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Computer analysis of vibrations of hoisting system

Dagmara Tejszerska
Department of Engineering Mechanics of Silesian Technical University,
Konarskiego str.18A, 44-100 Gliwice, Poland

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Model of longitudinal vibrations of mine hoist, treated as a discrete-continuous system is formulated. The model includes phenomena connected with variations of the load, carried by each rope and with sliding of the rope contacting with pulley. The effects of changes of length of both branches of rope, variations of their stiffness, internal damping, friction and diversification of parameters of individual ropes in multirope system are taken into account. General model equations, relations describing movement of elementary segments of ropes and of the whole system, and the method of solving the obtained equations are presented in the paper. Nonlinear system of partial and ordinary differential equations is solved numerically. Example results of numerical simulation, showing the possibilities of the formulated model and the program — are presented.

1. Introduction

Vibrations of nonlinear discrete-continuous systems occur very frequently in engineering practice and in scientific considerations. Nevertheless, they still need further studies, improving mathematical description of the process and the methods of numerical simulation. The modelling of vibrations of ropes of mine hoisting system is a typical example of such problems.

The published studies include, among others, analyses of emergency braking [1, 2, 16], conditions of sliding [11] and reasons of changes of tensions in ropes [3]. Simple, three mass models of the system were still used [5, 6]. The model of hoisting process, jointly describing the set of phenomena, the consideration of which is characteristic for the presented paper (including variations of length of rope branches, changes of loads, carried by individual rope, dependence of stiffness of rope on stresses, damping and possibility of sliding) — has not been found in the literature as yet.

The moving, longitudinally vibrating ropes, with consideration of different forms of their contact with other elements of the system — cages and pulley — are the subject of this study.

The phenomena listed below are analysed succesively, formulating the model of hoisting system vibrations:

- vibrations of the basic discrete element of rope, with consideration of its inertia, elasticity and damping characteristics;
- motion of the pulley and elements of the rope, contacting with it, with analyses of frictional coupling and changes of boundary conditions, resulting from contact of both objects;
- dynamic equilibrium of elements of balance rope, changing the direction of motion at the lowest level;
- motion of the cage, treated as a stiff body, connected with the elastic elements of rope.

The nonlinear system of partial and ordinary differential equations was obtained as the result of the physical model assumed and the appropriately formulated mathematical model. It was solved numerically, with the use of program, implementing the developed model on the computer. Numerical simulation of analyzed object was performed, with different values of important parameters.

2. MATHEMATICAL MODEL OF HOISTING ROPE VIBRATION

The differential equation describing longitudinal vibrations of a massive (ρ) elastic (EA) element of rope, viscous damping (ν) being taken into consideration has the form [4]:

$$\rho A\left[\frac{\partial^2(x+u)}{\partial t^2}\pm g\right]+EA\left[1+\nu\frac{\partial}{\partial t}\right]\frac{\partial^2 x}{\partial l^2}=0\,. \tag{1}$$

Dynamic force F_L in the rope is described by the relation

$$F_L=EA\left(1+\nu\frac{\partial}{\partial t}\right)\frac{\partial x}{\partial l}=0\,. \tag{2}$$

The finite difference equation describing the motion of a discrete element i of the rope, with the separation of displacements x, constituting the dynamic increase of the transportation displacement u, resulting from pulley movement — has the form:

$$\Delta m(\ddot{x}_i+\ddot{u})+\frac{EA}{\Delta L}\left(1+\nu\frac{\mathrm{d}}{\mathrm{d}t}\right)(2x_i-x_{i-1}-x_{i+1})\pm\Delta m g=0\,. \tag{3}$$

Between the number of discrete elements (n), their dimensions (ΔL) and the total length of both branches of rope connecting the cages (L^A+L^B) obvious relation exists:

$$\Delta L=\left(L^A+L^A\right)/n\,. \tag{4}$$

The index i_p of the discrete element of rope contacting actually with the pulley is expressed by the formula:

$$i_\mathrm{p}=\mathrm{INT}(L_n^\mathrm{A}/\Delta L_n)+1\,. \tag{5}$$

Diagram of consecutive positions of this element is presented in Fig. 1.

