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Combined time-stepping and Trefftz approach for nonlinear wave-structure interaction

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Generally, two approaches have been used to study the nonlinear wave-structure interaction in the context of offshore engineering in recent years. One is based on the Stokes perturbation procedure in the frequency domain and has been applied to weak-nonlinear problems. The other is based on a full nonlinear solution to the resulting wave field by a time-stepping procedure with boundary conditions applied on the moving free and body surfaces. In the present work an alternative solution method for nonlinear wave-structure interaction problems is proposed. The method is based on the evolution equations for the free-surface elevation and the free-surface potential, which are solved by the time-stepping procedure. The field problem is solved at each time step by the perturbation method combined with Trefftz approach. The method is applied to the study of the evolution of three-dimensional waves generated by a vertical circular cylinder oscillating in water of constant depth.

1. Introduction

The development of solution methods for hydrodynamic problems with nonlinear boundary conditions is essential for the understanding of the interaction of steep gravity waves with marine or offshore engineering structures. A literature survey indicates that, in general, two approaches have been used to study the nonlinear wave-structure interaction in recent years. One is based on the Stokes perturbation procedure in the frequency domain and has been applied to weak-nonlinear problems. The obtained solutions are correct to second order in the wave slope (see e.g. [3, 14, 19, 20, 34]) and for some aspects even third-order effects are analysed (see e.g. [6] and [24]). The other is based on a full nonlinear solution to the resulting wave field by a time-stepping procedure, in which the free-surface boundary conditions and the body boundary conditions are applied at the instantaneous free and body surfaces. The field problem is generated and solved by a direct BEM at each time step as both the free surface and the body move to the new positions (see e.g. [4, 7, 23, 38, 39]).

In most of the works based on a time-stepping procedure, the mixed Eulerian–Lagrangian approach is utilized. Its computational capabilities for two-dimensional problems are, since the seminal work of Longuet-Higgins and Cokelet (1976), now well established. However, the implementation of the mixed Eulerian-Lagrangian method in three dimensions is much more difficult. Reliable results for fully nonlinear problems are still rare and concern either simplified geometries [2, 31] or wave propagation without wave-body interaction [1, 39]. In the paper by Boo et al. a concept of a numerical wave tank has been developed while the study by Xü and Yue concerns the simulation of 3D deep-water overturning waves. A key to success in their simulations is the development of an efficient high-order BEM based on bi-quadratic curvilinear panel elements. However, the method is very CPU-time consuming and can hardly be applied to practical engineering problems. The total CPU-time required for a fundamental study of an overturning periodic wave amounts to 10 hours

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on a Cray Y/MP. According to Xü and Yue, the quantitative simulations of fully nonlinear wave-body interactions in 3D by their method would require about 100 hours of CPU-time on a Cray Y/MP. Thus, useful quantitative 3D fully-nonlinear simulations are generally feasible, nevertheless still inpractical. On the other hand, the less time consuming BEMs based on linear isoparametric elements have not proved to be completely succsessful. Indeed, the converged solutions in 3D have only been obtained for weak nonlinearities [7].

One can observe two research directions trying to overcome these difficulties. Firstly, the higher-order boundary element methods are developed, where B-spline functions are used to represent the geometry and potential [21]. Secondly, the so-called multipole accelerated, desingularized methods [32] may be applied. The use of B-splines offers the possibility of a continuous representation of the velocity potential and its derivatives. The description of the geometry can be practically exact. This improves considerably numerical efficiency.

In the desingularized method with multipole acceleration, the boundary integral equation is discretized by distributing fundamental singularities over an integration surface, which is slightly moved off the domain boundary (control surface). As a result, the kernel is nonsingular, so that no special treatment is needed when evaluating the integrals and simple numerical quadratures may be used. Additionally, the efficiency of an iterative solver can be improved significantly when accelerated by a multipole algorithm. Solutions obtained using multipole acceleration require O(N) effort and O(N) storage, whereas conventional methods require $O(N^2)$ effort and storage.

In the present work an alternative solution method for nonlinear wave-structure interaction problems is proposed. The method is based on the evolution equations formulated in the Eulerian description for the free-surface elevation and the free-surface potential, which are solved by time-stepping procedure. The field problem is solved at each time step by the perturbation approach. This yields a sequence of linear boundary value problems, which have to be solved at increasing order.

Such a methodology has already been applied in [5, 22] and [30] to study the interaction amoung propagating periodic gravity waves, and the diffraction of Stokes waves by a submerged circular cylinder. The considered problems were two-dimensional and periodic in space. The velocity potential was expanded in a Taylor series about the mean water level and substituted into free-surface boundary condition. This resulted in a sequence of linear boundary value problems for the components of the total velocity potential. In the absence of a body piercing the free-surface, the Fourier transformation could be utilized and the so-called high-order spectral method was developed to solve the sequence of linear boundary value problems.

In the present paper the much more complicated problem of three-dimensional radiating water waves due to forced oscillations of a free-surface piercing body is considered. In order to utilize the perturbation procedure, the free-surface potential and the velocity potential are expanded in Taylor series about the mean water level and about the position of the body at rest, respectively. This yields a sequence of linear boundary value problems, which are then solved by the Trefftz method with the use of complete systems of solutions. The boundary conditions are satisfied in the least-square sense by minimizing an error functional.

Although the order of the perturbation procedure may be much higher in numerical implementation than in analytical approach, it places the limit on the steepness of the free-surface elevation one can consider. Therefore, in practice, the method is suitable for the investigation of wave-structure interaction problems with moderate nonlinearities. Nevertheless, it allows the study of the evolution of two- or three-dimensional waves generated by an oscillating body in fluid. It can also be used to investigate the stability of steady travelling waves to small perturbations. Although the present formulation is confined to wave radiation problems due to forced oscillations of a body, it can also be extended to account for freely floating structures.

In the following section, the mathematical formulation of a three-dimensional radiation problem for water waves, together with an outline of the proposed solution method, is given. Then, the solution procedure for the obtained boundary value problem is presented. Approximate solutions are expressed by the series based on complete sets of functions and the least-squares technique of matching boundary conditions is used to calculate the unknown expansion coefficients. The next section deals with the implementation of the developed solution method to the radiation of nonlinear water waves due to the forced oscillations of a circular cylinder standing on the bottom in water of constant depth. Finally, preliminary results of computations are presented and discussed.

2. Mathematical formulation

Consider the radiation of nonlinear gravity waves in a fluid of constant depth h due to the forced oscillatory motion of a free-surface piercing body resting on the bottom. The origin of a fixed coordinate system is located at the undisturbed free-surface and the vertical z–axis is positive upward (see Figure 1).

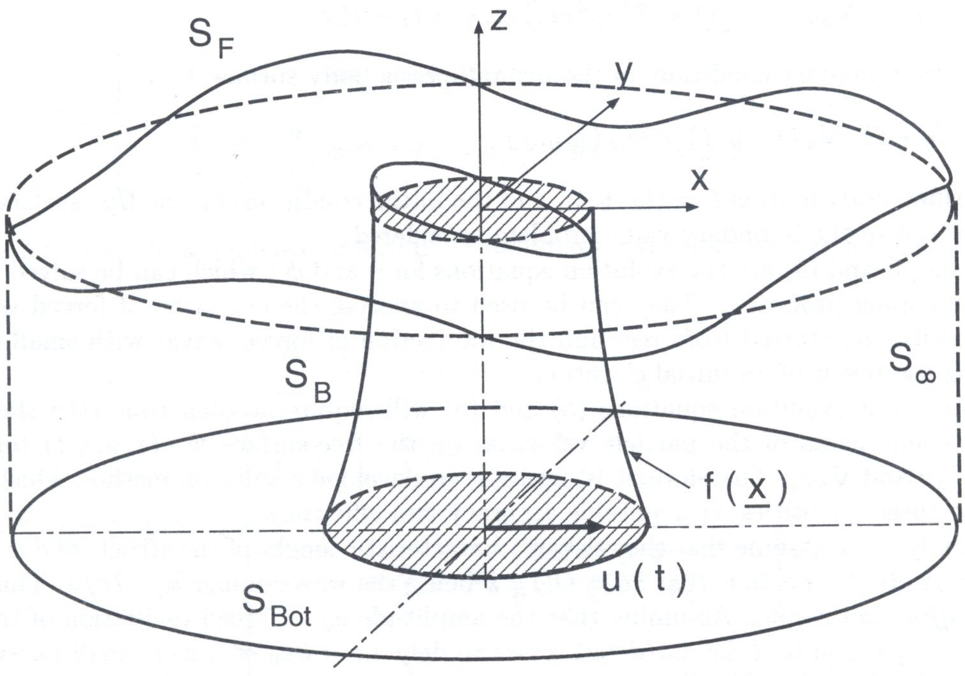

Fig. 1. Definition sketch

Under the assumption that the flow is irrotational and the fluid incompressible, there exists a velocity potential $\Phi(x, y, z, t)$ which satisfies the Laplace equation in the fluid domain Ω with the following boundary conditions:

– on the free-surface $z = \eta(x, y, t)$:

$$\eta_{,t} + \nabla_s\eta \cdot \nabla_s\Phi = \Phi_{,z} \;, \tag{1}$$

$$gz + \Phi_{,t} + \frac{1}{2}|\nabla\Phi|^2 = -P_a/\rho \;, \tag{2}$$

– on the instantaneous body surface $z = f(x, y, t)$:

$$f_{,t} + \nabla_s f \cdot \nabla_s\Phi = \Phi_{,z} \;, \tag{3}$$

– on the impermeable bottom $z = -h$:

$$\Phi_{,z} = 0 \;. \tag{4}$$

In the above formulae, subscripts after a comma denote the differentation with respect to the proper coordinate, ∇_s denotes the nabla operator with respect to horizontal (surface) coordinates, ρ is water density, g is the gravitational acceleration and P_a is the pressure on the free-surface. Moreover $f(x,y,t)$ will be consider as a sum $f(x,y,t) = f_0(x,y) + \xi(x,y,t)$ where $f_0(x,y)$ describes the rest position of the body.

