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Numerical analysis of a spectral photoelastic effect

Tadeusz Gajewski
Institute of Meteorology and Water Management
Podleśna 61, 01-673 Warsaw, Poland

Joanna Konwerska-Hrabowska
Institute of Physics, Warsaw University of Technology
"Hrabowski" Science Research Group
Gwardzistów 10/13, 00-422 Warsaw, Poland

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Interference effects in centers of "disk-like" solid cylinders of different photoelastic materials loaded by uniaxial forces acting along diameters was subject of study. An analysis of the intensity of light passing through the cylinder was carried out, and a few numerical models of the phenomenon were constructed and compared with the experimental results. The dispersive character of the "photoelastic constant" is shown and its consequence for the effect are emphasized. A computer aided spectrometer was specially constructed for the research as "the heart" of a semi-automatic measurement stand. The utilization of the effect for the construction of the optical force sensor is mentioned.

1. Introduction

The spectral photoelastic effect which has been discovered during the research on optical sensors [1, 2] consists in the appearance of a characteristic spectrum caused by interference effects observed in polarized light transmitted through the center of uniaxially compressed disk of photoelastic material.

The registered quantity is the intensity $I'(\lambda)$ of light passing through a medium of the photoelastic element (sensor) exposed to the force F. The theoretical shape of the observed spectrum is described in [2] by the following equation:

$$I'(\lambda) = \sin^2 \frac{8FC}{\lambda d}\,, \tag{1}$$

where λ is the light wavelength, d is the diameter of the disk, C is the photoelastic constant of the material of the disk compressed by force F.

The characteristic extremes are observed in the spectrum at such values of λ_{ex}, for which the derivative on F of the function (1) is equal to zero:

$$\frac{16FC}{\lambda_{\text{ex}} d} = N\pi\,, \quad \text{where} \quad N = 0, 1, 2, \ldots\,. \tag{2}$$

The number N denominates the following extremum. The number $m = N/2$ is attributed to the well-known notion in photoelasticity, of the order of an isochrome. Note that the above allows us to obtain the expression for the so-called model constant K [3]

$$K = \frac{F}{m} = \frac{\pi d}{8} \frac{\lambda_{\text{ex}}}{C}\,. \tag{3}$$

We obtained the value K_λ (at $\lambda = \text{const}$) in a calibration process.

The aim of our work is the numerical analysis of the conditions at which one can estimate the force F acting on a sensor when the spectral changes $I'(\lambda)$ are registered.

2. Experimental results

A number of series of measurements was performed in which all significant parameters of the optical sensor were changing: the kind of the material, the diameter d and the disk thickness t. Within every set of measurements the spectral change determining the relative light intensity I' as a function of the wavelength λ at the known value of the force F has been registered. The interval of light wavelengths was from 460 to 730 nm, the applied force values were from 0 to 200 kG and the following materials were used: polycarbonates, epoxides and glass.

Laboratory, computer-aided spectrometer was built for taking precise measurements at different loading conditions (Fig. 1) [4].

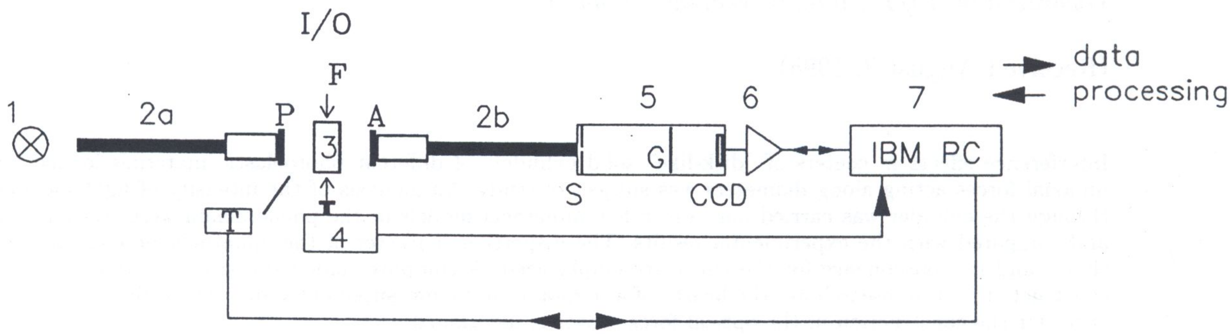


Fig. 1. Semi-automated stand for measurements of spectral photoelastic effects: 1 — light source, 2a and 2b — bunch of glass fibers, 3 — sample, 4 — force transducer, 5 — spectrometer with a dispersive element (grating G) and a detector (CCD line), 6 — amplifier, 7 — computer and a CCD controller card

