Computer Assisted Mechanics and Engineering Sciences, **17**: 51–58, 2010.

Evolutionary identification of laminates' granular parameters

Witold Beluch, Piotr Orantek
Department of Strength of Materials and Computational Mechanics
Silesian University of Technology, Konarskiego 18A, 44-100 Gliwice, Poland
e-mail: witold.beluch@polsl.pl

(Received in the final form November 12, 2010)

The paper deals with the identification of material constants in simple and hybrid laminates. It is assumed that identified constants are non-deterministic and can be described by means of different forms of the information granularity represented by interval numbers, fuzzy numbers or random variables. The Two-Stage Granular Strategy combining global (Evolutionary Algorithm) and local (gradient method supported by an Artificial Neural Network) optimization techniques is used to solve the identification problems. Finite Element Method in the granular form is used to solve the direct problem for laminates. Modal analysis methods are employed to collect measurement data for the identification process. Numerical examples presenting effectiveness of the strategy are enclosed.

Keywords: laminate, information granularity, identification, evolutionary algorithm, artificial neural network, interval numbers, fuzzy numbers, random variables

1. Introduction

In many engineering problems it is necessary to identify some parameters, such as material properties, geometrical parameters or boundary conditions. If the parameters of the system can not be determined precisely, the uncertain parameters which describe granular character of the data may be introduced. There exist different models of the information granularity, like interval numbers, fuzzy numbers, random variables or fuzzy random variables [3].

Laminates are composite materials made of many layers (plies). Reinforcement in form of fibers is typically located unidirectionally in each ply, but can vary in different plies. Laminates are characterized by their very high strength/weight ratio and possibility of obtaining required properties by manipulating the constituent materials, fiber orientation in particular layers and layer thicknesses.

The aim of the work is to identify elastic constants in laminate structures. Laminate elements are often produced individually or in short series, so the destructive identification methods are not allowed. Due to the technological process of the laminates' manufacturing, it is assumed that identified constants are non-deterministic. Three forms of the information granularity: interval numbers, fuzzy numbers or random variables are taken into account.

To solve the identification task for laminates, a special strategy, called Two-Stage Granular Strategy, is introduced. The strategy joins global and local optimization methods. Evolutionary Algorithm in a granular version is used as a global optimization method. Gradient optimization method supported by Artificial Neural Network is employed for local optimization. Special multi-level Artificial Neural Network (ANN) is used to perform the local optimization by means of the steepest descent method. Finite Element Method is used to solve the direct boundary-value problem for laminate structures. Modal analysis data (eigenfrequencies and frequency response diagrams) are collected as measurements necessary for identification procedure.

2. Formulation of the problem

Laminate structures are usually treated as orthotropic thin plates with four independent elastic constants: two Young's module E_1 and E_2, one shear modulus G_{12} and one Poisson's ratio ν_{12} [7]. The cost of the material increases if the improvement of material properties, such as strength or stiffness, is looked for. One of the solutions of this problem are hybrid laminates, with particular plies made of different materials. This attitude allows to find the balance between cost and the required properties of the laminate. In the present paper, hybrid laminates have the form of interply hybrids with internal plies made of cheaper/worse material and external plies, composed of the material being more expensive but having better properties [1].

The aim of the work is to identify elastic constants in simple and hybrid laminates. The identification tasks belong to inverse problems, which are mathematically ill-posed. The identification problem can be formulated as a minimization of the functional $J(\mathbf{x})$:

$$\min J(\mathbf{x}) = \sum_{i=1}^{N} [q_i - \hat{q}_i]^2, \tag{1}$$

where: $\hat{q}_i$ – the measured values of a state field; q_i – the values of the same state fields calculated from the numerical model.

A vector $\mathbf{x}$ describing identified parameters has one of the two forms [4]:

- for simple laminates:

$$\mathbf{x} = \{E_1, E_2, G_{12}, \nu_{12}\}, \tag{2}$$

- for hybrid laminates:

$$\mathbf{x} = \{E_1^1, E_2^1, G_{12}^1, \nu_{12}^1, \rho^1, E_1^2, E_2^2, G_{12}^2, \nu_{12}^2, \rho^2\}, \tag{3}$$

where ρ is the material density and superscripts denote the number of material in hybrid laminate.

