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2D shape optimization using genetic algorithm

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This paper presents an optimal design method of continuum structures by genetic algorithm. Profiles of the objects under consideration are represented by the spline functions and then, the chromosomes for the profiles are defined by the coordinates of the control points of the functions and the material code of the structures. The profiles and the material code are optimized by the genetic operations in order to determine the object satisfying the design objectives. The minimum weight design of the plate is considered as a typical example. The present method is applied to the problem in which the profile and the material of the objects are unknown.

Key Words: Genetic Algorithms(GAs), Boundary Element Methods(BEMs), Shape Optimization, Material Selection, Riesenfeld Spline Functions

1. Introduction

In the shape optimization problems, firstly, the objective functions and the constraint conditions are defined. Then, profiles of objects under consideration are optimized so that the objective functions are minimized by changing the design variables such as the shape parameters, the physical constants and so on. The problems are usually solved by applying the gradient-type search schemes using the derivatives of the objective functions and the constraint conditions with respect to the design variables (design sensitivities). The design sensitivity analysis schemes, however, encounter some difficulties in the actual optimization problem. For example, when the discrete-valued objective functions are defined, the design sensitivity analysis is not available. Besides, when the design sensitivities are weak (very insensitive), the system of equations often becomes ill-posed. Therefore, in these cases, we should use the search schemes without the sensitivities. Genetic algorithm (GA) is one of such search schemes and moreover, much more efficient than the random search, one of the most popular schemes without sensitivities [1, 2].

In this study, the minimum weight design of the continuum structures is considered as the numerical example and then, the shape parameters for the profiles of the objects and the material parameters such as the Young's modulus and the Poisson's ratio are taken as the design variables. Since the objective functions and the constraint conditions are the discrete-valued functions of the material parameters, the gradient-type search scheme is not available. By the way, the existing studies in the shape optimization schemes using the genetic algorithm mainly focus the truss structures [3, 4, 5, 6, 7]. Therefore, we firstly describes the shape optimization method for the continuum structures using the genetic algorithm. The profiles of the objects are represented by the spline functions and then, the chromosomes for the profiles are defined by considering as the genes the control points of the functions. The population is constructed by the individuals with such chromosomes. The genetic operations such as the selection, the crossover and the mutation are applied to the population in order to determine the individuals satisfying the deign objectives. Boundary element method is employed for estimating the objective functions and the constraint conditions. Since the boundary element method can solve the problems by the boundary discretization alone, the mesh generation is much simpler than the finite element method.

This paper is summarized as follows. In the Sec. 2, the shape representation scheme using spline functions and the boundary element method are described. In the Sec. 3, the genetic representation of the problem and the genetic operations employed in the present algorithm are defined. In the Sec. 4, the present method is applied to the minimum weight design of a continuum structure. In the Sec. 5, finally, we summarize the conclusions.

2. Boundary Representation and Numerical Analysis

2.1. Shape representation by spline functions

The boundary profiles of the objects under consideration are represented by the spline functions (Fig. 1).

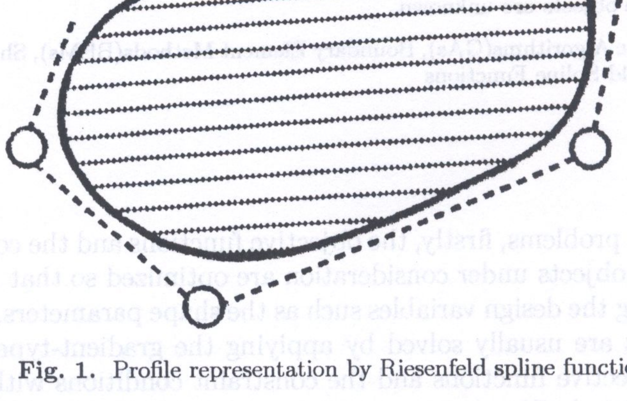


Fig. 1. Profile representation by Riesenfeld spline function

The arbitrary point on the boundary has the coordinates (x, y) so that

$$x = \sum_{i=1}^{N_c} x_i B_{i,k} \qquad y = \sum_{i=1}^{N_c} y_i B_{i,k} \tag{1}$$

where (x_i, y_i) are the coordinates of the control point of the functions. $B_{i,k}$ is the B-spline function of the order k and N_c the number of the control points.