Following Dommmermuth and Yue [5], we define the free surface potential

$$\phi_s(x,y,t) = \Phi(x,y,\eta(x,y,t),t) \ ,$$

where the free-surface is assumed to be continuous and single-valued. In terms of ϕ_s, the boundary conditions on the free-surface are respectively

$$\eta_{,t} = -\nabla_s\eta \cdot \nabla_s\phi_s + (1 + |\nabla_s\eta|^2)\Phi_{,z}(x,y,\eta,t) \ , \tag{5}$$

$$\phi_{s,t} = -g\eta - \tfrac{1}{2}|\nabla_s\phi_s|^2 + \tfrac{1}{2}(1 + |\nabla_s\eta|^2)\Phi_{,z}^2(x,y,\eta,t) - P_a/\rho \ . \tag{6}$$

The kinematic boundary condition on the instantaneous body surface

$$\xi_{,t} + \nabla_s(f_0 + \xi) \cdot \nabla_s\Phi(x,y,f) = \Phi_{,z}(x,y,f) \ , \tag{7}$$

with prescribed body motion $\xi = \xi(x,y,t)$, and the initial conditions on the free-surface $\phi_s(x,y,0)$, $\eta(x,y,0)$ complete the boundary value problem considered.

Equations (5) and (6) are the evolution equations for η and ϕ_s, which can be solved numerically by a time-stepping procedure. They can be used to analyse the evolution of forced waves due to the body oscillation started from rest and the interaction of forced waves with small disturbance waves being the result of an initial elevation.

The solution of evolution equations (5) and (6) will require at each time step the knowledge of vertical components of the particle velocities on the free-surface $\Phi_{,z}(x,y,\eta,t)$ together with gradients $\nabla_s\eta$ and $\nabla_s\phi_s$. Therefore, it is desirable to develope a solution method which enables us to calculate these quantities with suitable accuracy and efficiency.

In our analysis we assume that the relevant characteristic lenght of the structure d is comparable to the wavelength λ, and thus that $kd = O(1)$, k being the wavenumber $k = 2\pi/\lambda$. This is referred to as the *diffraction regime*. Assuming that the amplitude u_0 of forced oscillation of the structure is small in comparison to d, i.e. $u_0/d \ll 1$, one can define $\varepsilon = ku_0 \ll 1$ as a small parameter.

In order to solve the field problem at a time step t_i, it is assumed that Φ, η and ξ are $O(\varepsilon)$ quantities. Then, the velocity potential is expressed as a perturbation series up to a given order M

$$\Phi(x,y,z,t_i) = \sum_{m=1}^{M} \Phi^{(m)}(x,y,z,t_i) \ . \tag{8}$$

Here and hereinafter, $(\ )^{(m)}$ denotes a quantity of $O(\varepsilon^m)$.

Further, we expand each $\Phi^{(m)}$ evaluated on $z = \eta$ in a Taylor series about $z = 0$ and obtain

$$\phi_s(x,y,t_i) = \Phi(x,y,\eta,t_i) = \sum_{m=1}^{M} \sum_{k=0}^{M-m} \frac{\eta^k}{k!} \frac{\partial^k}{\partial z^k} \Phi^{(m)}(x,y,0,t_i) \ . \tag{9}$$

Similarly, one can apply to $\Phi^{(m)}$ evaluated on $z = f(x,y,t)$ a Taylor series expansion about $z = f_0(x,y)$

$$\Phi(x,y,f,t_i) = \sum_{m=1}^{M} \sum_{k=0}^{M-m} \frac{\xi^k}{k!} \frac{\partial^k}{\partial z^k} \Phi^{(m)}(x,y,f_0,t_i) \ . \tag{10}$$

Substituting equation (10) into boundary condition (7) one obtains

$$\xi_{,t}(\underline{x},t_i)+\nabla_s(f_0+\xi)\cdot\nabla_s\left\{\sum_{m=1}^{M}\sum_{k=0}^{M-m}\frac{\xi^k}{k!}\frac{\partial^k}{\partial z^k}\Phi^{(m)}(x,y,f_0,t_i)\right\}$$

$$=\sum_{m=1}^{M}\sum_{k=0}^{M-m}\frac{\xi^k}{k!}\frac{\partial^{k+1}}{\partial z^{k+1}}\Phi^{(m)}(x,y,f_0,t_i)\,. \tag{11}$$

At a given instant of time, ϕ_s, η, ξ and $\xi_{,t}$ may be considered as known, being prescribed at the collocation points (x_j, y_j, z_j) on the undisturbed free-surface and on the wetted surface of the body. Hence, equations (9) and (11) may be viewed as the boundary conditions for the unknown velocity potentials $\Phi^{(m)}$.

Collecting terms at each order in equations (9) and (11) one obtains a sequence of Dirichlet boundary conditions on the undisturbed free-surface $z=0$

$$\Phi^{(m)}(x,y,0,t_i)=\begin{cases}\phi_s(x,y,t_i) & \text{for}\quad m=1\,,\\ -\displaystyle\sum_{k=1}^{m-1}\frac{\eta^k}{k!}\frac{\partial^k}{\partial z^k}\Phi^{(m-k)}(x,y,0,t_i) & \text{for}\quad m=2,\,3,\,...,\,M.\end{cases} \tag{12}$$

Similar procedure leads to a sequence of Neumann boundary conditions on the body surface at rest $z=f_0(x,y)$

$$\frac{\partial}{\partial z}\Phi^{(m)}-\nabla_s f_0\cdot\nabla_s\Phi^{(m)}=\begin{cases}\dfrac{\partial\xi}{\partial t}\quad\text{for}\quad m=1\,,\\[2ex] \nabla_s f_0\cdot\nabla_s\left\{\displaystyle\sum_{k=1}^{m-1}\frac{\xi^k}{k!}\frac{\partial^k\Phi^{(m-k)}}{\partial z^k}\right\}\\[2ex] \qquad+\nabla_s\xi\cdot\nabla_s\left\{\displaystyle\sum_{k=0}^{m-2}\frac{\xi^k}{k!}\frac{\partial^k\Phi^{(m-k-1)}}{\partial z^k}\right\}\\[2ex] \qquad-\displaystyle\sum_{k=1}^{m-1}\frac{\xi^k}{k!}\frac{\partial^{k+1}\Phi^{(m-k)}}{\partial z^{k+1}}\qquad\text{for}\quad m=2,\,3,\,...,\,M.\end{cases} \tag{13}$$

Boundary conditions (12) and (13) can be rewritten as follows

$$\Phi^{(m)}\quad=u^{(m)}\quad\text{on}\quad z=0\,, \tag{14}$$

$$\frac{\partial\Phi^{(m)}}{\partial n}=v^{(m)}\quad\text{on}\quad z=f_0(x,y)\,,\quad m=1,2,...,M\,, \tag{15}$$

where $u^{(m)}$ and $v^{(m)}$ stand for the right hand sides of equations (12) and (13) divided by $(1+(\partial f_0/\partial x)^2+(\partial f_0/\partial y)^2)^{1/2}$.

Equations (14) and (15), together with Laplace equation and bottom boundary condition (4) define a sequence of linear boundary value problems for $\Phi^{(m)}$ in the domain $\Omega=\{z:\ -h\le z\le 0\cap z\ge f_0(x,y)\}$. These problems ought to be solved successively at increasing order. After this has been done, the vertical velocities of free-surface particles can be computed from

$$\Phi_{,z}(x,y,\eta,t_i)=\sum_{m=1}^{M}\sum_{k=0}^{M-m}\frac{\eta^k}{k!}\frac{\partial^{k+1}}{\partial z^{k+1}}\Phi^{(m)}(x,y,0,t_i)\,. \tag{16}$$

3. Solution of boundary value problems by Trefftz method

In order to solve the sequence of boundary value problems defined in the preceding section, one can use any suitable numerical method available for linearized wave-radiation problems (see e.g. [29]). However, in contrast to the standard formulation, the linear boundary value problems under consideration are characterized by an inhomogeneous Dirichlet boundary condition on the undisturbed free surface which extends to infinity. To make matters worse, the function prescribed at any time-step on the free surface may decay very slowly in space. In view of these features, the use of the standard solution methods like FEM, BEM, and hybrid methods is ineffective. So, it is desirable to seek for alternative solution methods, which might prove accurate and efficient enough for the time-stepping procedure. In the present work we try to use the Trefftz approach to solve the linear boundary value problems at every time-step. We expect it may prove superior in comparison to standard FEMs and BEMs.

