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Optimization of the block foundations

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Optimization of shape and size of foundation should consider the type of loading, numerous geotechnical and geological parameters as well as various safety and economic criteria. Dimensioning depends on various, often controversial, conditions and involves many uncertainties. Therefore we search compromise solutions using nondeterministic analysis based on mathematical logic. In our solution we use computer simulations and results of reliability theory.

1. STRUCTURAL OPTIMIZATION

Structural optimization seeks the selection of design variables to achieve within the limits placed on the geometry, the structural behavior, or other factors. Its goal is defined by the objective function for specified loading and environmental conditions. It is not possible to discuss structural optimization without entering the mathematical programming as follows:

Generally, the optimization problem of the structure has been restricted to the mathematical programming model as follows:

$$\text{minimize } \{f(\mathbf{X}),\ \mathbf{X} \in E,\ E \in R^n\}, \tag{1}$$

where: $\mathbf{X} = (x_1, x_2, \ldots, x_n)$ are the geometrical parameters that describe the structural configuration, E is the permissible domain determined by a set of design constraints such as functional constraints, architectural constraints, stress, displacement constraints and other conditions, $f(\mathbf{X})$ is the objective function determined by designer and R^n is the n-dimension Euclidean space.

The constraints and objective in model (1) are usually nonlinear. So it is expressed in terms of the global minimum of the function $f(\mathbf{X})$. In many methods a point of local minimum $\mathbf{X}'$ is usually sought, it means that there is a region around $\mathbf{X}$ in which $f(\mathbf{X}')$ is the minimum feasible point. It must be stated that there is no simple method to obtain a direct solution. Most of the approaches, which are effective for the solution of nonlinear structural design models, belong to two classes:

- Gradient searching techniques in which, the most studies presume an initial configuration coming from previous experience. It means that the gradients of its objective function with respect to the design variables $(\partial f/\partial x_j,\ j = 1, 2, \ldots, n)$ are computable from a initial structure. Then it should be indicated how that improved $(k+1)$-th design may be generated from a knowledge of the characteristics at k-th design step:

$$\begin{aligned} x_{k+1} &= x_k + \alpha_k d_k, \\ d_k &= -\nabla f(x_j), \qquad j = 1, 2, \ldots, n, \end{aligned} \tag{2}$$

  where α_k is scalar value required to minimize $f(x)$ in direction d_k.

- The second type of research approach have been widely presented in literature. These research techniques involve various methods which solve a sequence of linearized models or approximation of a nonlinear problems and lead to solution of the linear problems. For example, using the first term of a Taylor expansion we have: from a known value $f(x^0)$ of a function $f(x)$ at $\{x\} = \{x^0\}$ a new value $f(x^1)$ at $\{x\} = \{x^1\}$ is given by:

$$f(x^1) = f(x^0) + \nabla f(x^0) \cdot (\{x^1\} - \{x^0\}), \tag{3}$$

using the Taylor expansion the nonlinear programming problem (1) will be approximated as a linear programming problem.

Various techniques have been applied to problems in structural designing and a comprehensive review are given by Schmit and Fleury [8]. In all these approaches the design problem is formulated in such a way that the structural analysis is carried out separately for checking the design variables. Most of the studies show that great weight reductions can be obtained changing the geometry of the structure. This problem is to design the "best" shape of the structure varying certain design parameters.

2. STRUCTURAL SHAPE OPTIMIZATION

The interest with this problem has increased in the last decade, due to the possibility of computers using and the progress made in the field of mathematical programming (see [10]). Authors began by choosing a set of admissible locations in the design space and then linked each node to every other node to produce a fundamental structure. An optimum design was obtained by mathematical programming. In terms of iterative design the shape optimization problem can be stated as follows: Let X represents the subset of node coordinates x_n which is allowed to vary. Let $f(X)$ represents the weight of the structure, the problem is to find $x_n \in X$ which:

$$\text{minimize } f(X), \quad X \in E \tag{4}$$

the question is how to extend equation (4) and to bring it to an objective function which should be realistic but not overly complex. When one considers variations of the functional involved in the design problem, the effect of change in geometry must be predicted in addition to the effect of changes in the design variables. It appears that the given geometry of a structure can be improved in an automatic fashion. It was resolved by the graph theory in some simple situations. On the other hand this problem has been solved by the introduction of graph plotters in which more intuitive approaches may be used and CAD method is developed to determine the optimum solution. However, this approach was limited to structure with the single influence of a determined load vector on the shape of the structure (Fig. 1)

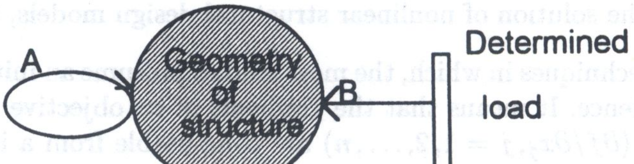

Fig. 1. A simple relation of the "STRUCTURE-MEDIUM" system. A is the internal relation of the geometry of the structure, B is the external relation of the structure

During the calculation of a structure founded on the soil, it must be remembered that its shape will not only depend on a given loading but also will be influenced by the surrounding media. Then the "structure-medium" interaction (Fig. 2) cannot be neglected.

A possibility to calculate the "structure-soil" interaction is offered by the substructure method together with the boundary elements method. To determine optimum dimensions of the structure, a number of factors must be taken into account, such as:

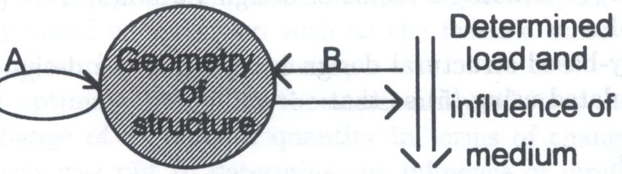

Fig. 2. Complex relation of the "STRUCTURE-SOIL" system

1. There is an interdependence between the shape of the structure and its surrounding soil medium. Any change in the environment will induce changes in the geometry of the structure, or even else any change in the shape of the structure will result in an increased or reduced influence on its environment. It is so called the geometric interaction of "structure-soil" system.