The identification procedure is usually performed for the data obtained from the structure response to the external excitation. In the present paper, the modal analysis data are used to collect necessary measurement data using the minimal number of sensor points. The eigenvalue problem for a laminate plate of length a in x direction, width b in y direction and thickness h in z direction, can be presented as [2]:

$$\rho h \omega^2 w = D_{11} w_{,xxxx} + 4D_{16} w_{,xxxy} + 2(D_{12} + 2D_{66}) w_{,xxyy} + 4D_{26} w_{,xyyy} + D_{22} w_{,yyyy} \tag{4}$$

where ρ is the mass density, ω is the eigenvalue vector, w is the deflection in the z-direction, and D_{ij} is the bending stiffness.

To avoid the ambiguity in the identification results, one can use the frequency response data in more complex problems. This attitude allows to collect many data with one (or a few) sensor points. In the present paper the accelerations in one sensor point are measured [12]. The acceleration measurements are especially convenient due to the small mass of the accelerometers in comparison with the displacement sensors. It is also possible to obtain the velocity and displacement signals by integration of the acceleration signal. The experimental data are simulated numerically in both cases.

The identified material constants are assumed to be non-deterministic as a result of the manufacturing process. The uncertainties are introduced to reduce the divergence between actual structures and their mathematical model.

3. Identification strategy

A Two-Stage Granular Strategy (TSGS) is used to identify laminates' material parameters [6]. A block diagram of the TSGS is presented in Fig. 1. It is assumed that identified parameters and measurements are non-precise and can be described by different variants of information granularity: interval numbers, fuzzy numbers or random variables. It is assumed in the stochastic case that the random variables are independent and they have Gaussian probability density functions.

The first stage of the TSGS is performed by means of the Granular Evolutionary Algorithm (GEA) to carry out the global optimization. In the GEA, the population of vectors $\mathbf{x}$ (individuals) is processed until the termination condition is satisfied. The GEA works similarly to the classic evolutionary algorithm [9], but special selection procedure, crossover and mutation operators are introduced to deal with granular genes [11].

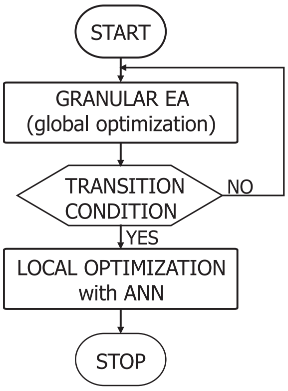

Fig. 1. Block diagram of the TSGS

Selection is based on the idea of tournament selection and allows comparison of two (or more) granular individuals. Granular arithmetical crossover and granular Gaussian mutation operators are introduced to create new individuals, being possible solutions of the identification problem. To calculate the granular fitness function it is necessary to solve the direct boundary–value problem for laminate structures. The Finite Element Method in granular form is used for calculation of direct problem for laminates [8], [10].

Each individual is made up of genes representing identified laminates' constants. Each gene x_i^j of the individual $\mathbf{x}^j$ consists of:

- for interval numbers: 2 values, representing edges of the interval:

$$x_i^j = \left[a(x_i^j), b(x_i^j)\right]; \tag{5}$$

- for fuzzy numbers: 4 values, representing edges of lower (L) and upper (U) α–cuts of the trapezoidal fuzzy number:

$$x_i^j = \left[a_L(x_i^j), b_L(x_i^j) a_U(x_i^j), b_U(x_i^j);\right] \tag{6}$$

- for random variables: 2 values, representing mean value m and standard deviation σ of the random variable:

$$x_i^j = \left[m(x_i^j), \sigma(x_i^j)\right]. \tag{7}$$

Due to the way in which the evolutionary algorithm works, it is usually possible to find the vicinity of the global optimum, though one may have problems with finding its precise value. To reduce this inconvenience, the TSGS switches to the local optimization stage after satisfying the transition condition. In this stage a special multi-level Artificial Neural Network (ANN) is used to perform the local optimization by means of the steepest descent method [5]. The back-propagation neural network with the sigmoid transition functions is applied.

The number of ANN levels depends on the number of values in each gene from the previous stage, e.g. 4 levels for fuzzy representation of genes. The number of inputs in each level is equal to the number of identified constants. The fitness function values in the granular form (interval, fuzzy or stochastic) are obtained as the output of ANN. As the fitness function is modelled closely to the optimum by the parabolic function for each design variable, only one hidden layer is necessary for the network. The number of neurons in the hidden layer depends on the degree of the difficulty of the fitness function.

