO offers also several ways of interactive control over the optimization process. The following interactive capabilities are under development:

- selection of proper algorithms, control parameters and solution strategies
- interrupting the optimization procedure, restarting from any iteration step
- adding/removing constraints, applying active/semi-active constraint strategy or linear approximations
- modifying status of variable/fixed design parameters, and stochastic/deterministic parameters

The user can control the accuracy of the optimization and β-point calculation by specifying a convergence parameter at each iteration step. After each iteration the program provides the information about the values of design parameters and objective function as well as the actual values of the reliability constraints and their derivatives. The user can check the trends of design parameters and constraints changes by plotting the appropriate graphs, sensitivity information can be also displayed. These capabilities of the program are specially helpful when choosing the right approximation and/or reduction strategy. They allow to decide whether the constraint should remain active or be deactivated. For active constraints, their values can be computed by accurate methods or approximated linearly (fixed gradients). The deactivated constraints may by completely neglected or their approximate values computed at each optimization step to examine correctness of the deactivation decision. More details on these methods are presented in the next section.

Graphical User Interface (GUI) for POLSAP-RBO is under development, supporting the modeling, initial and final steps of RBDO process. GUI enables access to all integrated applications and allows for interactive modifications of structural, stochastic and design models.

4. Interactive methodology for RBDO of large systems

4.1. Initial design/model reduction

The RBDO process for large-structural systems begins with the definition of a structural model. For this, advanced graphical modellers such as FEMGEN [8] or ABAQUS/Pre [1] may be used. All parameters describing geometry, layout, material properties and loading must be linked into groups, each represented by a selected deterministic or stochastic parameter. For stochastic parameters additional information on their distribution, moments and correlation should be provided. To complete parametric model of the structural problem, the design parameters are assigned to deterministic parameters or moments of stochastic parameters according to the scheme of Fig. 1. Variable or fixed type of design parameters must be specified.

To simulate the structural behaviour and define critical performance measures the initial finite element analysis should be carried out. Performances can be local such as stresses or displacements associated with critical points or elements as well as global such as the total weight or the eigenfrequency defining the overall structural behaviour.

Since the initial structural analysis has indicated critical performances, the information can be used for the definition of failure functions. To each failure function there corresponds a reliability index. Finally, the reliability indices as well as some additional performances and design parameters are used to define objective function and constraints. This completes the model definition for RBDO.

The initial step in the RBDO process aims at acquiring knowledge on the structural behaviour and trends of the performances. Since a lower number of parameters and constraints may result in a more stable and quickly convergent optimization process it is important to employ size reduction procedures to both the stochastic and design models.

Results of the initial reliability analysis may indicate some stochastic parameters as candidates for the reduction. A valuable information is given by the directional cosines of the $\boldsymbol{u}^*$ point showing the importance of particular stochastic variables for the β–value. The stochastic parameters corresponding to relatively low cosines values may be reduced to deterministic parameters [12].

If the normalized sensitivity of the objective function and all the constraint functions with respect to a variable design parameter are relatively low, this parameter can be redefined as a fixed design parameter. Since the sensitivities may differ significantly when the design is changed, the validity of such a reduction should be checked after several optimization steps or at the final RBDO step. Impact of the reduction of a number of stochastic and variable design parameters as well as performances depends on the DSA method employed. Using the adjoint approach the number of parameters has a very small influence on efficiency. The gradients for a large number of parameters are computed roughly at the same computational cost as for few parameters. What really affects the cost is the number of performances. Therefore, using the adjoint approach (as in this paper), one should consider the reduction of constraints rather than parameters. The opposite situation is for the direct differentiation DSA approach. Here, the number of performances has no significant influence on the computational cost.

Ill-conditioning of optimization problem is another important aspect. If possible, constraints as well as variable design parameters should be normalized to have values in the same order of magnitude.

Due to a high cost of RBDO, an initial crude deterministic optimization may be performed aimed at approaching a vicinity of the optimum. At that stage approximate algorithms should rather be used. For such an optimization process design values $\boldsymbol{x}^*$ of the stochastic parameters should be interpreted as follows

$$x_i^* = \gamma(x_i^\mu \pm \eta x_i^\sigma) \qquad\qquad i = 1, \ldots, N \tag{6}$$

where x_i^μ and x_i^σ are the mean value and standard deviation, η is a factor defining the characteristic value (usually $\eta = 2.7 \div 3.0$), and γ is the partial safety coefficient (can be assumed as $\gamma = 1.0 \div 1.2$).

4.2. Strategies for solving RBDO

As it was discussed in section 2, the solution of RBDO requires a large amount of computational time. To increase efficiency and avoid problems with convergence of the optimization process the interactive design should rather be carried out. Several strategies may be proposed. Most of them may also be applied in the framework of automated RBDO procedures; however, better results are typically obtained when they are used in the interactive RBDO.

Strategies discussed in this paper can be divided into integration, approximation and reduction approaches. Integration strategies aim at time savings through a closer integration of embedded procedures of RBDO; approximation strategies, instead of using accurate but costly values, use the first order predictions or crude, not fully iterated values of performances; reduction strategies try to reduce the design model through the elimination at each step of non significant constraints and design parameters. All these strategies are compared in a benchmark study with the standard RBDO approach.

4.2.1. Standard optimization

The standard RBDO procedure is obtained by the formal integration of optimization algorithm with the reliability analysis and design sensitivity analysis codes. No special methods speeding up the computation process are used. This strategy is commonly used in the automated RBDO. It was used in some first RBDO systems such as CARBOS [4]. Critical for its efficiency is the choice of a DSA algorithm. The worst choice is the finite differences method. It is a very popular method due to its simplicity but the serious shortcoming of the method is the uncertainty as to the choice of a perturbation step size of the design variables as well as a low computational efficiency. It requires one original solution followed by at least one perturbed solution for each design variable. For realistic, large-scale problems the computational cost may then become prohibitively high.

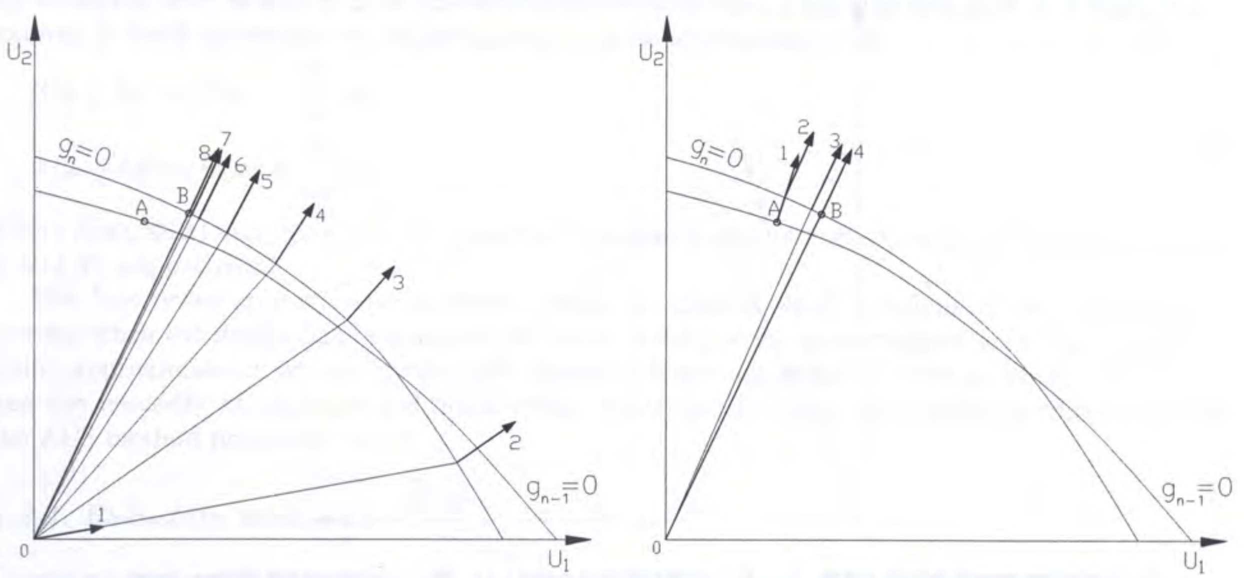
The best DSA algorithms are considered to be the continuum and the discrete methods [11]. The latter is used in this paper. The standard approach combining SQP method for optimization, FORM and discrete DSA, will also be used in the benchmark studies and referred to as STD.