2. The geometry of the structure is determined by a series of parameters mutually dependent on each other (relation A — Fig. 2). Even the best solution for a single parameter will not suffice to attain a satisfactory functional effect for the whole system of parameters of the structure so that the general properties of the structure cannot be explained reviewing each of its parameters separately, one by one.

3. At the beginning of every structural design problem, there is a great deal of uncertainty related to load and strength of an actual structure. It is considered by repeated experiments in a frequency sense, i.e. chance can be measured. This is an approach based on so called objective probability point of view. Thus, an optimum engineering design requires the methodology of the reliability theory. It is now widely recognized that any non statistical approach generally represents a vague view only on the structural design problem. Reliability theory, aimed at using probability for safety evaluation of a structure and structural optimization, must be a decision-making process in an uncertain situation.

4. The optimal design is defined as a geometry of structure having not only least weight but also minimum influence from its surrounding medium or maximum stability and maximum *reliability* as well as *meeting the safety criteria* and *economic feasibility* of *"structure-soil"* system. Conflicting design requirements make it difficult to set up a single criterion. The difficulty lies in the problem of multi-criterial evaluation and option, and the ability to deal with thousands of parameters mutually dependent on each other. It is not a usual type of the mathematical programming problem as mentioned above.

3. Reliability-based structural optimization

Because the implied safety factor has a much greater effect on the cost weight of structure, it is now generally recognized that structural optimization must cope with uncertainties. The initial work of Charnes and Cooper [1] described the stochastic optimization model using the chance constraint programming technique. The authors' idea is to expand the objective and constraint function around the corresponding mean value. However, for the case of nonlinear function of the random variables, when the linearization takes place at the mean values of the random variables, errors may be introduced at increasing distances from the linearization points by neglecting higher order terms. Other stochastic optimization technique was presented for the structural optimization under uncertainty conditions. It can be defined as follows:

Let us consider the set of safety checking equations for structural design problem, which are presented in the form:

$$g_i(\mathbf{X}, \mathbf{Y}) \geq 0, \quad i = 1, 2, \ldots, m, \tag{5}$$

where $\mathbf{X} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$ denotes a vector of design variables, $\mathbf{Y} = (y_1, y_2, \ldots, y_r)^{\mathrm{T}}$ denotes a vector of random variables.

Generally, probability-based structural design is to define the design rules, in which the limit state probability is calculated using (5) so that:

$$P_f - P_f^0 \Rightarrow \text{minimum}, \tag{6}$$

where P_f^0 is the target limit state probability which is specified by the regulatory authority. Assuming that a structure may fail in one of the failure conditions:

$$P_f = P\{(g_1(\mathbf{X},\mathbf{Y}) < 0) \vee (g_2(\mathbf{X},\mathbf{Y}) < 0) \vee \ldots \vee (g_m(\mathbf{X},\mathbf{Y}) < 0)\} \tag{7}$$

with the second-moment approach, the first-order second-moment safety index is given by:

$$S_i = m_{gi}/d_{gi} \tag{8}$$

where m_{gi}, d_{gi} denote the first-order approximations of the mean value and standard deviation of g_i. If the random variables are not normally distributed and $g_i(.)$ is nonlinear, advanced second moment can be used in which, the safety index S is defined by M. Shinozuka [9] as follows:

$$\begin{aligned} S &= \text{minimize } (\boldsymbol{\lambda}^{\mathrm{T}}\mathbf{A}\,\boldsymbol{\lambda})^{1/2}, \\ \boldsymbol{\lambda} &= (\lambda_1, \lambda_2, \ldots, \lambda_r)^{\mathrm{T}}, \\ \lambda_i &= (y_i - m_{yi})/d_{yi}, \end{aligned} \tag{9}$$

where $\mathbf{A}$ is the square matrix, it denotes the inverse of a correlation matrix of the random variables $\mathbf{Y}$. From the central limit theorem we have the normally distributed g_i from that the failure probability P_f is given by:

$$P_f = 1 - \int_{-\infty}^{S} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}t^2\right) \mathrm{d}t. \tag{10}$$

Generally, the reliability-based optimization problem just defined is a deterministic problem in its formulation (mathematical programming method) but the constraints or the objective function involve the failure probabilities and are implicit to the design variables as follows: The failure probability or reliability $(1 - P_f)$ is used in the objective function, that is:

$$\begin{aligned} &\text{minimize } \{P_f = P_f(\mathbf{X},\mathbf{Y})\}, \\ &\text{minimize } \{C = C^0 + P_f C_f\}, \\ &\text{maximize } \{U_t = B - C^0 - EL\}, \end{aligned} \tag{11}$$

where, C^0 is the initial cost, C_f is the cost of failure of the structure, B is the benefit derived from the existence of the structure, EL is the expected loss due to failure, U_t is the total utility function of the structure. The failure probability is also used as a constraint as follows:

$$P_f(\mathbf{X},\mathbf{Y}) \le P_f^0, \tag{12}$$

where P_f^0 is the given data, which is the target limit state probability. On the other hand, a structural reliability-based optimization problem can be formulated as:

$$\underset{\mathbf{X}\in E,\mathbf{Y}\in L}{\text{maximize}}\ \mathbf{S}^*, \tag{13}$$

$$\mathbf{S}^* = \text{minimum}\,(S_1, S_2, \ldots, S_m), \tag{14}$$

$$L = \{\mathbf{Y} \mid u_{\mathbf{Y}}(\mathbf{Y}),\quad u_{\mathbf{Y}}(\mathbf{Y}) \text{ is any probability distribution}\}, \tag{15}$$

where $S_1, S_2, \ldots$ are determined by (8).

As a result, mathematical programming method is being developed, different techniques being used in the reliability-based optimization such as the feasible directions method, sequential quadratic programming and stochastic programming.