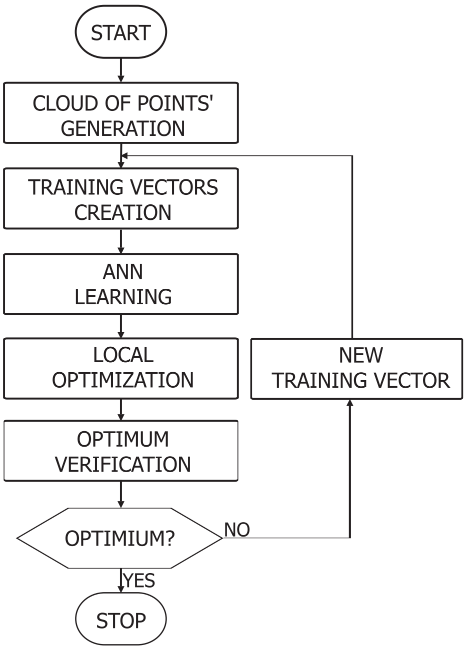

Fig. 2. Block diagram of the local identification stage

ANN is also employed for approximation of the fitness function sensitivity. The best individuals from the first stage are used as the training vectors. After training of the multilevel ANN, the local optimization process is performed. If the termination condition is not fulfilled, the considered point is re-calculated using granular version of Finite Element Method and added to the training vector set (Fig. 2). Otherwise, the considered point is treated as a result of the identification process.

4. Numerical examples

4.1. Identification of simple laminate

A rectangular simple laminate plate of dimensions presented in Fig. 3a is considered. The aim of the identification is to find four elastic constants for the laminate plate.

Each ply has the same thickness $h_i = 0.002$ m. The stacking sequence of the laminate is (0/45/90/-45/0/90/0/90)s, where s denotes symmetry. To solve the direct problem the plate is divided into 200 4-node plane finite elements. Each chromosome $ch_j(x)$ of the population consists of 4 genes x_i, which have the form depending on the assumed granularity variant. The interval and stochastic granularity cases are considered for simple laminates.

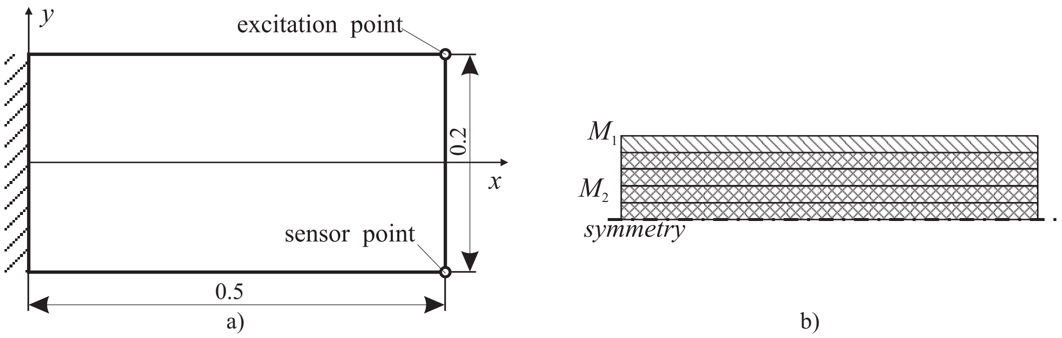

Fig. 3. The laminate plate: a) location of excitation and sensor points; b) materials location in hybrid laminate

In order to collect the measurements, the frequency response diagram is obtained in the interval case. The plate is excited in one point by the the sinusoidal signal (Fig. 3a). The frequency of the excitation varies from 100Hz to 2000Hz. 200 samples of the acceleration amplitudes at one sensor point are measured. The first 10 eigenfrequencies ω_i of the plate are the measurement data in stochastic case. In this case, measurements were repeated 200 times to collect the necessary data.

The parameters of GEA are: the number of chromosomes $n_{ch} = 100$ (interval case) or $n_{ch} = 200$ (stochastic case), the number of generations $n_g = 100$ (interval case) or $n_g = 400$ (stochastic case), the arithmetic crossover probability $p_{ac} = 0.2$, the Gaussian mutation probability $p_{gm} = 0.4$. The number of iterations in the second stage of TSGS is assumed to be 1000 in interval case or 1800 in stochastic case. The variable ranges, actual values and identification results are collected in Tables 1 and 2 for interval and stochastic cases.