2.1.1. Comparison of shape representation scheme

When the Genetic algorithm is applied to the shape optimization of the continuum structures, the following algorithm is often employed [8, 9]. The objects under consideration are divided into small square cells and then, binary parameters are specified to the cells so that the parameter is taken as 0 for the empty cell and 1 for the occupied one. The chromosomes for the profiles are defined by the parameters. The population is constructed by the individuals with the chromosomes and then, the genetic operations are applied to the population. By considering the cells as finite elements, the finite element method is applied for estimating the objective functions and the constraint conditions. This scheme, however, has some difficulties. The computational accuracy of the numerical analysis and the obtained final profiles are dependent on the size of the cells. There are, besides, another difficulty related to the mutation operation. In the mutation operation, the values of the genes are changed stochastically. Since the cells are, in general, very small, the mutation dose not affect the shape and the topology of the objects.

In order to overcome these difficulties, we adopted the boundary profile representation by Free-Form Deformation (FFD) in the previous study [10]. In the FFD [11], the grid is taken so as to cover the profiles of the objects and then, the profiles are distorted by moving the grid points from their initial positions. The chromosomes for the profiles are defined by the position vectors of the grid points. This scheme can represent the profiles of the objects by the relatively short chromosomes but, unfortunately, has some difficulties. Firstly, it is very difficult to modify only small parts of the profiles. Secondly, there are some restrictions for specifying the boundary conditions. For overcoming these difficulties, this paper adopts the representation scheme of the boundary profile by the Riesenfeld spline functions. Since, in the Riesenfeld spline functions, the coordinates of the control points are taken as the coefficients of the functions, the computational cost is much cheaper than the other spline functions.

2.2. Boundary element method

We shall explain briefly the boundary element method for the two-dimensional elastic problem [12]. The governing equation and the boundary conditions without the body forces are given as:

$$\sigma_{ij,j} = 0 \quad (\text{in } \Omega) \tag{2}$$

and

$$\begin{aligned} u_i &= \bar{u}_i \quad (\text{on } \Gamma_u) \\ t_i &= \bar{t}_i \quad (\text{on } \Gamma_t) \end{aligned} \tag{3}$$

where u_i, t_i and σ_{ij} denote the displacement, the traction and the stress components of the two-dimensional elastic problem. $(\ )_{,j}$ denotes the partial differentiation in the x_j-direction. Ω, Γ_u and Γ_t are the domain occupied by the object under consideration, its displacement- and traction-specified boundaries, respectively.

By introducing the Kelvin functions as the weight function and applying the Gauss-Green formula, the governing equation is transformed to the boundary integral equation:

$$Cu_i = \int_\Gamma (u^*_{ij}t_j - t^*_{ij}u_j)d\Gamma \tag{4}$$

where C is the constant parameter depending on where the source point is placed. u^*_{ij} and t^*_{ij} denote the fundamental solutions of the displacements and the traction components, respectively. Discretizing Eq. (4) with the boundary elements, we have

$$\mathbf{Hu} = \mathbf{Gt} \tag{5}$$

where $\mathbf{u}$ and $\mathbf{t}$ denote the nodal potential and flux vectors and $\mathbf{H}$ and $\mathbf{G}$ the coefficient matrices, respectively. By applying the boundary conditions, the above equation is solved for the boundary unknowns.

2.2.1. Boundary element method for shape optimization problem

The present method is considered to be one of the successively shape modification method. When the finite element method is employed for numerical analysis, the shape modification distorts the finite element mesh heavily and thus, the computational accuracy may become worse. For overcoming this difficulty, the mesh generation must be carried out automatically after each shape modification. Its computational cost, however, is expensive and the automatic system for the mesh generation is not yet perfect. On the other hands, the boundary element method can solve the problem by the boundary discretization alone when the objects under consideration are governed by linear and homogenous differential equations. Therefore, the automatic generation of the boundary element mesh is much simpler than the finite element method. We, finally, can conclude that the boundary element method is more suitable for the present method.