The sensitivity of the optimum design to the uncertainty of its parameters creates another problem. It provides a change of a response quantity in terms of change of design variable. In this approach, the emphasis was put to determine the influence of input statistical parameters, including distribution function and coefficient of variation on optimal design variables and cost. Other approach suggests the possibility to obtain a formulation for probabilistic optimal design. Frangopol [3] explored in depth the sensitivity of overall reliability and optimum solution. The results indicate its usefulness for practical aspects. However, sensitivity analysis of the mutual relation among the design variables of the structure can also be handled for shape optimization problem.

4. Pareto-optimal concept used in structural optimization

Recent advances in structural optimization resulted in the development of techniques based on nonlinear programming. However, there often exist many practical problems, which involve several objectives to be considered by the designer. These objectives are usually conflicting mutually with one another. An approach for solving this problem seems to be multi-objective nonlinear programming, which arose from the solution of economical problems. It can be formulated as follows:

$$\underset{\mathbf{X}\in E}{\text{minimize}}\ \{F(\mathbf{X}) = (f_1(\mathbf{X}), f_2(\mathbf{X}), \ldots, f_k(\mathbf{X}))\}. \tag{16}$$

Ordinarily in multiobjective optimization a possibility exists for trade-off among objectives i.e. a change in design may result in improvement according to one or more objectives only at the expense of worsening as measured by others. The earliest work reporting the consideration of this problem in mathematical programming appears to be that of Kuhn and Tucker [7]. The progress in consideration the multiobjective optimization was summarized by Stadler [12]. A method for generating a Pareto optimal set for multiobjective optimization capable of being formulated as serial stage-state discrete dynamic programs was presented by M.A. Rosenman and J.S. Gero [6]. The development of a computer program, which provides the designer with useful trade-off information from which sound decisions can be made for linear multiobjective problem was described by Dlesk and Liebman [4]. The multiobjective optimization represented in design of a clamped-hinged beam was determined by Adali [11] by approximating the area by linear splines. The numerical results for a number of design examples were presented in the form of graphs of optimal trade-off curves. Usually there is no unique solution which would give an optimum for all the objectives simultaneously. For this problem, a sound logic and universally accepted solution concept do not exist yet. Thus a new concept, different from that used in scalar optimization, is used in solution of the multiobjective optimization problem. Generally speaking, the multiobjective optimization seems to be the mathematical model of the decision making problem involving more than one decision maker. For this type of decision problem, a good decision should not be dominated by other alternatives in the sense that there should not be other feasible alternatives which yield a greater satisfaction for decision maker. It was stated that a good decision must be a non dominated solution.

5. Shape optimization of block foundation

Foundations for transmission line structures must satisfy the same basis criteria that must be met for all types of foundations: they must be stable, and have an adequate factor of safety or level of reliability against failure. They must not move excessively, which could impair the function of the foundation. They must be economical, or at least cost effective for the particular type of structure.

Here, we have to solve a difficult problem, in which the non smoothness (or non differentiable), discontinuities and multiobjective are characters of the real foundation optimization problem.

In conventional optimization methods, the standard process to avoid the difficulties caused by the multiple objectives and the non differentiability is to scale the objective functions by some simple procedure (sum them up together) and then regularize the non differentiable functions by some appropriate smoothing technique like penalization methods. These simplifications may facilitate the numerical solution of the foundation optimization problem, however they may confuse the relationship between the mathematical model and the real system. In this case, we have proposed an approach based on simulation and optimization technique. By means of simulation and optimization we can calculate the "best-fit" model of the "foundation–soil" system. It is based on numerical simulation techniques as follows:

5.1. Shape representation of the foundation

Any geometry of the foundation exists within an embedding space with dimensionally equal to the minimum number of coordinates required to describe the structure of the foundation. We assume that the foundation can be represented by the coordinates of a set of (n) specific points, in which their horizontal (**HX**) and vertical (**VX**) coordinates are described by the ordered sets:

$$\mathbf{HX} = (x_1, x_3, \ldots, x_{2n-1}), \tag{17}$$

$$\mathbf{VX} = (x_2, x_4, \ldots, x_{2n}). \tag{18}$$

An example of ordered coordinates of two points A and B is furnished in Fig. 3. From the geometrical point of view, all coordinates are treated equally, even if they have different mechanical interpretations. We have a model of the foundation, which is presented by its boundary line. The construction operation in the boundary representation of the foundation ensures that the boundary is well formed in the sense that it is closed. The method for creating and manipulating the geometry in the boundary representation of the foundation includes sweeping, which define the boundary line of the foundation by moving a path in space, and "tweaking" which performs local operations on the geometry but leaves the topology of the model intact. The geometry of the foundation (G) is specified by vector of $2n$ independent components, and is obtained by:

$$G = \mathbf{X} \subset \mathbf{HX} \otimes \mathbf{VX}, \tag{19}$$

where: $\otimes$ represents the Cartesian product. Now, the following function may be introduced for calculation of the new proposed variables as:

$$h = x^i = x^i_l + (x^i_u - x^i_l)\psi^i, \quad i = 1, 2, \ldots, 2n, \tag{20}$$

where x^i_u, x^i_l are the maximum and minimum limits of variable x^i and ψ^i is a random number characterized by uniform distribution in the interval $(0, 1)$ – it seems an evolutionary factor.

The computation of the foundation geometry is, in fact, the derivation of its explicit geometric representation from complete but implicit geometric input data. Thus, the first stage requires the preparation of a subprogram to generate parameters for the foundation in terms of the chosen design variables. The simulation of the Monte Carlo method is used for simultaneous determination of the objective space Q using the following formulations:

Firstly, because each point $x \in E$ is defined by an appropriate point $\psi^i \in \Psi$ through the function h, therefore

$$h : \Psi \to E, \tag{21}$$

where E is defined by the all restraining conditions. From the other side, we have:

$$\mathbf{F} : E \to Q, \quad \mathbf{F} = (f_1, f_2, \ldots, f_k), \tag{22}$$

from (21) and (22) we obtained:

$$F \circ h : \Psi \Rightarrow Q. \tag{23}$$

5.2. Determining the Pareto-optimal solution

Let us denote E the decision space, while Q is the criteria space. An element $\mathbf{F}_i \in Q$ is called the outcome of a decision. In our work the concept of a partial relation is generally used to define the optimality. It is a fundamental approach based on the concept of Pareto optimality.