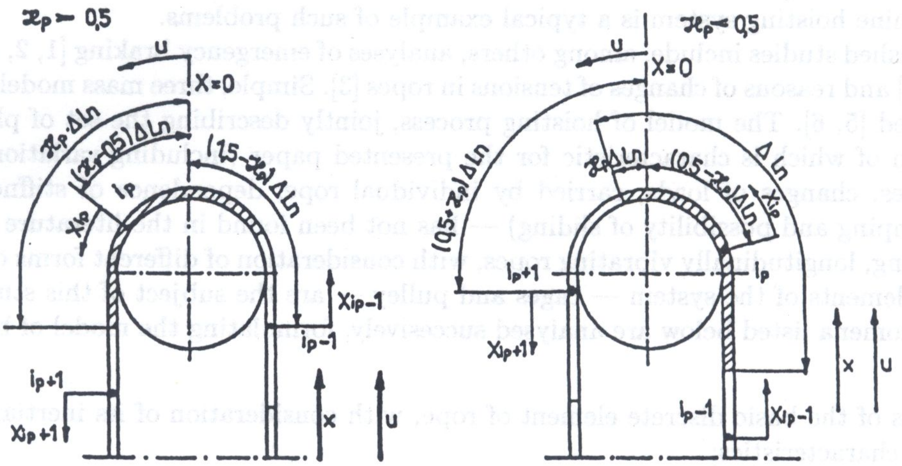

Fig. 1. Diagram of consecutive positions of the discrete element of rope contacting with pulley

The rate of displacement of the element i_p in relation to the pulley is denoted as κ_p:

$$\kappa_\mathrm{p}=i_\mathrm{p}-L_n^\mathrm{A}/\Delta L_n\,. \tag{6}$$

The equations of the dynamic equilibrium of discrete elements, situated in the vicinity of pulley, obtained taking into account the passing of the rope along the pulley and changes of length of both of its branches, have the form presented below (with subscript n for values Δm, EA, ΔL, concerning lifting rope):

- for element $i_p - 1$:

$$0 = \Delta m_n(\ddot{x}_{i_p-1} + \ddot{u}) + \left(\frac{EA}{\Delta L}\right)_n^A \left(1 + \nu\frac{\mathrm{d}}{\mathrm{d}t}\right)(x_{i_p-1} - x_{i_p-2})$$

$$+\begin{cases} \Delta m_n g(1+\kappa_p) + \left(\frac{EA}{\Delta L}\right)_n^A \left(1 + \nu\frac{\mathrm{d}}{\mathrm{d}t}\right)(x_{i_p-1} - x_{i_p}) & \text{for} \quad \kappa_p \le 0.5\,, \\ \Delta m_n g(2-\kappa_p) + \left(\frac{EA}{\Delta L}\right)_n^A \left(1 + \nu\frac{\mathrm{d}}{\mathrm{d}t}\right)\frac{x_{i_p-1}}{1.5-\kappa_p} & \text{for} \quad \kappa_p \ge 0.5\,, \end{cases}$$

- for element i_p (with multiplication by $|\kappa_p - 0.5|$, to avoid indeterminacy when $\kappa_p = 0.5$):

$$0 = |0.5 - \kappa_p|\Delta m_n(\ddot{x}_{i_p} + \ddot{u})$$

$$+\begin{cases} \Delta m_n g\frac{0.5-\kappa_p}{0.5} + \left(\frac{EA}{\Delta L}\right)_n^A \left(1 + \nu\frac{\mathrm{d}}{\mathrm{d}t}\right)[(0.5-\kappa_p)(x_{i_p} - x_{i_p-1}) + x_{i_p}] \\ \qquad\qquad \text{for} \quad \kappa_p \le 0.5\,, \\ -\Delta m_n g\frac{\kappa_p-0.5}{0.5} + \left(\frac{EA}{\Delta L}\right)_n^A \left(1 + \nu\frac{\mathrm{d}}{\mathrm{d}t}\right)[x_{i_p} + (\kappa_p-0.5)(x_{i_p} - x_{i_p+1})] \\ \qquad\qquad \text{for} \quad \kappa_p \ge 0.5\,, \end{cases}$$

- for element $i_p + 1$:

$$0 = \Delta m_n(\ddot{x}_{i_p+1} + \ddot{u}) + \left(\frac{EA}{\Delta L}\right)_n^B \left(1 + \nu\frac{\mathrm{d}}{\mathrm{d}t}\right)(x_{i_p+1} - x_{i_p+2})$$

$$+\begin{cases} -\Delta m_n g(1+\kappa_p) + \left(\frac{EA}{\Delta L}\right)_n^B x_{i_p+1}\frac{1}{0.5+\kappa_p} & \text{for} \quad \kappa_p \le 0.5\,, \\ -\Delta m_n g(2-\kappa_p) + \left(\frac{EA}{\Delta L}\right)_n^B (x_{i_p+1} - x_{i_p}) & \text{for} \quad \kappa_p \ge 0.5\,. \end{cases} \tag{7}$$

The equalization of the velocities of rope and pulley is assumed at the top point of the pulley, when sliding does not occur. Appropriate linearization was introduced to eliminate step changes of parameters, when the centre of element passes along this conventional point of contact with the pulley (specially of Young's modulus, depending on stresses in both branches of the rope) — which would act as an apparent excitation of vibrations.