3. Optimization algorithm

3.1. Optimization problem and fitness function

We shall consider the minimum weight design of the continuum structures. The objective function of the problem is given as:

$$\frac{\rho A}{\rho_0 A_0} \to \min \tag{6}$$

where ρ and A denote the density of the material and the area of the object under consideration. The subscript 0 denotes the best individual in the initial population.

The constraint condition of the maximum stress is given as:

$$\sigma_c \geq \sigma_{\max} \tag{7}$$

where $\sigma_{\max}$ and σ_c denote the maximum principal stress on the boundary nodes and the reference value of the material specified by user, respectively.

The constraint condition of the maximum displacement is given as:

$$u_c \geq u_{\max} \tag{8}$$

where $u_{\max}$ and u_c denote the maximum displacement and the reference value specified by user, respectively.

The constraint condition for the crossing of the boundary is given as:

$$g_c = 0 \tag{9}$$

where

$$g_c = \begin{cases} 0 & \text{when the boundary is not crossing.} \\ 1 & \text{when the boundary is crossing.} \end{cases} \tag{10}$$

Besides, the body forces are neglected in this study, which is also important constraint condition of the optimization problem.

The coordinates of the control points of the spline functions (x_i, y_i) and the code of the materials k_m are taken as the design variables. The constraint conditions for the design variables are given as

$$\left.\begin{array}{rcl} 0 \leq & k_m & \leq k_{\max} - 1 \\ x_{\min} \leq & x_i & \leq x_{\max} \\ y_{\min} \leq & y_i & \leq y_{\max} \end{array}\right\} \tag{11}$$

where $k_{\max}$ denotes the maximum number of the materials as the candidates. Then, $x_{\min}$, $x_{\max}$, $y_{\min}$ and $y_{\max}$ denote the minimum and the maximum values of x_i and y_i, respectively.

In the genetic algorithms, it is usual that the constraint conditions are included into the fitness functions by introducing the penalty functions. In the present method, however, the populations are constructed by the individuals satisfying the constraint conditions alone. Therefore, the fitness function dose not include the constraint conditions;

$$f = 1 - \frac{\rho A}{\rho_0 A_0} \tag{12}$$

3.2. Genetic representation and genetic operations

3.2.1. Genetic representation of profiles

We shall introduce the new coordinates of the control points (m_i, n_i) in order to define the chromosomes of the profiles (Fig. 2). m_i and n_i are the integer numbers taken as $0 \leq m_i \leq M$ and $0 \leq n_i \leq N$, respectively. The coordinates (x_i, y_i) are related to the coordinates (m_i, n_i) as follows:

$$\left.\begin{aligned} x_i &= x_{\min} + (x_{\max} - x_{\min})\frac{m_i}{M} \\ y_i &= y_{\min} + (y_{\max} - y_{\min})\frac{n_i}{N} \end{aligned}\right\}. \tag{13}$$