4.2.2. Integration strategies

Considering the deeply embedded architecture of RBDO, significant time savings may result from closer integration of computational loops. From all FEA, DSA, REL and OPT modules, necessary for RBDO, only FEA and DSA are usually fully integrated and provided in commercial packages.

REL and OPT loops can be considered as a two-level optimization procedure. A closer integration of these procedures may result in the considerable improvement of efficiency [7]. Further research in this direction is needed.

A minimum integration requires that information computed at previous iterations is not wasted. In particular, it applies to the computation of the β-values starting always from the mean values of stochastic variables. A straightforward improvement of the standard approach (STD) is to use as starting points the design points $\boldsymbol{x}^*$ taken from the last optimization iteration. This idea is schematically shown in Fig. 4 for two random variables U_1 and U_2. Limit state functions $g_n(\boldsymbol{u}) = 0$ and $g_{n-1}(\boldsymbol{u}) = 0$ correspond to reliability constraint at the current and previous iteration, respectively. Point A is the previous $\boldsymbol{u}^*$ point while B is the current one. The arrows symbolize gradients of limit state functions while the straigh lines represent the line search directions. It can be seen (Fig. 4) that the choice of A, instead of 0, as the starting point may substantially reduce the number of β-point search iterations. Obviously, the number of iterations will be lower if points A and B are close to each other, which is the case when the design changes in subsequent iterations are small.



Starting point – mean values (point 0)
8 gradient calls
7 line search

Starting point – last $\boldsymbol{x}^*$ point (point A)
4 gradient calls
3 line search

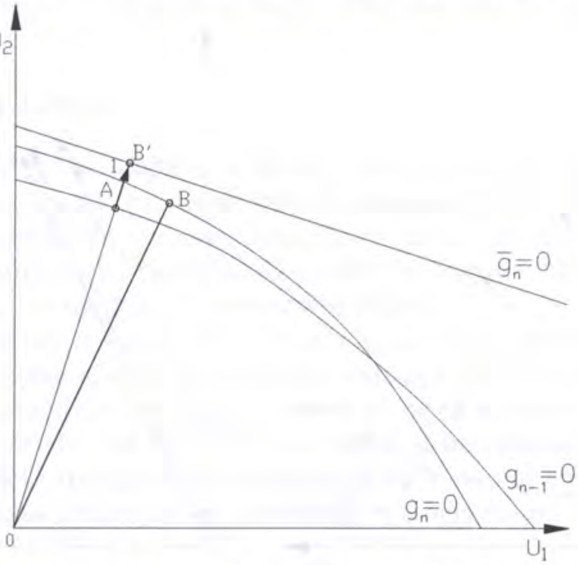
Rys. 4. Using the last $\boldsymbol{x}^*$ point as a starting point in β computation algorithm

The results of using this technique in benchmark studies are shown in the Table 2, column ROI (REL/OPT Integration). It is seen that the computation cost was reduced significantly (929 function calls compared with 1529 in the standard approach) which may lead to the conclusion that the STD approach should never be used. For the first RBDO iteration, the predicted design point x^*, defined according to (6) may be used as the starting point instead of the mean values. If the initial, crude deterministic optimization was performed the resulting optimum design would be the best choice for a starting point for reliability evaluations.

Another possibility of improving the efficiency of RBDO is a closer integration of REL and DSA algorithms. It would be adventageous to solve a system reliability problem or a number of componental reliability analysis problems at a single design sensitivity analysis call. It implies that in terms of the computational efficiency the reliability performance becomes equal to deterministic one. It would allow to eliminate a considerable number of DSA/FEA calls; however, all reliability problems need to have the same starting solution. Such possibility is given by MVFO method. However, the MVFO applied in the benchmark studies brought severe problems with the process convergence. After several case studies the method was recognized as not suitable for RBDO of large systems. Nevertheless, the idea of providing the original solution and design gradients in just one call for all reliability performances can be employed applied at the first optimization iteration, when all componental reliability problems start from the same mean-value or predicted x^* design.

4.2.3. Approximation strategies

At initial optimization stages it is enough to use rough estimates for reliability indices. Accuracy of β-value estimations must be increased toward the end of reliability-based optimization. Such procedures were tested by Maglaras and Nikolaidis [14]. The Rackwitz/Fiessler algorithm for evaluation of the β-point is an iterative procedure which updates directional cosines of the most probable failure point. It was originally proposed to use information obtained from only one iteration to produce an approximation of β–point. The approximated β-values are used in the optimization procedure. This method has been shown to be very efficient for small problems.



A – previous exact design point B – exact design point B' – approximate design point

$\bar{g}$ – limit state function linerized at A 1 gradient call 1 limit state function call

Rys. 5. Modified Mean Value approach to reliability problem

A similar approximation method, called the Modified Mean Value (MMV) method and based upon MVFO, was tested in this paper. The main idea behind this approach was to linearize the limit state function not at the mean value and not at the exact x^* point (which requires iterations) but at design point from the last optimizer step (point A in Fig. 5). By employing such a technique, only one function call and one gradient call is required to calculate the value of the reliability constraint. The efficiency of approximate methods is growing when a starting point is selected close to the optimum. This is because the design, and therefore the corresponding limit state surfaces, do not change drastically in the last stages of optimization, which means that the assumptions underlying the approximate methods are valid. Unfortunately, when applying MMV to practical design problems with highly nonlinear limit state functions, the low accuracy of the method may often lead to the lack of convergence. This was the case in the example presented in this paper.

Instead of procedures discussed above a simple approximation method controlling the β–point convergence parameter ϵ can be proposed. For example, in COMREL the convergence criterion is defined as

$$\begin{aligned} & u_i^{k+1} - u_i^k \leq \epsilon \qquad \text{for all stochastic parameters in } U \text{ space and} \\ & |g(\boldsymbol{u}^*)| \leq \epsilon \end{aligned} \tag{7}$$

It is suggested to set the parameter ϵ to a relatively high value for the initial iterations and then reduce it when the optimization process develops. This can be done automatically assuming a priori some descent of the value of the parameter ϵ throughout iterations in the optimization procedure. Better results can be achieved with some user interaction, which makes it possible to better adjust the value of ϵ along with the optimization process development. Results of this approach, called Adaptive Convergence Parameter approach (ACP) are presented in Table 2, column ACP. It is seen that the same results as for standard approach STD (constant ϵ value) are obtained with the number of function and gradient calls smaller by some 15%. Therefore, it is recommended to combine ACP with some other RBDO strategies.

