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Finite element analysis of fractured tibia treated with ZESPOL stabilizer

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Some aspects of applying the external stabilization method to the treatment of selected cases of tibia fractures make the subject of this paper. ZESPOL, used as an external stabilizer, was selected from among many other methods. In order to define the state of deformations and stresses existing in a fractured tibia stabilized with ZESPOL, an unconventional quantitative model was prepared. Finite element analysis was applied to the strength analysis of the whole system. Some final results and computations are presented.

1. Introduction

The use of finite element analysis (FEA) in the orthopedic biomechanics was started after the FEA method had been well established in traditional engineering applications. Rybicki et al. [15] and Brekelmans et al. [1] were probably among the first to use FEA for stress calculation in the femur. Since their two-dimensional analysis was made, in the last decade an enormous progress has been achieved in applying the finite element method to the bone and joint mechanics, obtaining a high degree of sophistication.

The structural elements that are subjected to analysis include the bone, articular cartilage, and the intervertebral disk. These diphase biological materials are heterogeneous and anisotropic, with time-dependent non-linear behaviours. Metals, ceramics, graphite, and polymers are commonly used in the fixation of a bone fracture and a prosthetic joint replacement. To fix joint implants to the bone, polymethylmethacrylate is used as a grouting agent (bone cement). Because of the structural and material complexities, simplified assumptions must be made to obtain practical solutions.

The most common stress analysis in the orthopedic biomechanics includes the traditional limit-value problem where body and surface forces or displacements are well defined. With elastodynamic problems, the ill-defined energy function associated with non-linear material properties has created new challenges in applying the finite-element analysis to the orthopedic biomechanics. Interface loosening has been a significant problem in the field of joint prosthetics. Unknown limiting conditions and the associated failure criteria require innovative modelling and computational methods. Simple two-dimensional models, using either the technique of an effective modulus or the approach of a reinforcing side plate (spanning element) approach, were tried to solve the three-dimensional problem. Axial-symmetry models using ring elements have proved to be useful for selected geometric shapes. The virtually three-dimensional model appears to be most attractive, but its benefit must be carefully weighed in relation to the increased costs involved.

It is important to determine the amount of stress distribution in the bone; its biological remodelling appears to be related to the stress location [8, 16]. The femur has been studied extensively because of its common involvement in the joint disease and a full-hip replacement. Both two and three-dimensional finite-element models were used [1, 2, 10, 13, 15, 17]. The effective FEM models for tibia, with an external stabilizer in particular, are, however, missing.

In spite of a continuous development in the operative treatment of fractures, the treatment of leg fractures appears to be one of the more challenging problems of the present-day traumatology. Among numerous methods of osteosynthesis, the ZESPOL device used as the external stabilizer has found a broad application to the treatment of such fractures [11, 12].

A tibia forms a 'support pole' transferring static and dynamic loads. It is the aim of a traumatologist to reconstitute the main function of the tibia through an anatomic reduction and a stable osteosynthesis. However, the type of a stabilizer used should take into account both biological and mechanical properties of the healing bone.

The tibia fractures (Fig. 1) can be classified as follows:

- transverse fracture,
- short-oblique fracture,
- long-oblique fracture,
- spiral-oblique fracture,
- multifragmental fracture.

Therefore, a need to assess a deformation of an indirect fragmental fracture and the stress distribution in the system of the fractured bone with the external stabilizer installed is well motivated. An evaluation of such a stress distribution in a 'living' bone is practically impossible. So far, the research basing on models or anatomic specimens allows to describe undergoing changes in a rather approximate way.