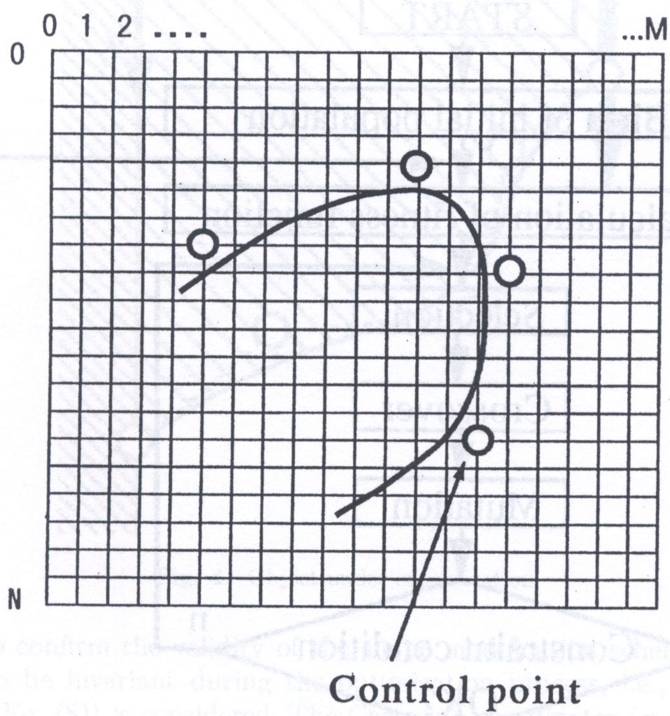


Fig. 2. (m_i, n_i) coordinates

By taking (m_i, n_i) and the material code k_m as the genes, the chromosomes are defined as

$$k_m m_1 n_1 m_2 n_2 \cdots m_{N_c} n_{N_c} \tag{14}$$

where N_c denotes the total number of the control points. Therefore, the length of the chromosome is $2N_c + 1$.

3.2.2. Selection

The present method adopts the ranking selection scheme. The ranking of each individual is specified according to the magnitude of the fitness function of the individual. While the highest selection rate is given to the best individual, the worst individual has the lowest selection rate so that the worst individual has, at least, one offspring individual. The selection rates of the other individuals are determined by the linear interpolation between them. Besides, the elitist scheme is also employed in order that the best individual at each generation survives at the next generation.

3.2.3. Crossover

The uniform crossover scheme is employed.

3.2.4. Mutation

In the mutation scheme, the values of the genes are modified arbitrarily.

3.3. Present algorithm

The algorithm of the present method is shown in Fig. 3. This is the same algorithm as that in our previous study [10].

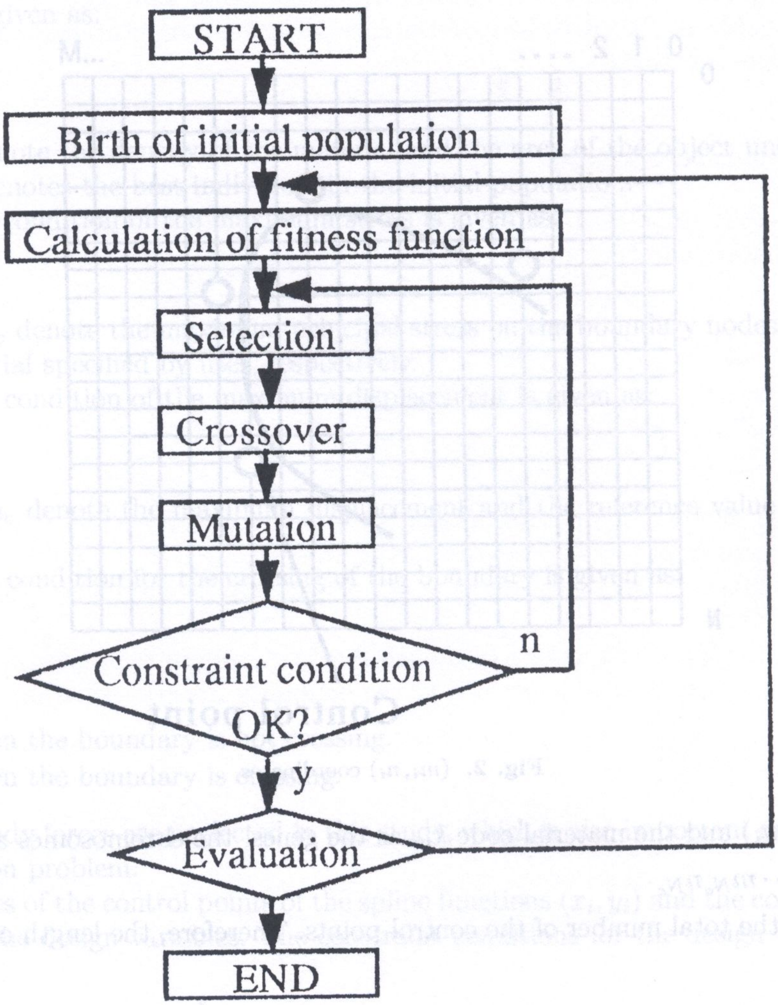


Fig. 3. Present algorithm

Step.1 An initial population is constructed by the individuals satisfying the constraint conditions alone.
Step.2 The fitness functions of the individuals are estimated.
Step.3 The selection operation selects two individuals as parents.
Step.4 The crossover and the mutation operations generate individuals as children from the parents.
Step.5 If the children satisfy the constraint conditions, they are added to the new population. If they do not satisfy, they are not added and then, the process goes to Step.3.
Step.6 If the new population is constructed by the individuals satisfying the constraint conditions alone, the process goes to the Step 2.