Another type of approximation strategy is the First Order Approximation approach (FOA). First order approximations allow to predict values of performances using their actual values and sensitivity information. Performances at perturbed designs may be obtained at a very low price without carrying out any new finite element or reliability analyses. FOA may be applied at the optimization level as well as at at the reliability analysis level. The reliability indices β and performances Ψ (such as stresses or displacements), can be approximated as

$$\begin{aligned} \beta(\boldsymbol{x}+\delta\boldsymbol{x}) &= \beta(\boldsymbol{x}) + \frac{\partial\beta}{\partial\boldsymbol{x}}\delta\boldsymbol{x} \\ \Psi(\boldsymbol{x}+\delta\boldsymbol{x}) &= \Psi(\boldsymbol{x}) + \frac{\partial\Psi}{\partial\boldsymbol{x}}\delta\boldsymbol{x} \end{aligned} \tag{8}$$

where $\beta(\boldsymbol{x})$, $\Psi(\boldsymbol{x})$ and $\beta(\boldsymbol{x}+\delta\boldsymbol{x})$, $\Psi(\boldsymbol{x}+\delta\boldsymbol{x})$ are the known and perturbed values of the performances β and Ψ, respectively.

The first order approximations should rather be used at final iterations of the optimization process when the design has become already more stable, or for constraints of very low sensitivity. Using approximations too early, when the design changes significantly, leads to wrong predictions and the necessity of applying additional costly iterations. FOA can be combined efficiently with the ACS method presented below.

4.2.4. Reduction strategies

Carrying out the RBDO process with the adjoint DSA approach it is critical to keep the number of constraints as low as possible. The Active Constraint Strategy (ACS), known from the advanced optimization procedures should be employed. If the constraint (with some additional ε-value) is not violated and its value is growing or its sensitivity is very low, it is eliminated from the design problem. Validity of such reduction should be checked after several iterations or at the final iteration of RBDO. Application of ACS depends on general trends of the design and whether the constraint value converges from positive or negative values. Therefore, better results are obtained when the user, equipped with capabilities to observe the trends of design parameters, the objective function and constraints, interactively makes decisions concerning the set of active reliability constraints.

POLSAP-RBO offers some combinations of the ACS and FOA:

- standard ACS which neglects the constraint
- semi-active ACS neglecting the constraint, but controlling its value by FOA (constant gradients) during the optimization steps. If this value drops below a critical value, the constraint should be again included as active into considerations. This strategy was used in the benchmark studies and was referred to as ACS
- FOA which includes the constraint into the optimization problem; however, its value is approximated linearly (section 4.2.3) whereas the gradient is kept constant.

Significant time savings are possible also with the direct DSA approach. In this case it is critical to keep the number of variable design parameters as low as possible. Reduction of variables into fixed design parameters is also natural at final design steps when continuous values of variables should be fixed taking the closest discrete values from a catalogue of available cross-sections or materials.

In RBDO the reduction of stochastic model, discussed in section 4.1, should also be considered. POLSAP-RBO offers the following capabilities of modifying properties of parameters:

- switch of variable/fixed design parameter
- switch of stochastic/deterministic parameter

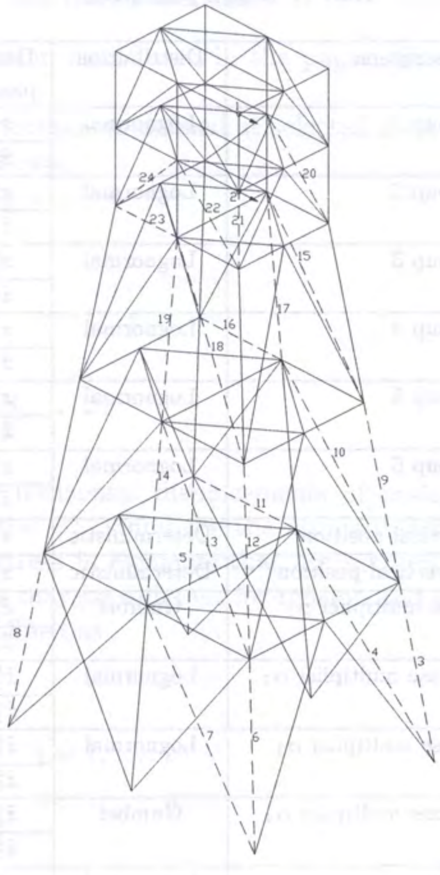
FOA approach combined with semi-active ACS was used in the benchmark studies. Extended studies showed that approximate methods (section 4.2.3), applied too early may cause divergence of the optimum process. Better results are obtained when they are applied after some stabilization of design.

5. Benchmark studies

For benchmark studies of RBDO methods a rather complicated example of the offshore jacket subjected to four load cases has been selected, see Fig. 6. The jacket consists of 139 tubular elements. Sizing and configuration RBDO minimizing structural volume (cost) under 24 componental reliability constraints is considered. Stochastic properties of the system are modelled by 11 stochastic parameters. Linear elastic theory is employed and the structural behaviour is described by selected displacement and stress performance measures.

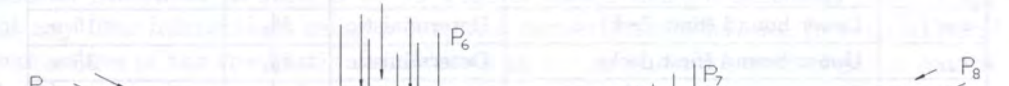
The structural elements are divided into 6 groups. The first group comprises outer braces of the 1st, 2nd and 3rd deck. The second group comprises non-corner legs up to the 3rd deck. The third group comprises vertical diagonals up to the 3rd deck, inner braces of the 1st, 2nd and the 3rd deck and non corner legs starting from the 3rd deck upwards. The fourth group comprises vertical diagonals starting from the 3rd deck upwards and also all elements of the 4th and 5th deck. The fifth group comprises corner legs up to the 3rd deck. Finally the sixth group comprises corner legs starting from the 3rd deck upwards. The jacket is fixed at the 6 bottom nodes. The in-plane dimensions at the fixation and at the top are 22m × 16m and 11m × 8m, respectively. Each vertical section of the structure up to the 3rd deck is 25m high; from the 3rd deck upwards each section is 11m high, resulting in the total height of 97m. To model the effect of a top deck stiffening caused by the top platform, artificially high Young modulus of the material in the top deck elements is used. The Young modulus in the remaining elements is $2.1\,10^8$ kN/m^2.

The external loads acting on the structure are represented by 4 load cases shown schematically in Fig. 7. The first load case is supposed to simulate the waves action and consists of horizontal forces applied to every left hand-side node of all 5 decks along their larger in-plane dimension. The values of the forces are $P_1 = 4500$ kN, $P_3 = 2500$ kN, $P_2 = P_4 = P_5 = 3000$ kN, respectively. The



Rys. 6. Offshore jacket, 1, 2 – displacement constraints, 3-24 – elements with stress constraints

second load case simulates the loading of the top platform. The value of each force within this load case is $P_6 = 5500$ kN. The third load case simulates an extra load acting on the top platform. It is represented by three vertical forces of the magnitude $P_7 = 2000$ kN loading the right edge of the top deck. Finally, the fourth load case stands for a wind load. The values of the forces, acting as shown in Fig. 7, are the same and equal $P_8 = 1000$ kN.