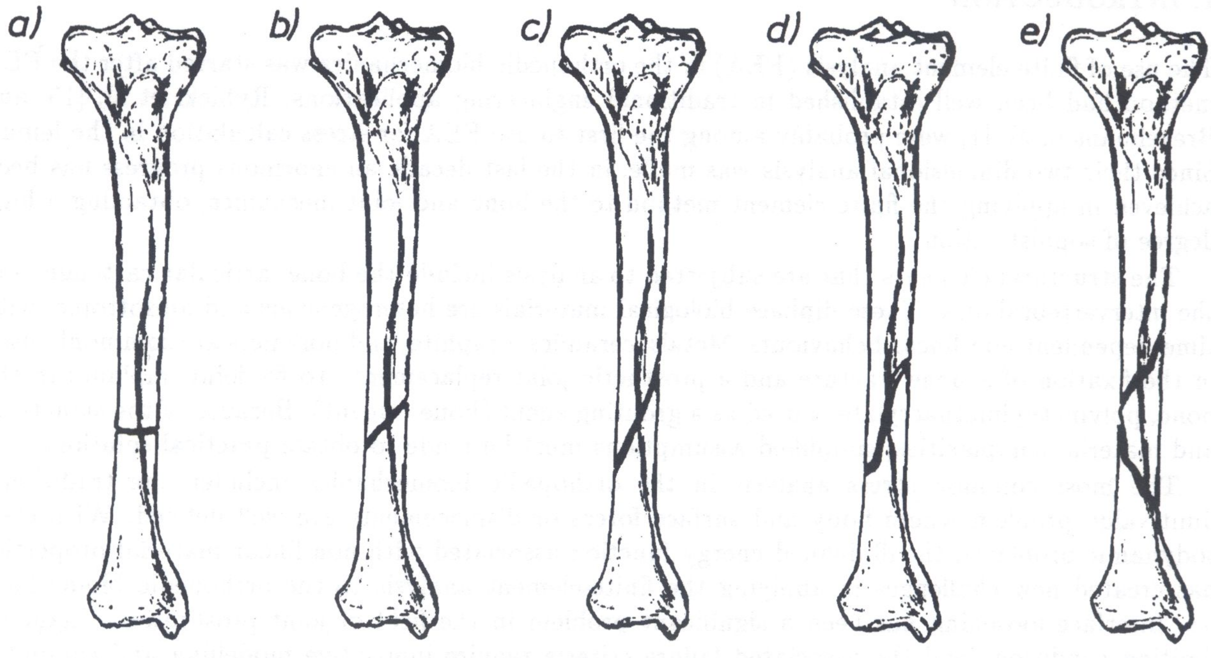


Fig. 1. The most frequently occurring tibia fracture: (a) transverse fraction, (b) short-oblique fraction, (c) long-oblique fraction, (d) spiral-oblique, (e) multifragmental fraction

This paper presents a quantitative model representing a system of a fractured bone with a stabilizer. The finite element method was chosen from among the other existing techniques. Using this method [14, 18], a numerical simulation of the selected phase in the treatment was presented. It is another aim of this research to make an attempt to calibrate a suitable rigidity of the ZESPOL stabilizer according to the fracture type and the healing phase.

2. ZESPOL AS A TIBIA FRACTURE TREATMENT METHOD

As a result of the research on optimizing the plate osteosynthesis [11], a new design of the plate stabilizer named ZESPOL was invented (the stabilizer name, ZESPOL, is an acronym formed from the Polish words corresponding to 'Polish osteosynthesis').

The core of the system is based on binding the plate connecting the bone fragments with screws through nuts washers so that the plate is not in a direct contact with the bone. This enables to avoid some complications related to the application of the common AO osteosynthesis principles [11].

The elasticity of the ZESPOL device allows to conduct dynamic osteogenesis of the fracture through inducing micromovements appropriate for each stage of healing.

The following two applications of ZESPOL were chosen from the others:

- six-hole ZESPOL plate as a clasp contact stabilizer,
- six-hole ZESPOL plate as a clasp neutralising stabilizer for the AO single-screw osteosynthesis.

In both cases the stabilizer is placed above the skin as an external osteosynthesis. Figures 2 and 3 show both the stabilizers.

The elasticity of osteosynthesis, the lack of plate pressure on the bone lead to a fairly quick bone consolidation of a diverse form. We may distinguish: a self-induced consolidation, forming the so-called bone scar, and a primary consolidation, appearing in two forms — a lacunar consolidation and a contact consolidation, the latter being based on the bone tissue growth along the vessels. The ZESPOL stabilizer enables both the self-induced and the primary consolidation. The limited axial mobility is a condition for forming the bone scar and this mobility is possible due to the elasticity of the osteosynthesis. The most frequent consolidation of the bone under the plate follows the primary consolidation scheme. This in turn has been the condition for the similarity of the elasticity of the bone-stabilizer unit and that of a normal bone. Under the condition of numerous