4. Numerical example

4.1. Confirmation of validity of shape modification scheme

A plate is considered as a numerical example (Fig. 4). The objective of the design is to minimize the weight of the plate by changing its lower profile. The boundary profile is controlled by 7 control points.

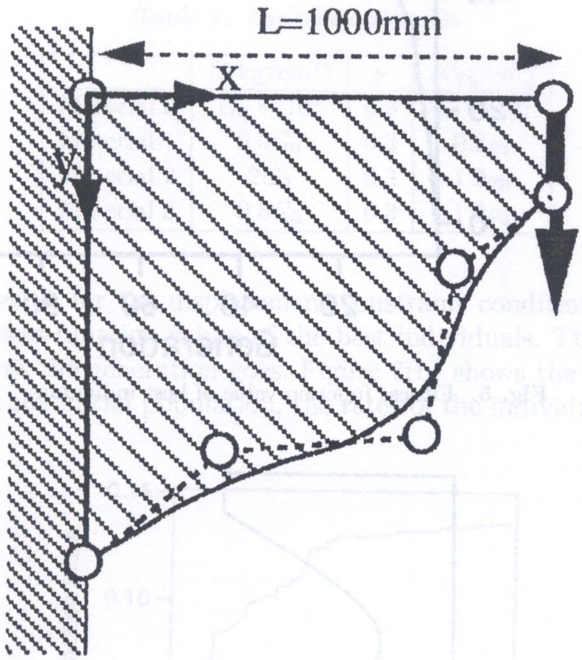


Fig. 4. Object under consideration

Firstly, in order to confirm the validity of the shape modification scheme, the material of the plate is considered to be invariant during the optimization process, i.e., $k_{\max} = 1$. The stress constraint condition (Eq. (8)) is considered. The Cartesian coordinates (x, y) is taken as shown in Fig. 4 and then, the constraint conditions for the design variables are specified as:

$$\left.\begin{array}{rcccl} -0.2L & \leq & x_i & \leq & 1.2L \\ -0.2L & \leq & y_i & \leq & 1.2L \end{array}\right\}.$$

The parameters are taken as follows:

Shear modulus G	10^6kg/cm^2
Poisson's ratio ν	0.3
Reference stress σ_c	300kg/cm^2
Population size	100
Crossover rate	1.0
Mutation rate	0.05
(M, N)	$(64, 64)$

In the following numerical results, the numerical analyses at the same initial population are performed 10 times and then, the average values of their results are shown.

Figure 5 shows the fitness function values of the best individuals. The performance of the best individuals is improved monotonously because the elitist scheme is employed. Figure 6 shows the profiles of the best individuals at first, 10th and 100th generations. The profiles of the individuals are improved as the generation goes.