Rys. 7. Load cases

Tab. 1. Design parameters

Param.	Description	Distribution	Design param.	Initial value
X_1	Cross-sec. group 1	Lognormal	x_1^μ	0.113391 m^2
			$\tilde{x}_1^\sigma$	0.0113391 m^2
X_2	Cross-sec. group 2	Lognormal	x_2^μ	0.113391 m^2
			$\tilde{x}_2^\sigma$	0.0113391 m^2
X_3	Cross-sec. group 3	Lognormal	x_3^μ	0.113391 m^2
			$\tilde{x}_3^\sigma$	0.0113391 m^2
X_4	Cross-sec. group 4	Lognormal	x_4^μ	0.113391 m^2
			$\tilde{x}_4^\sigma$	0.0113391 m^2
X_5	Cross-sec. group 5	Lognormal	x_5^μ	0.113391 m^2
			$\tilde{x}_5^\sigma$	0.0113391 m^2
X_6	Cross-sec. group 6	Lognormal	x_6^μ	0.113391 m^2
			$\tilde{x}_6^\sigma$	0.0113391 m^2
x_7	First deck vertical position	Deterministic	x_7^d	25 m
x_8	Second deck vertical position	Deterministic	x_8^d	50 m
X_9	First load case multiplier α_1	Gumbel	$\tilde{x}_9^\mu$	1.0
			$\tilde{x}_9^\sigma$	0.15
X_{10}	Second load case multiplier α_2	Lognormal	$\tilde{x}_{10}^\mu$	1.0
			$\tilde{x}_{10}^\sigma$	0.1
X_{11}	Third load case multiplier α_3	Lognormal	$\tilde{x}_{11}^\mu$	1.0
			$\tilde{x}_{11}^\sigma$	0.2
X_{12}	Fourth load case multiplier α_4	Gumbel	$\tilde{x}_{12}^\mu$	1.0
			$\tilde{x}_{12}^\sigma$	0.2
X_{13}	Young's modulus	Lognormal	$\tilde{x}_{13}^\mu$	2.10 10^8 kN/m^2
			$\tilde{x}_{13}^\sigma$	1.05 10^7 kN/m^2
x_{14}	Admissible disp. of the 5th deck corner node	Deterministic	$\tilde{x}_{14}^d$	0.35 m
x_{15}	Admissible disp. of the 4th deck corner node	Deterministic	$\tilde{x}_{15}^d$	0.30 m
x_{16}	Admissible stress	Deterministic	$\tilde{x}_{16}^d$	300000 kN/m^2
x_{17}	Admissible reliability index	Deterministic	$\tilde{x}_{17}^d$	3.7
x_{18}	Lower bound (cross-section)	Deterministic	$\tilde{x}_{18}^d$	0.04082 m^2
x_{19}	Upper bound (cross-section)	Deterministic	$\tilde{x}_{19}^d$	0.70869 m^2
x_{20}	Lower bound (first deck)	Deterministic	$\tilde{x}_{20}^d$	15 m
x_{21}	Upper bound (first deck)	Deterministic	$\tilde{x}_{21}^d$	35 m
x_{22}	Lower bound (second deck)	Deterministic	$\tilde{x}_{22}^d$	40 m
x_{23}	Upper bound (second deck)	Deterministic	$\tilde{x}_{23}^d$	60 m

Stochastic properties of the system are described by stochastic parameters involving:

- Young modulus of material in all the elements except those in the top deck
- cross-sectional areas for the groups of elements
- load case multipliers $\alpha_1 - \alpha_4$

Moreover, a set of configuration and bounding deterministic parameters is chosen, see Table 1. Variable design parameters used in RBDO problem were taken as:

- mean values of the cross-sectional areas of all groups

- vertical position of the first and the second deck

According to the uniform notation introduced in the paper, the entire set of parameters can be presented as in Table 1.

Displacement and stress performances defined at selected locations were used to define 24 limit state functions. They are as follows:

$$
\begin{aligned}
g_1 &= 1 - \frac{|q_{44}|}{\tilde{x}_{14}^d} \\
g_2 &= 1 - \frac{|q_{36}|}{\tilde{x}_{15}^d} \\
g_i &= 1 - \frac{|\sigma_i|}{\tilde{x}_{16}^d} \qquad i = 3, \ldots, 24
\end{aligned}
\tag{9}
$$

where q_{44} and q_{36} denote the horizontal displacements of nodes 44 and 36 (labeled as 1 and 2 in Fig. 6) respectively, $\tilde{x}_{14}^d$ and $\tilde{x}_{15}^d$ stand for the admissible displacements (see Table 1), σ_i, $i = 3, \ldots, 24$ denote the axial stress in elements that are marked with dashed line in Fig. 6, and $\tilde{x}_{16}^d$ is the admissible stress that may be different for tension and compression.

The optimization problem is defined as

$$
\begin{aligned}
\min \quad & V(\boldsymbol{x}^d, \boldsymbol{x}^\mu) \\
\text{s.t.} \quad & \beta_i \geq \tilde{x}_{17}^d && i = 1, \ldots, 24 \\
& \tilde{x}_{18}^d \leq x_i^\mu \leq \tilde{x}_{19}^d && i = 1, \ldots, 6 \\
& \tilde{x}_{20}^d \leq x_7^d \leq \tilde{x}_{21}^d \\
& \tilde{x}_{22}^d \leq x_8^d \leq \tilde{x}_{23}^d
\end{aligned}
\tag{10}
$$

where V is the structure's volume and β_i are reliability indices related to limit state functions (9), $\tilde{x}_{17}^d$ is the admissible β value equal to 3.7.

Preliminary reliability analysis showed that β-values corresponding to the limit state functions defined above vary from -0.85 to 10.46, see Table 2. The initial volume of the structure is 289.606 m^3.

In the benchmark studies several strategies were compared:

- *standard approach* – STD (column 2, Table 2)
  The results, presented in Figs. 8 to 15 were obtained using traditional RBDO approach i.e. without any user interaction, using a constant convergence tolerance ($\epsilon = 0.001$) and applying the mean values of the stochastic variables as an initial guess for each β–point search. We shall consider these results as a reference when comparing the computational cost with interactive optimization strategies.
- *adaptive convergence parameter* – ACP (column 3 Table 2)
  The results were obtained by the adaptive convergence parameter method described in section 4.2.3.
- *REL/OPT integration* – ROI (column 4, Table 2)
  The results were obtained by ROI approach (cf. section 4.2.2). The ROI approach was combined with ACP, which allowed to reduce the number of limit state function calculations by nearly 40% and the number of limit state function gradient calls by 41% with respect to standard method. The ROI + ACP improvements of the STD approach were also used in the framework of the automated RBDO as they were also employed in the interactive RBDO of our benchmark example.
- *interactive semi-active constraint strategy* – ACS (column 5, Table 2)
  The results were obtained by the interactive RBDO, employing the ACS approach (see. section

4.2.4). Considering the information about the values and trends of changes of the reliability constraints it was possible to choose and eliminate the constraints that were not active, and assumed as not affecting the optimal project. After the second iteration step the constraints 8 and 9 (see Fig. 6) were fixed, after the next step additional constraints: 3, 4, 5, 12 and 13 were decided to be fixed, and finally after the fourth optimizer iteration the constraints 2, 14, 16, 17, 18, 22, 23 and 24 were deactivated. As can be seen from Table 2, the 'pure' ACS approach allowed to reduce the number of the limit state function and gradient calls by some 55% comparing to the standard approach.