The main idea of the Trefftz method is to approximate the exact solution of a boundary value problem by a series based on complete sets of functions which satisfy exactly the governing differential equation, but do not necesserily satisfy the prescribed boundary conditions.

Since the pioneering work of Trefftz [37], the method has undergone a long period of development and modifications. The main progress has been achieved during the last twenty years, which have been marked with such milestones like:

- formulation of the criterion of c-completeness with respect to the metric of suitable spaces of boundary values ([9, 10, 11]),
- development of complete systems of solutions ([8, 12, 13]),
- the application of complete systems to the finite element method ([15, 16, 17, 18, 42]).

The Trefftz method has been used in many fields (for detailed survey see e.g. [33]). However, there exists only a few examples of using it in wave-structure interaction problems (see [25, 26, 27, 28, 36]).

Following Trefftz's idea we express an approximate solution of the boundary value problem of order m in the form

$$\Phi^{(m)}(x,y,z,t_i) = \sum_{n=1}^{N_m} a_n^{(m)}(t_i)\, T_n^{(m)}(x,y,z)\ , \tag{17}$$

where different sets of solution functions $\{T_n^{(m)}\}$ may be used for each order of approximation. The bases $\{T_n^{(m)}\}$ satisfy the Laplace equation and additionally the bottom boundary condition (4). On the far field surface at infinity (S_∞) the boundary condition is

$$\nabla\Phi^{(m)} \to 0 \quad \text{as} \quad |r| = (x^2+y^2+z^2)^{1/2} \to \infty\ .$$

Due to the fact that only a finite amount of the free-surface can be modelled, this far field condition is usually treated by either an approximate radiation condition, an absorbing beach or it is simply ignored and the boundary is left open. In our work, the fluid domain is bounded by a circular cylinder S_∞ (see Figure 1) on which either a solid boundary condition

$$\frac{\partial\Phi^{(m)}}{\partial n} = 0 \tag{18}$$

or an approximation for an open boundary

$$\Phi^{(m)} = 0 \tag{19}$$

may be imposed. The latter is valid provided that S_∞ is taken to be sufficiently large so that no wave disturbances reach S_∞. Alternatively, one might consider a time-dependent Sommerfeld type radiation condition (see [40]).

Under these assumptions, the boundary value problem has been reduced to satisfying the boundary conditions on the immersed body surface $z = f_0(x,y)$, on the undisturbed free-surface $z = 0$ and on the far-field boundary S_∞. There are many ways of fitting boundary conditions in the Trefftz method (for survey see [41]). In the present work we apply direct least-squares fitting by minimizing the following error functional

$$J(\Phi^{(m)}) = \int_{S_F} \left|\Phi^{(m)} - u^{(m)}\right|^2 \mathrm{d}S + w^2 \int_{S_B} \left|\frac{\partial \Phi^{(m)}}{\partial n} - v^{(m)}\right|^2 \mathrm{d}S + \int_{S_\infty} \Psi\left(\Phi^{(m)}, \frac{\partial \Phi^{(m)}}{\partial n}\right) \mathrm{d}S\,, \tag{20}$$

where the choice of the integrand $\Psi(\cdot)$ in the third integral depends on the far-field boundary condition considered. Accordingly to equation (18), $\Psi(\cdot) = |\Phi^{(m)}|^2$ for an open boundary whereas in the case of a solid boundary $\Psi(\cdot) = w^2 \left|\dfrac{\partial \Phi^{(m)}}{\partial n}\right|^2$. The integrals with partial derivatives of the potential function are multiplied by some positive weight w^2, which has to be properly chosen.

Equating the first variation of the functional (20) to zero yields the following system of linear algebraic equations

$$\mathbf{K}^{(m)} \cdot \underline{a}^{(m)} = \underline{R}^{(m)} \tag{21}$$

for the unknown expansion coefficients $a_n^{(m)}$ of the approximate solution (17).

The elements of the matrices $\mathbf{K}^{(m)}$ and $\underline{R}^{(m)}$ are

$$K_{ij}^{(m)} = \int_{S_F} T_i^{(m)} T_j^{(m)} \mathrm{d}S + w^2 \int_{S_B} \frac{\partial T_i^{(m)}}{\partial n}\frac{\partial T_j^{(m)}}{\partial n} \mathrm{d}S + \int_{S_\infty} \Gamma(T^{(m)}, \frac{\partial T^{(m)}}{\partial n})\, \mathrm{d}S\,, \tag{22}$$

$$R_i^{(m)} = \int_{S_F} T_i^{(m)} u^{(m)} \mathrm{d}S + w^2 \int_{S_B} \frac{\partial T_i^{(m)}}{\partial n} v^{(m)} \mathrm{d}S\,. \tag{23}$$

Here, like in equation (20), the integrand $\Gamma(\cdot)$ depends on the far-field boundary considered. For an open boundary $\Gamma(\cdot) = T_i^{(m)} T_j^{(m)}$. For a solid boundary $\Gamma(\cdot) = w^2 \dfrac{\partial T_i^{(m)}}{\partial n}\dfrac{\partial T_j^{(m)}}{\partial n}$. The matrix $\mathbf{K}^{(m)}$ is real and symmetric. If one chose a set of solution functions $\{T_n\}$ to be independent of the order m of approximation, the matrix $\mathbf{K}^{(m)}$ would be computed and factorized only once for the whole sequence of boundary value problems. Nevertheless, it is only the right-hand-side vector $\underline{R}^{(m)}$ that is updated during the time-stepping procedure.

It should be mentioned here that the above formulation of the least-squares procedure holds only for the global approximation, in which one set of T-complete functions is used in the whole fluid region. It might sometimes be necessary to divide the fluid domain into subdomains and to approximate the solution in each subdomain separately. This would lead to the formulations of the so-called finite least-squares (or frameless) T-elements which have recently been applied to two-dimensional wave-diffraction problems (see [33]). In the present work, however, this approach will not be considered.

After the expansion coefficients $a_n^{(m)}$ have been determined, the vertical velocities of the particles on the free surface can be calculated analytically

$$\Phi_{,z}(x,y,\eta,t_i) = \sum_{m=1}^{M} \sum_{k=0}^{M-m} \frac{\eta^k}{k!} \sum_{n=1}^{N_m} a_n^{(m)}(t_i) \frac{\partial^{k+1}}{\partial z^{k+1}} T_n^{(m)}(x,y,0) \tag{24}$$

at the collocation points $(x_j, y_j, 0)$ on the undisturbed free-surface. Unfortunately, the calculation of the gradients $\nabla_s \eta$ and $\nabla_s \phi_s$ is much more complicated.

Applying a gradient operator to relation (9) yields

$$\nabla_s \phi_s = \sum_{m=1}^{M} \nabla_s \Phi^{(m)} + \nabla_s \eta \sum_{m=1}^{M} \sum_{k=1}^{M-m} \frac{\eta^{k-1}}{(k-1)!} \frac{\partial^k \Phi^{(m)}}{\partial z^k} + \sum_{m=1}^{M} \sum_{k=1}^{M-m} \frac{\eta^k}{k!} \frac{\partial^k}{\partial z^k} \nabla_s \Phi^{(m)}\,, \tag{25}$$

where all the functions and derivatives have to be calculated at collocation points on the undisturbed free-surface at time $t = t_i$. This can be done analytically for $\nabla_s \Phi^{(m)}$

$$\nabla_s \Phi^{(m)}(x_j, y_j, 0, t_i) = \sum_{n=1}^{N_m} a_n^{(m)} \nabla_s T_n^{(m)}(x_j, y_j, 0) \, , \tag{26}$$

but not for $\nabla_s \eta$, since no analytical expression for $z = \eta(x, y, t_i)$ is avaluable.

In order to calculate $\nabla_s \eta$ one can either interpolate $\eta(x, y, t_i)$ among neighbouring collocation points or use the Fourier transform to calculate the derivatives at collocation points directly. In the present work, the mixed approach for axisymmetric problems is applied, i.e. the derivatives with respect to the radial coordinate r are approximated by central differences whereas the FFT technique is used for the derivatives with respect to the angular coordinate θ.

After $\nabla_s \eta$, ∇_s and $\Phi_{,z}$ have been determined for $t = t_i$, the free-surface elevation for the next time-step can be calculated via numerical integration of evolution equations (5) and (6).

4. Radiation of water waves by an oscillating circular cylinder

In this section we implement the developed solution method to the radiation of water waves by a circular cylinder (radius R) standing on the bottom in water of constant depth h. The fluid domain is bounded by a circular cylinder (radius R_∞) on which one of the far-field boundary conditions (18) and (19) is imposed (see Figure 2). The problem seems to be simple enough, however, to the authors' knowledge, neither a full-nonlinear solution nor the complete solution correct to second order in weak-nonlinear approximation have been found up to now.