A Pareto-optimal solution is qualitatively defined as one where any improvement of one criteria can be achieved only by permitting the deterioration of another. It is generally formulated as:

Assume that $\mathbf{X}^0 \in \mathbf{E}$ is the solution of problem (16) such as:

$$\mathbf{F}(\mathbf{X}^0) = \text{minimum}\ \mathbf{F}(\mathbf{X}), \tag{24}$$

where $\mathbf{F}(\mathbf{X}^0) = \{f_1(\mathbf{X}^0), f_2(\mathbf{X}^0), \ldots, f_k(\mathbf{X}^0)\}$, $\mathbf{X}^0 = (x_1^0, x_2^0, \ldots, x_{2n}^0)$. There is no such other point $\mathbf{X}^+ \in E$ for which:

$$\mathbf{F}(\mathbf{X}^+) \succ \mathbf{F}(\mathbf{X}^0), \quad \mathbf{X}^+ \neq \mathbf{X}^0, \tag{25}$$

where the symbol $\succ$ denote the preferred relation. It is read as "better" or "predominates", etc. It is seen that $\succ$ is a partial ordering over Q. From (24), (25) we have:

$$\bigwedge_{\mathbf{X}^0, \mathbf{X}^+ \in E} \{\mathbf{F}(\mathbf{X}^+) \succ \mathbf{F}(\mathbf{X}^0) \Rightarrow \mathbf{X}^+ = \mathbf{X}^0\}. \tag{26}$$

From this definition we can see that $\mathbf{X}^0 \in E$ is a Pareto-optimal solution. There is no feasible solution that could provide a simultaneous improvement for all the objectives. For convenience, we will denote the set of all Pareto-optimal solutions by P.

The Pareto-optimal solution set (P): Assume that the relative importance between the two criteria is known:

$$(\mathbf{F}^i \neq \mathbf{F}^j) \Leftrightarrow \{(f_1(\mathbf{X}^i) \neq f_1(\mathbf{X}^j)) \vee (f_2(\mathbf{X}^i) \neq f_2(\mathbf{X}^j))\}, \tag{27}$$

$$(F^i \succ F^j) \Leftrightarrow \{(f_1(X^i) \succ f_1(X^j)) \wedge (f_2(X^i) \succ f_2(X^j))\}. \tag{28}$$

Thus in trade-off relationship among two objective functions we have:

$$\bigwedge_{X^i, X^j \in E} \ \bigwedge_{l, r \in (1,2)} \{(f_r(\mathbf{X}^i) \succ f_r(\mathbf{X}^j)) \wedge (f_l(\mathbf{X}^i) \succ f_l(\mathbf{X}^j))\}, \quad l \neq r. \tag{29}$$

It is evident from (28) and (29) that these points $\mathbf{X}^i, \mathbf{X}^j$ will not be compared. From definition (25) we have $\mathbf{F}(\mathbf{X}^i), \mathbf{F}(\mathbf{X}^j)$, which are the Pareto-optimal solutions. Formally, a point $\mathbf{F}(\mathbf{X}^0) \in Q$ is Pareto-optimal if:

$$\left(\neg \bigvee_{\mathbf{F}(\mathbf{X}^0) \in \mathbf{Q}} \ \bigwedge_{\mathbf{F} \in \mathbf{Q}} \{\mathbf{F}(\mathbf{X}^0) \succ \mathbf{F}(\mathbf{X})\}\right) \wedge \left(\bigwedge_{r \in (1,2)} \ \bigvee_{\mathbf{X}^0 \in E} \{f_r(\mathbf{X}^0) \succ f_r(\mathbf{X})\}\right). \tag{30}$$

From (30) for all the points $\mathbf{F}(\mathbf{X}^i) \in Q$ which is not a Pareto-optimal solution, we always have $\mathbf{F}(\mathbf{X}^j) \in P \subset Q$ so $\mathbf{F}(\mathbf{X}^j) \succ \mathbf{F}(\mathbf{X}^i)$ and they can be expressed by:

$$\bigwedge_{\mathbf{F}(\mathbf{X}^i) \notin P \subset Q} \{(\mathbf{F}(\mathbf{X}^j) \in P \subset Q) \Rightarrow \mathbf{F}(\mathbf{X}^j) \succ \mathbf{F}(\mathbf{X}^i)\}. \tag{31}$$

Usually, Pareto-optimal solutions for multiobjective optimization problem consist of infinite numbers of points, in general not all of them are obtained in a practical numerical calculation. In this work we will define a Pareto-optimal solution set by eliminating all points that do not belong to it, with the following "refinement" operation.

Abstract treatment of the problem:

1. The objective point selected is $\mathbf{F}(\mathbf{X}^1)$ and the point compared is $\mathbf{F}(\mathbf{X}^2)$: $\mathbf{F}(\mathbf{X}^1), \mathbf{F}(\mathbf{X}^2) \in Q$,

$$\mathbf{F}(\mathbf{X}^1) = \{f_1(\mathbf{X}^1), f_2(\mathbf{X}^1), \ldots, f_k(\mathbf{X}^1),\ k \geq 2\},$$

$$\mathbf{F}(\mathbf{X}^2) = \{f_1(\mathbf{X}^2), f_2(\mathbf{X}^2), \ldots, f_k(\mathbf{X}^2),\ k \geq 2\}.$$

Notation: instead of $(\mathbf{X}^1, \mathbf{F}(\mathbf{X}^1)) \succ (\mathbf{X}^2, \mathbf{F}(\mathbf{X}^2))$ we write $F(X^1) \succ F(X^2)$.