The equation of the dynamic equilibrium of winding machine and pulley, with discrete elements of rope i_p, passing actually along it — is obtained taking into consideration the driving moment M, forces F_L in $k = 1, \ldots, N_n$ ropes and inertial moments of pulley and driver reduced on the axis of pulley J_{kp}:

$$\frac{M}{R} = \frac{J_{kp}\ddot{u}}{R^2} + \sum_{k=1}^{N_n}\Delta m_n(\ddot{u} + \ddot{x}_{i_p,k}) + N_n\Delta m_n g\frac{0.5-\kappa_p}{0.5}$$

$$+\begin{cases} \sum_{k=1}^{N_n}\left(1+\nu\frac{\mathrm{d}}{\mathrm{d}t}\right)_k/\Delta L_n\left[EA_n^{\mathrm{A}}(x_{i_\mathrm{p}}-x_{i_\mathrm{p}-1})-x_{i_\mathrm{p}+1}\frac{EA_n^{\mathrm{B}}}{0.5+\kappa_\mathrm{p}}\right] & \text{for}\quad \kappa_\mathrm{p}\le 0.5\,, \\ \sum_{k=1}^{N_n}\left(1+\nu\frac{\mathrm{d}}{\mathrm{d}t}\right)_k/\Delta L_n\left[EA_n^{\mathrm{B}}(x_{i_\mathrm{p}}-x_{i_\mathrm{p}+1})-x_{i_\mathrm{p}-1}\frac{EA_n^{\mathrm{A}}}{1.5-\kappa_\mathrm{p}}\right] & \text{for}\quad \kappa_\mathrm{p}\ge 0.5\,. \end{cases} \tag{8}$$

The difference of the forces in rope on both sides of pulley is the cause of the phenomenon of sliding. Difference of strains, depending on forces, causes the shortening or elongation of every element of rope before leaving the pulley and its shifting along the part or its full perimeter [5, 8, 14]. The sliding occurs when the ratio of forces in ropes on both sides of pulley exceeds the value determined by equation of Euler–Eytelwein:

$$F_{\max}/F_{\min}=\exp\left(\mu\alpha\right), \tag{9}$$

in which μ denotes the coefficient of friction between rope and pulley, and α the wrapping angle.

Increase of the ratio of forces stops, when it reaches the value determined by equation (9). The influence of factors tending to its increase is compensated by sliding [12]. The relationship between the displacement of the rope in respect to the pulley, caused by sliding δ_{sl} — and the changes of forces ΔF in both branches of rope has the form:

$$\Delta F=\delta_{\mathrm{sl}}(EA/L)\,. \tag{10}$$

Thus the value of the sliding displacement of rope δ_{sl} is equal to:

$$\delta_{\mathrm{sl}}=\frac{F^A-F^B\exp\left(\mu\alpha\right)}{(EA/L)^A+(EA/L)^B\exp\left(\mu\alpha\right)}\,, \tag{11}$$

where superscripts A, B denote the lifting and falling branches.

Value δ_{sl} is introduced into terms describing the forces in ropes in equations of dynamic equilibrium of pulley (8) and of the discrete elements of rope, situated in its vicinity (7).

A coefficient κ_r, characterizing the rate of displacement of discrete element j_r of the balance rope, passing over its lower loop, in relation to its lower point, necessary for analysing the equilibrium of this element — was introduced:

$$\kappa_\mathrm{r}=j_\mathrm{r}-L_\mathrm{w}^\mathrm{B}/\Delta L_\mathrm{w}\,, \tag{12}$$

where subscript "w" denotes the balance rope.

Forces acting on the lower elements of the balance rope are presented in Fig. 2:

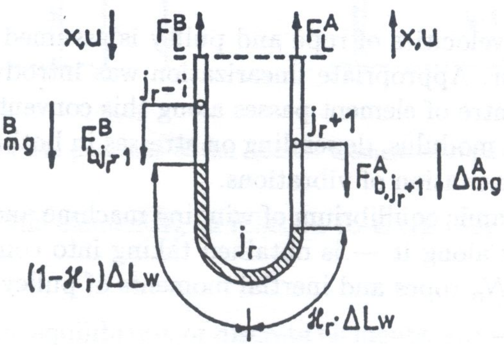

Fig. 2. Forces acting on lower elements of balance rope

Equations of dynamic equilibrium of the lower elements of the balance rope have the form:

- for element $j_r - 1$:

$$\left(\frac{EA}{\Delta L}\right)_{\mathrm{w}}^{B} \left(1 + \nu_{\mathrm{w}} \frac{\mathrm{d}}{\mathrm{d}t}\right) (x_{j_r-1} - x_{j_r-2}) + [\Delta m_{\mathrm{w}} + (1 - \kappa_{\mathrm{r}}) \Delta m'_{\mathrm{w}}] (\ddot{x}_{j_r-1} + \ddot{u} - g) = 0 ,$$