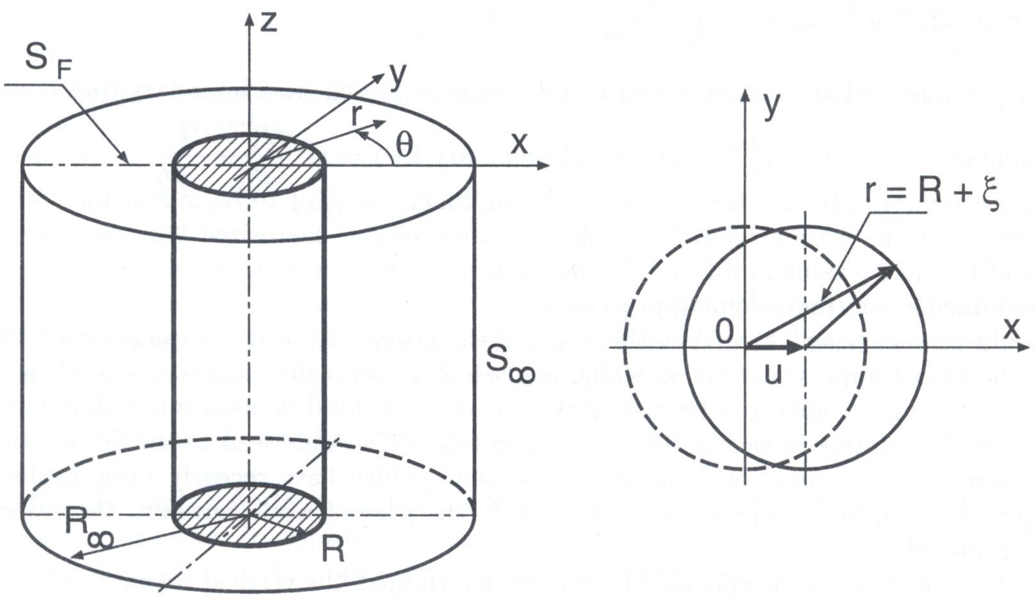

Fig. 2. Cylinder configuration and the instantaneous cylinder surface definition

The forced oscillation of the cylinder axis in the direction of x-axis are described by the following displacement function

$$u(t) = -u_0 \cos \Omega t \qquad \text{for} \quad t \geq 0 \, . \tag{27}$$

Assuming that $u_0 < R$, one can express the instantaneous cylinder surface (see Figure 2) in cylindrical co-ordinates as

$$r = f(\theta, t) = u(t) \cos \theta + \sqrt{R^2 - u^2(t) \sin^2 \theta} \, . \tag{28}$$

The position of the body at rest is simply $r = R$ and $\xi(\theta,t) = f(\theta,t) - R$. Finally, we assume that the motion is started from rest with no waves on the free-surface, so that the initial conditions on $z = 0$ are

$$\eta(x,y,0) \equiv 0\,, \quad \phi_s(x,y,0) \equiv 0\,. \tag{29}$$

All the relations derived in the preceding section have to be reformulated in cylindrical coordinates. Particularly, the boundary condition on the instantaneous cylinder surface has the following form

$$\frac{\partial \xi}{\partial t} + \frac{1}{r^2}\frac{\partial \xi}{\partial \theta}\frac{\partial \Phi}{\partial \theta} = \frac{\partial \Phi}{\partial r} \qquad \text{on} \quad r = f(\theta,t)\,. \tag{30}$$

In order to transform it onto the cylinder surface at rest, one needs a Taylor series expansion of the velocity potential about $r = R$:

$$\Phi(r,\theta,z,t) = \sum_{m=1}^{M}\sum_{k=0}^{M-m}\frac{\xi^k}{k!}\frac{\partial^k}{\partial r^k}\Phi^{(m)}(R,\theta,z,t)\,. \tag{31}$$

Noting that $\nabla_s f_0 \equiv 0$, and that $\frac{1}{r^2}\frac{\partial \xi}{\partial \theta}\frac{\partial \Phi}{\partial \theta}$ has to be transformed into $\frac{1}{(R+\xi)^2}\frac{\partial \xi}{\partial \theta}\frac{\partial \Phi}{\partial \theta}$, one obtains eventually a sequence of Neumann boundary conditions on $r = R$ corresponding to relations (13):

$$\begin{aligned}
\frac{\partial \Phi^{(1)}}{\partial r} &= \frac{\partial \xi}{\partial t}\,,\\
\frac{\partial \Phi^{(2)}}{\partial r} &= \frac{1}{R^2}\frac{\partial \xi}{\partial \theta}\frac{\partial \Phi^{(1)}}{\partial \theta} - \xi\frac{\partial^2 \Phi^{(1)}}{\partial r^2}\,,\\
\frac{\partial \Phi^{(3)}}{\partial r} &= \frac{1}{R^2}\frac{\partial \xi}{\partial t}\left(\frac{\partial \xi}{\partial \theta}\right)^2 + \frac{1}{R^2}\xi\frac{\partial \xi}{\partial \theta}\frac{\partial^2 \xi}{\partial \theta\partial t} + \frac{1}{R^2}\frac{\partial \xi}{\partial \theta}\frac{\partial \Phi^{(2)}}{\partial \theta}\\
&\quad - \frac{2\xi}{R^3}\frac{\partial \xi}{\partial \theta}\frac{\partial \Phi^{(1)}}{\partial \theta} - \xi\frac{\partial^2 \Phi^{(2)}}{\partial r^2} - \frac{\xi^2}{2}\frac{\partial^3 \Phi^{(1)}}{\partial r^3}\,,\\
\frac{\partial \Phi^{(m)}}{\partial r} &= \frac{1}{R^2}\frac{\partial \xi}{\partial \theta}\frac{\partial}{\partial \theta}\left\{\sum_{k=0}^{m-2}\frac{\xi^k}{k!}\frac{\partial^k \Phi^{(m-k-1)}}{\partial r^k}\right\} - \sum_{k=1}^{m-1}\frac{\xi^k}{k!}\frac{\partial^{k+1}\Phi^{(m-k)}}{\partial r^{k+1}}\\
&\quad - \frac{2\xi}{R}\sum_{k=0}^{m-2}\frac{\xi^k}{k!}\frac{\partial^{k+1}\Phi^{(m-k-1)}}{\partial r^{k+1}} - \frac{\xi^2}{R^2}\sum_{k=0}^{m-3}\frac{\xi^k}{k!}\frac{\partial^{k+1}\Phi^{(m-k-2)}}{\partial r^{k+1}} \qquad \text{for} \quad m \geq 4\,.
\end{aligned} \tag{32}$$

4.1. Approximating functions

The next important step is the choice of a basis for a series approximation (17) of the velocity potential $\Phi^{(m)}(r,\theta,z,t_i)$. As the first option one might choose the complete set of solution functions defined in an open domain in terms of spherical harmonics:

$$\{T_{ln}\} = \left\{r^{-(l+1)}P_l^n(\cos\psi)e^{in\theta}\,;\; l = 0,1,\ldots,\; -l \leq n \leq l\right\}, \tag{33}$$

where $P_l^n(\cos\psi)$ are associated Legendre functions. Obviously, these functions do not satisfy the bottom boundary condition (4), so their application to our solution procedure would inevitably require an additional term in the functional (20) associated with integration over the bottom boundary (S_{Bot}). Moreover, in order to be incorporated in our solution procedure, this function set should be expressed in terms of cylindrical coordinates. As a result, the most general approximation of the velocity potential could have the form

$$\Phi^{(m)} = \sum_{n=0}^{N}\sum_{l=n}^{L}\Big\{(a_{ln}^{(m)}\cos n\theta + b_{ln}^{(m)}\sin n\theta)(r^2+z^2)^{l/2}$$
$$+(c_{ln}^{(m)}\cos n\theta + d_{ln}^{(m)}\sin n\theta)(r^2+z^2)^{-(l+1)/2}\Big\}\cdot \beta_{ln}P_l^n\Big(\frac{z}{\sqrt{r^2+z^2}}\Big)\,, \tag{34}$$

where $\beta_{ln} = \left(\frac{(l-n)!}{(l+n)!}\right)^{1/2}$ and $a_{ln}^{(m)}, b_{ln}^{(m)}, c_{ln}^{(m)}, d_{ln}^{(m)}$ are unknown expansion coefficients.

In the present work, however, we make use of a simplified solution which satisfies the bottom boundary condition (4). We express, at the present, the velocity potential $\Phi^{(m)}$ in terms of cylindrical functions as follows

$$\Phi^{(m)}(r,\theta,z,t_i) = \sum_{n=0}^{N}\Big\{\Big[a_{0n}^{(m)}J_n(k_0r) + a_{1n}^{(m)}Y_n(k_0r)\Big]\cos n\theta$$
$$+\Big[b_{0n}^{(m)}J_n(k_0r) + b_{1n}^{(m)}Y_n(k_0r)\Big]\sin n\theta\Big\}Z_0(z)$$
$$+\sum_{n=0}^{N}\sum_{l=2}^{L}(a_{ln}^{(m)}\cos n\theta + b_{ln}^{(m)}\sin n\theta)K_n(|k_{l-1}|r)Z_{l-1}(z)\,, \tag{35}$$

where J_n, Y_n are Bessel functions of the first and second kind respectively, and K_n are modified Bessel functions of the first kind.

The functions $Z_l(z)$, $(l = 0, 1, ...)$ given by

$$Z_l(z) = \frac{\cosh[k_l(z+h)]}{\cosh(k_lh)} \tag{36}$$

fulfil a linearized boundary condition on the free-surface, together with the bottom boundary condition (4), and k_l are real and imaginary roots of the dispersion relation

$$\omega^2/g = k\tanh(kh)\,.$$

Equation (35) resembles an exact steady-state solution to the linearized, three-dimensional problem of wave propagation in water of constant depth h

$$\Phi(r,\theta,z,t) = \Big\{\sum_{n=0}^{\infty}(a_{0n}\cos n\theta + b_{0n}\sin n\theta)H_n(k_0r)Z_0(z)$$
$$+\sum_{n=0}^{\infty}\sum_{l=1}^{\infty}(a_{ln}\cos n\theta + b_{ln}\sin n\theta)K_n(|k_l|r)Z_l(z)\Big\}\,e^{-i\omega t}\,. \tag{37}$$

Since we are dealing with a transient problem, the Hankel functions have been replaced in equation (35) by a linear combination of J_n and Y_n.

