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Optimization of cylindrical shells subjected to pitting corrosion

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The paper presents modelling of optimization process of thin-walled structures such as vertical cylindrical reservoirs subjected to pitting corrosion. The problem is formulated in terms of nonlinear mathematical programming. The function which is a product of its constituents is accepted as the optimization criterion. The choice of an optimal thickness of the reservoir shell along the height is determined from the conditions of its equal reliability.

1. Introduction

The surfaces of reservoir under pressure of a liquid and those operating in a corrosive medium are often subjected to pitting corrosion. In this case it is rather important to carry out an estimation of the strength and reliability of the structure, because the development of at least one corrosion pit which gives rise to a substantial hole causes the loss of its service capability and leads to serious emergency consequences. On the other hand, as mentioned in [1], reservoires occupy the leading place in the volume of construction and in the amount of steel used for it. For this reason the problems of increasing their strength decreasing the weight of metal and the amount of labour for their manufacture are urgent. The development of a probability model of failure for thin-walled structures due to pitting corrosion was presented in [2]; it was supposed that the initial pitting size distribution along the surface of the structure and the time of appearance of the first hole are normal variates. Optimal design of type of structures using the criterion of average expected utility was discussed in [3].

2. Formulation of the operation problem

The paper presents the modelling of a comprehensive approach to the design of optimal structures of this class from the utility point of view and the degree of the relation to its components. The product of the average expected utility function [4]: $U(\mathbf{X}) = B(\mathbf{X}) - H_1(\mathbf{X}) - L(\mathbf{X})$ and characteristic coefficients of its constituents is accepted as an optimization criterion for the cylindrical shell

$$F = U(\mathbf{X})\mu(K,B)\mu(K,H_1)\mu(K,L), \tag{1}$$

where $\mathbf{X}$ is a vector of variable parameters (from the set of structure states K); $B(\mathbf{X})$ — average income expected from the service during its design period of life T with regard to a possible failure at the moment of time $t_{\text{fail}} < T$; $H_1(\mathbf{X})$ — the initial cost, $L(\mathbf{X})$ — damage due to the failure of structure $\mu(K,B)$, $\mu(K,H_1)$ and $\mu(K,L)$ are characteristic coefficients or membership functions of the design (their values vary within the range [0,1]) which define the degree of the compliance of the structure with the present optimal parameters of the income, initial cost and loss, respectively. Modelling of this type of membership functions depends on the priority of every constituent factor of the utility function. In this case they are equivalent, and the membership functions are accepted as follows:

$$\mu(K,B)=\begin{cases}0, & \text{if } B\le B_{\min},\\ 1/2+1/2\sin\{\pi[B-(B_{\rm opt}+B_{\min})/2]/(B_{\rm opt}-B_{\min})\}, & \text{if } B_{\min}\le B\le B_{\rm opt},\\ 1, & \text{if } B>B_{\rm opt}.\end{cases}$$

$$\mu(K,L)=\begin{cases}1, & \text{if } L\le L_{\rm opt},\\ 1/2-1/2\sin\{\pi[L-(L_{\rm opt}+L_{\max})/2]/(L_{\max}-L_{\rm opt})\}, & \text{if } L_{\max}\le L\le L_{\max},\\ 0, & \text{if } L>L_{\max}.\end{cases}$$

$$\mu(K,H_1)=\begin{cases}1, & \text{if } H_1\le H_{\rm opt},\\ 1/2-1/2\sin\{\pi[H_1-(H_{\rm opt}+H_{\max})/2]/(H_{\max}-H_{\rm opt})\}, & \text{if } H_{\rm opt}\le H_1\le H_{\max},\\ 0, & \text{if } H_1>H_{\max},\end{cases}$$

where $B_{\min}, L_{\max}$ and $H_{\max}$ are minimum and maximum permissible values, $B_{\rm opt}, L_{\rm opt}$, and $H_{\rm opt}$ are suboptimal values of the income, loss and initial cost of the tank adopted by the designer, respectively.