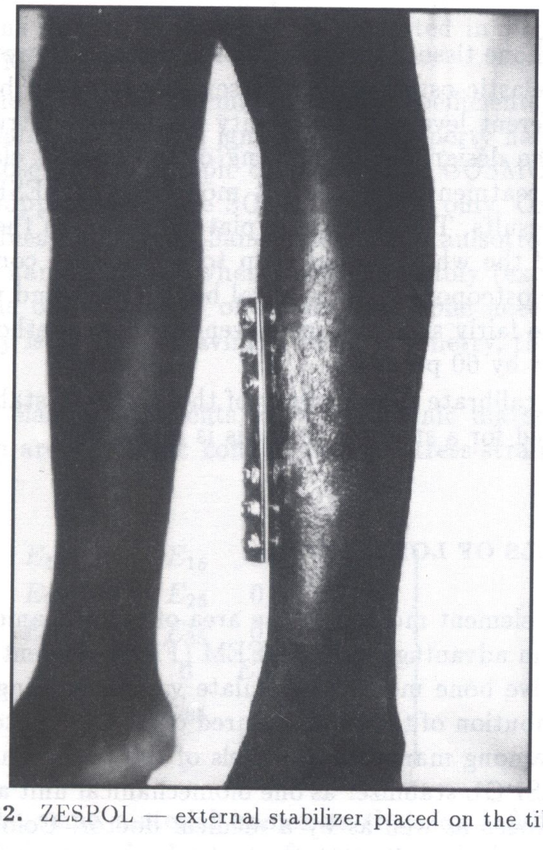
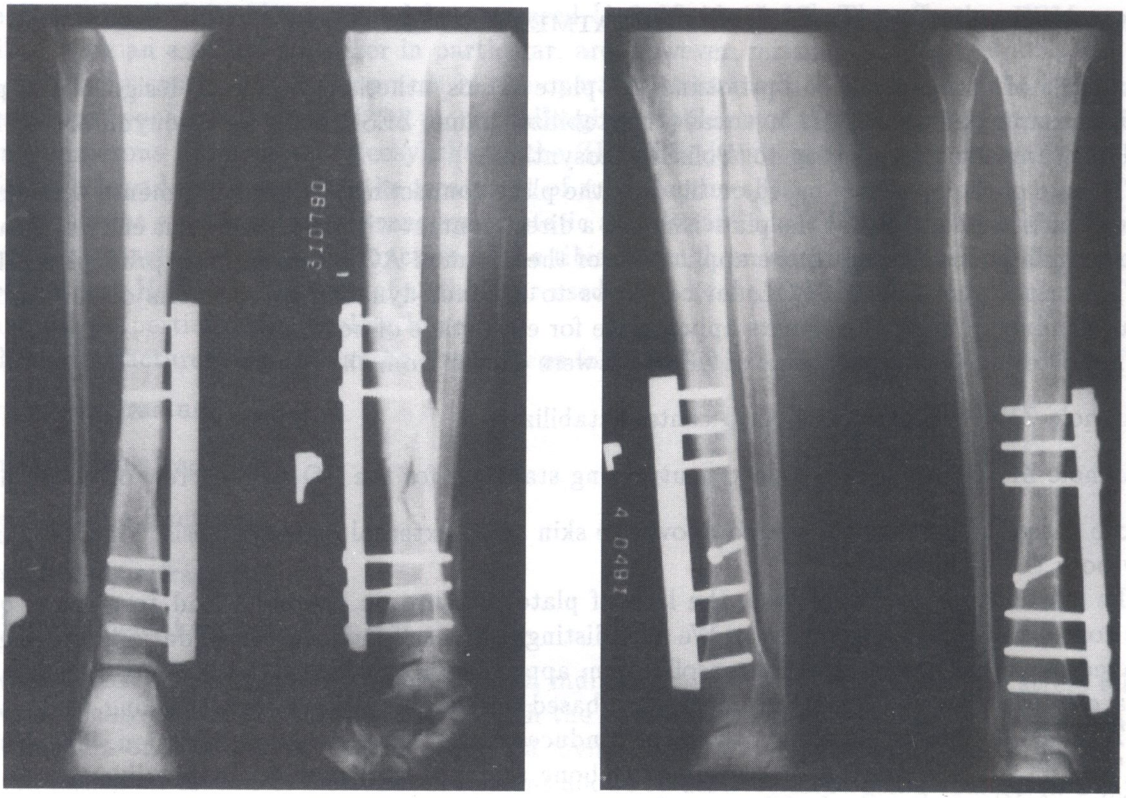


Fig. 2. ZESPOL — external stabilizer placed on the tibia



(a) (b)

Fig. 3. Examples of applying ZESPOL stabilizer in the tibia fracture treatment: (a) without pulling screw; (b) with pulling screw

variables characterising the bone tissue, the process of fitting these two quantities is not easy. The development of the equally elastic osteosynthesis is severely impeded by the anisotropic structure of the bone (mainly on different levels), a variability of the bone structure induced by age and other general conditions. The design and generating of the equally elastic osteosynthesis should considerably facilitate the treatment. The present model of the ZESPOL stabilizer has already improved the bone healing results. The lack of the plate pressure on the bone, non-strenuous bone threads and the elasticity of the whole system help to avoid some common complications in the AO plate osteosynthesis. No osteoporosis, no cortical bone lesions and no fractures after removing the plate were observed. The fairly small bioagressiveness of the method caused a decrease in the rate of complications at least by 60 per cent.

In order to appropriately calibrate the elasticity of the ZESPOL stabilizer, suitable for different stages of consolidation, a need for a strength analysis is arising.

3. Quantitative models of long bones

The application of the finite element method in the area of biomechanics is an exciting part of the present-day science. The main advantage of using FEM (Finite element method) is a possibility to directly apply the quantitative bone model to simulate various loading stresses so as to obtain a deformation or a stress distribution of the tibia pictured on the computer screen [14, 18]. Therefore FEM is very highly valued among many other models of the bone behaviour under load stresses. FEM applied to the tibia-ZESPOL stabilizer as one biomechanical unit allows to assess the strength of this unit both by an engineer as well as by a medical doctor. Colour pictures generated by a computer enhance a general understanding of the model.