Table 1. Simple laminate identification results: interval numbers

		Min	Max	Actual	Stage 1	Stage 2
E_1 [Pa]	a	2.00E10	5.00E10	3.82E10	3.84E10	3.82E10
	b	2.00E10	5.00E10	3.90E10	3.93E10	3.90E10
E_2 [Pa]	a	4.20E09	10.80E9	8.23E09	7.93E09	8.23E09
	b	4.20E09	10.80E9	8.31E09	8.36E09	8.31E09
ν_{12}	a	0.20	0.40	0.257	0.249	0.257
	b	0.20	0.40	0.263	0.265	0.263
G_{12} [Pa]	a	1.00E09	8.00E09	4.10E09	4.11E09	4.10E09
	b	1.00E09	8.00E09	4.18E09	4.17E09	4.18E09

Table 2. Simple laminate identification results: random variables

		Min	Max	Actual	Stage 1	Stage 2
E_1 [Pa]	m	2.00E10	6.00E10	3.86E10	3.92E10	3.86E10
	σ	0.00E9	0.30E9	0.12E9	0.11E9	0.12E9
E_2 [Pa]	m	4.00E9	9.00E9	8.28E9	8.14E9	8.28E9
	σ	0.00E9	0.30E9	0.20E9	0.17E9	0.20E9
ν_{12}	m	0.00	0.50	0.26	0.27	0.26
	σ	0.00	0.10	0.02	0.04	0.02
G_{12} [Pa]	m	2.00E9	6.00E9	4.14E9	4.07E9	4.14E9
	σ	0.10E8	0.70E8	0.50E8	0.22E8	0.50E8

4.2. Identification of hybrid laminate

A hybrid, symmetrical laminate plate of the same shape and dimensions like in a simple laminate case is considered. The aim of the identification is to find four elastic constants and the density for each of the two materials in the laminate.

Table 3. Hybrid laminate identification results: fuzzy numbers, material M_1 (outer)

		Min	Max	Actual	Stage 1	Stage 2
E_1 (Pa)	a_L	1.50E10	2.52E11	1.78E11	1.72E11	1.78E11
	b_L	1.50E10	2.52E11	1.84E11	1.83E11	1.84E11
	a_H	1.52E10	2.50E11	1.80E11	1.68E11	1.80E11
	b_H	1.52E10	2.50E11	1.82E11	1.85E11	1.82E11
E_2 [Pa]	a_L	4.20E09	2.60E10	9.50E09	9.87E09	9.50E09
	b_L	4.20E09	2.60E10	1.11E10	1.04E10	1.11E10
	a_H	4.25E09	2.58E10	9.80E09	9.74E09	9.80E09
	b_H	4.25E09	2.58E10	1.08E10	9.86E09	1.08E10
ν_{12}	a_L	0.19	0.41	0.275	0.273	0.275
	b_L	0.19	0.41	0.285	0.284	0.285
	a_H	0.21	0.39	0.277	0.249	0.277
	b_H	0.21	0.39	0.283	0.281	0.283
G_{12} [Pa]	a_L	9.70E08	8.05E09	7.08E09	7.11E09	7.08E09
	b_L	9.70E08	8.05E09	7.20E09	7.33E09	7.20E09
	a_H	9.75E08	8.00E09	7.10E09	7.00E09	7.10E09
	b_H	9.75E08	8.00E09	7.18E09	7.43E09	7.18E09
ρ [kg/m^3]	a_L	9.70E02	3.03E03	1.55E03	1.58E03	1.55E03
	b_L	9.70E02	3.03E03	1.70E03	1.62E03	1.70E03
	a_H	9.75E02	3.00E03	1.60E03	1.60E03	1.60E03
	b_H	9.75E02	3.00E03	1.65E03	1.69E03	1.65E03

Table 4. Hybrid laminate identification results: fuzzy numbers, material M_2 (inner)

		Min	Max	Actual	Stage 1	Stage 2
E_1 [Pa]	a_L	1.50E10	1.50E11	3.82E10	3.74E10	3.82E10
	b_L	1.50E10	1.50E11	3.90E10	4.12E10	3.90E10
	a_H	1.52E10	1.51E11	3.84E10	3.79E10	3.84E10
	b_H	1.52E10	1.51E11	3.88E10	3.92E10	3.88E10
E_2 [Pa]	a_L	4.00E09	1.60E10	8.23E09	8.21E09	8.23E09
	b_L	4.00E09	1.60E10	8.31E09	8.22E09	8.31E09
	a_H	4.20E09	1.58E10	8.25E09	8.28E09	8.25E09
	b_H	4.20E09	1.58E10	8.29E09	8.23E09	8.29E09
ν_{12}	a_L	0.19	0.41	0.257	0.273	0.257
	b_L	0.19	0.41	0.263	0.278	0.263
	a_H	0.21	0.39	0.259	0.249	0.259
	b_H	0.21	0.39	0.261	0.272	0.261
G_{12} [Pa]	a_L	9.70E08	8.05E09	4.10E09	4.01E09	4.10E09
	b_L	9.70E08	8.05E09	4.18E09	4.54E09	4.18E09
	a_H	9.75E08	8.00E09	4.12E09	4.21E09	4.12E09
	b_H	9.75E08	8.00E09	4.16E09	4.35E09	4.16E09
ρ [kg/m^3]	a_L	9.70E02	3.03E03	1.78E03	1.85E03	1.78E03
	b_L	9.70E02	3.03E03	1.83E03	1.89E03	1.83E03
	a_H	9.75E02	3.00E03	1.78E03	1.81E03	1.78E03
	b_H	9.75E02	3.00E03	1.83E03	1.84E03	1.83E03