Definition 1: Let E and Q are by preference ordered sets. A mapping F from E to Q is called an order of homomorphism if

$$\mathbf{X}^1 \succ \mathbf{X}^2 \Rightarrow \mathbf{F}(\mathbf{X}^1) \succ \mathbf{F}(\mathbf{X}^2). \tag{32}$$

2. **Definition 2:** Let us $(f_r(\mathbf{X}^i) \succ f_r(\mathbf{X}^j)) = TRUE,\ \neg(f_r(\mathbf{X}^i) \succ f_r(\mathbf{X}^j)) = FALSE$ we have:

$$\neg\left(f_r(\mathbf{X}^i) \succ f_r(\mathbf{X}^j)\right) \Leftrightarrow \left(f_r(\mathbf{X}^i) \prec f_r(\mathbf{X}^j)\right). \tag{33}$$

Comparing the objective points, the following three cases may occur:

$$\begin{aligned}
a &: \bigwedge_{r\in K} \{f_r(\mathbf{X}^i) \succ f_r(\mathbf{X}^j),\ K = 1,2,\ldots,k\},\\
b &: \bigwedge_{r\in K} \{f_r(\mathbf{X}^i) \prec f_r(\mathbf{X}^j),\ K = 1,2,\ldots,k\},\\
c &: \bigvee_{l\in K} \left\{ \bigwedge_{r\in K, r\neq l} \{(f_r(\mathbf{X}^i) \succ f_r(\mathbf{X}^j)) \wedge (f_l(\mathbf{X}^i) \prec f_l(\mathbf{X}^j))\} \right\}.
\end{aligned} \tag{34}$$

In the first case (a) we know for example that the point $\mathbf{X}^j$ does not belong to the Pareto-optimal solution set. However, it should also be mentioned that the point $\mathbf{X}^i$ cannot be regarded as belonging to Pareto solution set, because it should be compared with other points located in the permissible space Q. Accordingly, for the second case (b) the point $\mathbf{X}^i$ does not belong to the Pareto solution set. In the third case (c) both points $\mathbf{X}^i, \mathbf{X}^j$ may possibly belong to the Pareto set (but it is not certain). So it is difficult to determine a point belonging to the Pareto set. It is easier however, to determine points that do not belong to the Pareto set. Let NP denote a set of points that do not belong to Pareto-optimal solution set, then:

$$NP \cup P = Q. \tag{35}$$

From (32), (33) and taking into account de Morgan principle:

$$\neg\{(f_r(\mathbf{X}^i) \succ f_r(\mathbf{X}^j)) \wedge (f_l(\mathbf{X}^i) \succ f_l(\mathbf{X}^j))\} \Leftrightarrow \{(\neg(f_r(\mathbf{X}^i) \succ f_r(\mathbf{X}^j))) \vee (\neg(f_l(\mathbf{X}^i) \succ f_l(\mathbf{X}^j)))\} \tag{36}$$

the procedure is easily derived as follows:

$$(\mathbf{F}(\mathbf{X}^i) \in NP) \Leftrightarrow \{(f_1(\mathbf{X}^i) \succ f_1(\mathbf{X}^j)) \vee (f_2(\mathbf{X}^i) \succ f_2(\mathbf{X}^j)) \vee \ldots \vee (f_k(\mathbf{X}^i) \succ f_k(\mathbf{X}^j))\} = FALSE, \tag{37}$$

$$(\mathbf{F}(\mathbf{X}^j) \in NP) \Leftrightarrow \{(f_1(\mathbf{X}^i) \succ f_1(\mathbf{X}^j)) \wedge (f_2(\mathbf{X}^i) \succ f_2(\mathbf{X}^j)) \wedge \ldots \wedge (f_k(\mathbf{X}^i) \succ f_k(\mathbf{X}^j))\} = TRUE. \tag{38}$$

From that $NP \cup P = Q$ we have:

$$P = Q/NP. \tag{39}$$

The comparison of objective functions is based on the value of relative deviation $Z(X)$ from the extreme solution $f^0(x)$:

$$\bigwedge_{f_r(\mathbf{X})\in P} Z_r(\mathbf{X}) = \frac{| f_r(\mathbf{X}) - f_r^0(\mathbf{X}) |}{f_r^0(\mathbf{X})}. \tag{40}$$

For finding solutions characterized by the minimum value of the relative deviation from the extreme solutions we assume that:

$$Z_r(\mathbf{X}^i) < Z_r(\mathbf{X}^j) \Leftrightarrow f_r(\mathbf{X}^i) \succ f_r(\mathbf{X}^j), \quad r = 1, 2, \ldots, k. \tag{41}$$

For seeking the preference solution $(\mathbf{X}^*, \mathbf{F}(\mathbf{X}^*))$, calculations are carried out according to formula (40) for each objective $i = 1, 2, \ldots, k$ and for $X = X^j$, a vector $\mathbf{Z}_{ij}$ is obtained as:

$$\mathbf{Z}_{ij} = \{Z_{j1}(\mathbf{X}), Z_{j2}(\mathbf{X}), \ldots, Z_{jk}(\mathbf{X})\}. \tag{42}$$

The calculations are performed for m variants: thus a rectangular $(m \times k)$ matrix $\mathbf{Z}$ is obtained:

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{12} & \ldots & Z_{1k} \\ Z_{21} & Z_{22} & \ldots & Z_{2k} \\ \ldots & \ldots & \ldots & \ldots \\ Z_{m1} & Z_{m2} & \ldots & Z_{mk} \end{bmatrix}. \tag{43}$$

Subsequently, the matrix $\mathbf{\Phi}$ can be obtained from:

$$\mathbf{\Phi} = \begin{bmatrix} \phi_{11} & \phi_{12} & \ldots & \phi_{1k} \\ \phi_{21} & \phi_{22} & \ldots & \phi_{2k} \\ \ldots & \ldots & \ldots & \ldots \\ \phi_{m1} & \phi_{m2} & \ldots & \phi_{mk} \end{bmatrix}, \tag{44}$$

where: $\phi_{jr} =| Z_{jr} - \max\ Z_{jr} |$, $r = 1, 2, \ldots, k$. We regard the matrix $\mathbf{\Phi}$ as a payment table of a game for two. We select from each row the elements: $\max\limits_r \phi_{jr} = \sup\{\phi_{jr}, r = 1, 2, \ldots k\}$. The calculations are carried out for $j = 1, 2, \ldots, m$ yielding:

$$\text{maximize } \mathbf{\Phi} = \begin{pmatrix} \max\limits_r \phi_{1r} \\ \max\limits_r \phi_{2r} \\ \ldots \\ \max\limits_r \phi_{mr} \end{pmatrix}, \quad r = 1, 2, \ldots, k. \tag{45}$$