Each individual distribution of a change of light intensity, i.e. the shape of the function $I'(\lambda)$, corresponds to the fixed value of force F. This function is represented by a numerical signal containing a set of 1024 numbers (pixels; 13 μm $\times$ 13 μm size). Spectral response of the CCD sensor is given by the producer — in this case we have a high sensivity from blue (400 nm) up to Near Infrared (1100 nm). The electrical signal from each pixel is proportional to the intensity of the incident light. The signals from the CCD line, after pre-amplification in an amplifier (6) are transferred to a CCD-controller (a computer (7) card). Specialized software allows collection, storage and visualization of the spectra. The magnitude of the acting force F is measured independly in the laboratory stand by a calibrated force transducer (4).

For every force F acting on the sample made of photoelastic material, the spectrum shows unique pattern of the fringes (e.g. Fig. 2).

When the pressure on the sensor is increased, the effect of an apparent shift of the observed band toward the longer wavelengths takes place.

Before starting our measurements, we have performed the calibration of the sensor consisting in registration of the force F ascribed to the isochrome with the known value of the order m observed by means of interference filters at some fixed light wavelength λ. It turned out that the magnitude of the model constant K (3), expressing the value of the force ΔF which induces the change of the order m of isochrome by unity at the wavelength λ, depends on the light wavelength λ at which it has been registered and therefore we have denoted the quantity, similar to K, by:

$$K_\lambda = \frac{\Delta F}{\Delta m} \text{ [kg/isochrome order]} \quad \text{at} \quad \lambda = \text{const}\,. \tag{4}$$

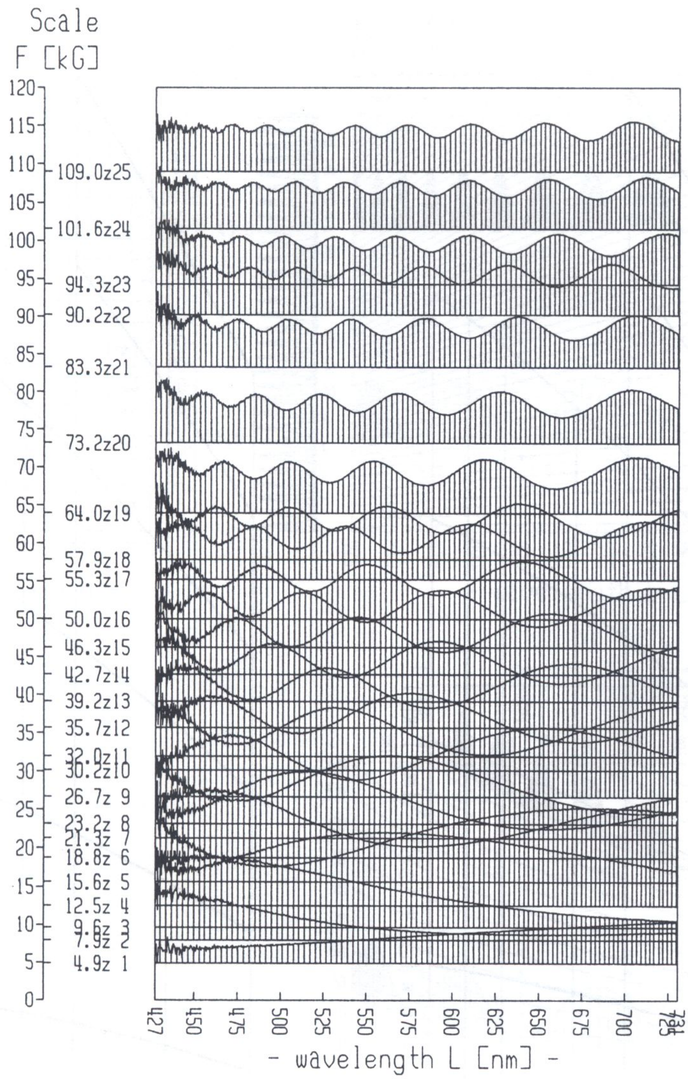


Fig. 2. Typical set of the spectra observed in the center of the disk uniaxially compressed by different forces F

The dispersive character of the model constant K_λ noted by us, is illustrated in Fig. 4a, Fig. 6a (for polycarbonate) and in Fig. 5a and Fig. 6b (for epoxide S3).