- for element $j_r + 1$:

$$\left(\frac{EA}{\Delta L}\right)_{\mathrm{w}}^{A} \left(1 + \nu_{\mathrm{w}} \frac{\mathrm{d}}{\mathrm{d}t}\right) (x_{j_r+1} - x_{j_r+2}) + (\Delta m_{\mathrm{w}} + \kappa_{\mathrm{r}} \Delta m_{\mathrm{w}}) (\ddot{x}_{j_r+1} + \ddot{u} + g) = 0 . \tag{13}$$

In the time instant, when the successive element j of the balance rope takes place in its lower loop ($j = j_r$) — substitute value $\Delta m'_{\mathrm{w}}$ is calculated, to obtain unchanged force, acting on the preceding element of the rope $j_r - 1$:

$$\Delta m'_{\mathrm{w}} = \Delta m_{\mathrm{w}} \frac{g - \ddot{x}_{j_r} - \ddot{u}}{g - \ddot{x}_{j_r-1} - \ddot{u}} . \tag{14}$$

At the same time a new element $j_r + 1$ is subtracted from the discrete element of rope, which passed the loop (obtaining index $j_r + 2$) — the velocity and acceleration of which are determined as:

$$\dot{x}_{j_r+1} = \dot{x}_{j_r+2} , \quad \ddot{x}_{j_r+1} = \ddot{x}_{j_r+2} . \tag{15}$$

The displacement of this element is calculated, considering the elongation due to gravitation and actual acceleration:

$$x_{j_r+1} = x_{j_r+2} - \frac{\Delta m_{\mathrm{w}} (g + \ddot{u} + \ddot{x}_{j_r+2})}{(EA/\Delta L)_{\mathrm{w}}} . \tag{16}$$

Modulus of elasticity E, occuring in above presented equations, displays parabolic dependence on stress σ, having the form [7]:

$$E = E_0 + E_1 \sigma + E_2 \sigma^2 . \tag{17}$$

In closed kinematic chain, formed by the hoisting system — the role of links, connecting the lifting and balance ropes is fulfilled by the cages, modelled as massive particles. The equation of dynamic equilibrium of cage, having mass m_{s} and dynamic displacement x_{s} takes the form:

$$m_{\mathrm{s}} (\ddot{x}_{\mathrm{s}} + \ddot{u}) + \sum_{k=1}^{N_n} \left[\frac{EA}{\Delta L/2} \left(1 + \nu \frac{\mathrm{d}}{\mathrm{d}t}\right)\right]_k (x_{\mathrm{s}} - x_{i=1,k})$$
$$+ \sum_{l=1}^{N_{\mathrm{w}}} \left[\frac{EA}{\Delta L/2} \left(1 + \nu \frac{\mathrm{d}}{\mathrm{d}t}\right)\right]_l (x_{\mathrm{s}} - x_{j=n_{\mathrm{w}},l}) \pm m_{\mathrm{s}} g + F_{\mathrm{t}} . \tag{18}$$

The drag force F_{t}, acting on the cage, can be described by three components, connected with sliding friction, viscous friction (proportional to the velocity of the cage) and aerodynamic drag (proportional to its square):

$$F_{\mathrm{t}} = \left[C_{0,\mathrm{t}} + C_{2,\mathrm{t}} (\dot{x}_{\mathrm{s}} + \dot{u})^2\right] \operatorname{sign}(\dot{x}_{\mathrm{s}} + \dot{u}) + C_{1,\mathrm{t}} (\dot{x}_{\mathrm{s}} + \dot{u}) . \tag{19}$$

Linearized form of this dependence was introduced, by substituting coefficient $C_{1,\mathrm{t,z}}$, calculated in every time step, basing on extrapolated value of velocity $|\dot{x}_{\mathrm{s}} + \dot{u}|_0$:

$$\begin{aligned} F_{\mathrm{t}} &= C_{1,\mathrm{t,z}} (\dot{x}_{\mathrm{s}} + \dot{u}_{\mathrm{s}}) , \\ C_{1,\mathrm{t,z}} &= \frac{C_{0,\mathrm{t}}}{|\dot{x}_{\mathrm{s}} + \dot{u}_{\mathrm{s}}|_0} + C_{1,\mathrm{t}} + C_{2,\mathrm{t}} |\dot{x}_{\mathrm{s}} + \dot{u}_{\mathrm{s}}|_0 . \end{aligned} \tag{20}$$