On the basis of the multiobjective optimization vector $(\mathbf{X}^* : \mathbf{F}(\mathbf{X}^*) \in P)$ is assumed as the solution of our problem when it satisfies the condition:

$$\mathbf{\Phi}(\mathbf{X}^* = \min_j \max_r \phi_{jr}(\mathbf{X}), \quad j = 1, 2, \ldots, m. \tag{46}$$

For the control of the optimization process using Monte Carlo simulation, in numerous examples, the entropy coefficient $(SP1)$ is introduced.

$$SP1 = \text{ent}(\mathbf{X})/\text{Max}\{\text{ent}(\mathbf{X})\}, \tag{47}$$

$$\text{ent}(\mathbf{X}) = -\sum_1^n P_i(\mathbf{X} \in E) \ln P_i(\mathbf{X} \in E), \tag{48}$$

where: P_i denotes a probability of the design variable $\mathbf{X} \in E$. Different results with this aspect are shown in Figs. 4, 6.

6. Calculation example

Figure 3 submits the calculation scheme Input data: the cohesion of the soil: $c = 0$, the soil density $\gamma = y_1$ and the friction angle $\varphi = y2$ taken as random parameters (y_i) with a triangular distribution $u(y)$:

$$u(y_i) = \begin{cases} (y_i - m_{yi} + \sqrt{6}d_{yi})/6d_{yi}^2 & \text{for } \ y_i \in (m_{yi} \div \sqrt{6}d_{yi} - m_{yi}), \\ (-y_i + m_{yi} + \sqrt{6}d_{yi})/6d_{yi}^2 & \text{for } \ y_i \in (m_{yi} \div m_{yi} + \sqrt{6}d_{yi}), \\ 0 & \text{for } \ y_i \notin (m_{yi} - \sqrt{6}d_{yi} \div m_{yi} + \sqrt{6}d_{yi}), \end{cases} \tag{49}$$

where m_{yi}, d_{yi} are the means and deviations of y_i, which are given as: $m_{y1} = 16.5$ kN/m^3, $d_{y1} = 1.92$ kN/m^3, $m_{y2} = 35°$, $d_{y2} = 3°$, foundation material parameter $\gamma_b = 22$ kN/m^3; External load: $H = 44$ kN; $V = 64$ kN; The foundation geometry is defined by the following conditions: $0.45 \le x1_1 \le 1.0$; $0.85 \le x2_1 \le 1.25$; $0.85 \le x2_1 \le 1.25$; $1.5 \le x3_2 \le 3.0$; $0.75 \le x4_1 \le 1.25$; $1.0 \le x5_1 \le 2.5$; $1.3 \le x5_2 \le 4.8$; $2.0 \le x6_2 \le 4.0$; $x1_1 = x3_1 = x7_1$; $x3_2 = x4_2$; $x5_1 = x6_1 = x8_1$; $0.2 \le x2_1 - x1_1 \le 0.6$; $0.3 \le x4_1 - x3_1 \le 0.6$; $0.3 \le x5_2 - x4_2 \le 0.8$; $0.5 \le x6_2 - x4_2 \le 1.0$; $x2_1 - x1_1 \le x4_1 - x3_1$; $x5_2 - x4_2 \le x6_2 - x3_2$. Objective functions are included: minimize material volume of the foundation

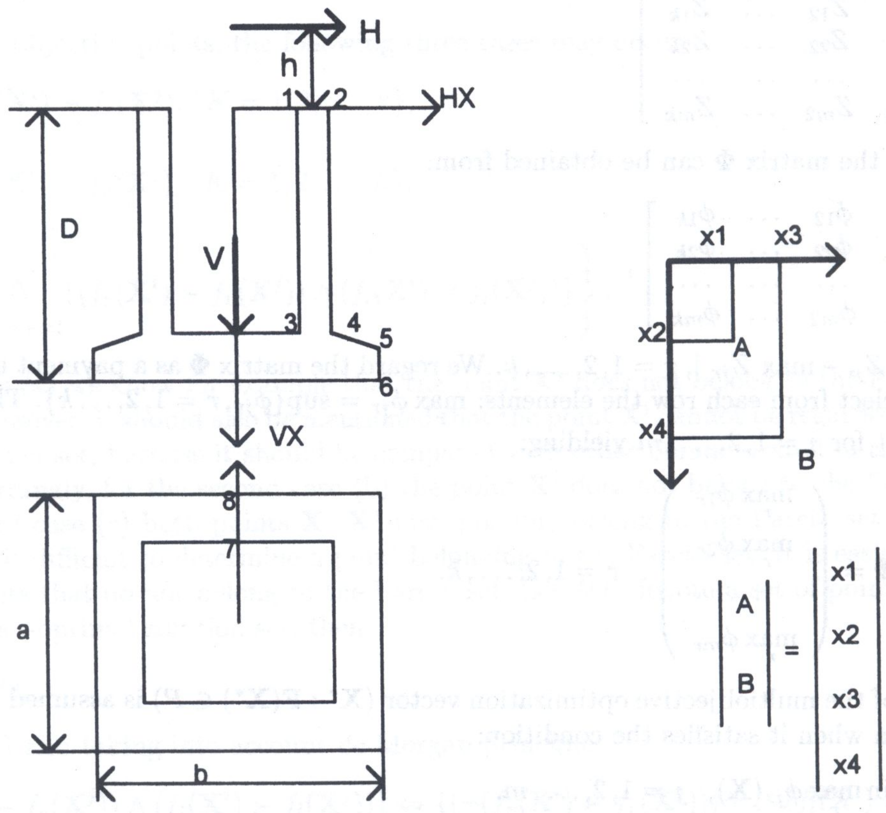

Fig. 3. Computation scheme of foundation: a) The foundation scheme, b) Determination of coordination of characteristic points, where $x1 = xA_1$, $x2 = xA_2$, $x3 = xB_1$, $x4 = xB_2$

$(f1)$; maximize safety factor of the foundation $f2, f2 = M_l/M_h$ ($f2 > 1.2$ and $f2 \to$ maximum), where M_l denotes limit moment, M_h is the moment of the force H, see [5]; minimize volume of needed earthworks $(f3)$ and maximize the reliability of the foundation $f4, f4 = 1 - P_f$ ($f4 > 0$ and $f4 \to$ maximum), P_f jest the failure probability. The compromise solution set is shown in Table 1. Figures 5, 7 show some results of the numerical simulation of the shape of the foundation.