3. Numerical modelling

Several computer programs have been developed and used for modelling the dependence between the parameters F, m and λ of the calibration process.

The phenomenon of photoelasticity, discovered by Seebeck (1813) was mathematically described by Maxwell in 1890:

$$n_1 - n_2 = C(\sigma_1 - \sigma_2)\,, \tag{5}$$

where: n_1 and n_2 — the indices of refraction, caused by the stresses σ_1 and σ_2, along the two principal axes associated with the stresses; C — the relative stress-optic coefficient dependent on the kind of the material. Equation (5) is known as the fundamental equation of photoelasticity [3].

The differences in values of refractive indices n_1^λ and n_2^λ for light passing through birefringent material of thickness t, manifest themselves in relative linear retardation δ^λ between two components of the light having wavelength λ:

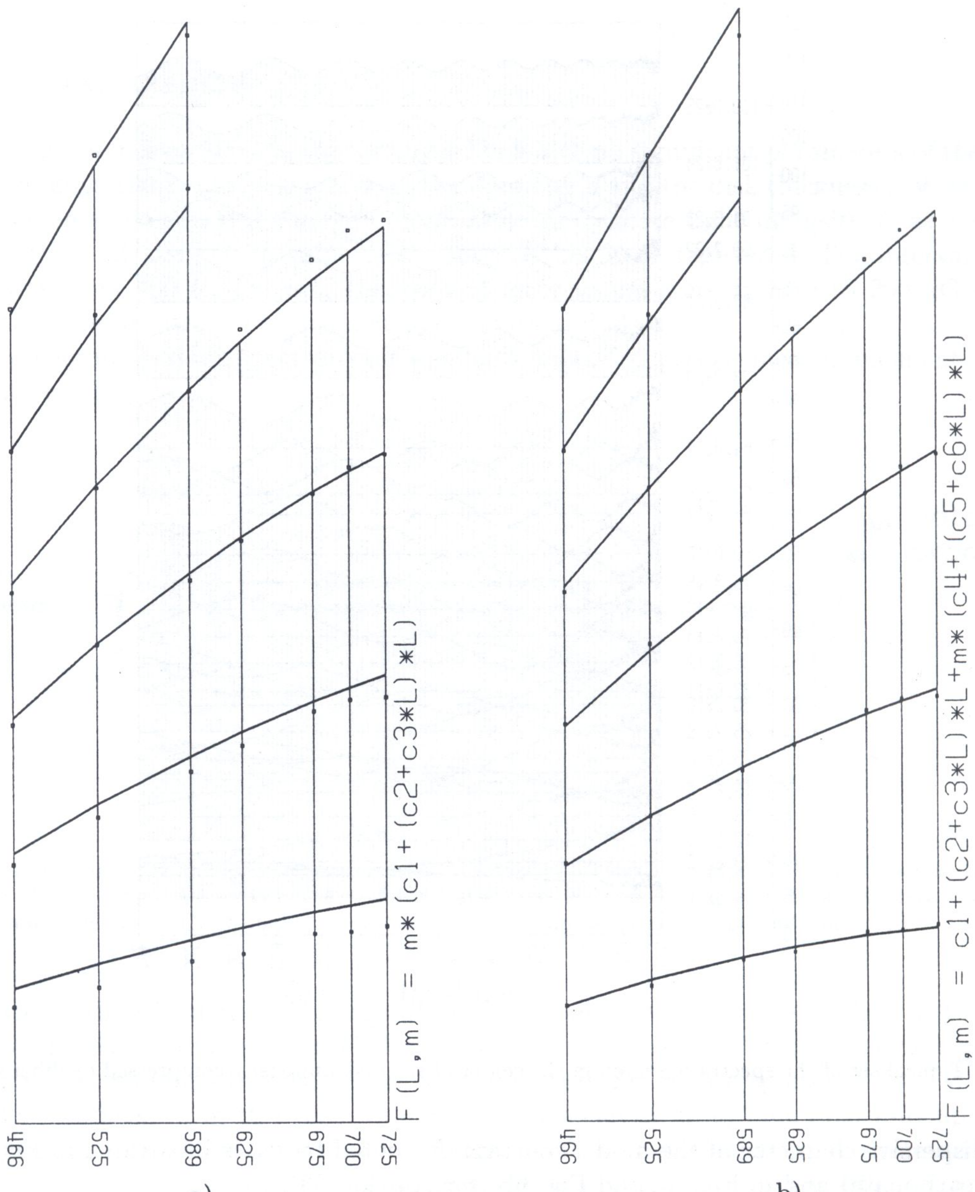


Fig. 3. Theoretical lines — the dependence $F = f(\lambda)$ for different m $(0, 1, 2, \ldots)$ — and experimental points — for two numerical models: a) $F = mK_1(\lambda)$; b) $F = K_0(\lambda) + mK_1(\lambda)$

$$\delta^\lambda = m\lambda = t(n_1^\lambda - n_2^\lambda)\,. \tag{5a}$$

It should be emphasized that Equation (5) is satisfied for a given wavelength λ, because of the material dispersion: $n = n(\lambda)$, and dependence of the measured photoelastic constant C on the wavelength λ: $C = C(\lambda)$. So, Eq. (5) can be rewritten in the following form:

$$n_1^\lambda - n_2^\lambda = C(\lambda)(\sigma_1 - \sigma_2)\,. \tag{5b}$$

In the center of the disc of diameter d and thickness t, loaded by diametrically applied force F — the normal stresses σ_1 and σ_2 are given by [6]:

$$\sigma_1 = \frac{2F}{\pi t d}; \quad \sigma_2 = -\frac{6F}{\pi t d}; \quad \text{so the difference:} \quad \sigma_1 - \sigma_2 = \frac{8F}{\pi t d}\,. \tag{6}$$

