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Traffic network design by cellular automaton-based traffic simulator

Eisuke Kita[1,2], Wataru Nanya[3], Yukiko Wakita[4], Tatsuhiro Tamaki[5]

[1] *Graduate School of Information Science*
Nagoya University, Japan
e-mail: kita@is.nagoya-u.ac.jp

[2] *Graduate School of System Informatics*
Kobe University, Kobe 657-0013, Japan

[3] *KOMATSU*
Tokyo 107-8414, Japan

[4] *Institute of Innovation for Future Society, Nagoya University*
Nagoya 464-8601, Japan

[5] *Okinawa National College of Technology*
Nago 905-2171, Japan

Braess pointed out that adding a new road to overcome a traffic congestion could cause a new traffic congestion leading to the reduction of the traffic flow in the whole traffic network, which is called Braess' paradox. The aim of this study is to formulate a traffic network design algorithm to increase the traffic flow in a traffic network. The objective function is the traffic flow of the whole traffic network and the route selection at the corners is considered as design variable. The traffic flow is estimated by a traffic flow simulator based on the cellular automaton model. A simple traffic network is considered as a numerical example. At different traffic densities, the traffic network is optimized to maximize the traffic flow. The results show that the optimized traffic network depends on traffic density. The situation presented by Braess' paradox could disappear at high traffic density.

Keywords: traffic network design, cellular automaton, optimization, Braess' paradox.

1. Introduction

Traffic congestions cause both financial losses for transportation system and environmental pollution. One of the most popular ways to overcome traffic congestion is to increase traffic flow capacity of the traffic network. It is well known, however, that the construction of new roads sometimes reduces the total traffic flow capacity of the whole traffic network. One of the most popular studies in this field is Braess' paradox [1–5]. The paradox states that adding a new road to overcome traffic congestion could cause another traffic congestion and thus reduce the traffic flow of the whole traffic network. On the other hand, Nagurney pointed out that Braess' paradox disappears at high traffic density [6]. The aim of this study is to discuss the relationship between Braess' paradox and traffic density.

In the study, the traffic road network design is formulated first. The objective function is the traffic flow of the whole traffic network and the route selection at the corners is taken as a design variable. Traffic flow is estimated by a traffic simulator. In the traffic flow simulator, vehicle behavior is controlled by local rules of the cellular automaton model [7–9] and velocity is controlled according to the optimal velocity model [7, 10]. The simple traffic network is considered as numerical example. At different traffic densities, the traffic network is optimized to maximize the traffic flow. The results show that the optimized traffic network depends on traffic density. Therefore, the relationship between Braess' paradox and traffic density can be discussed using the numerical results.

The remaining part of this paper is organized as follows. In Sec. 2, Braess' paradox and cellular automaton traffic flow simulation are introduced. In Sec. 3, cellular automaton simulation model is defined. In Sec. 4, the optimization problem is defined. In Sec. 5, the algorithm is applied to a simple numerical example. The conclusions are summarized in Sec. 6.

2. Background

2.1. Braess' paradox

Braess' paradox tells us that a newly constructed road link causes the reduction of the traffic capacity between two points [3].

The example of Braess' paradox is illustrated in Fig. 1a. 3000 vehicles move from point A to point B through the route ACB or the route ADB. The travel time on the road link is shown in Table 1. The travel time on links AC and DB depends on the number of vehicles moving along the road link N, which is $N/100$ (s). The travel time on links AD and CB is constant, 40 s. The travel times on the routes ACB and ADB are referred to as t_{ACB} and t_{ADB}, respectively. Assuming that the numbers of the vehicles moving along the routes ACB and ADB are N_1 and N_2, respectively, the travel times on the routes ACB and ADB are given as follows:

$$t_{\text{ACB}} = \frac{N_1}{100} + 40, \tag{1}$$

$$t_{\text{ADB}} = \frac{N_2}{100} + 40. \tag{2}$$

The average travel time $\bar{t}_1$ is estimated as follows:

$$\bar{t}_1 = \frac{1}{2}(t_{\text{ACB}} + t_{\text{ADB}}) = \frac{N_1 + N_2}{200} + 40. \tag{3}$$