Figure 4 shows a trade-off relationship among the four objectives (in the compromise solution set (in the table 1) with entropy coefficient $SP1 = 3$). Figure 6 shows a trade-off relationship among the four objectives (in the nine compromise solutions with the entropy coefficient $SP1 = 2.5$). Here, the entropy coefficient $SP1$ affects the difference between the compromise solution sets (Figs. 4–6). It would be studied in the future.

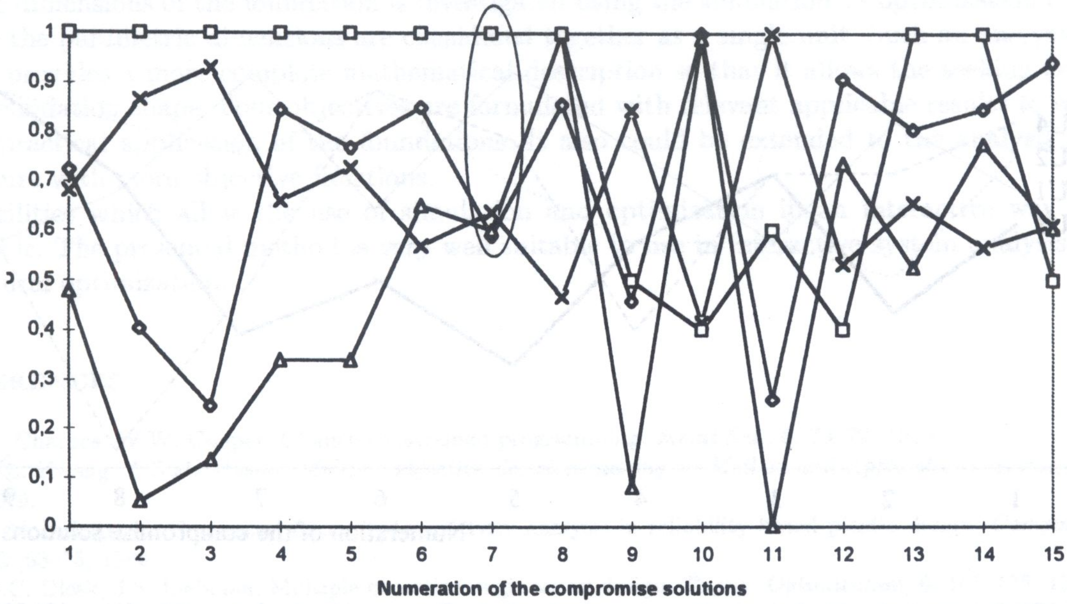

Fig. 4. Trade-off relationship between four objectives and the Pareto-optimal solution "0" ($f1^* = 231.6945$, $f2^* = 1.8978$, $f3^* = 107.4863$, $f4^* = 1.0$) with the entropy coefficient $SP1 = 3$. Where: $-\diamond-$, $U_1 = 1 - Z_1$; $-\times-$, $U_2 = 1 - Z_2$; $-\triangle-$, $U_3 = 1 - Z_3$; $-\square-$, $U_4 = f_4$; $Z_i, i = 1, 2, 3$ are calculated by Eq. (40)

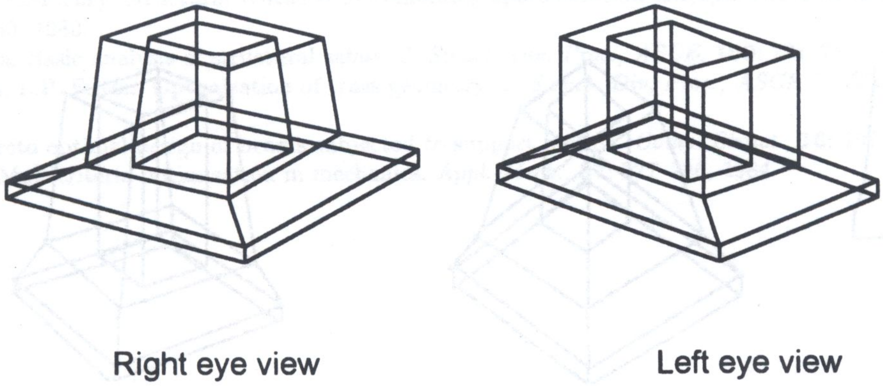

Fig. 5. Stereographic projection of the foundation

Table 1. The compromise solution set $(1, 2, 3, \ldots, 15)$

	f_1	f_2	f_3	f_4		f_1	f_2	f_3	f_4
1	209.914	2.0384	116.646	1.0	**9**	253.128	2.5168	147.229	0.5
2	262.199	2.5888	149.297	1.0	**10**	164.255	1.2396	76.788	0.4
3	288.329	2.7697	142.781	1.0	**11**	285.931	2.9781	153.202	0.6
4	189.562	1.9691	127.237	1.0	**12**	179.121	1.5881	96.849	0.4
5	202.279	2.1693	127.284	1.0	**13**	196.069	1.9544	112.956	1.0
6	189.404	1.6963	103.092	1.0	**14**	189.190	1.6898	93.763	1.0
7	231.694	1.8978	107.486	1.0	**15**	173.293	1.8123	106.923	0.5
8	187.921	1.3928	76.665	1.0					