For long bones, the spatial model, which should accurately reflect the geometric shape of the bone and its anisotropy as well, is the most effective.

For anisotropic materials, there are no planes of symmetry and the elastic moduli are different in any direction. In three dimensions, the stress-strain relation for anisotropic materials involves 21 elastic constants [19, 20] as shown in the equation below:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} & E_{15} & E_{16} \\ & E_{22} & E_{23} & E_{24} & E_{25} & E_{26} \\ & & E_{33} & E_{34} & E_{35} & E_{36} \\ & & & E_{44} & E_{45} & E_{46} \\ & \text{sym} & & & E_{55} & E_{56} \\ & & & & & E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{1}$$

or, in a shorter notation,

$$\{\sigma\} = [\mathbf{D}]\{\varepsilon\} \tag{2}$$

where

$\{\sigma\}$ – stress vector,
$[\mathbf{D}]$ – elasticity matrix,
$\{\varepsilon\}$ – strain vector.

The strength analysis of the tibia was performed by using volume units, SOLID. In the first stage, the discrete model of the tibia was developed — see Fig. 4.

The strength calculations for the tibia model as presented in Fig. 4 will be carried out with FEM using the COSMOS/M system.

Please note that only the first three thermal expansion coefficients can be input into the COSMOS/M system, the remaining three being ignored. The property names of these coefficients considered are the same as those for orthotropic coefficients. In COSMOS/M, the anisotropic material property is currently supported for the SOLID elements only. One can input the coefficients of either the material stiffness or a compliance matrix for anisotropic materials. All 21 material constants have unique names so that when handled suitably (explained hereinafter), one can describe materials with one or two planes of symmetry. If one needs to model materials which are monoclinic, transversely isotropic or having a cubic symmetry, this feature becomes very useful.

One can also input the elastic coefficients for a monoclinic material (which has one plane of symmetry) for which there are 13 elastic constants. The stress-strain equation for a monoclinic material is shown below:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & 0 & E_{15} & 0 \\ & E_{22} & E_{23} & 0 & E_{25} & 0 \\ & & E_{33} & 0 & E_{35} & 0 \\ & & & E_{44} & 0 & E_{46} \\ & \text{sym} & & & E_{55} & 0 \\ & & & & & E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}. \tag{3}$$

One needs to specify the same material names as that for a fully anisotropic material and to input the non-zero coefficients only. In case of a transversely isotropic material, the material