Since $N_1 + N_2 = 3000$, the above equation is estimated as follows:

$$\bar{t}_1 = \frac{N_1 + N_2}{200} + 40 = \frac{3000}{200} + 40 = 55. \tag{4}$$

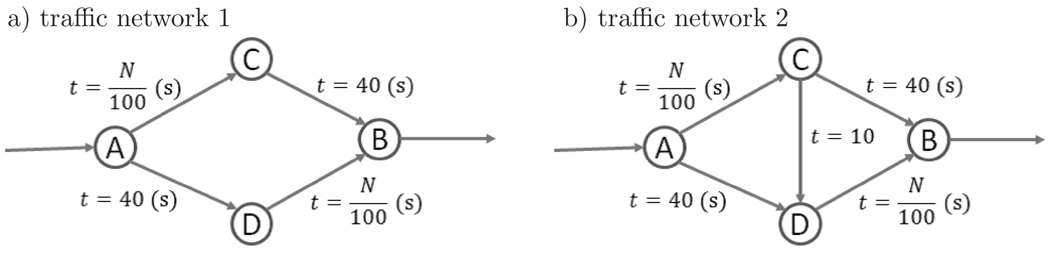

Fig. 1. Braess' paradox.

Table 1. Travel time.

Road Link	Travel time
AC, DB	$N/100$
AD, CB	40
CD	10

A new road link CD is added between point C and point D. Since the link CD is a one-way road, vehicles can move only from point C to point D. The travel time on the link CD is short, 10 s, as shown in Fig. 1b. Assuming that the numbers of the vehicles moving along the route CB, AD and CD are N_1, N_2 and N_3, respectively, the travel times on the routes ACB, ADB and ACDB are given as follows:

$$t_{\mathrm{ACB}} = \frac{N_1 + N_3}{100} + 40, \tag{5}$$

$$t_{\mathrm{ADB}} = 40 + \frac{N_2 + N_3}{100}, \tag{6}$$

$$t_{\mathrm{ACDB}} = \frac{N_1 + N_3}{100} + 10 + \frac{N_2 + N_3}{100}. \tag{7}$$

The average travel time $\bar{t}_2$ is estimated as follows:

$$\bar{t}_2 = \frac{1}{3}(t_{\mathrm{ACB}} + t_{\mathrm{ACDB}} + t_{\mathrm{ADB}}) = \frac{N_1 + N_2 + 2N_3}{150} + 30. \tag{8}$$

Since $N_1 + N_2 + N_3 = 3000$, the above equation is estimated as follows:

$$\bar{t}_2 = \frac{N_1 + N_2 + 2N_3}{150} + 30 = \frac{N_3}{150} + 50. \tag{9}$$

It is shown that the average travel time $\bar{t}_2$ depends on N_3. When N_3 is small, $\bar{t}_2 < \bar{t}_1$. As N_3 increases, the average travel time $\bar{t}_2$ also increases. Once N_2 exceeds 750, $\bar{t}_2 \geq \bar{t}_1$. In the case of high traffic density shown in Fig. 1, the average travel time becomes the shortest when the link CD is not available. This is one of the examples for the situation described by Braess' paradox.

2.2. Cellular automaton – based traffic flow simulation

A finite automaton model was presented by Von Neumann in the 1940s [11, 12]. Von Neumann discussed with Ulam and the concept of cellular automaton (CA) model was formulated.

The cellular automaton model is one of discrete mathematical models. Space is divided square cells and the time axis is expressed with discrete time steps. The state variable at each cell is updated at each time step according to the local rules from the state variables at the cell and its neighboring cells. Time evolution of the state variables simulates natural and social phenomena.

Wolfram discussed in his book [13] the relationship between the state behavior and the local rules in the one-dimensional CA model, and identified four classes:

Class 1. Nearly all initial patterns evolve quickly into a stable, homogeneous state.

Class 2. Nearly all initial patterns evolve quickly into stable or oscillating structures.

Class 3. Nearly all initial patterns evolve in pseudo-random or chaotic manner.

Class 4. Nearly all initial patterns evolve into structures that interact in complex and interesting ways, including formation of local structures that are able to survive for long periods of time.

According to Wolfram's work, the model named as "rules 184 CA" is considered to be the first application of the CA model to traffic flow simulation.

In the rule 184 CA model, the vehicle velocity is invariant. Therefore numerous models are presented to address this issue; e.g., the Nagel-Schrekenberg (Na-Sch) model [14], the optimal velocity model and other. The Na-Sch model accelerates a vehicle when the vehicle is behind another vehicle in front (loading vehicle). When the vehicle meets the frontal one, it decelerates randomly.