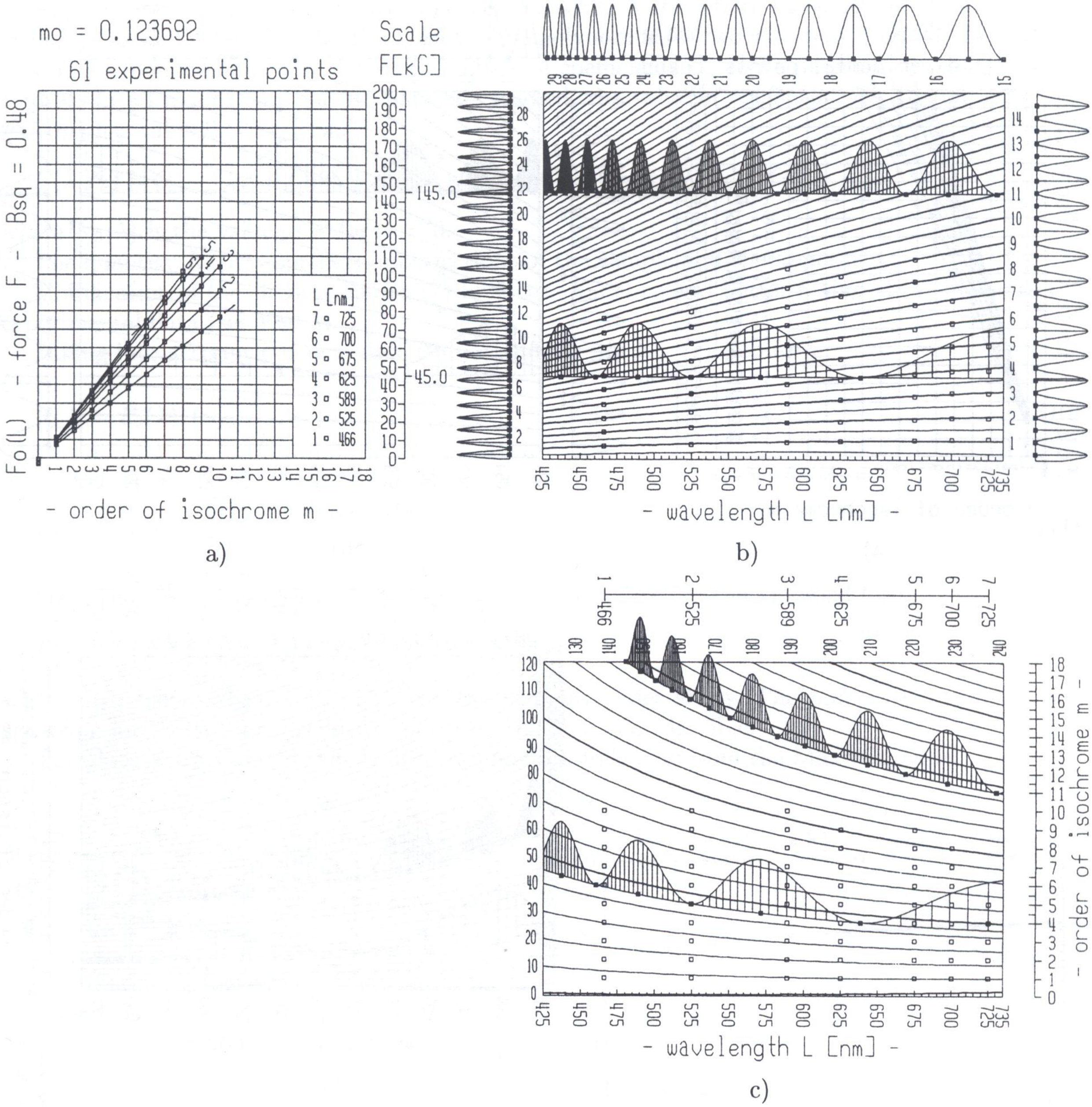


Fig. 4. Numerical model for polycarbonate ($d = 3.98$ cm, $t = 0.42$ cm): a) Experimental points $F = f(m)$ for different $\lambda =$ const, and theoretical lines (calibration process); b) Numerical model of the spectrum for two forces $F = 45$ kG and $F = 145$ kG, isolines $m =$ const and experimental points; c) Isolines $F =$ const (at 10 kG steps), theoretical spectrum for the two forces as above and the experimental points

Taking into account the relations (3), (5) and (6) we can write the equation linking parameters F, m and λ:

$$F(\lambda, m) = \frac{cm\lambda}{C} = mK_\lambda\,, \tag{7}$$

where $c = \pi d/8$. Equation (7) is analogous to Eq. (3). As follows from our laboratory and computer experiments, the well-known photoelastic constant $C = c\lambda/K_\lambda$ is no longer a "constant", according to the relation (5b), but turns out to be a dispersive quantity depending on λ. Therefore we can denote it as $C(\lambda)$ (Fig. 6).

Numerical values of the quantities F and m as functions of λ were determined using a sufficiently large set of experimental points (up to 120) for each series of measurements. In the analysis of