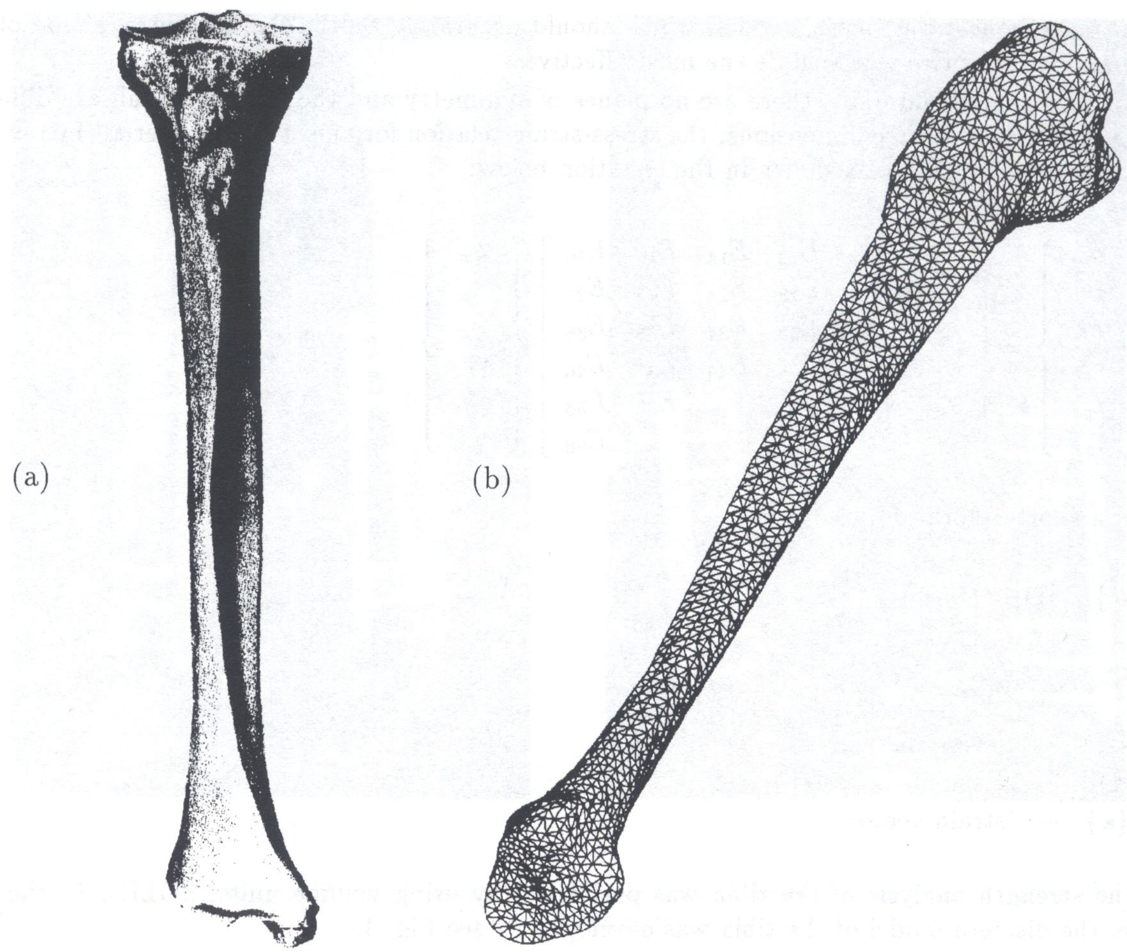


Fig. 4. Tibia: (a) general view, (b) elementary meshes of the human tibia

exhibits a rotationally elastic symmetry round one of the coordinate axes. In this case the number of independent constants is reduced to 5, and the stress-strain relations are shown below:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{12} & 0 & 0 & 0 \\ & E_{11} & E_{23} & 0 & 0 & 0 \\ & & E_{33} & 0 & 0 & 0 \\ & & & \frac{1}{2}(E_{11}-E_{12}) & 0 & 0 \\ & \mathrm{sym} & & & E_{55} & 0 \\ & & & & & E_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}. \tag{4}$$

In case of linearly elastic materials with a cubic symmetry, the properties along the principal directions are identical. A material with a cubic symmetry has only 3 independent elastic constants. The stress-strain equation for such a material is shown below:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{12} & 0 & 0 & 0 \\ & E_{11} & E_{12} & 0 & 0 & 0 \\ & & E_{11} & 0 & 0 & 0 \\ & & & E_{44} & 0 & 0 \\ & \mathrm{sym} & & & E_{44} & 0 \\ & & & & & E_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}. \tag{5}$$

For a fully isotropic material, the form of the stress-strain equations is the same as that for a transversely isotropic material, but there are only two independent constants: E — Young's modulus and ν — Poisson's ratio. Therefore one can use the input format for anisotropic materials to describe isotropic materials by properly introducing the elastic coefficients. The most general of the isotropic stress-strain relations including thermal effects is shown below:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

$$\times \left(\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} - (T - T_0) \begin{Bmatrix} \alpha \\ \alpha \\ \alpha \\ 0 \\ 0 \\ 0 \end{Bmatrix} \right). \tag{6}$$

Three dimensional stress-strain relations are applied by default to all solid elements, and one does not need to use any other instruction.

The modelling of virtual material properties for the tibia would require to determine all the twenty-one material constants for both compact and spongeous bone tissues. Since the material properties of the tibia are strongly anisotropic, so the elasticity matrix $[\mathbf{D}]$ from Eq. (1) should be applied to the discrete model. Fragmentary information only on material properties of human bones, these being bone preparations of the deceased only, may be found in the specialistic literature. As is known, the material properties of a living bone are diametrically opposed to those of a bone from a preparation. The bone properties also depend on the man's age, sex, way of nourishment and many other factors.

The material data to simulate the tibia unit were taken from the referenced literature [3, 4, 5, 7, 9] as follows:

- compact bone tissue: $E = 5000 \div 20000$ MPa and $\nu = 0.32 \div 0.35$,
- spongeous bone tissue: $E = 6 \div 250$ MPa and $\nu = 0.47$ (simulated values vary in the referenced literature).

The main feature of a mechanical ability of bones is their high anisotropy (Table 1), arising from their construction and structure. It is noticed, in an accessible literature, that there is a

Table 1. Material constants obtained from the strength tests on the tibia tissue

Bone tissue type	Layer	E [MPa]	G_{zx} [MPa]	G_{zy} [MPa]	Density [kg/m^3]
Compact bone tissue	a	20000	5090	3450	1850
	b	18000	5260	3500	
	c	17900	5320	4010	
	d	16300	4490	3060	
	e	18800	5220	4620	
	f	20600	4690	3120	
	a÷f	18700	4310	4310	
Spongeous bone tissue	g	250	100	100	800