The car-following models are also very popular algorithms for controlling vehicle velocity. The car-following models define the vehicle velocity or acceleration rate as the function of the velocity or distance to the leading vehicle [15–18]. The optimal velocity model is another car-following model [7, 10]. The vehicle acceleration rate is defined as follows:

$$\ddot{x} = \alpha(F(\Delta x) - \dot{x}). \tag{10}$$

Since x denotes the vehicle position, $\ddot{x}$ and $\dot{x}$ denote the acceleration rate and the velocity, respectively. The Δx is the distance between the following vehicle and the leading vehicle. The function F is referred to the optimal velocity function.

In the simulator employed in this study, the vehicle behavior is controlled by the local rules and the velocity is controlled with the optimal velocity model. The movement of the vehicles is controlled with a uniform random number. Therefore, this model is called “stochastic velocity model” [19, 20].

3. Traffic flow simulator

3.1. Simulation process

In this study, the traffic simulation model based on the cellular automaton is employed [21, 22]. The object's domain is discretized into cells of 3 m in width and 3 m in height.

The vehicle occurrence at each cell is taken as the state variable of the cell, which is updated by behavior, velocity and movement local rules. The behavior local rule changes the moving direction of the vehicle and determines the cell which each vehicle would like to occupy at the next time-step. The velocity local rule changes vehicle velocity according to distance from the vehicle to the nearest vehicle ahead. The local movement rule determines that the vehicle moves to the next cell.

The simulation process is summarized as follows:

1. Initialize time-step t.
2. Update t by $t + 1$.
3. Estimate the distance from each vehicle to the nearest vehicle ahead.
4. Determine vehicle behavior according to the behavior rule.
5. Change vehicle velocity according to the velocity rule.
6. Determine vehicle movement according to the movement rule.
7. If $t < T$, go to step 2.

The value T is the maximum simulation time-step. One time-step is 0.1 s in real time.

3.2. Object under consideration

The object domain is shown in Fig. 2. Points A and B denote the branching points of roads, and points E and F denote the meeting points of roads. The object domain is discretized into cells of 3 m in width and 3 m in height. The cell size depends on the vehicle size. Especially the cell length is determined from the average length of vehicles.

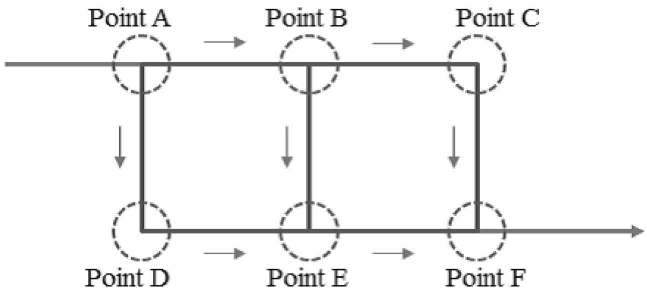

Fig. 2. Illustration of object domain.

3.3. State variable

The vehicle occurence at each cell is taken as the state variable of the cell, which is updated by the following behavior, velocity and movement local rules.

3.4. Local movement rule

The movement rule determines whether a vehicle moves to the next cell, that is, the cell that the vehicle would like to occupy at the next time-step. The rules are summarized as follows [21, 22]:

1. A threshold P_0 is calculated from the vehicle velocity V and the maximum velocity $V_{\max}$ as follows:
$$P_0 = \frac{V}{V_{\max}}, \tag{11}$$
where the maximum velocity $V_{\max}$ is the maximum value in all vehicles' maximum velocities in the simulation.

2. A uniform random number p is generated from 0 to 1.

3. If $p < P_0$ the vehicle moves to the next cell.

3.5. Behavior local rule

The behavior rule has two roles. One is to change vehicle's moving direction and another is to determine the next cell, that is, the cell that a vehicle will occupy at the next time-step. In order to define the behavior rule, the neighboring cells are called S_1, S_2, S_3, S_4 and S_5 (Fig. 3).