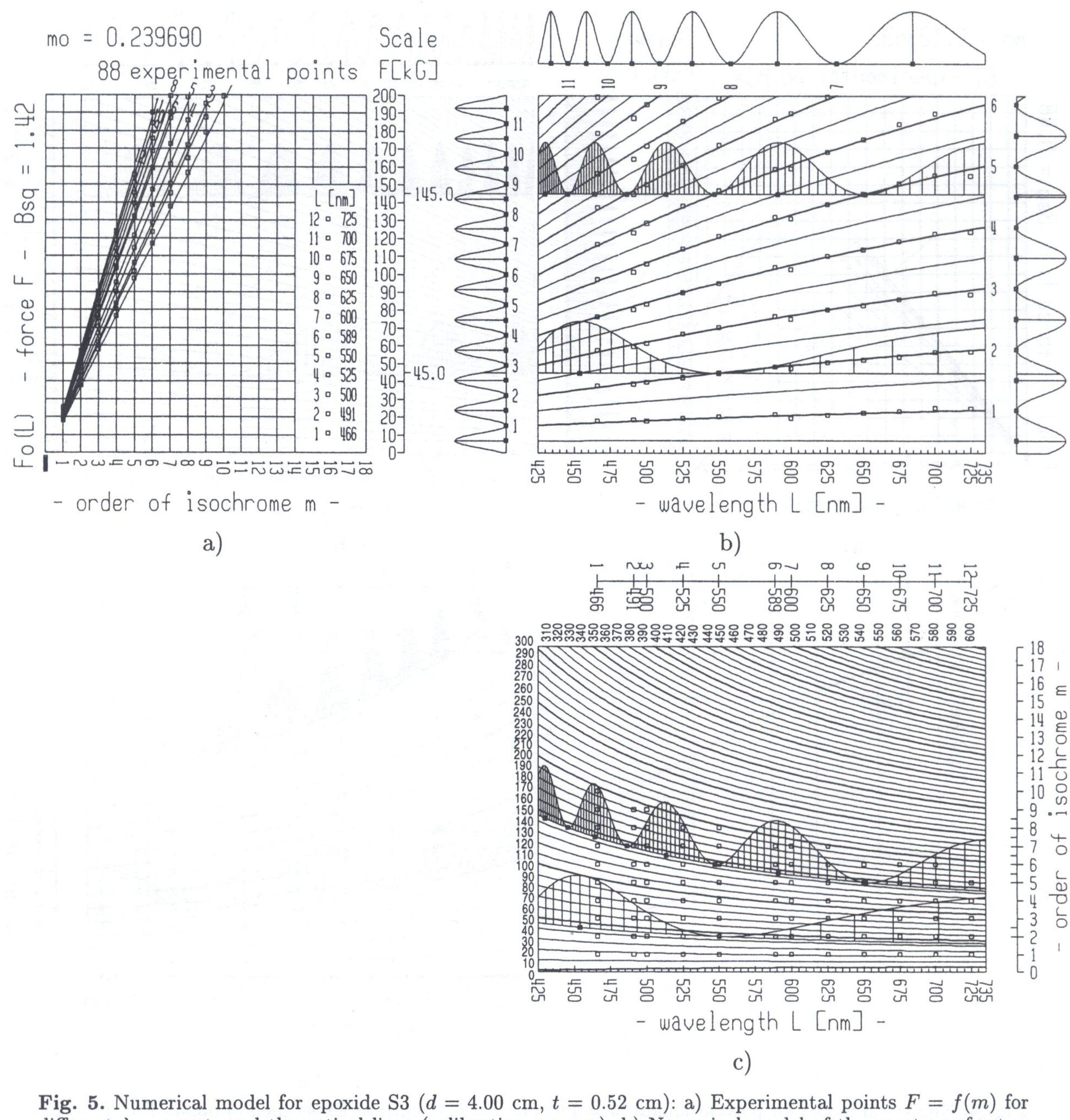


Fig. 5. Numerical model for epoxide S3 ($d = 4.00$ cm, $t = 0.52$ cm): a) Experimental points $F = f(m)$ for different $\lambda =$ const, and theoretical lines (calibration process); b) Numerical model of the spectrum for two forces F=45 kG and F=145 kG, isolines m =const and experimental points; c) Isolines F=const (with a step 10 kG), theoretical spectrum for the two forces as above and the experimental points

experimental data and their graphical representation, the method of least squares was applied. The quantity $C(\lambda)$ has been fitted to the polynomials of order $n = 0$, 1 and 2. The minimum value of root-mean square deviation from experimental points was achieved at $n = 2$. So, we have

$$C(\lambda) = C_1 + C_2\lambda + C_3\lambda^2 . \tag{8}$$

Now, using the dispersion relation (8) one can determine the unknown forces when the relevant spectral changes are registered as described in the paper [5]. It should be stressed at the same time that one has to determine the position of extremum λ_{ex} of the function (1) corresponding to the lines with definite number N, and to identify this number and the isochrome order $m = N/2$ (2). But the observed spectra are disordered and it is necessary to filter them effectively before starting to automatic identification of extreme positions.

Approximations of the dependence between F, m and λ other than those defined in Eq. (7) were examined as well. They have been represented by polynomials of two variables, λ and m, at different sets of the mononomials and different numbers of coefficients C_1 to be determined (changes from 1 to 9). In all models and for all materials studied, the value of coefficient B_{sq} is minimal when $F(\lambda, m)$ has the following form:

$$F(\lambda, m) = C_1 + C_2\lambda + C_3\lambda^2 + m(C_4 + C_5\lambda + C_6\lambda^2) = K_0(\lambda) + mK_1(\lambda)\,, \tag{9}$$

where $K_0(\lambda)$ is the value of F for $m = 0$.