3. Method of solving model equations

The set of linear algebraic equations was obtained by the connection of dynamic equilibrium equations of every element of the model and by substitution of time derivatives by adequate finite difference expressions [9], based on parabolic splines of time transients of every dynamic displacement $x_{i,k}, x_{j,l}, \ldots$, in every time step Δt. The transient of the chosen parameter (denoted below generally as x) can be written in the form:

$$x(\tau) = a\tau^2 + b\tau + c, \qquad \text{for } 0 < t < \Delta t. \tag{21}$$

Considering the condition of continuity of velocity at the boundaries of the neighbouring time steps Δt and denoting boundary values by indexes "o" and $'$, we can obtain:

$$x(\tau) = \Big[(x' - x^{\mathrm{o}} - \dot{x}^{\mathrm{o}}\Delta t)\,/\Delta t^2\Big]\,\tau^2 + \dot{x}^{\mathrm{o}}\tau + x^{\mathrm{o}}. \tag{22}$$

Average values of displacements and velocities in the time step Δt, introduced into equations of dynamic equilibrium, are expressed by values on its edges, applying integration of expressions (21). Acceleration and velocity at the end of a time step are also determined:

$$\begin{aligned}
x_{\mathrm{sr}} &= \frac{1}{\Delta t}\int_0^{\Delta t} x(\tau)\,\mathrm{d}\tau = \frac{x'}{3} + \frac{2x^{\mathrm{o}}}{3} + \frac{\dot{x}^{\mathrm{o}}\Delta t}{6}, \\
\dot{x}_{\mathrm{sr}} &= \frac{x' - x^{\mathrm{o}}}{\Delta t}, \qquad \ddot{x} = \frac{2(x' - x^{\mathrm{o}} - \dot{x}^{\mathrm{o}}\Delta t)}{\Delta t^2}, \\
\dot{x}' &= \frac{2(x' - x^{\mathrm{o}})}{\Delta t} - \dot{x}^{\mathrm{o}}.
\end{aligned} \tag{23}$$

Method of parabolic splines was chosen as the most convenient for the purpose of this work, after an analysis of the precision, time of calculation and stability of some methods of integration of the equations [15].

The general shape of the obtained set of equations (24) — shows a structure of sparse matrix and arrangement of blocks, describing its particular parts. Elements of matrix are corrected in every time step, depending on variations of the elasticity modulus of rope, connected with changes of stresses. The column matrix of free elements is calculated also in every time step. It contains values of displacements and velocities at the beginning of time step and actual values of excitation of vibrations.

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The set of equations (24) is solved numerically for every time step. The method of triangular reduction is used, with full utilization of informations about blocks having zero values. Matrix is diagonally dominating. The values of displacements x' and velocities $\dot{x}'$ at the end of time step are obtained, and treated as starting values for the next time step. The actual values of forces, stresses and accelerations are then calculated. The stationary positions of cages and tensions in ropes are calculated before the beginning the simulation of vibrations. They are necessary — their lack would introduce the disturbance in the form of non existing dynamic excitation during the starting period of motion. The calculations are based on equations describing stationary state, simpler, but similar to the general dynamic equations [13] and are performed iteratively.

The length of time step is automatically chosen, basing on the period of natural vibrations, calculated approximately from highest frequency of the system, consisting of three bodies — massive pulley and two cages, connected by elastic ropes, having masses included into bodies considered.

The mathematical model formulated provided the basis for the elaboration of the computer program, simulating coupled longitudinal and transverse vibrations of a multirope hoisting system. Program consists of segments, serving for the calculation, preparation of input data and graphical presentation of results on the screen and printer. Segments of the program are written in FORTRAN and TURBO PASCAL.

The model and computer program developed enable the simulation of vibrations of a hoisting system, having optional design and process parameters to be made. The kinematic excitation of vibrations in steps of rectangular or trapezoid shape of start-up acceleration and braking deceleration is treated as basic. The vibratons forced dynamically, by the effective moment of the winding machine are also simulated.

The verification of the model, basing on the published experimental results [10] and the set of numerical experiments was performed with the use of the program. Examples of obtained graphs, showing the possibilities of the program and sensivity of the model to the variations of important parameters [13], are presented below.