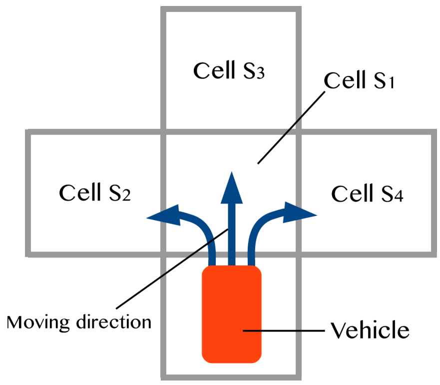

Fig. 3. Cells near cell occupied with vehicle.

A vehicle determines the next cell according to the following rules:

1. When the cell S_2 is unoccupied and the cell S_4 is occupied by another vehicle, the vehicle takes the current cell as the next cell (Fig. 4a). This is the case shown in Fig. 2, where the vehicle waits for the vehicle approaching from point D to point E or from point E to point F when the vehicle moves from point B to point E or from point C to point F. This rule means that a vehicle waits for the vehicle approaching from the right-hand side, and it is called "the right-hand side rule".

2. When the cell S_1 is occupied by another vehicle, the vehicle takes the current cell as the next cell. This is the case where the neatest cell in front is occupied by another vehicle.

3. Otherwise, the vehicle takes the nearest cell in front as the next cell. (Fig. 4b).

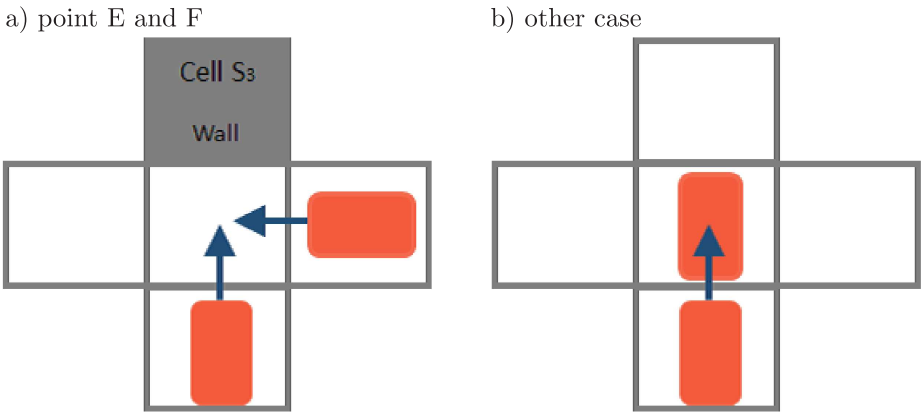

Fig. 4. Behavior rules for the determination of next cell.

A vehicle changes its moving direction according to the following rules:

1. When the cell S_2 is occupied by walls and a vehicle would like to turn to the right at the cell S_1, the vehicle turns to the right by 90 degrees at the cell S_1 after reaching the cell S_1 (Fig. 5a). This is the case where the vehicle turns to the right at point A or point B in Fig. 2.

2. When the cells S_2 and S_3 are occupied by walls, the vehicle turns to the right by 90 degrees at the cell S_1 after reaching the cell S_1 (Fig. 5b). This is the case where the vehicle is at point C in Fig. 2.

3. When the cells S_3 and S_4 are occupied by walls, the vehicle turns to the left by 90 degrees at the cell S_1 after reaching the cell S_1 (Fig. 5c). This is the case where the vehicle is at point D in Fig. 2.

4. When the cell S_3 is occupied by a wall and the cells S_2 and S_4 are not occupied, a vehicle turns to the left by 90 degrees at the cell S_1 after reaching the cell S_1 (Fig. 5d). This is the case shown, in Fig. 2, where the vehicle reaches point E from point B or point F from point C.

5. Otherwise, the vehicle does not change the moving direction at the cell S_1.

3.6. Local velocity rule

The vehicle velocity is controlled according to the optimal velocity model [10], which is given by