Tab. 2. Comparison of RBDO approaches

		Optimization results				
	Initial value	STD	ACP	ROI	ACS	ACS + FOA
	1	2	3	4	5	6
Vol.	289.606 m^3	275.175 m^3	275.175 m^3	274.733 m^3	275.046 m^3	274.744 m^3
x_1^μ	0.113391 m^2	0.058738 m^2	0.058738 m^2	0.058738 m^2	0.058747 m^2	0.058717 m^2
x_2^μ	0.113391 m^2	0.058656 m^2	0.058656 m^2	0.058645 m^2	0.058433 m^2	0.058485 m^2
x_3^μ	0.113391 m^2	0.112871 m^2	0.112871 m^2	0.112871 m^2	0.112884 m^2	0.112564 m^2
x_4^μ	0.113391 m^2	0.060130 m^2	0.060130 m^2	0.059301 m^2	0.060133 m^2	0.060137 m^2
x_5^μ	0.113391 m^2	0.253096 m^2	0.253096 m^2	0.253096 m^2	0.252723 m^2	0.252949 m^2
x_6^μ	0.113391 m^2	0.078885 m^2	0.078885 m^2	0.078954 m^2	0.078883 m^2	0.078895 m^2
x_7^d	25.00 m	25.55 m	25.55 m	25.55 m	25.54 m	25.47 m
x_8^d	50.00 m	49.58 m	49.58 m	49.58 m	49.58 m	49.56 m
β_1	1.99	3.82	3.82	3.81	3.81	3.81
β_2	1.98	3.97	3.97	3.96	3.93 / 3.96 fc	3.93 / 3.96 fc
β_3	1.49	4.80	4.80	4.80	4.15 / 4.80 fc	4.15 / 4.80 fc
β_4	2.70	4.00	4.00	4.00	3.84 / 4.00 fc	3.84 / 4.00 fc
β_5	1.73	4.85	4.85	4.85	4.22 / 4.85 fc	4.22 / 4.85 fc
β_6	-0.85	3.70	3.70	3.70	3.70	3.70
β_7	2.14	3.70	3.70	3.70	3.70	3.70
β_8	3.06	5.71	5.71	5.71	3.88 / 5.71 fc	3.88 / 5.72 fc
β_9	2.96	6.98	6.98	6.98	4.15 / 6.99 fc	4.15 / 6.99 fc
β_{10}	3.39	3.70	3.70	3.70	3.70	3.70 / 3.66 fc
β_{11}	6.10	3.70	3.70	3.70	3.70	3.70
β_{12}	0.70	5.86	5.86	5.86	4.89 / 5.87 fc	4.89 / 5.87 fc
β_{13}	3.45	3.78	3.78	3.78	3.77 / 3.78 fc	3.79 / 3.74 fc
β_{14}	6.94	4.48	4.48	4.48	4.63 / 4.63 fc	4.64 / 4.64 fc
β_{15}	3.85	3.70	3.70	3.70	3.70	3.71 / 3.67 fc
β_{16}	7.10	3.92	3.92	3.77	3.77 / 3.92 fc	3.77 / 3.92 fc
β_{17}	10.46	6.18	6.18	6.18	6.23 / 6.21 fc	6.23 / 6.21 fc
β_{18}	4.09	3.75	3.75	3.75	3.75 / 3.75 fc	3.70 / 3.72 fc
β_{19}	6.39	3.70	3.70	3.70	3.70	3.70 / 3.70 fc
β_{20}	7.46	3.70	3.70	3.70	3.70	3.70 / 3.70 fc
β_{21}	8.21	3.70	3.70	3.70	3.70	3.70 / 3.70 fc
β_{22}	7.81	3.92	3.92	3.72	3.83 / 3.92 fc	3.83 / 3.92 fc
β_{23}	7.16	3.73	3.73	3.73	3.73 / 3.73 fc	3.73 / 3.73 fc
β_{24}	6.93	4.04	4.04	3.96	4.03 / 4.04 fc	4.03 / 4.04 fc

OPT func. calls	7	7	7	7	7
OPT grad. calls	6	6	6	6	6
REL func. calls	1529	1295	929	691	622
REL grad. calls	1217	1016	716	525	476
Comparative time	1.0	0.84	0.60	0.44	0.40

- *interactive first order approximation FOA combined with semi-active constraint strategy* – ACS + FOA (column 6, Table 2)
  Additional reduction of the computation time can be obtain by combining ACS with FOA method, Figs. 16 to 19. The user intervention points of changing the status of the reliability constraint are marked with a flag. The letter L on the flag denotes activating of FOA strategy which means that from this iteration on the value of the constraint is calculated using the linear approximation. The letter C denotes the point of deactivating the constraint and setting the control over it by linear approximation of its value, (see Fig. 17).

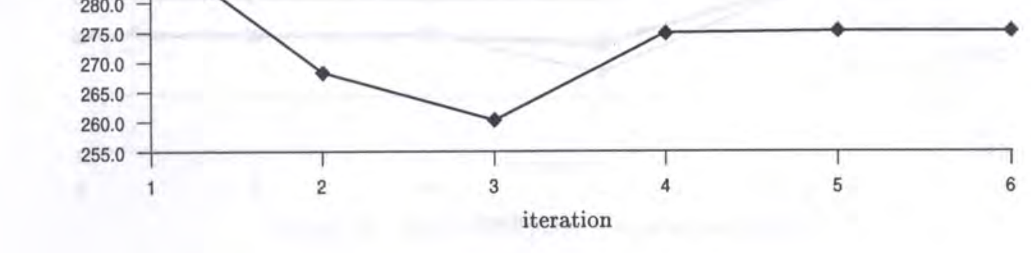
Let us analyze Figs. 16 to 19 in a more detail. After the second iteration it was already possible to observe certain trends of the constraint value changes. Reliability indices β_{10}, β_{13}, β_{15}, β_{18}, corresponding to the stress type limit states in vertical diagonals (see Fig. 6) were all close to 3.7 and their sensitivities with respect to design parameters were low. Therefore, it was decided to approximate their values by FOA. Moreover, there were still two vertical diagonals with stress type constraints (constraints 4 and 7), violated at that instant, so it was concluded that they played the decisive role in controlling the stress in this group of elements. Constraints 8 and 9, imposed on the stresses in corner leg elements, were deactivated. Because of an early stage of the optimization process it was decided to control their values by linear approximation (dashed lines in Fig. 17).

At the third iteration step the value of the design parameter x_5^{μ} (cross-sec. of group 5) was changed substantially which resulted in the increase of the values of many constraints, especially of the displacement type and those associated with the stress in corner legs. So, the decision about deactivating the constraints 3, 4, 5 and 12 was made.

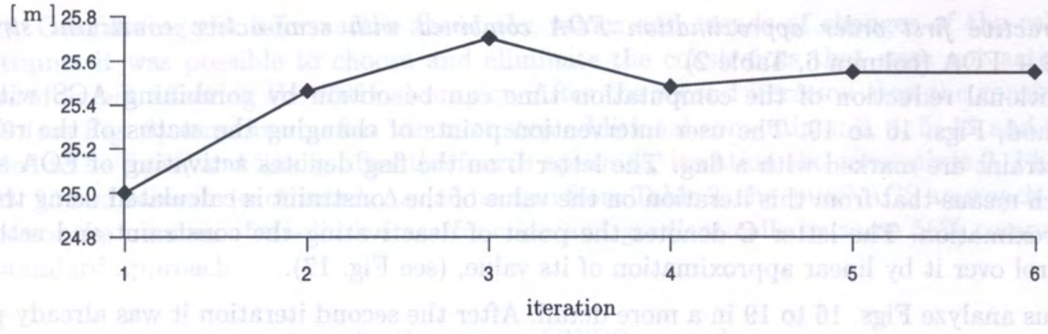
The next, fourth, iteration brought the increase of all the constraints satisfying almost all of them. It can be seen in the graphs that the displacement constraint 1 is more critical then constraint 2 and that they change in accordance with the constraint no. 6, which is the stress type constraint in the bottom corner leg element. The constraint 2 was eliminated. Considering the stabilization of the design (the decks position changed very little during the first four iterations) and the fact that most of the constraints are satisfied it was decided to deactivate or use FOA method for all the not violated constraints, excluding the displacement constraint 1. The optimal project was found in the following two iteration steps.

Results of the interactive ACS + FOA approach show that the computation cost was reduced by nearly 60%, comparing to the standard approach. Of course one can always choose other interaction strategy. The better understanding of the structural behavior can certainly help in making more appropriate decisions concerning the constraint elimination.