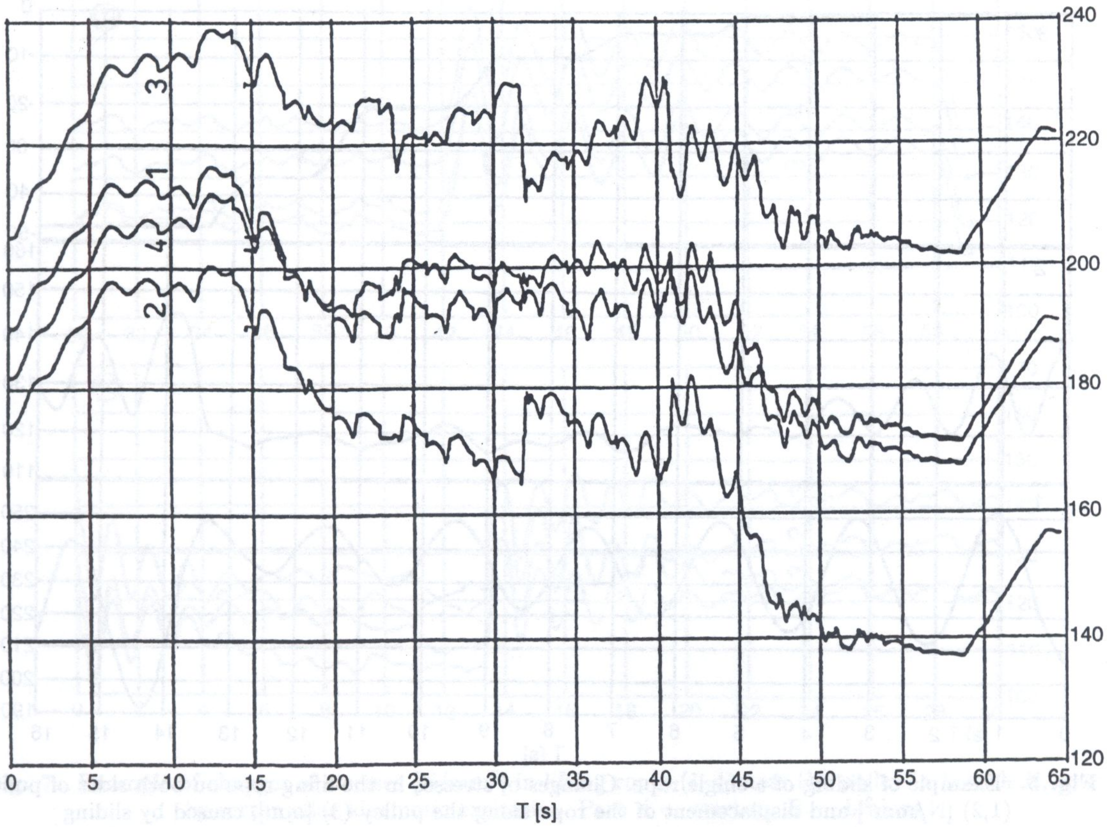

Fig. 3. Example of changes of stresses in lifting ropes (1–4) [N/mm^2], in uprising branch A, near the pulley, caused by kinematic excitation, of trapezoid shape