$$\ddot{x}(t) = a \cdot \{V(\Delta x) - \dot{x}(t)\}. \tag{12}$$

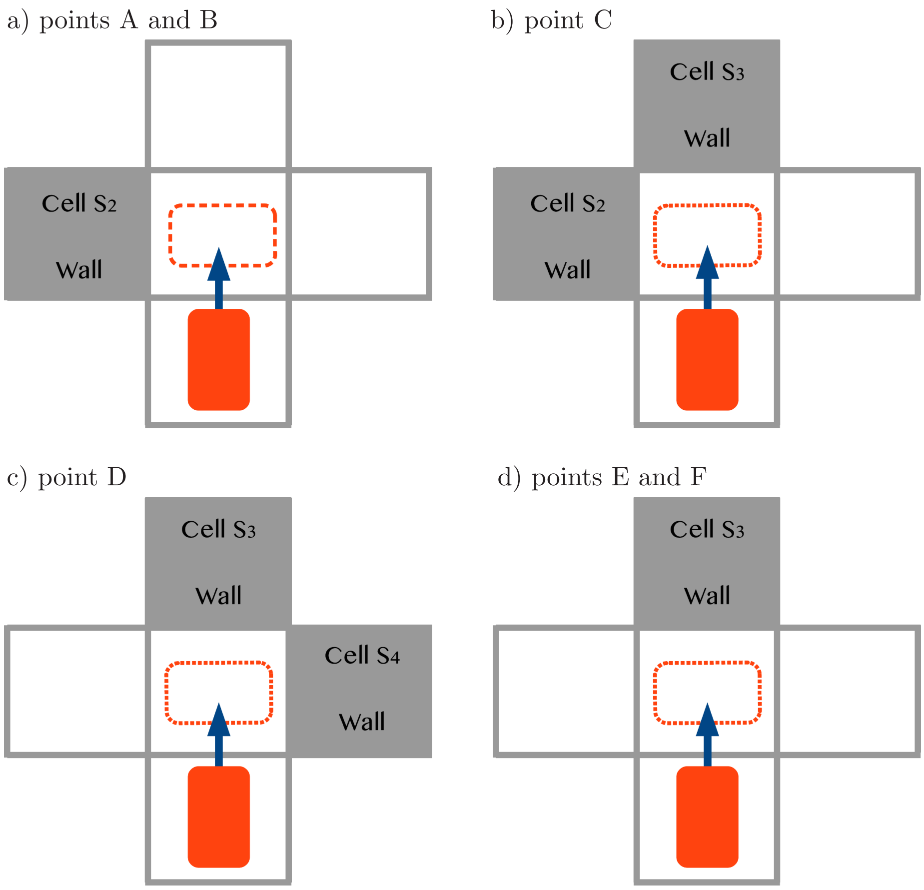

Fig. 5. Behavior rules for the change of vehicle moving direction.

The parameter a and the variable Δx denote the sensitivity and the distance between the vehicle and its nearest vehicle in front, respectively. The function $V(\Delta x)$ is called the optimal velocity function and it is given by

$$V(\Delta x) = \frac{v_{\max}}{2}[\tanh(\Delta x - x_c) + \tanh(x_c)]. \tag{13}$$

The parameters $v_{\max}$ and x_c denote the maximum vehicle velocity and the safe following distance, respectively.

4. Traffic network design

A traffic network is composed of many short road links, which denote the roads between the intersections.

The traffic flow q is estimated as follows:

$$q = \frac{N}{t}, \tag{14}$$

where the values N and t denote the total vehicle number and the estimation time of the traffic flow simulation, respectively.

The objective function f is defined as follows:

$$f = q \to \max. \tag{15}$$

The variable z_j denotes the availability of the road link j. The design variable vector is defined as follows:

$$\mathbf{z} = \{z_1, z_2, \cdots, z_n\}. \tag{16}$$

The variable n denotes the total number of road links. The variable z_j is defined as follows:

$$z_j = \begin{cases} 0 & \text{The road link } j \text{ is not available.} \\ 1 & \text{The road link } j \text{ is available.} \end{cases} \tag{17}$$

The optimization problem is summarized as follows:

$$f = q \to \max, \tag{18}$$

$$\mathbf{z} = \{z_1, z_2, \cdots, z_n\}, \quad z_j \in \{0, 1\}. \tag{19}$$

The above problem is the 0-1 integer-number planning problem. Replacing the integer variable with the real variable leads to the relaxed problem as follows:

$$f = q \to \max, \tag{20}$$

$$\vec{z} = \{z_1, z_2, \cdots, z_n\}, \quad 0 \le z_j \le 1. \tag{21}$$

This problem is solved by the steepest descent method.