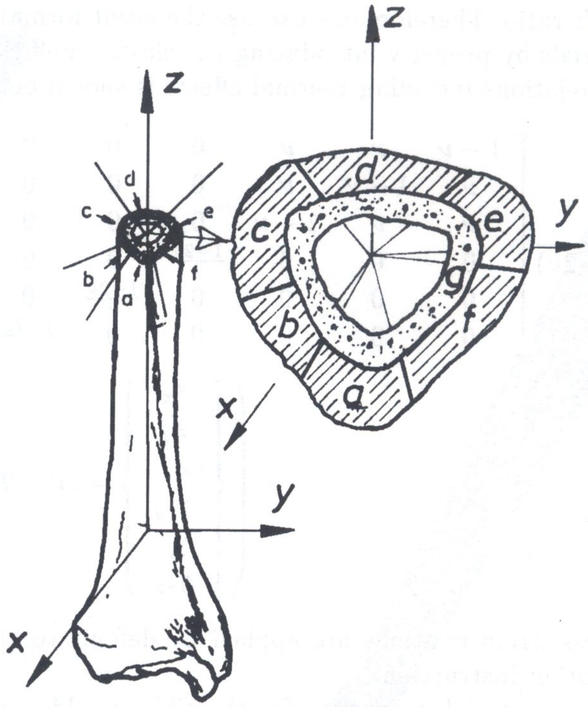


Fig. 5. Axes of a tibia from which samples for the material were taken

Table 2. Description material

	Material type	E_x [Pa]	G_{xy} [Pa]	ν_{xy}	Density [kg/m^3]
1	Bone tissue	0.50E+10 0.20E+11	0.30E+10 0.53E+10	0.32÷0.35	0.19E+04
2	Spongeous bone tissue	0.25E+09	0.10E+09	0.47	0.80E+03
3	Nickel	0.21E+12	0.79E+11	0.31	0.85E+04
4	Ceramic porcelain	0.10E+12	0.10E+12	0.22	0.25E+04
5	Glass	0.63E+11	0.10E+12	0.22	0.30E+04
6	Rubber	0.61E+07	0.29E+07	0.49	0.10E+04
7	Nylon 6/10	0.83E+10	0.32E+10	0.28	0.14E+04

comparatively large range of the Young's modulus of elongation and Poisson's ratio as well as density. The differentiation of these values results, among other things, from the fact that samples are taken from different places and studied at a different time; the anisotropy property is not always considered, and there is a problem of a personal physiology which influences many factors: way of nourishment, mode of life and the like.

The material constants (Table 1) were determined, on the basis of the research made on the tibia preparation, in many different layers, as shown in Fig. 5.

Comparison of some material characteristics of human bones with characteristics of such materials as: nickel, ceramic porcelain, rubber was made in Table 2.

It results out of material characteristics for the selected materials presented that the mechanical properties of human bones are close to those of glass and nylon. This is why the first endoprostheses, which were manufactured of alloy steel with a high nickel content fulfilled the strength criteria only but their density was too high. Nowadays, owing to the materials technology, materials have been

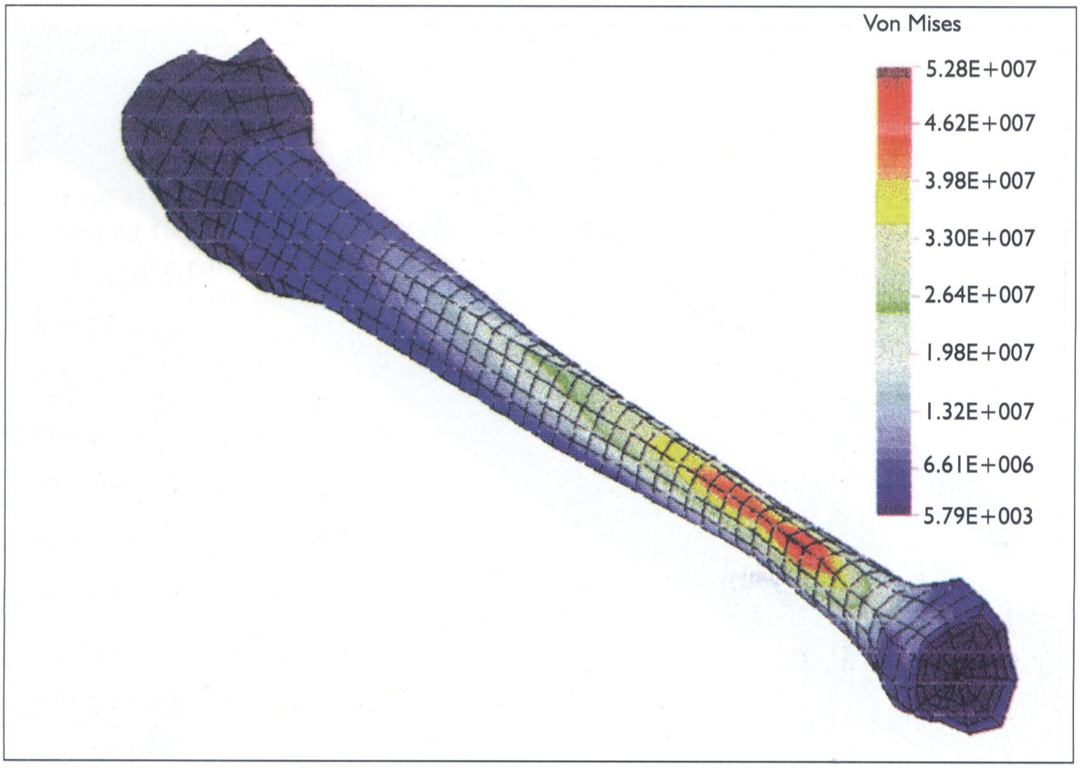


Fig. I. A healthy tibia under the pressure when standing on one leg — geodesic isolines of stress intensity according to Huber-Mises, [MPa]