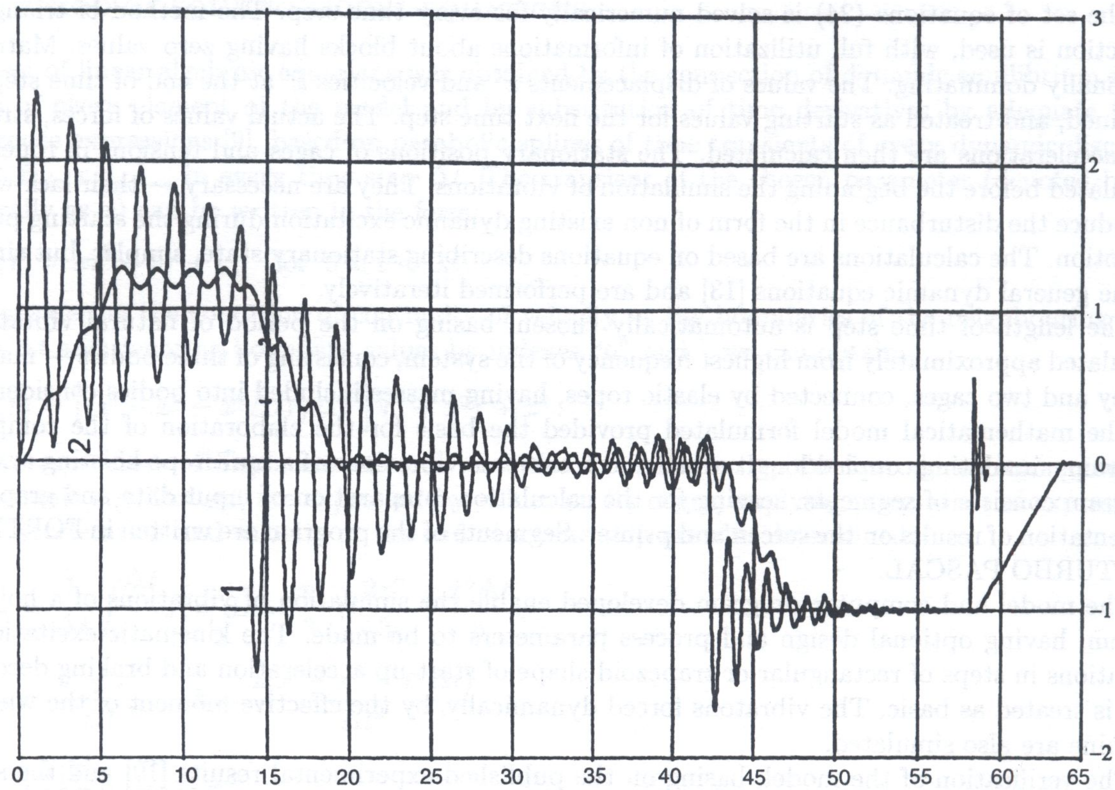

Fig. 4. Comparison of longitudinal acceleration of the lifted cage [m/s^2], caused by kinematic excitation, of rectangular (1) and trapezoid (2) form

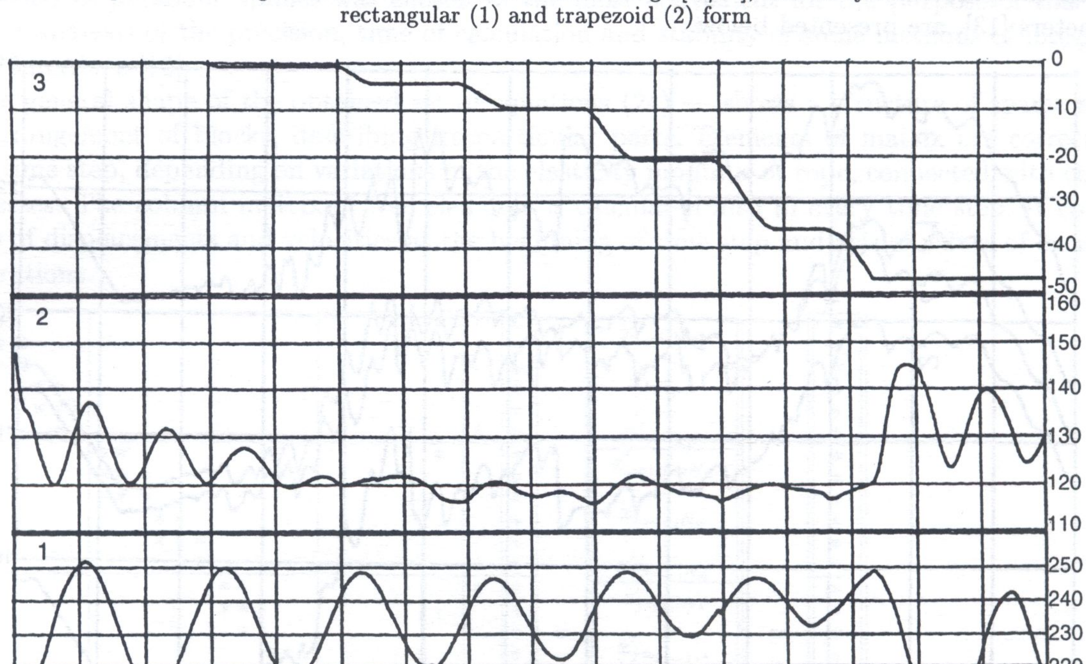

Fig. 5. Example of sliding of a single rope. Changes of stresses in the lifing rope on both sides of pulley (1,2) [N/mm^2] and displacement of the rope along the pulley (3) [mm], caused by sliding