5. Numerical example

5.1. Object under consideration

The object under consideration is shown in Fig. 6. In Fig. 6a, the road is represented by the thick solid line and the moving direction of vehicles is described by the arrows along the road. The road

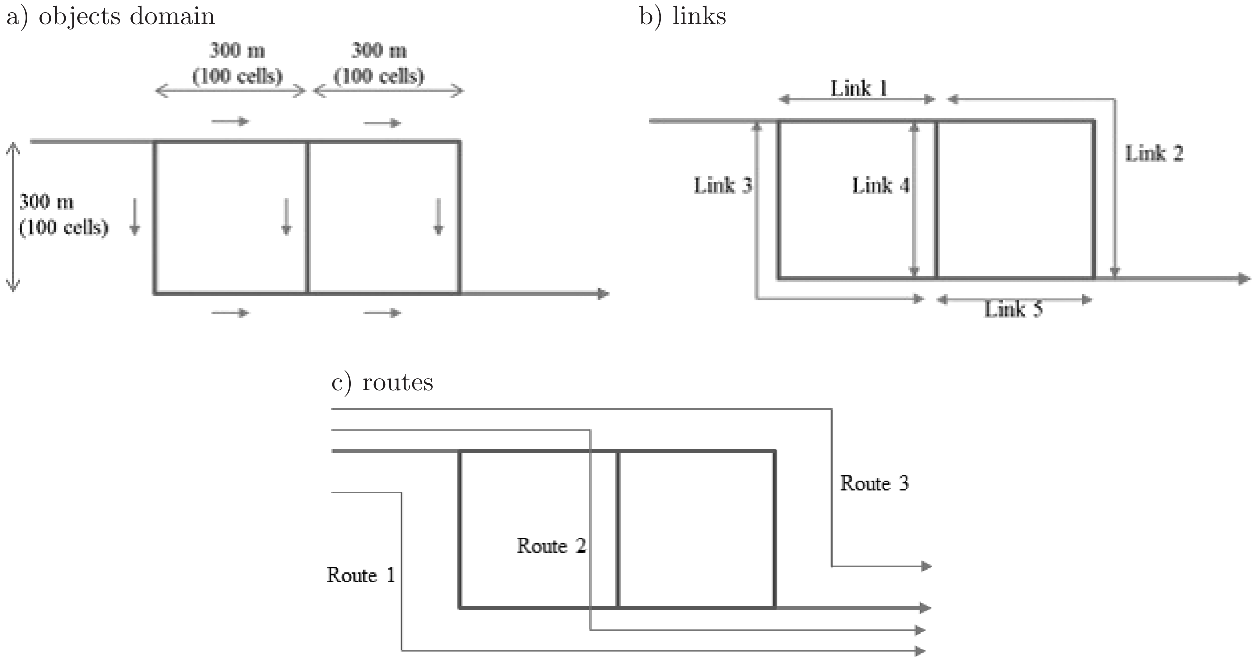

Fig. 6. Object under consideration (traffic network).

link number is shown in Fig. 6a. The cell representation of the traffic network is shown in Fig. 7. Maximum velocity at road links is listed in Table 2. The maximum velocity at road link 4 is specified as 30 m/s, which is more than twice faster than at the other road links. This velocity condition is defined according to Braess' paradox. All vehicles move from the entrance at the left-top to the exit at the right-bottom. After the vehicles go out of the exit, they come in from the entrance. Each vehicle takes randomly one of the three routes shown in Fig. 6a. Vehicle density, which means the number of the initially distributed vehicles on the whole traffic network, is specified as 12, 30, 60, 90, 120, 180, 240 and 300. Then, 10 000 time-step simulation is performed 10 times at each vehicle density. The average traffic flow, which is defined as the average value of the traffic flow q with respect to 10-times simulation, is shown in Table 3. The vehicles are randomly distributed in the whole traffic network before the simulation. The average traffic flow seems to depend on the traffic density and the initial distribution of vehicles. Therefore, 10-times simulations are performed at the different vehicle distributions of the same vehicle density. Table 3 shows that the traffic flow increases according to the increase of vehicle density from 12 to 90 and that the traffic flow decreases when the vehicle density exceeds 90. Thus, it is assumed that a traffic congestion occurs near the vehicle density of 90. Therefore, the traffic flow simulations at the densities of 80 and 240 are considered as the traffic flows of the low and high traffic densities, respectively.