The purpose of the optimization task is to find the vector of the structure parameters $\mathbf{X}_{\rm opt}$ that maximizes the function (1) with limitation of the reliability:

$$[F=U(\mathbf{X})\mu(K,B)\mu(K,L)]\mu(K,L)]\Rightarrow\max,\quad P(T)\ge P_*,\tag{2}$$

where $P(T)$ — the reliability, P_* — the value of assumed reliability.

The vector of variable parameters $\mathbf{X}$ is assumed as follows: $\mathbf{X}=\{x_1,\ x_2\}^{\rm T}=\{n,\ T\}^{\rm T}$, where n — the number of constant thickness belts (sheets) into which the reservoir is divided; T — designed service life of the reservoir. The reservoir is under the influence of the hydrostatic internal pressure of a liquid, corrosive medium (such as petroleum products) of density ρ. The general reservoir dimensions: radius R and height H are known.

3. Calculation methods

Let us show the determination of the values included in (2). Income and loss due to the failure of the structure will be modelled according to [3]:

$$B(\mathbf{X})=\int_0^T B^0(t)p_{\rm fail}(t)\,{\rm d}t,\tag{3}$$

where: $B^0(t)=b(1-e^{-rt})/r$ — income; b — annual income in the case there is no failure; $r=\ln(1+r')$; r' — interest on the capital;

$$p_{\rm fail}(t)=P'_{\rm fail}(t)=[1-P(t)]'\tag{4}$$

is density of the failure probability of the structure during the period of its service.

The average value of losses (or a loss due to failure) up to the present time is modelled similarly:

$$L(\mathbf{X})=\int_0^T L^0(t)p_{\rm fail}(t)\,{\rm d}t,\tag{5}$$

where: $L^0(t)=L_te^{-rt}$ — losses; L_t — total damage evaluated before the structure entered into service.

The initial cost of the structure is determined as [1]:

$$H_1(\mathbf{X}) = C_3 + C_{\rm m}, \tag{6}$$

where $C_3 = C_{\rm om} + C_i + C_{\rm p}$ — manufacturing cost; $C_{\rm om} = 1.07(\sum^n C_{{\rm np}i} k_{{\rm np}i} G_i + 1.5G)$ — cost of the materials; $C_{{\rm np}i}$ — wholesale price of sheets for the i-th part of the cost of main structure; $G_i = 2\pi R h_i H\gamma/n$ — weight of the i-th part; $C_i = C_{\rm ob} + C_{\rm cb} + C_{\rm n} \cong 0.62\sqrt{Gn}$ — cost of the manufacture structure; $C_{\rm ob}$, $C_{\rm cb}$, $C_{\rm n}$ — cost of machining, erection welding and rolling; $C_{\rm p}$ — cost of structure, FOB destination $C_{\rm p} = 1.1406[(C_{\rm om} + C_i)1.0054 + 2.66]$; $C_{\rm m} = 0.641\sqrt{Gn}$ — cost of assembling the reservoir.

Now let us determine the function of the shell reliability included in (2). The reliability of the shell structure is the probability of a random event consisting in the fact that no pitting formation will exceed the permissible level — in this case, the thickness of the sheet — during the present period of its operation $0 \le t \le T$. The equation of corrosion is modelled in the form:

$$\mathrm{d}l_i/\mathrm{d}t = \alpha + \beta\sigma_i, \tag{7}$$

where l_i — current depth of pitting; α and β — constant coefficients; σ_i — effective stress in the i-th sheet. The solution of the equation (7) if: $t = 0$, $l_i = l_{0i}$, is as follows:

$$l_i = l_{0i} + (\alpha + \beta\sigma_i)t = l_{0i} + b_i t.$$

If we take the initial depth of pitting l_{0i} as a random value with a normal density $q(l_{0i}) = \exp[-(l_{0i} - \bar{l}_{0i})^2/2\sigma^2_{l_{0i}}]/\sqrt{2\pi}\sigma_{l_{0i}}$ and take into account that b_i is a constant value, then l_i is a fixed random value. In this case the probability that no pitting will exceed the thickness of the i-th sheet h_i (taken as a constant value) is determined as follows [5]:

$$P_i^* = 1/\sqrt{2\pi\sigma_{l_{0i}}} \int_{-\infty}^{h_i - b_i t} \exp[-(l_{0i} - \bar{l}_{0i})^2/2\sigma^2_{l_{0i}}]\,\mathrm{d}l_{0i} = 0.5 + \Phi(a_i),$$

where $a_i = (h_i - b_i t - \bar{l}_{0i})/\sigma_{l_{0i}}$, $\Phi(a_i) = 1/\sqrt{2\pi}\int_0^{a_i} e^{-z^2/2}\,\mathrm{d}z$ — Laplace integral.

If we denote the total number of pitting formations on the surface of the whole reservoir by N and assume the thickness along the height of the shell in such a way that the reliability of each sector is constant (equally reliable), then the reliability of the whole reservoir is determined by the expression:

$$P = (P_i^*)^N.$$

Assuming the permissible level of reliability P_* and the total number of pitting formations, it is easy to calculate the required values of thickness h_i complying with the principle of equal reliability of the reservoir. In this case: $P_i^* = (P_*)^{1/N}$ and $a = a_i = (h_i - b_{it} - \bar{l}_{0i})/\sigma_{l_{0i}} = \text{const}$, where a is found from the condition:

$$(P^*)^{1/N} = \Phi(a).$$

Taking $\bar{l}_{0i} = c_1 h_i$, $\sigma_{l_{0i}} = c_2 h_i$, we get:

$$a = (h_i - c_1 h_i - b_i t)/c_2 h_i, \quad b_i = \alpha + \beta\sigma_i. \tag{8}$$

As the effective stress in the i-th sheet is $\sigma_i = \rho H i R/n h_i$, the unknown quantities h_i are determined as follows: $h_i = K + \sqrt{k^2 + p}$, $i = 1, \ldots, n$, where $k = \alpha t/2(1 - c_1 - c_2 a)$; $p = \beta H i \rho R t/n(1 - c_1 - c_2 a)$.

Having determined the function of reliability and having substituted the value $p_{\rm fail}$ in (3) and (5), obtained according to (4), we can find the expressions of the income and the damage due to the failure of the structure:

$$B(\mathbf{X}) = bN\left\{[0.5 - \Phi(a)] - e^{-b_1+\alpha_1^2/2}\left[0.5 - \Phi(a - a_1)\right]\right\}/r. \tag{9}$$

$$L(\mathbf{X}) = L_T N e^{-b_1+a_1^2/2}[0.5 - \Phi(a - a_1)], \tag{10}$$

where $b_1 = rh_i(1 - c_1)/b_i$, $a_1 = rc_2h_i/b_i$. It is necessary to note that $a_1 = \text{const}$ and $b_1 = \text{const}$ at any value of its results from the Eq. (8).

Having calculated the current value of $B(\mathbf{X}), H_1(\mathbf{X})$ and $L(\mathbf{X})$, we turn our attention directly to the reservoir optimization.

In case of complex, multiextremal tasks of nonlinear programming such as (2), it is advisable to use one of the effective algorithms of the random search method [6]. This algorithm, based on the global random search, uses the idea of the controllable selection of test points and multiple lowering to the local extreme. The algorithm and the program differ from a similar kind of algorithms by the method of modelling the prospective direction of the search. The process of the random search realized in the program is described in the form:

$$\overline{\mathbf{X}}_\xi^{(k+p)} = \overline{\mathbf{X}}_0^{(k)} \pm \overline{S}\sum -; \quad \overline{S} = \begin{cases} \gamma_1 S, & \text{if} \quad F\left(\overline{\mathbf{X}}_\xi^{(k+P)}\right) < F\left(\overline{\mathbf{X}}_0^{(k)}\right); \\ \gamma_2 S, & \text{if} \quad F\left(\overline{\mathbf{X}}_\xi^{(k+L_p)}\right) \geq F\left(\overline{\mathbf{X}}_0^{(k)}\right). \end{cases}$$