Another form of an approximate solution can be proposed, if different dispersion relations

$$\frac{(m\Omega)^2}{g} = k_l^{(m)}\tanh(k_l^{(m)}h) \tag{38}$$

for each order m of approximation are considered. This results in the different sets of real and imaginary roots

$$\{k_l^{(m)}\} = \{k_0^{(m)},\ k_l^{(m)} = i\kappa_l^{(m)}\ ,\ l = 0,1,\ldots\}\,.$$

Bearing in mind, that higher-order solutions to a wave radiation problem will contain terms resulting from the interaction of propagating and evanescent, lower-order modes, one can propose the following, extended form of an approximate solution

$$\begin{aligned}\Phi^{(m)}(r,\theta,z,t_i) &= \sum_{j=1}^{m}\sum_{n=0}^{N}\Big\{\Big[a_{0n}^{(j)}J_n(k_0^{(j)}r)+a_{1n}^{(j)}Y_n(k_0^{(j)}r)\Big]\cos n\theta \\ &+\Big[b_{0n}^{(j)}J_n(k_0^{(j)}r)+b_{(l+1)n}^{(j)}Y_n(k_0^{(j)}r)\Big]\sin n\theta\Big\}\,Z_0^{(j)}(z) \\ &+\sum_{j=1}^{m}\sum_{n=0}^{N}\sum_{l=2}^{L}(a_{ln}^{(j)}\cos n\theta+b_{ln}^{(j)}\sin n\theta)K_n(|k_{l-1}^{(j)}|r)Z_{l-1}^{(j)}(z)\,. \end{aligned} \tag{39}$$

It should be emphasized again that the unknown expansion coefficients in equation (39) have to be computed by simultaneous minimization of the boundary errors both on the cylider surface and on the water free-surface. In order to enhance the accuracy of the approximation one can supplement the solution (39) with an additional velocity potential $\Phi_D^{(m)}$ which satisfies the homogeneous Dirichlet boundary condition on the free-surface and the bottom boundary condition. This function can again be expressed as a series of modified Bessel functions

$$\Phi_D^{(m)}(r,\theta,z,t_i)=\sum_{n=0}^{N}\sum_{l=1}^{L1}(e_{ln}^{(m)}\cos n\theta+f_{ln}^{(m)}\sin n\theta)K_n(\kappa_l r)Z_l(z)\,, \tag{40}$$

where $Z_l(z)=\sin(\kappa_l z)$, and $\kappa_l=\dfrac{2l-1}{2h}\pi$. As a result, the approximate solutions to our boundary value problems can be described by the sum of the potential function (39) and the function (40). This general form has been implemented in the developed computer program.

As one may notice, the form of the approximate solutions (35), (39) and (40) has been chosen, to some extent, by intuition. Strictly speaking, their basis does not posess the completeness property as it would be in the case of a steady-state solution (37). However, the cylindrical harmonics, being better suited for our axisymmetric problems than the spherical harmonics (34), deserve undoubtedly a thorough numerical study. Therefore, it is the performance of this approximation that will be examined in the next section.

4.2. Numerical examples

Before proceeding to the solution of a nonlinear wave radiation problem, it is essential to establish the accuracy and convergence characteristics of the procedure described in Section 2 and 3. The convergence study should be carried out on three levels of the procedure. The most important is the convergence study of the Trefftz approximation for each order m of the perturbation procedure at each time t_i. Then, the convergence of the perturbation procedure to a nonlinear solution at each time instant t_i should be investigated. Finally, the convergence of the time-stepping procedure to a steady-state solution at each control point of the fluid domain ought to be checked.

This is an enormous research task, which we have not completed yet. In what follows, we present some preliminary results of the convergence study for the Trefftz approximation defined by equation (39). The convergence is studied up to the second order of the perturbation procedure.

The accuracy of the approximation can be assessed directly by comparison of the prescribed boundary values with the calculated ones at each time-step. For this purpose, the following boundary norms are defined:

- an absolute error in $L^2(\Gamma)$-norm for the body surface $\Gamma = S_B$ and the free-surface $\Gamma = S_F$

$$e_{S_B}^{(m)}(t_i) = \left(\int_{S_B} \left| \frac{\partial \Phi^{(m)}}{\partial n} - v^{(m)} \right|^2 \mathrm{d}S \right)^{1/2}, \quad e_{S_F}^{(m)}(t_i) = \left(\int_{S_F} \left| \Phi^{(m)} - u^{(m)} \right|^2 \mathrm{d}S \right)^{1/2},$$

where $\Phi^{(m)}$ and $\dfrac{\partial \Phi^{(m)}}{\partial n}$ are computed whereas $u^{(m)}$ and $v^{(m)}$ are prescribed at $t = t_i$,

- a relative error in $L^2(\Gamma)$-norm

$$er_{S_B}^{(m)}(t_i) = \left(\frac{\displaystyle\int_{S_B} \left| \frac{\partial \Phi^{(m)}}{\partial n} - v^{(m)} \right|^2 \mathrm{d}S}{\displaystyle\int_{S_B} \left| v^{(m)} \right|^2 \mathrm{d}S} \right)^{1/2}, \quad er_{S_F}^{(m)}(t_i) = \left(\frac{\displaystyle\int_{S_F} \left| \Phi^{(m)} - u^{(m)} \right|^2 \mathrm{d}S}{\displaystyle\int_{S_F} \left| u^{(m)} \right|^2 \mathrm{d}S} \right)^{1/2}.$$

The convergence of the solution may depend on several factors. To the most important belong: system parameters, excitation parameters, number of T-function used in the series, values of the weight w^2 used in error functional and the accuracy of the integration procedure. For our test example, only the influence of the weight and the number of T-functions will be shown.

Consider a vertical cylinder with prescribed horizontal harmonic motion along x-axis, starting from $t = 0$ with the velocity $v = 0$ in otherwise still water of depth h. The parameters of the test system are as follows: cylinder radius $R = 0.09$ m, far-field cylinder radius $R_\infty = 1.7$ m, water depth $h = 1$ m, frequency of cylinder oscillation $f = 3$ Hz, amplitude of cylinder oscillation $u_0 = 0.008$ m. The wave length corresponding to the frequency of cylinder oscillation is $\lambda = 0.17$ m, so, the radius $R_\infty = 10\lambda$. The small parameter $\varepsilon = 0.3$ and the induced waves may be considered as steep. The evolution eqations are integrated with the use of a variable-order variable-step Adams method.

First of all we examine the influence of the weight on the boundary error. In the computations we set the following parameters of the approximating series (39): N = 2, m = 2, L = 20, L1 = 5 (for $\Phi_D^{(m)}$). Figures 3 and 4 show the plots of an absolute error $e_\Gamma^{(1)}(t)$ for the first-order solution $\Phi^{(1)}$ on the cylinder surface and on the free-surface, respectively. As one might expect, increasing weight reduces the error on the cylinder boundary increasing at the same time the error on the free-surface. Therefore, an optimum value of w can be found for each computational example. In Figures 5 and 6 the plots of the relative errors $er_\Gamma^{(1)}(t)$ are presented. In view of the error definition and the applied initial conditions, the plots for $er_{S_F}^{(1)}$ start from infinity for $t = 0$. Then the curves decrease rapidly reaching their minimum and, finally, oscillate about a mean level. In Figures 7 and 8 the absolute errors $e_\Gamma^{(2)}(t)$ for the second-order solution $\Phi^{(2)}$ are shown. Their mean values are smaller in comparison to the first-order results.

The mean errors can be reduced by the increase of the number of T-functions in the approximation series. Figures 9 and 10 show this for three values of L in the first-order approximation. The number of the functions for the supplementary potential $\Phi_D^{(m)}$ is kept constant (L1 = 5). One can see that it is quite easy to reduce the error on the cylinder surface (Figure 9). In contrast, a saturation effect can be observed for the free-surface boundary error (Figure 10). The reduction of the mean errors in the second-order approximation is even more difficult to achieve (see Figures 11 and 12). The errors are stable and small, but can hardly be controlled by the change of the number of T-functions.

It is crucial for the long-time simulation to keep the computational errors below a certain limit. This can be achieved by simultaneous increase of the number of T-functions and the proper choice of the weight w.