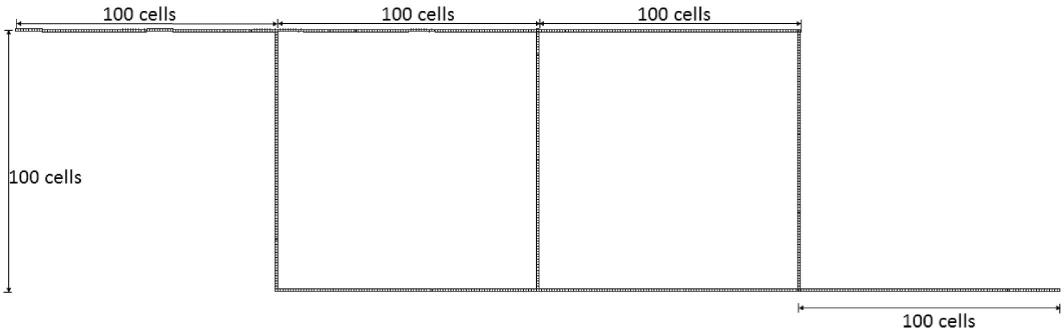

Fig. 7. Cell representation of traffic network.

Table 2. Maximum velocity on road links.

Link No.	1	2	3	4	5
Max. Velocity [m/s]	12	12	12	30	12

Table 3. Traffic flow on simulations.

Vehicle density (Vehicles)	12	30	60	90
Average traffic flow (Vehicles/s $\times 10^4$)	67.5	192.4	497.3	509.8
Vehicle density (Vehicles)	120	180	240	300
Average traffic flow (Vehicles/s $\times 10^4$)	508	396	353.8	143.3

5.2. Low traffic density

The traffic flow at the traffic density of 60 is considered as a case of the low traffic density. The availabilities of links 2, 3 and 4 are taken as the design variables. The relaxed problem given by Eqs. (20) and (21) is solved. The final traffic network is shown in Fig. 8. In the optimized traffic network, link 4 is not available. The traffic flow at different traffic network is compared in Table 4a. The label "Nothing" means that all road links are available. The traffic flow in the case of absence of links 2 or 3 is smaller than in the case of presence of all the links (Fig. 4). The traffic flow in

Fig. 8. Optimized traffic network.

Table 4. Traffic flow.

(a) Traffic density = 60				
Unavailable link No.	Nothing	Link 2	Link 3	Link 4
Average traffic flow (Vehicles/s $\times 10^4$)	497.3	376.5	286.1	553
(b) Traffic density = 240				
Unavailable link No.	Nothing	Link 2	Link 3	Link 4
Average traffic flow (Vehicles/s $\times 10^4$)	353.8	152.3	143	185

the case of absence of link 4 is 11.2% larger than in the case of presence of all links (Fig. 4). This result reveals that Braess' paradox appears at the low traffic density.

5.3. High traffic density

The traffic flow at the traffic density of 240 is considered as a case of high traffic density. The availabilities of links 2, 3 and 4 are taken as the design variables. The relaxed problem given by Eqs. (20) and (21) is solved. When the traffic flow is maximized, it is concluded that all road links are necessary. The traffic flow at a different traffic network is shown in Table 4b. The results show that the traffic flow in case of absence of any road link is smaller than in the case of presence of all road links (Fig. 4). At a high traffic density, the traffic flow on all road links almost reaches the maximum capacity of the whole traffic network. Since the absence of any road link causes the reduction of the traffic capacity of the whole traffic network, the traffic network of all road links is the optimal one.

6. Conclusions

In the traffic network design, it is reported that a newly constructed road sometimes causes a traffic congestion at another place and, as a result, the total road capacity is reduced. This is called Braess' paradox. The algorithm to identify the unnecessary road links causing the traffic congestion is presented in this study. The algorithm is applied to simple numerical examples. The results show that the proposed algorithm identifies the unnecessary road links in order to overcome the situation of Braess' paradox. Additionally, the optimized traffic network depends on the traffic density. At low traffic density, the algorithm identifies the unnecessary road links. At the highest traffic density, the algorithm does not.

It is pointed out in [6] that Braess' paradox disappears at high traffic density. Since at low traffic density, the traffic flow is smaller than the maximum capacity of the traffic network, the average velocity of the traffic flow depends on the topology of the traffic network. Therefore, the change of the traffic network topology can improve the capacity. Since at high traffic density, the traffic flow is almost equal to the maximum capacity of the traffic network, the traffic network including all road links is the best. The maximum traffic flow does not depend on the topology of the traffic network. Therefore, Braess' paradox disappears at high traffic density.

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