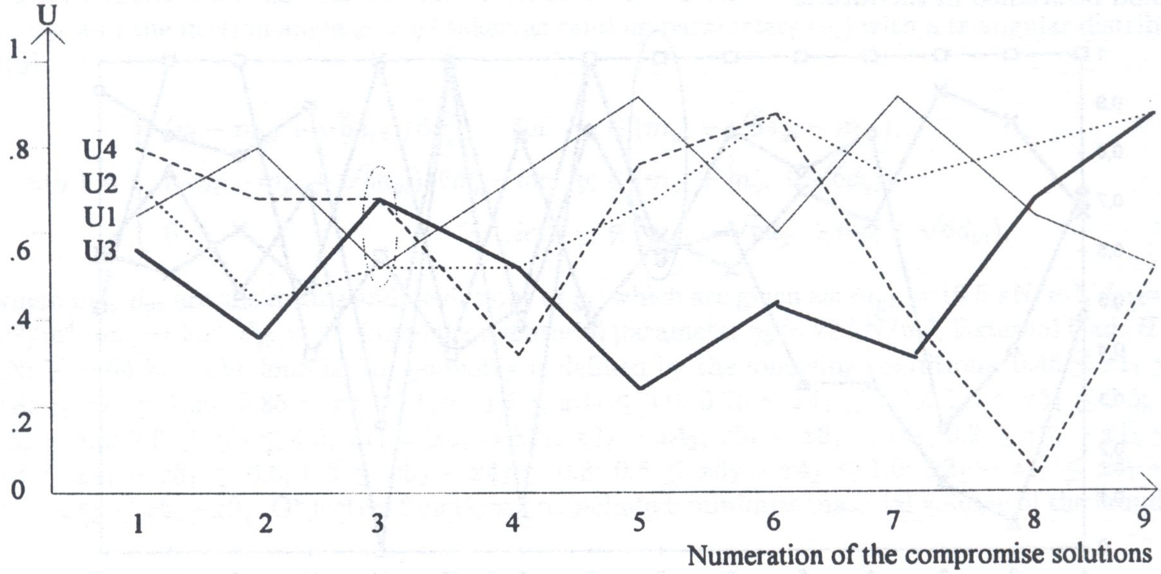

Fig. 6. Trade-off relationship between four objectives and the Pareto-optimal solution "0" ($f1^* = 189.5616$, $f2^* = 1.9690$, $f3^* = 127.2372$, $f4^* = 0.6$) with the entropy coefficient $SP1 = 2.5$

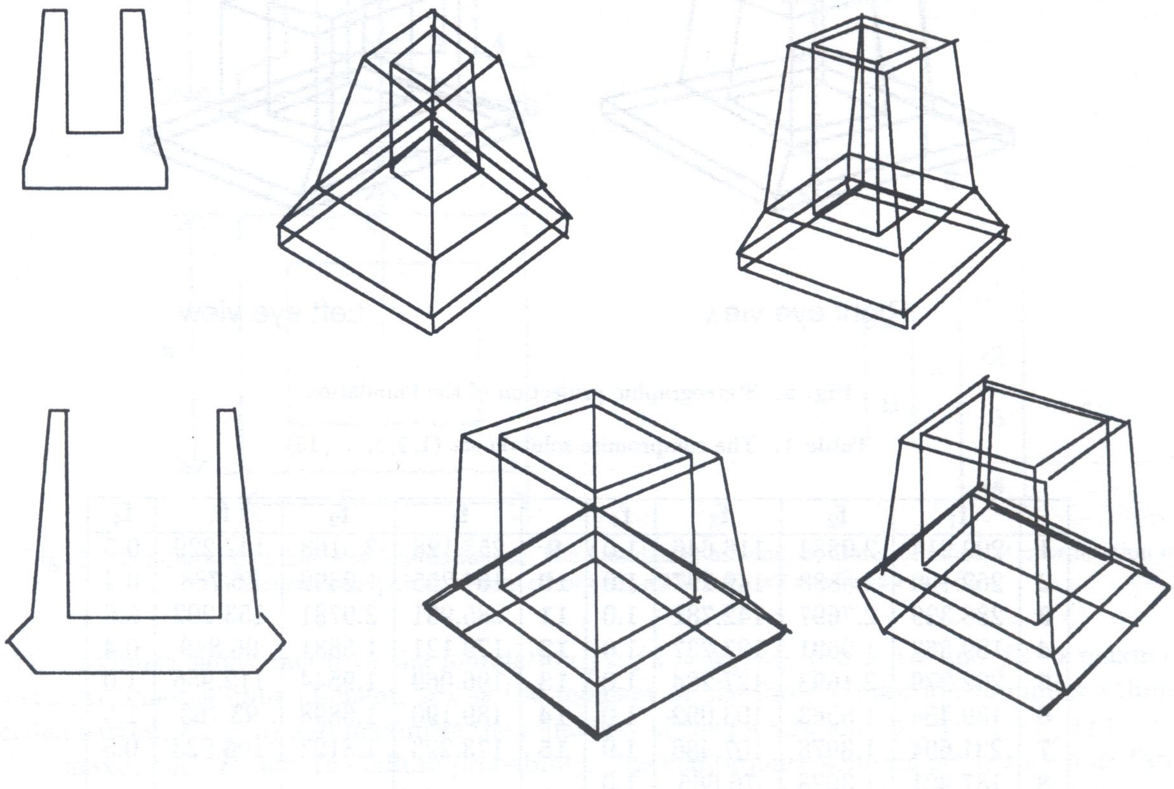

Fig. 7. Two typical structures with their axonometric projection which geometry is compared

7. Conclusions

In our study on the optimization of the foundation geometry, effect of interaction between the parametric dimensions of the foundation is investigated using the simulation — optimization technique where the parametric dimensions are considered together as a single unit. Such an analysis in this study provides a more complete mathematical description so that it allows the seeking process of new foundation shape. Four objectives are formulated with relevant applicable results to cover the most practical application of the foundations. It also could be extended to the analysis of other structure with more objective functions.

Facilities which allow the use of simulation and optimization in an interactive way may be available. The presented method is very well suitable to use in interactive system analysis and for structural optimization.

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