The main interest of this work is the performance study of the developed method, rather than the investigation of the nonlinear hydrodynamic problem itself. However, some results of the hydrodynamic analysis are also presented in Figures 13 and 14. They show the instantaneous free-surface

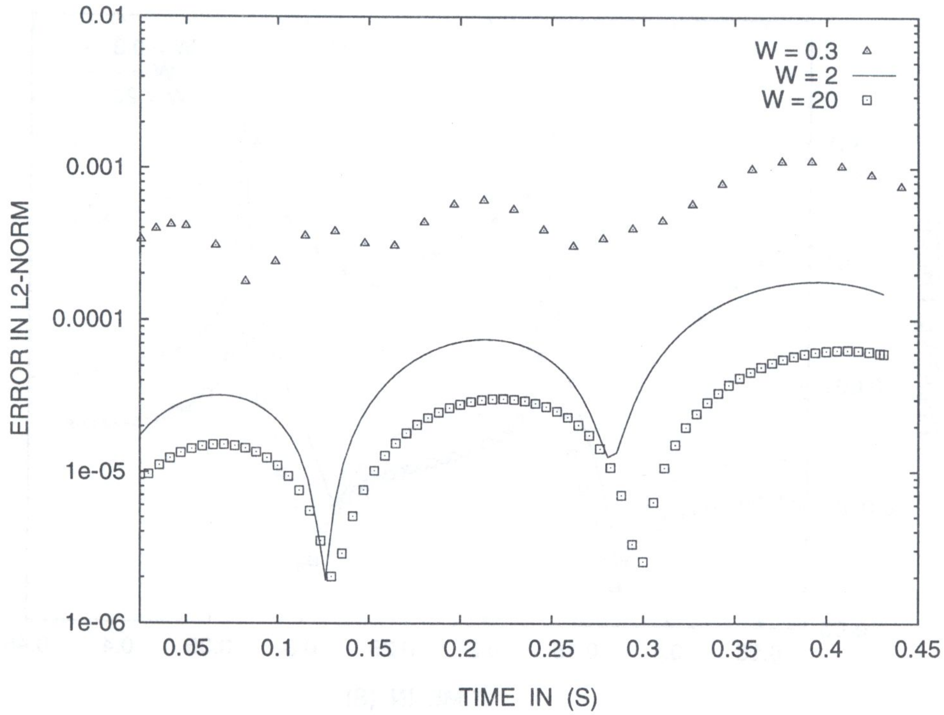

Fig. 3. Error in L^2-norm on the cylinder surface for various weights w in the first-order approximation

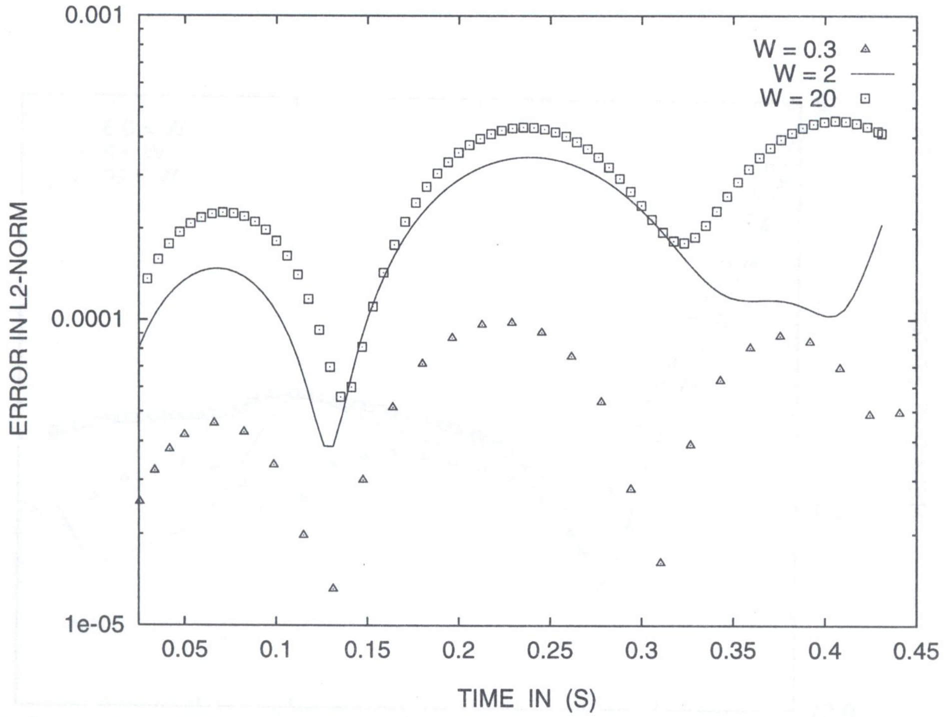

Fig. 4. Error in L^2-norm on the water free-surface for various weights w in the first-order approximation

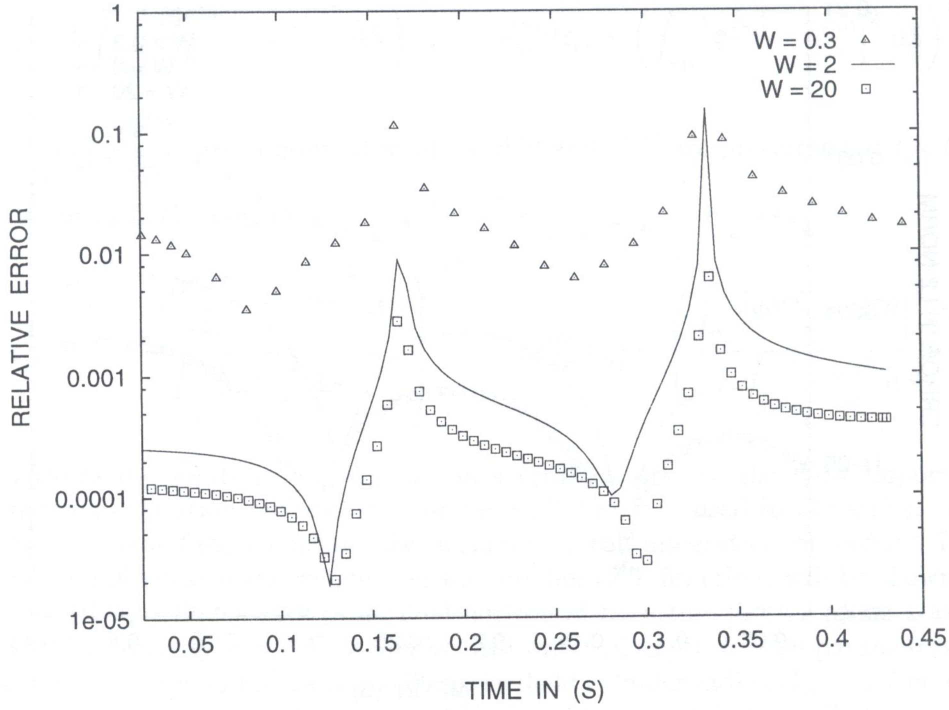

Fig. 5. Relative error on the cylinder surface for various weights w in the first-order approximation

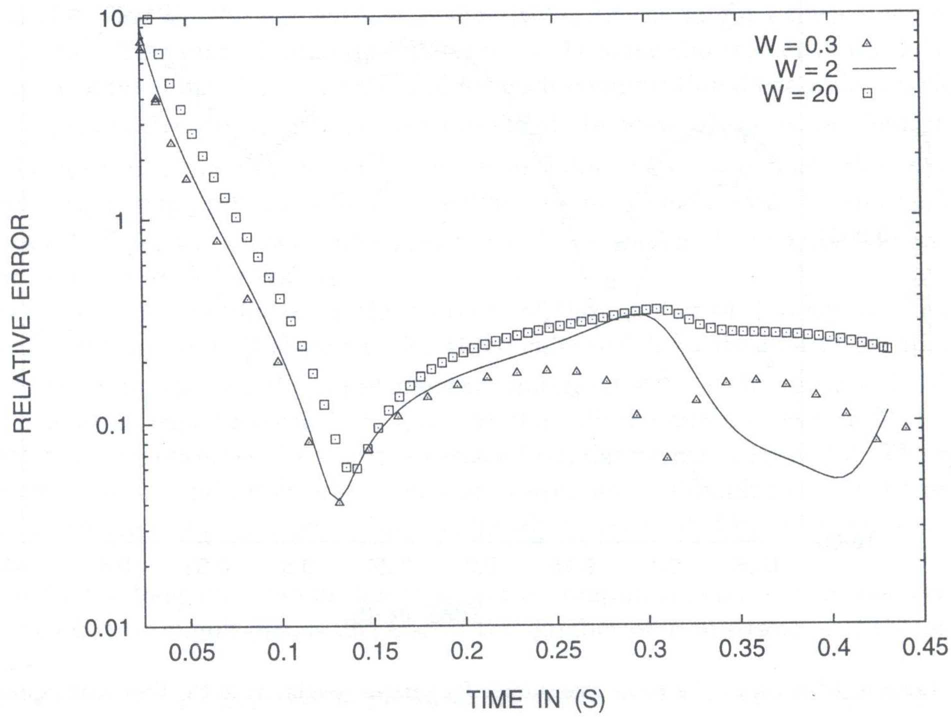

Fig. 6. Relative error on the water free-surface for various weights w in the first-order approximation