Here $\sum -$ is a single random uniformly distributed vector; $\gamma_1 \geq 1; \gamma_2 < 1; \gamma_1\gamma_2 > 1$ — constants of tension (contraction) of the search hypercube $S; p = \{1, 2, \ldots, L_p\}$ — number of random realizations of vector $\overline{\mathbf{X}}_\xi$ at constant $\overline{S}$; signs $\pm\overline{S}\sum -$ denote the realization of the double return of the test random point $\overline{\mathbf{X}}_\xi; \overline{\mathbf{X}}_0^{(k)}$ — parameters corresponding to the lowest value obtained at k-th stage of the search $F\left(\overline{\mathbf{X}}_0^{(k)}\right)$.

4. Design Example

As an illustration let us consider the optimization of the reservoir model having the following initial data: $R = 2$ m; $H = 4$ m; $N = 48$; $\alpha = 0.06$ cm/y; $\rho = 0.8 \cdot 10^3$ kg/m^3; $c_1 = 0.2$; $c_2 = 0.01$; $P_* = 0.99$; $b = 10^4$; $L_T = 10^4$; $r' = 10\%$; $B_{\text{opt}} = 1200$; $B_{\min} = 800$; $L_{\max} = 10$; $L_{\text{opt}} = 5$; $H_{\max} = 250$; $H_{\text{opt}} = 180$.

Four values of different coefficients of correlation between the corrosion and stress β have been modelled: $\beta_1 = 0.085$ cm^3/T·year; $\beta_2 = 0.17$ cm^3/T·year; $\beta_3 = 0.24$ cm^3/T·year; $\beta_4 = 0.34$ cm^3/T·year. The assumed ranges of the variable parameters are: $2 \leq n \leq 10$; 5 years $\leq T \leq 40$ years.

The results of the numerical experiment on the optimization of reservoir shell are given in the Table 1.

Table 1

β $\left[\frac{\text{cm}^3}{\text{T}\cdot\text{y}}\right]$	n	T [y]	h_1 [cm]	h_n [cm]	$B(T,\mathbf{X})$	$H_1(\mathbf{X})$	$L(T,\mathbf{X})$	$U(T,\mathbf{X})$	β_3 $\left[\frac{\text{cm}^3}{\text{T}\cdot\text{y}}\right]$	$\mu(K,B)$	$\mu(K,L)$	$\mu(K,H_1)$	F
0.085	10	27.93	2.191	2.194	969.77	217.62	7.08	745.06	0.085	0.3824	0.6304	0.4413	79.263
0.17	10	27.993	2.197	2.2	969.45	218.25	7.11	744.08	0.17	0.3812	0.6212	0.4272	75.27
0.25	10	28.094	2.205	2.21	969.42	219.16	7.11	743.14	0.25	0.381	0.6204	0.407	71.52
0.34	10	28.08	2.204	2.213	968.5	219.21	7.2	742.09	0.34	0.3776	0.5936	0.406	67.53

5. Conclusion

It is evident that the increase of the coefficient β has little influence on the optimal thickness of the reservoir (this is connected with the fact that the effective stresses in it are small). Thus, the optimal service life of the reservoir is $T = 28$ years, when the number of constant thickness belts n in all variants achieves its maximum. Due to this fact, the change of thickness h_i along the height found from the principle of equal reliability of the reservoir shell, is practically insignificant.

The use of the principle of equal reliability allowed to eliminate in optimization the limitation of reliability given in the statement (2). In conclusion, we can note that the type of characteristic functions and the limits set in them can be corrected in the course of search for optimal decisions.

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