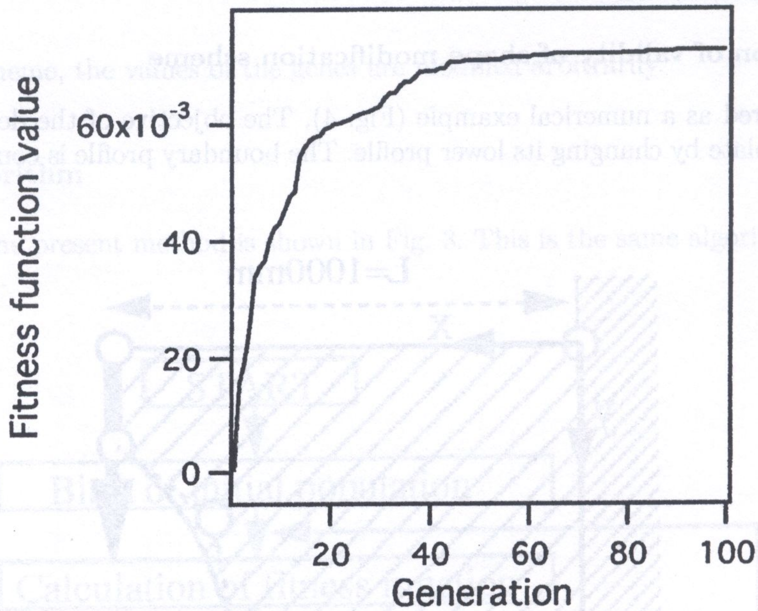


Fig. 5. Fitness function value of best individual

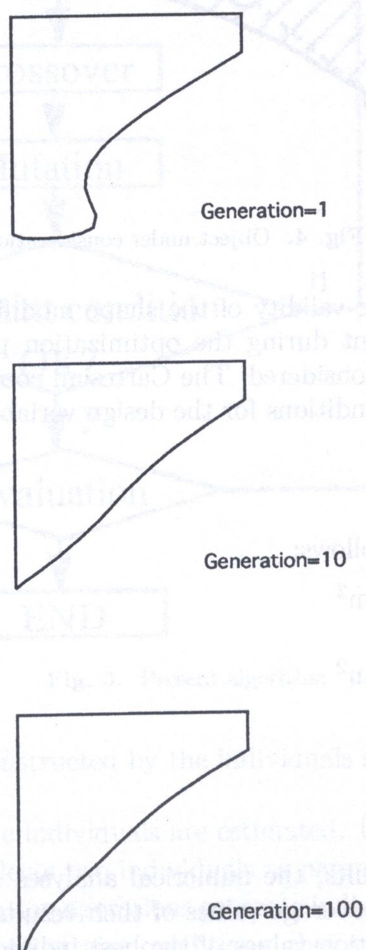


Fig. 6. Profiles of best individuals

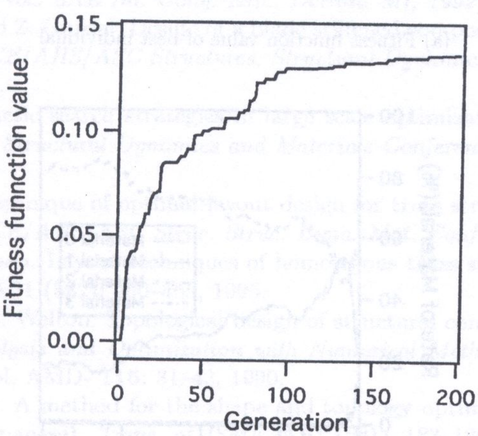
4.2. In case that profile and material are unknown

Next, we shall consider that the boundary profile and the material of the object are unknown. The object under consideration and the parameters are identical to the first example (Fig. 4). The material of the object can be selected from four candidates shown in Table 1. The material 0 is the standard one. The material 1 is lighter and weaker than the material 0. The material 2 is heavier and stiffer than the material 0. The material 3 is weaker and heavier than the material 0.

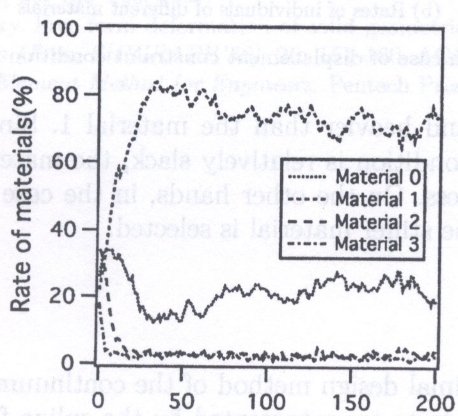
Table 1. Candidate materials

	$G(\mathrm{kg/cm^2})$	ν	$\rho(\mathrm{kg/m^3})$
Material 0	$G_0 = 10^6$	0.3	$\rho_0 = 1.0$
Material 1	$0.5G_0$	0.3	$0.9\rho_0$
Material 2	$2G_0$	0.3	$1.2\rho_0$
Material 3	$0.5G_0$	0.3	$1.2\rho_0$