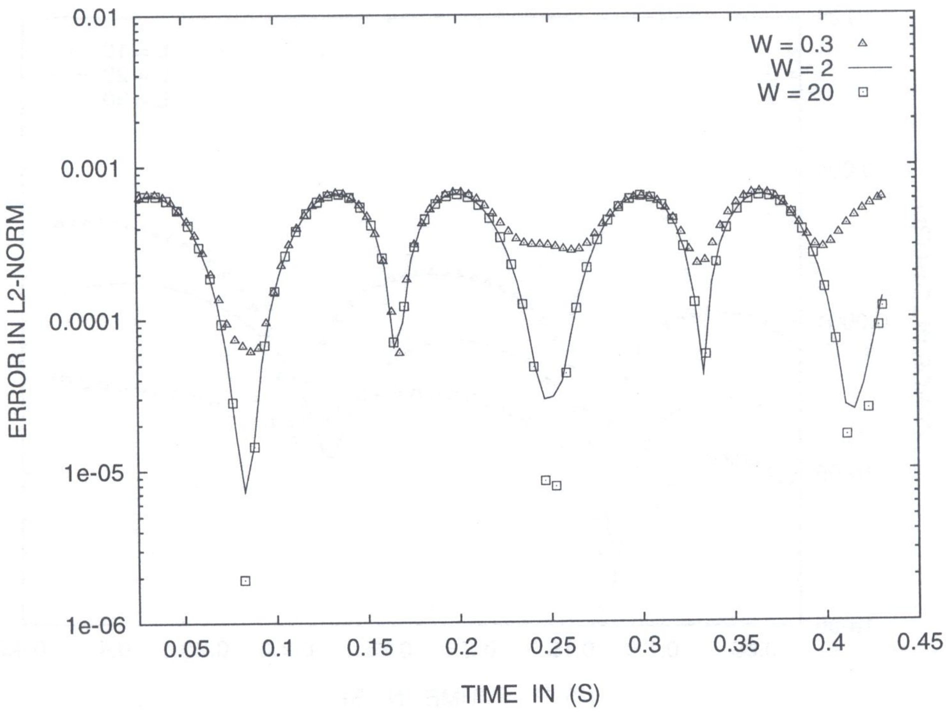

Fig. 7. Error in L^2-norm on the cylinder surface for various weights w in the second-order approximation

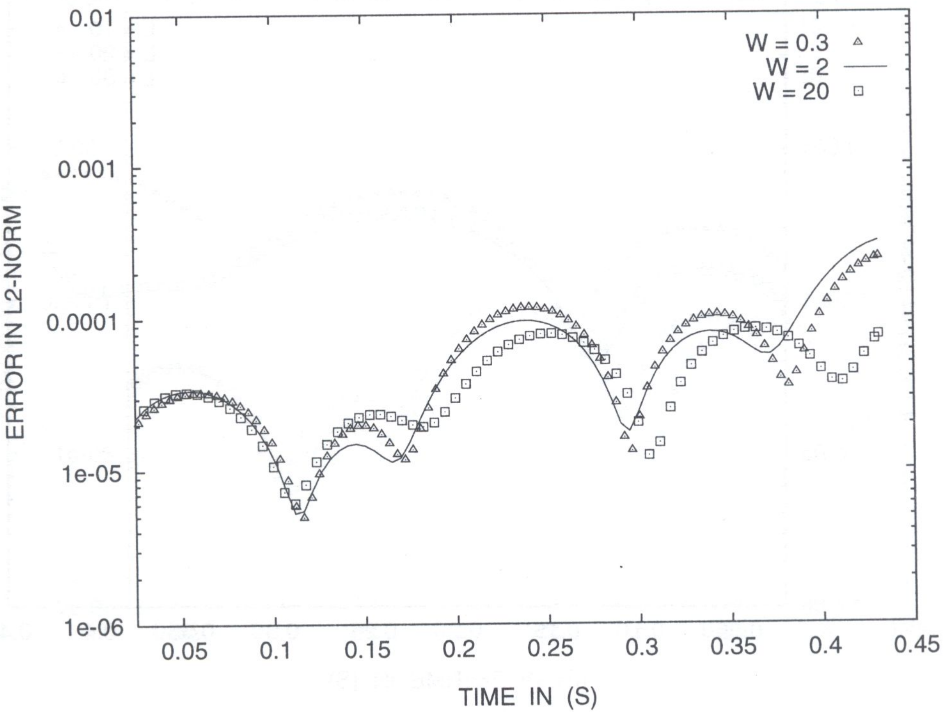

Fig. 8. Error in L^2-norm on the water free-surface for various weights w in the second-order approximation

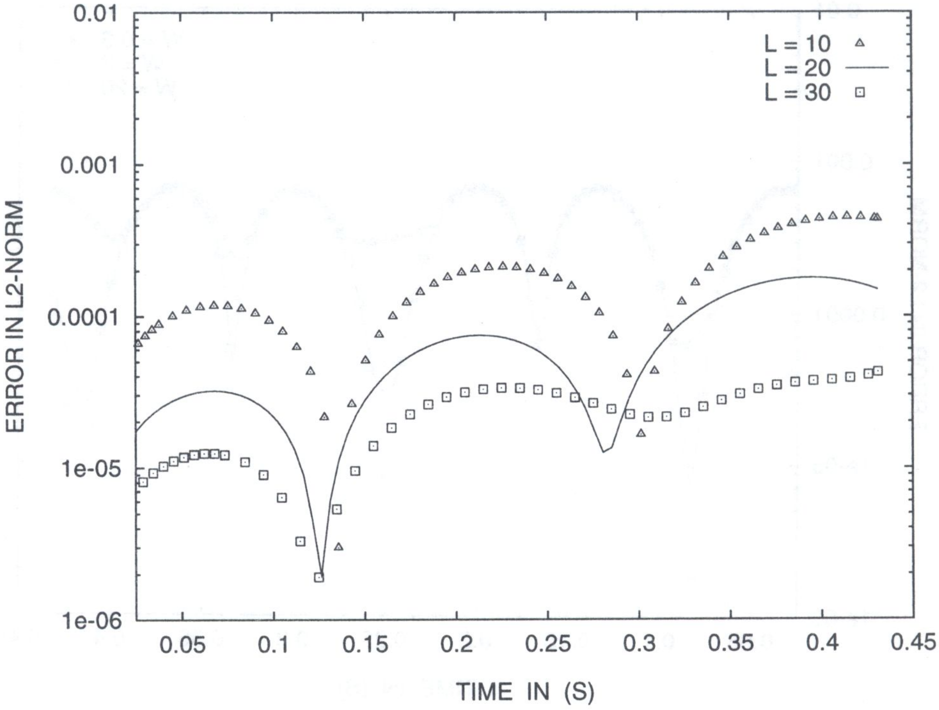

Fig. 9. Error in L^2-norm on the cylinder surface for various numbers of T-functions in the first-order approximation

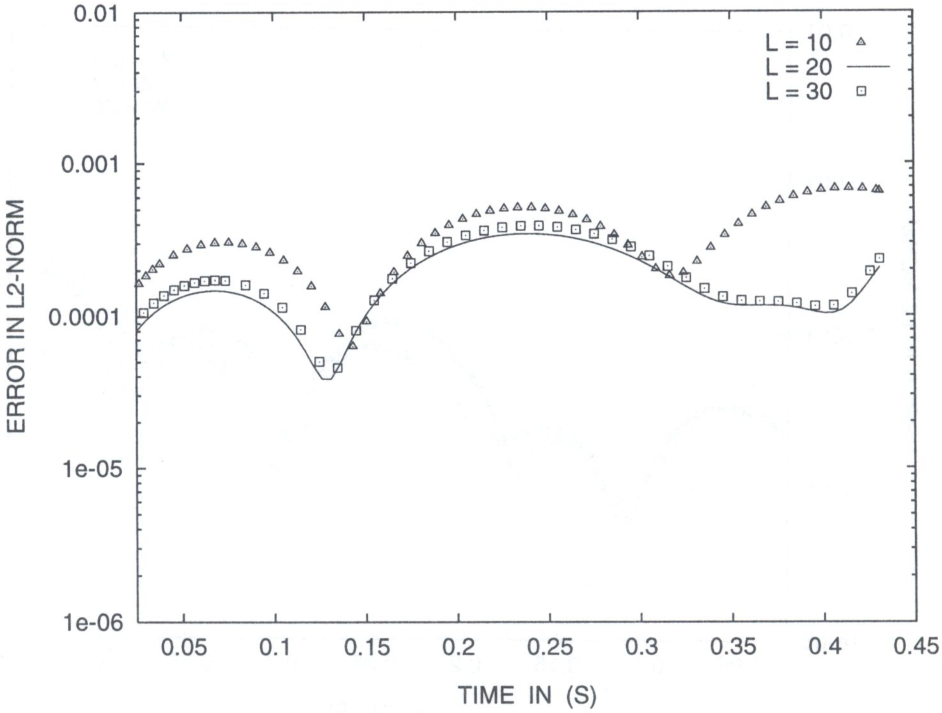

Fig. 10. Error in L^2-norm on the water free-surface for various numbers of T-functions in the first-order approximation