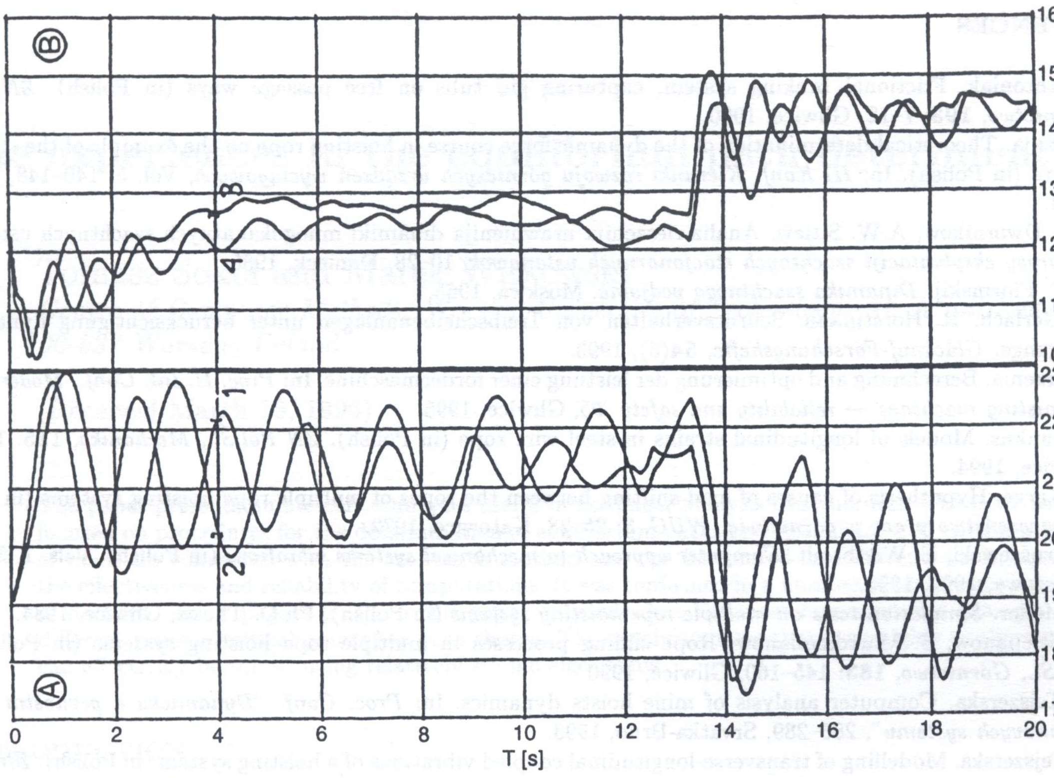

Fig. 6. Comparison of stresses in the lifting rope near the pulley, during startup, in the rising (part A) and falling branch (B), having the basic and the double value of coefficient E_0 of Young modulus. Curve (2,4) — $E_0 = 65$, (1,3) — $E_0 = 130$ kN/mm^2

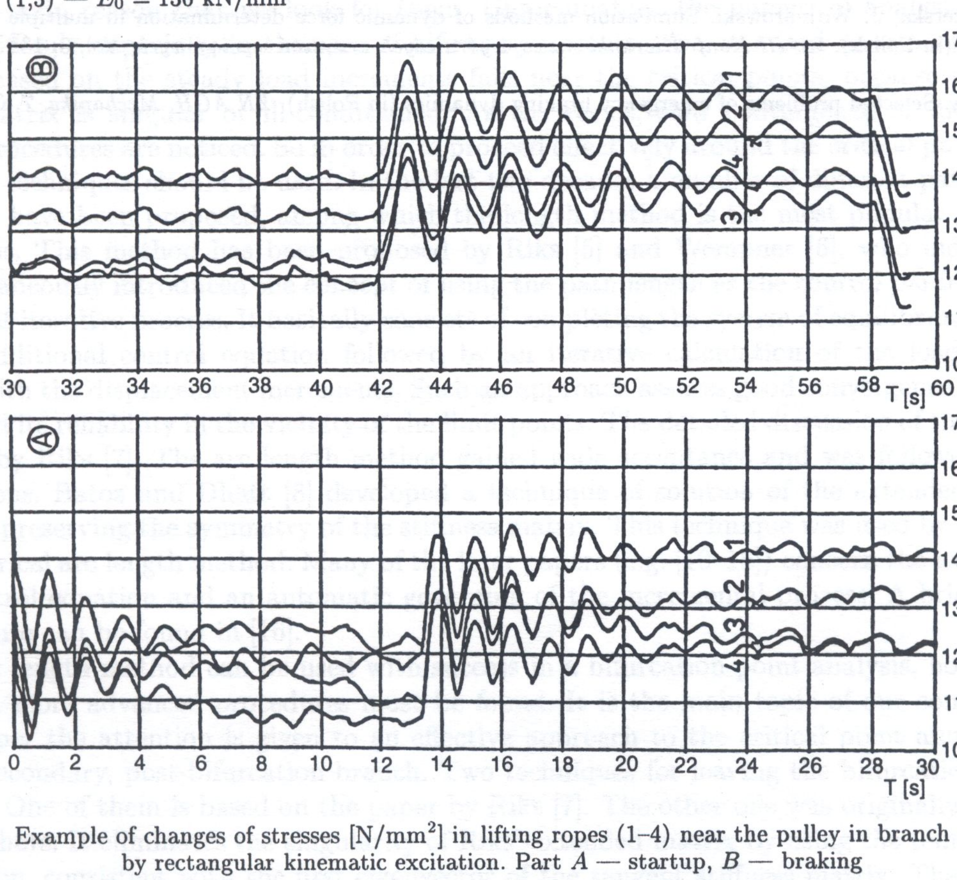

Fig. 7. Example of changes of stresses [N/mm^2] in lifting ropes (1–4) near the pulley in branch B, caused by rectangular kinematic excitation. Part A — startup, B — braking

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