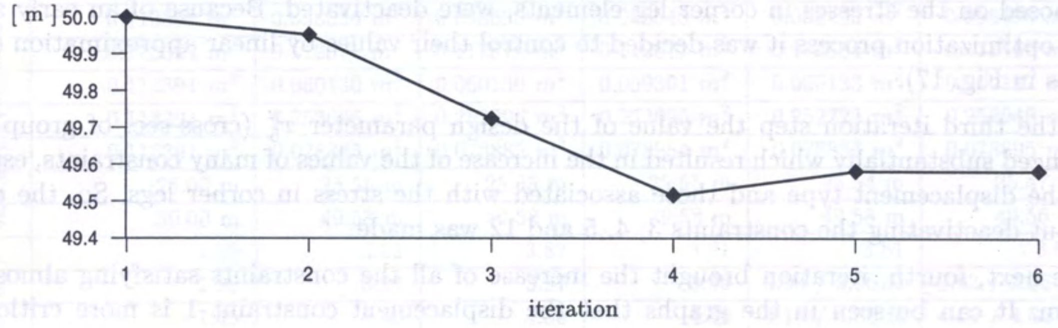
To validate the results it was necessary to check the real values of the non-active and approximate reliability constraints for the final design. In Table 2 these values are labeled fc (final check). Note, that constraints 10 and 15 turned out to be slightly violated, but in this case it was not necessary to perform additional optimization step to satisfy them. The designer can easily make them feasible using the reliability index sensitivity information from the final check.



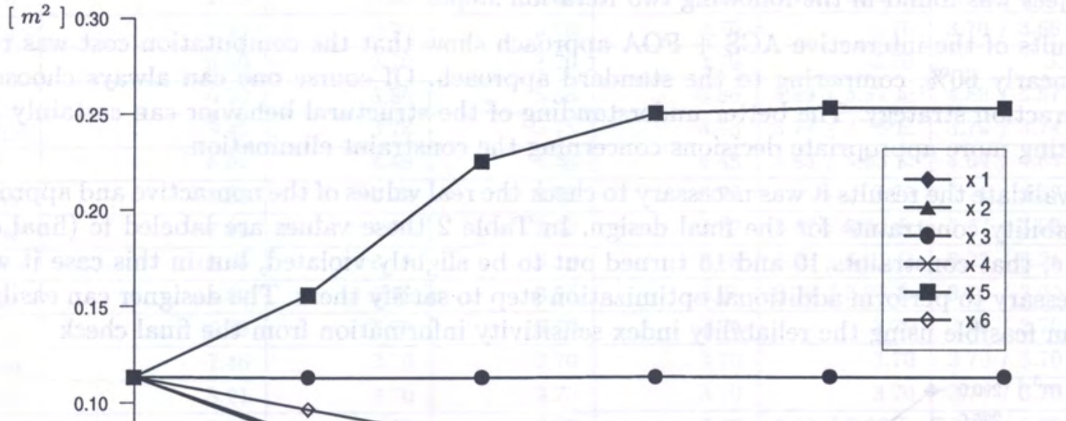
Rys. 8. Volume of the structure



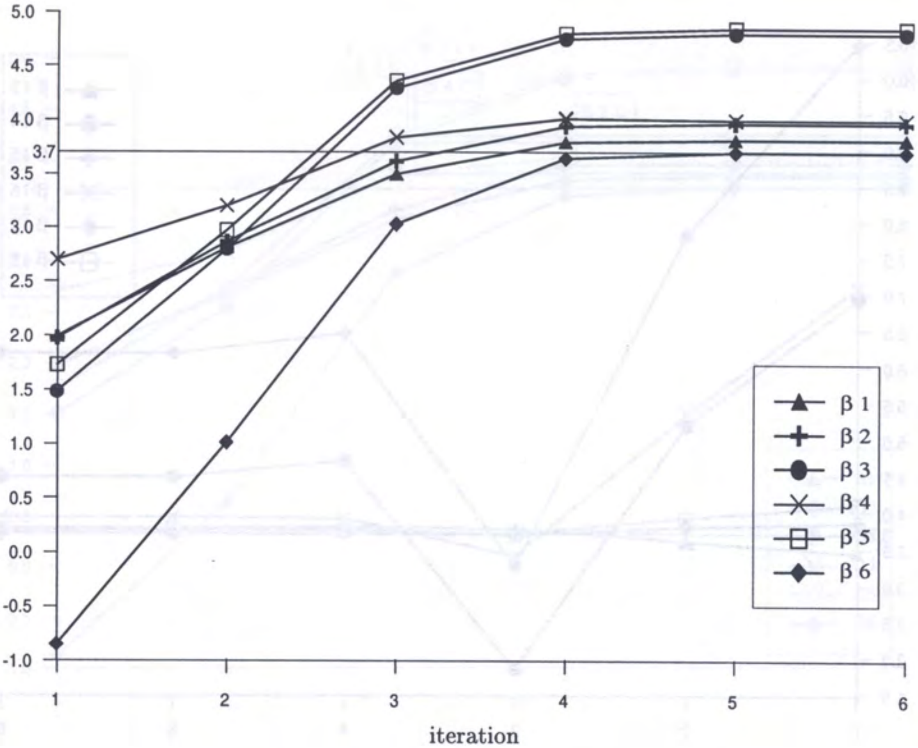
Rys. 9. Shape parameter – 1st deck position



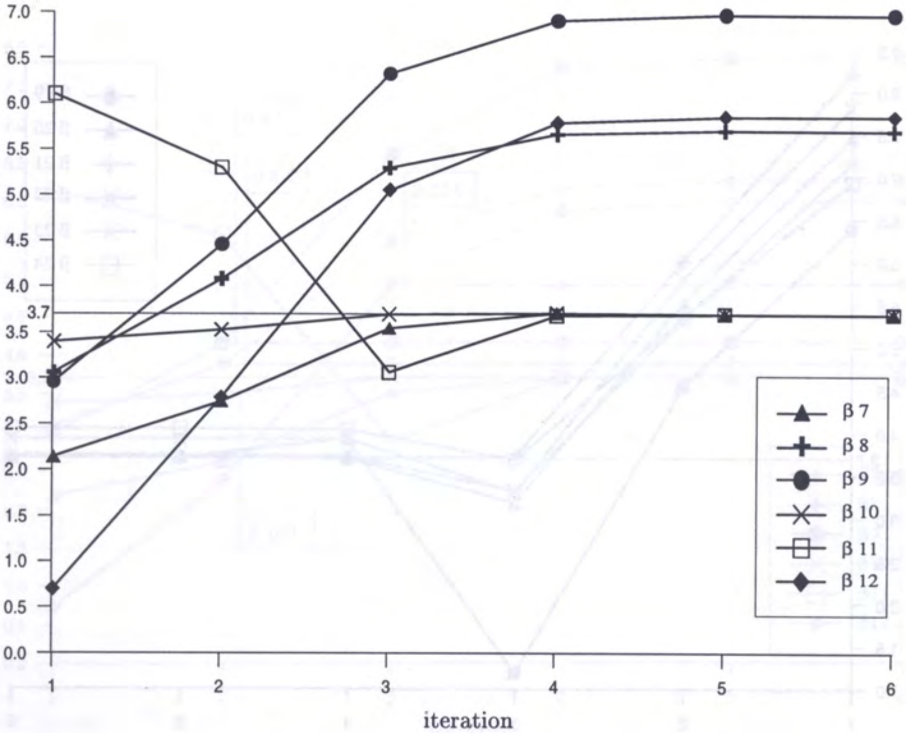
Rys. 10. Shape parameter – 2nd deck position