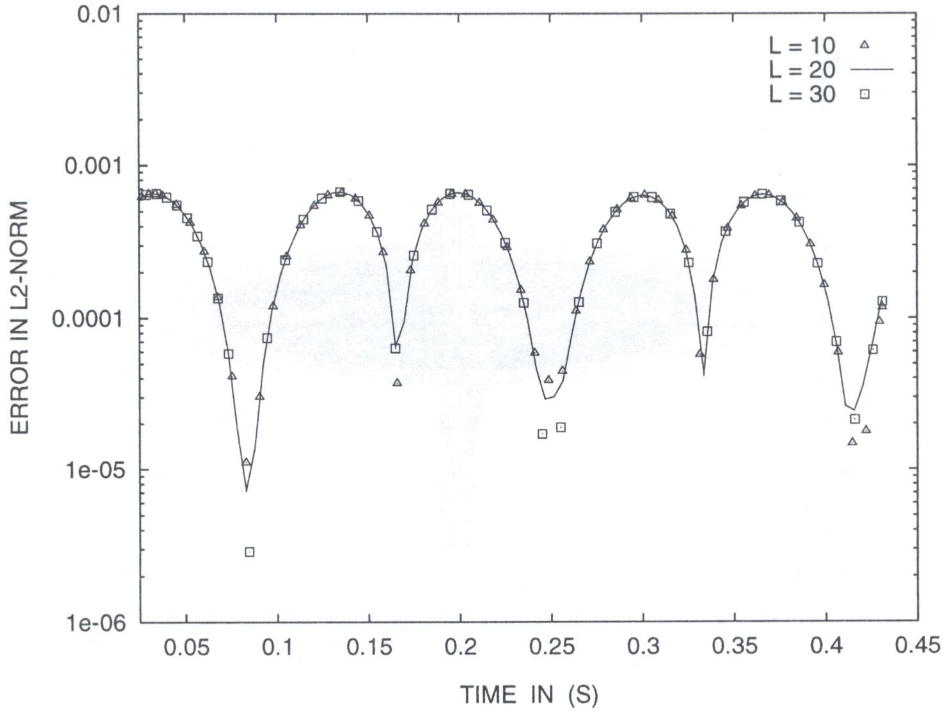

Fig. 11. Error in L^2-norm on the cylinder surface for various numbers of T-functions in the second-order approximation

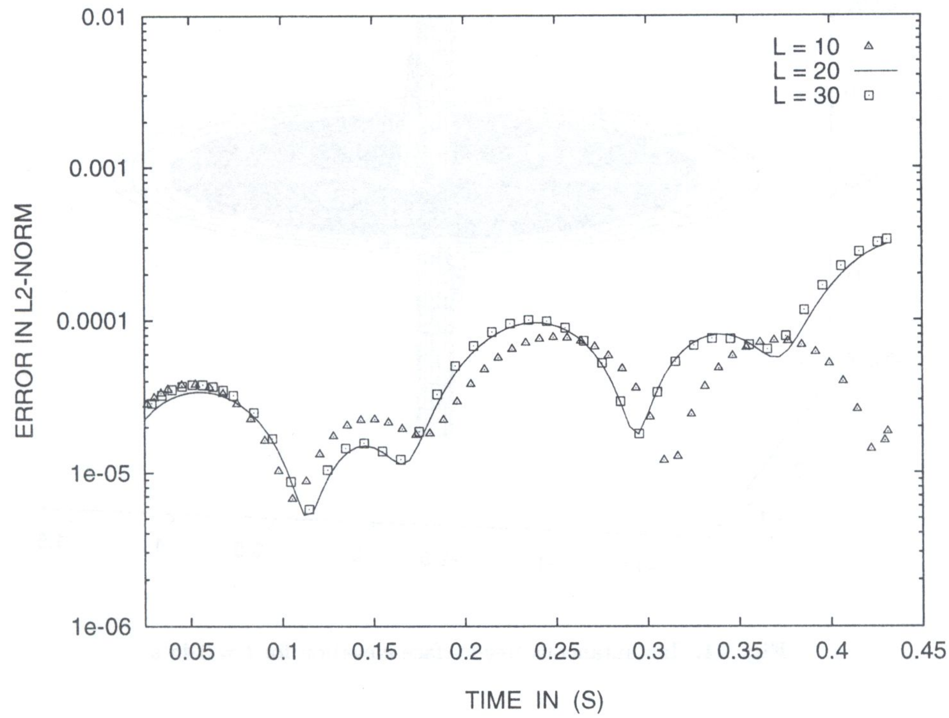

Fig. 12. Error in L^2-norm on the water free-surface for various numbers of T-functions in the second-order approximation

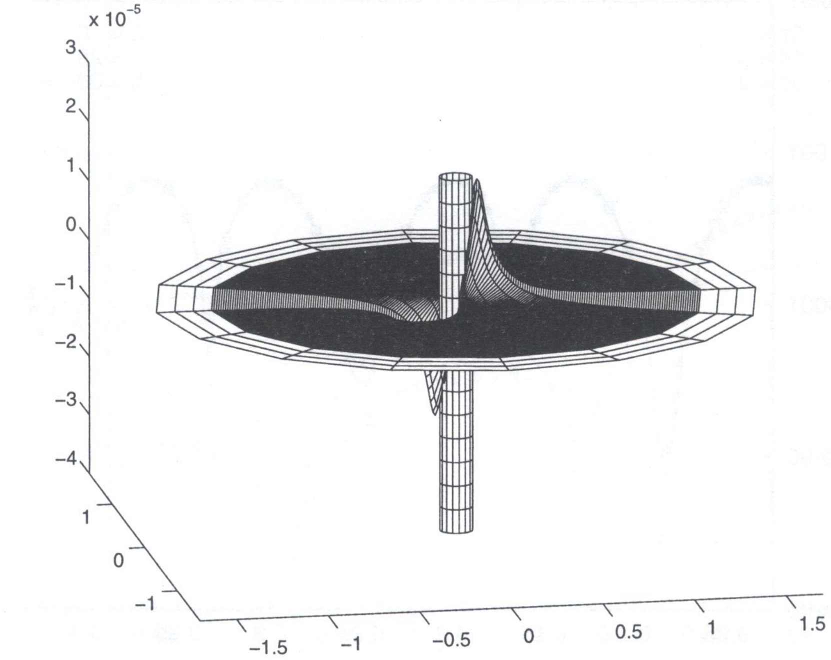

Fig. 13. Instantaneous free-surface elevation for $t = 0.008$ s

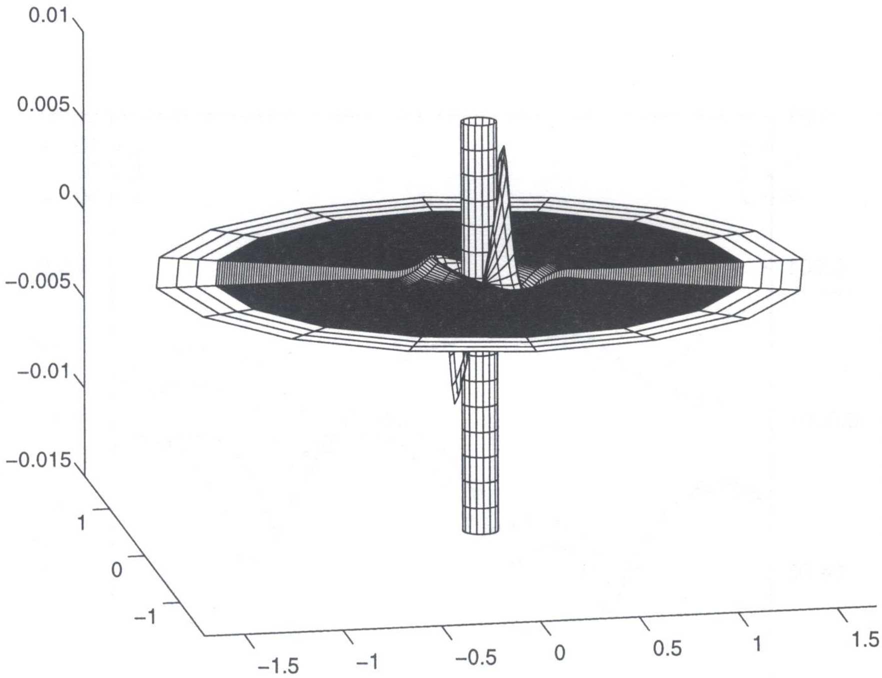

Fig. 14. Instantaneous free-surface elevation for $t = 0.45$ s

elevation plots for our computational example. Figure 13 shows the water free-surface immediately after the simulation has been started. Figure 14 shows the results after 1.5 periods. One may notice that the wave steepness for $\varepsilon = 0.3$ is rather high.

5. Concludings remarks

In the paper, a new method for the nonlinear wave-structure interaction problems has been developed. It combines the time-stepping approach with Trefftz's concept of solving of linear boundary value problems. The method has been implemented to an axisymmetric wave-radiation problem for an oscillating vertical cylinder in water of constant depth.

In the paper, the main attention is devoted to the performance analysis of the Trefftz approach. Several approximation series utilizing various T-function sets have been proposed to solve a sequence of linear boundary value problems. The accuracy of the approximation by means of cylindrical harmonics has been studied thoroughly. Only one set of approximation functions has been used in the whole fluid domain.

Although the applied solution functions do not posses the completeness property, the satisfactory convergence has been obtained and the results of preliminary computations are promising. However, the growth in the boundary errors has also been observed during long-time simulations.

The computational efficiency of the mehod is very high. The total CPU-time required for typical second-order simulation up to 3 oscillation periods amounts to 1 hour on a Convex SPP-1600. It can be further improved by the optimization of the computer code.

In order to keep the boundary errors below an acceptable limit, the subdivision of the fluid domain into subdomains (frameless Trefftz elements) may be necessary. It is expected, that much better results may also be obtained with the use of spherical harmonics combined with domain subdivision. Although the numerical assessment of the method is far from being complete, it may serve as a foundation for future works.

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