The stacking sequence of the laminate is: (0/15/-15/45/-45)s. The external and internal plies of the laminate are made of materials M_1 and M_2, respectively (Fig. 3b). Each ply has the same thickness $h_i = 0.002$ m. The first 10 eigenfrequencies of the plate are considered as the measurements. It is assumed that measurements are not precise and can be modeled by means of fuzzy numbers. The plate is divided into 200 4-node plane finite elements to solve the direct problem.

Each chromosome consists of 10 genes representing identified constants. The parameters of GEA are: the number of chromosomes $n_{ch} = 200$, the number of generations $n_g = 400$, the arithmetic crossover probability $p_{ac} = 0.2$, the Gaussian mutation probability p_{gm}=0.4. The number of iterations in the second stage of TSGS is equal to 2500. The variable ranges, actual values and identification results are collected in Tab. 3 for material M_1 and in Tab. 4 for material M_2.

5. Final conclusions

The Two-Stage Granular Strategy applied for the identification of material constants in laminates is presented. The strategy works with different models of information granularity. In the present paper, interval numbers, fuzzy numbers and random variables are used, but it is also possible to incorporate other granularity models, like approximate sets or fuzzy random variables.

The granular version of the evolutionary algorithm is used in the first stage to perform the global optimization. Gradient-based optimization method supported by special multilevel artificial neural network is employed in the second stage. The Finite Element Method in granular form is used to solve the direct problems for laminates. The modal analysis techniques are employed to collect the measurement data necessary for the identification.

A few numerical examples show the efficiency of such a hybrid method for identification of simple and hybrid laminates. Application of the strategy is not limited to the presented problems and authors hope that TSGS can be applied in various identification and optimization tasks with non–precise parameters.

Acknowledgement

The research is financed from the Polish science budget resources in the years 2008-2011 as the research project.

References

[1] S. Adali, A. Richter, V. E. Verijenko and E. B. Summers. Optimal design of symmetric hybrid laminates with discrete ply angles for maximum buckling load and minimum cost. *Composite Structures* **32**: 409–415, 1995.
[2] S. Adali, V.E. Verijenko. Optimum stacking sequence design of symmetric hybrid laminates undergoing free vibrations. *Composite Structures*, **54**: 131-138, 2001.
[3] A. Bargiela, W. Pedrycz. *Granular Computing: An introduction.* Kluwer, Boston/Dordrecht/London, 2002.
[4] W. Beluch. Evolutionary Identification and Optimization of Composite Structures. *Mechanics of Advanced Materials and Structures*, **14**(8): 677-686, 2007.
[5] T. Burczyński, P. Orantek. The evolutionary algorithm in stochastic optimization and identification problems. In: Arabas, J. (Ed.) *Evolutionary Computation and Global Optimization 2007*, pp. 309-320, Warsaw, 2006.
[6] T. Burczyński, P. Orantek. Uncertain Identification Problems in the Context of Granular Computing. In: A. Bargiela, W. Pedrycz (Eds.), *Human-Centric Inforamtion Processing, SCI182*, pp. 329-350, Springer Verlag, Berlin Heidelberg, 2009.
[7] J. German. *The basics of the fibre-reinforced composites' mechanics* [in Polish]. Publ. of the Cracow University of Technology, Cracow, 2001.
[8] M. Kleiber, T. Hien. *The Stochastic Finite Element Method.*. John Wiley & Sons, New York, 1992.
[9] Z. Michalewicz. *Genetic Algorithms + Data Structures = Evolutionary Programs.* Springer, Berlin, 1996.
[10] D. Moens, D. Vandepitte. Fuzzy Finite Element Method for Frequency Response Function Analysis of Uncertain Structures. *AIAA Journal*, **40**(1): 126-136, 2002.
[11] P. Orantek. The optimization and identification problems ofstructures with fuzzy parameters. *3rd European Conference on Computational Mechanics ECCM-2006*, CD-Edition, Lisbon, 2006.

[12] T. Uhl. *Computer-Aided Identification of Constructional Models* [in Polish]. WNT, Warsaw, 1997.