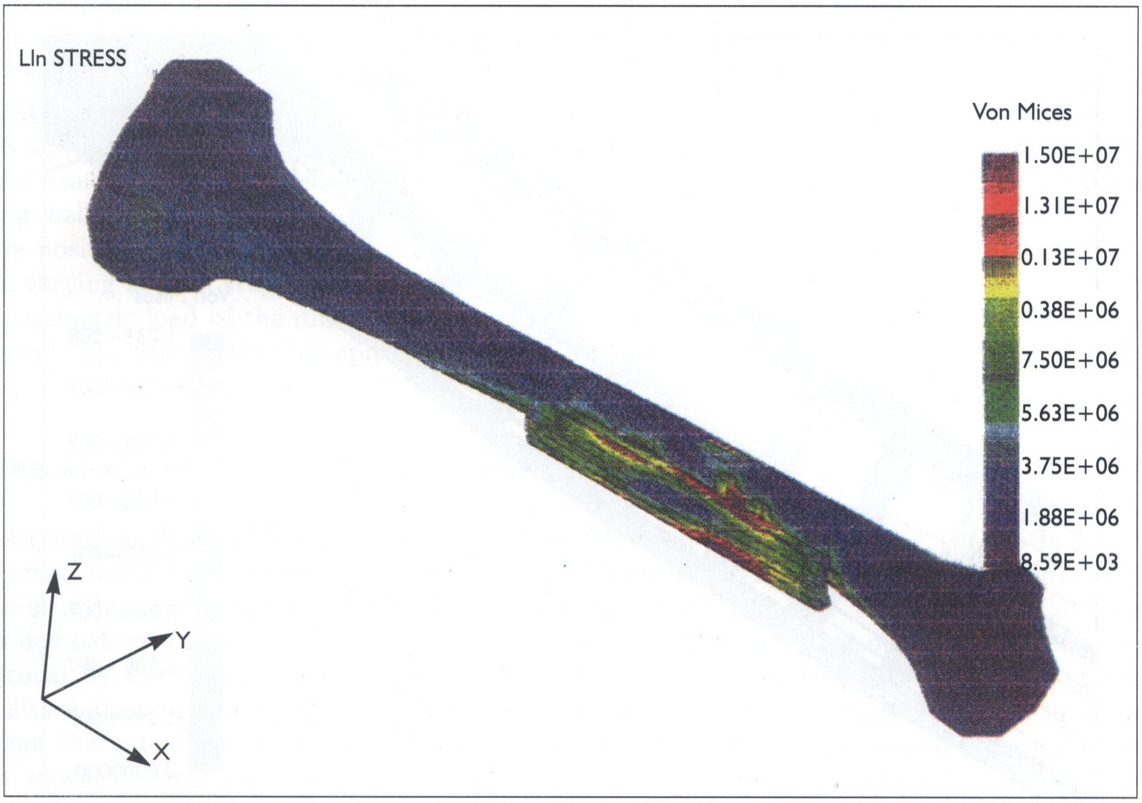


Fig. II. A fractured tibia with the ZESPOL stabilizer when standing on one leg — geodesic isolines of stresses intensity according to Huber-Mises, [MPa]