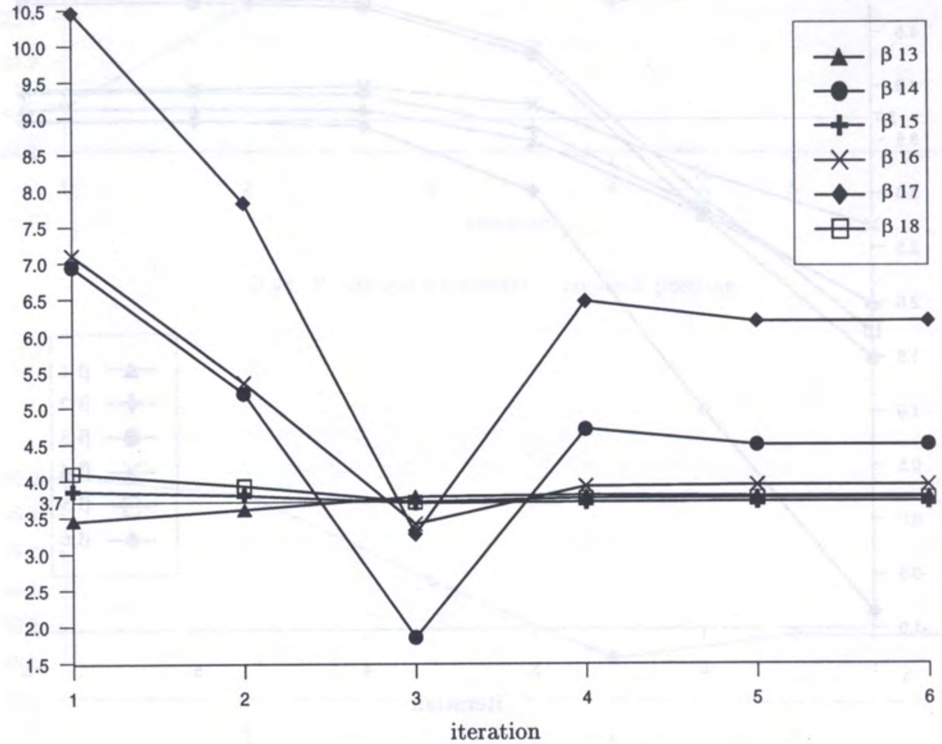
Rys. 11. Size parameters – cross-sections



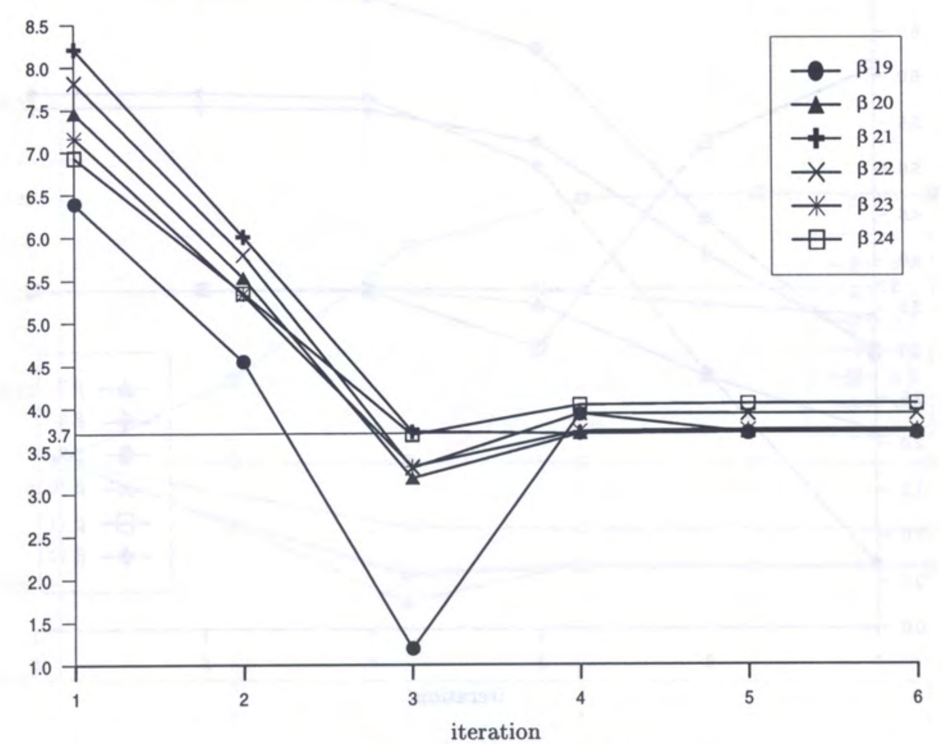
Rys. 12. Reliability indices – constraints 1-6



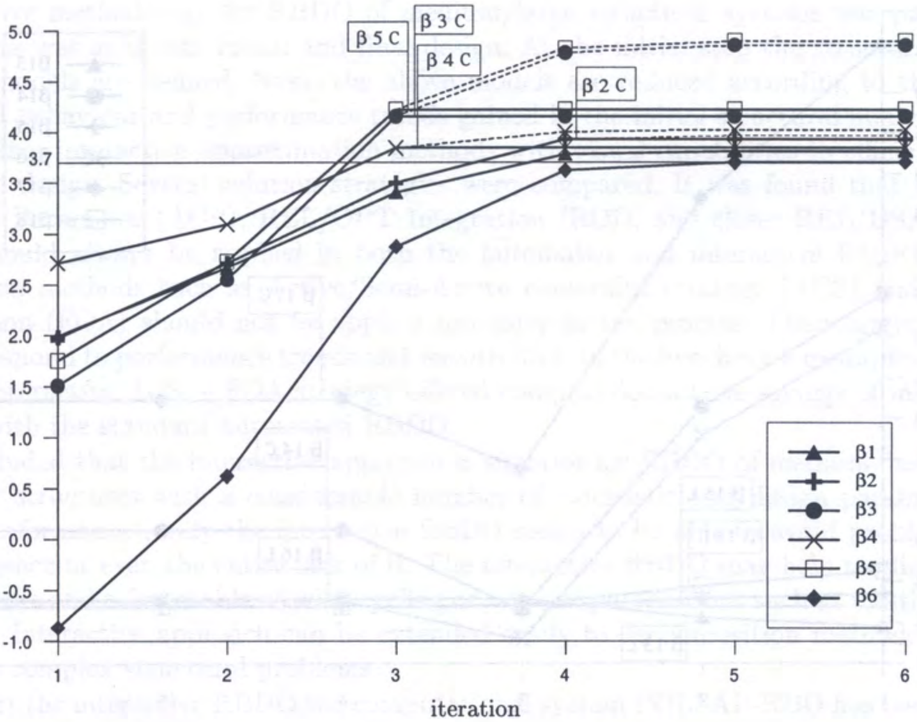
Rys. 13. Reliability indices – constraints 7-12



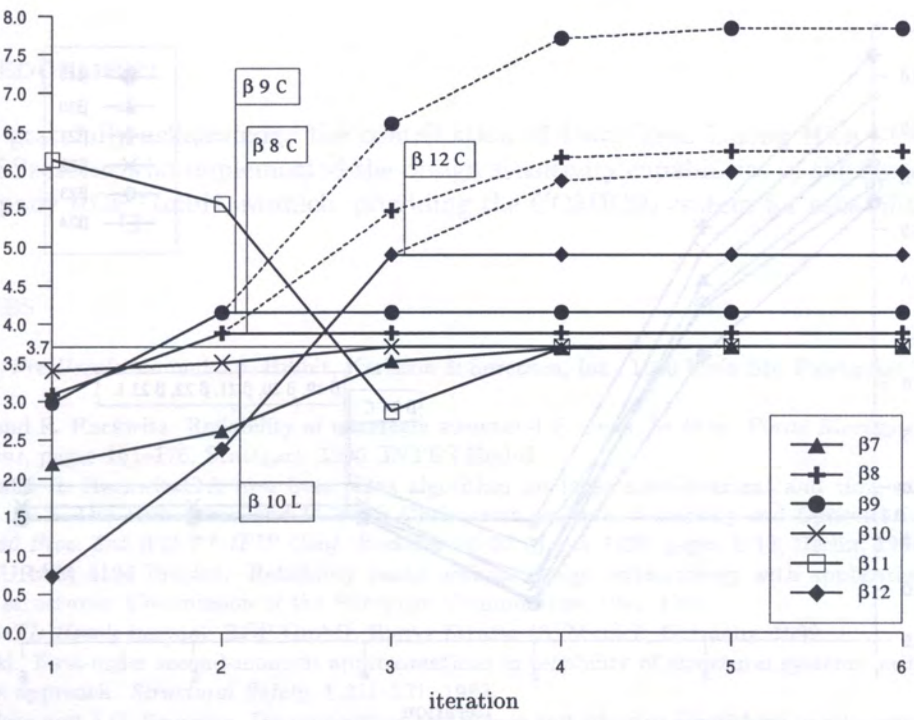
Rys. 14. Reliability indices – constraints 13-18