The quantity $K_0(\lambda)$ expresses imperfections of the sample and the apparatus.

Modification of the classical model (7) by factor $K_0(\lambda)$ caused better fitting of the model to the experimental data (see Fig. 3).

Taking into account our postulate (9) one can modify Eq. (7) into form:

$$F_{\lambda,m} = F_0(\lambda) + \frac{cm\lambda}{C(\lambda)} \tag{10}$$

or

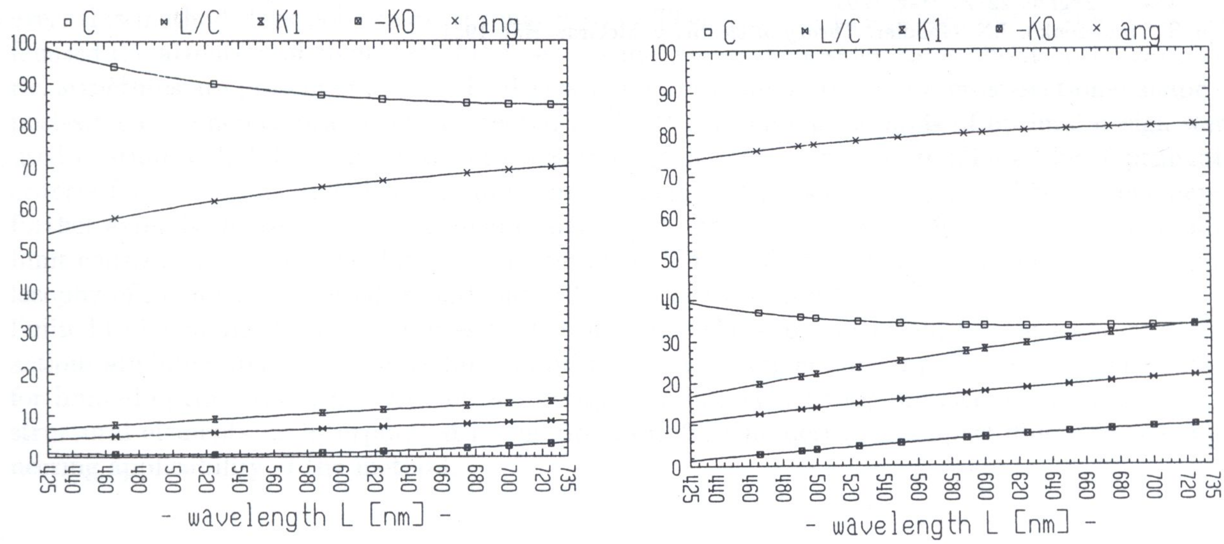
$$F_{\lambda,m} = c(m - m_0)\frac{\lambda}{C(\lambda)}\,. \tag{11}$$

From the above follows that expression (1) can be written in the modified form:

$$I'(\lambda) = \sin^2[\pi(m - m_0)]\,, \tag{12}$$

where $m_0\pi$ means the initial phase for the samples under study. The numerical models of the spectrum for the two materials are shown in Fig. 4b,c and Fig. 5b,c.

This does not appear in the classical formulation following from the Maxwell model.



a) b)

Fig. 6. The dependences: C, λ/C, K_1, $-K_0$ vs. λ: a) for polycarbonate ($d = 3.98$ cm, $t = 0.42$ cm) b) for epoxide S3 ($d = 4.00$ cm, $t = 0.52$ cm)

4. Conclusion

Our calculations and relevant graphical analysis show that it is reasonable to perform the calibration of a sensor according to the modified formula (7).

The relation (9) confirms our note about nonlinearity of the dependence of K on λ. Moreover, in this way a nonlinear component, independent of m, is introduced to the expression for F.

The relations (10) and (11), unlike Eq. (7), admit non-zero value of the force F at $m = 0$, which compensates the influence of the initial state of the material of the sample or/and the initial positions of the polarisers (see Figs. 4 and 5).

The graphical presentation is very helpful revealing the relations between the parameters and so suggesting other possible models.

Optical sensors of the new type will be available for use in the evaluation and registration of stress changes occurring inside objects under extreme conditions. At the same time, they will have to satisfy the stringent requirements of reliability and full service automation.

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