Firstly, the reference value for the displacement constraint condition is taken as $u_c = 0.5\mathrm{cm}$. Figure 7(a) shows the fitness function values of the best individuals. The performance of the best individuals are improved as the generation goes. Figure 7(b) shows the rates of the individuals of the different materials. In the initial population, the rates of the individuals of the different mate-



(a) Fitness function value of best individual

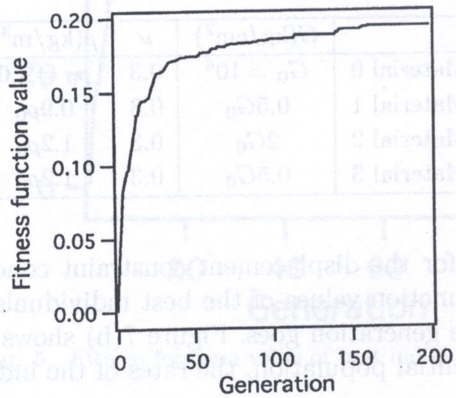


(b) Rates of individuals of different materials

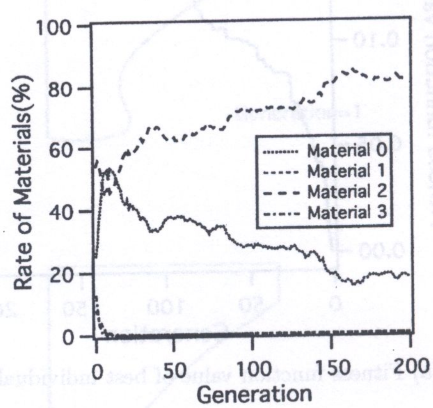
Fig. 7. In case of displacement constraint condition $u_c = 0.5\mathrm{cm}$

rials are identical. As the generation goes, the individuals of the material 2 and 3 decrease rapidly and almost zero after 30th generation. On the other hands, the individuals of the material 1 increases.

Secondly, the reference value is taken as $u_c = 0.25$cm. Figure 8(a) shows that the performance of the best individuals is improved monotonously as the generation goes. Figure 8(b) shows that the individuals of the material 1 and 3 decreases rapidly and then, almost zero after 20th generation. On the other hands, the individuals of the material 2 increase.



(a) Fitness function value of best individual



(b) Rates of individuals of different materials

Fig. 8. In case of displacement constraint condition $u_c = 0.25$cm

The material 2 is stiffer and heavier than the material 1. Since, in the case of $u_c = 0.5$cm, the displacement constraint condition is relatively slack, the material is selected according to the density rather than the stiffness. On the other hands, in the case of $u_c = 0.25$cm, the constraint condition is rigid and thus, the stiffer material is selected.

5. Conclusion

This paper presented the optimal design method of the continuum structures by the genetic algorithms. The profiles of the objects are represented by the spline functions and then, the chromosomes of the objects are defined by taking as the genes the coordinates of the control points of the spline functions and the material code. The objective functions and the constraint conditions are estimated by the boundary element method.

The plate under a shear force was considered as a numerical example. The design objective was to minimize the weight of the plate. Firstly, the material was assumed to be invariant and only boundary profile of the plate was optimized in order to confirm the validity of the shape modification method. The numerical result was satisfactory. Then, the present method was applied to the problem in which the profile and the material of the objects are unknown. In this case, the numerical results, especially, the material selection, are dependent on the magnitude of the reference value of the displacement constraint. If the condition is relatively slack, the material is selected according to the density rather than the stiffness. On the other hands, the material is determined by the stiffness if the condition is relatively rigid.

Finally, we should point out that the present method has some disadvantages to be overcome. One of the great difficulties is the computational cost. In the present method, the boundary element analysis must be performed repeatedly for estimating the fitness function values of the individuals, which often increases the computational cost dreadfully. Therefore, we have been developing the new algorithm for reducing the cost.

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