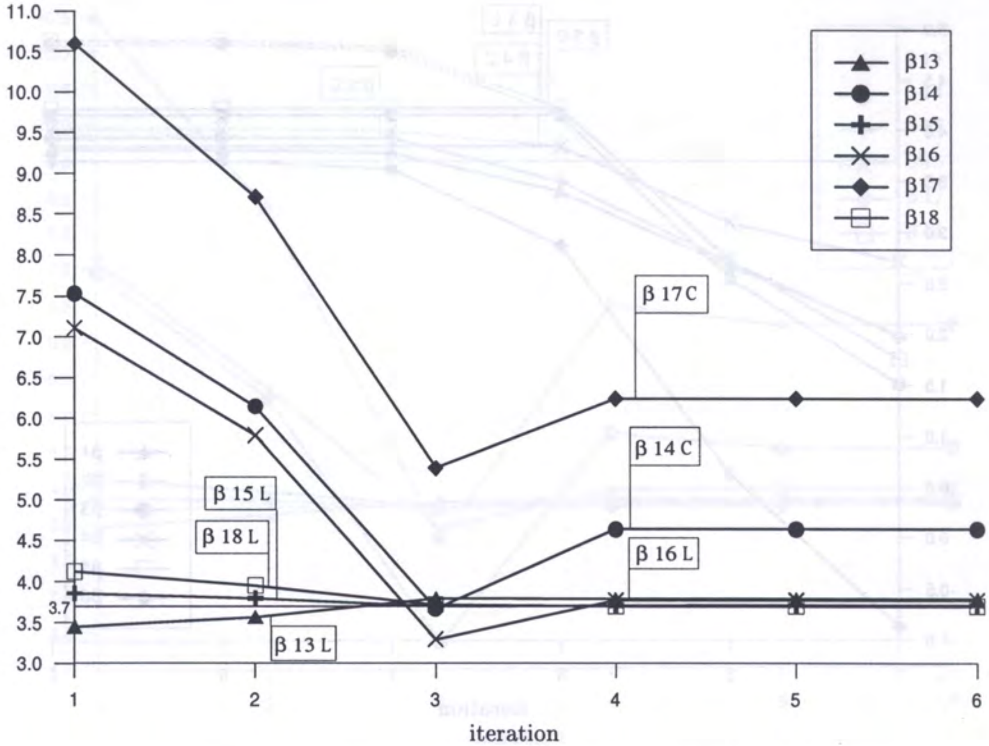
Rys. 15. Reliability indices – constraints 19-24



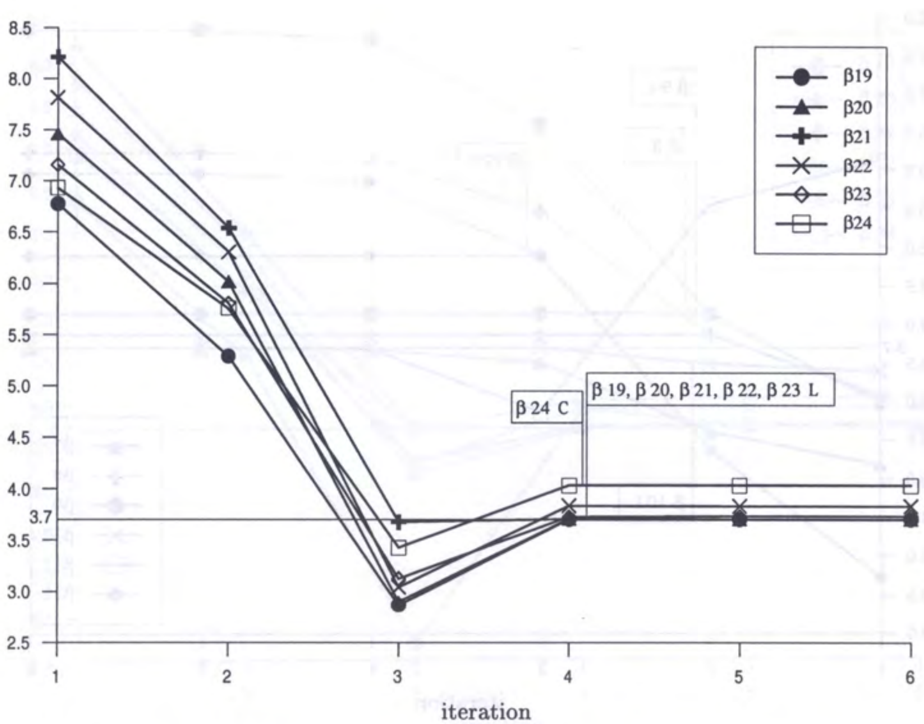
Rys. 16. Reliability indices – constraints 1-6. Interactive approach.



Rys. 17. Reliability indices – constraints 7-12. Interactive approach.



Rys. 18. Reliability indices – constraints 13-18. Interactive approach.



Rys. 19. Reliability indices – constraints 19-24. Interactive approach.

6. Conclusions

The interactive methodology for RBDO of medium/large structural systems was presented. The design process was split into initial and final design. At the initial step the structural, stochastic and design models are defined. Next, the above models are reduced according to the knowledge on structural behaviour and performance trends gained in the initial structural analysis. The final design combines interactive approximation methods with visual capabilities to efficiently generate the optimum design. Several solution strategies were compared. It was found that the Adaptive Convergence Parameter (ACP), REL/OPT Integration (ROI), and closer REL/DSA Integration strategies should always be applied in both the automated and interactive RBDO. Interactive approximation methods such as Active/Semi-Active constraint strategy (ACS) and First Order Approximation (FOA) should not be applied too early in the process. Their application should strictly correspond to performance trends and sensitivities. In the benchmark example of an offshore jacket, the interactive ACS + FOA strategy offered computational time savings of more then 60% comparing with the standard automated RBDO.

It is concluded that the interactive approach is superior for RBDO of medium/large structural systems. For structures with a considerable number of stochastic and design parameters as well as critical performances, only the interactive RBDO seems to be able to avoid problems with the slow convergence or even the entire lack of it. The interactive RBDO may help to eliminate model definition errors and solve problems with cycling or non-unique solutions such as multiple β-points, for instance. Interactive approach can be extended easily to decomposition methods required for solving large complex structural problems.

To support the interactive RBDO the computational system POLSAP-RBO has been developed. It is equipped with capabilities of modifying stochastic and design models, changing status of design parameter from variable to fixed and vice versa, eliminating constraints in Active and Semi-Active Constraint Strategy (ACS) and applying first order approximations in the FOA strategy. The system contains graphical visual capabilities, allowing to display structural performances and to present sensitivity results.

Acknowledgement

The authors gratefully acknowledge the contribution of Prof. Tran Duong Hien of the Technical University of Szczecin who implemented the design sensitivity capabilities in the system POLSAP, and the company RCP GmbH, Munich, providing the COMREL system for reliability analysis.

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