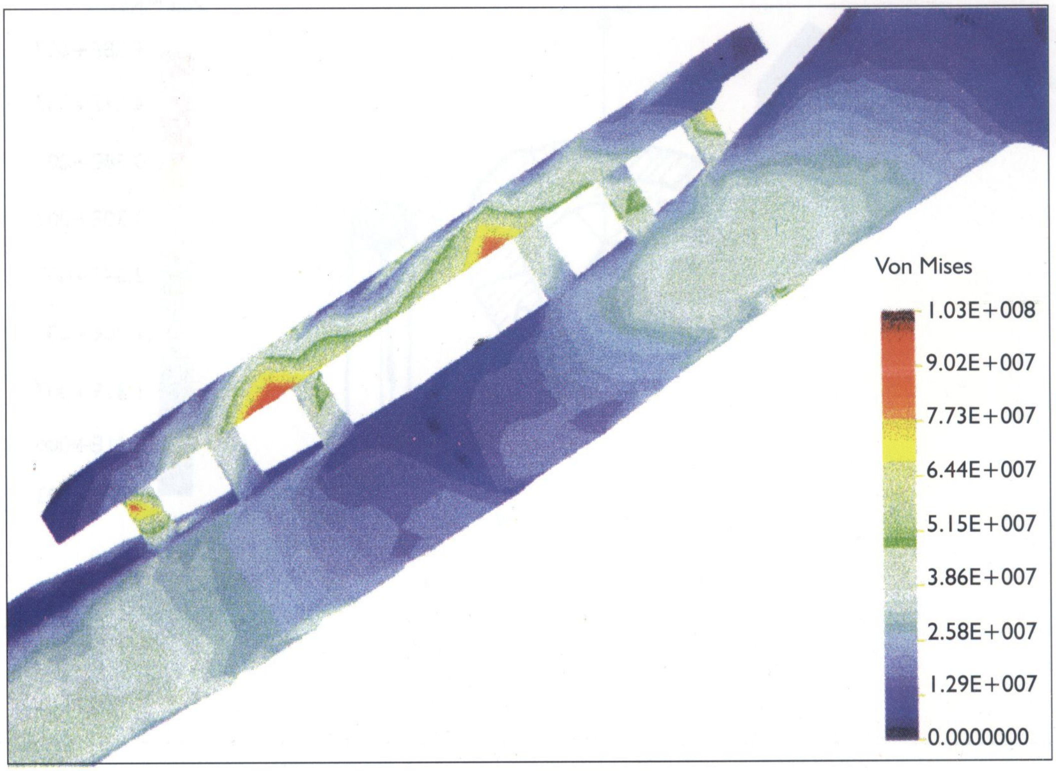


Fig. III. Huber-Mises stress intensity [Pa] in a tibia with a transverse fracture

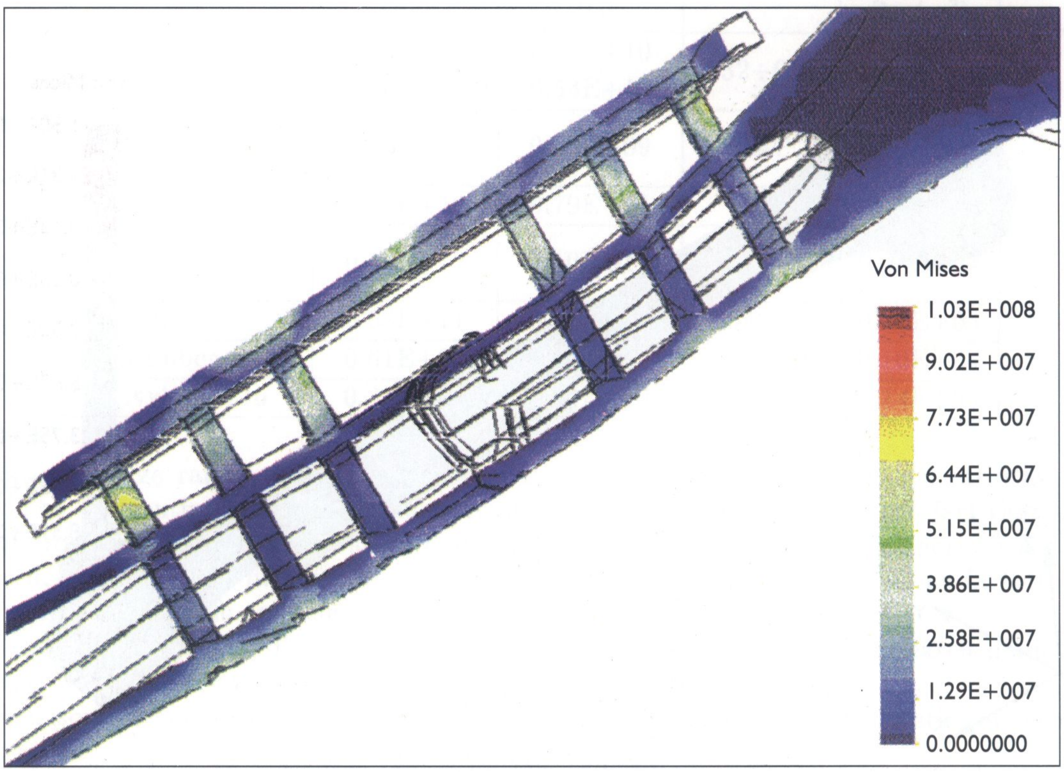


Fig. IV. Huber-Mises stress intensity [Pa] over the longitudinal section of a tibia with the ZESPOL stabilizer worked out which show the same density as that of a human bone and which meet the strength criteria.

In the tibia model the variables of the material in different intersections were taken into consideration. The differences between the compact and the spongeous bone tissues were included through assigning those values to the SOLID elements which model corresponding parts of the bone. The ZESPOL plate was also modelled using the SOLID elements which, in turn, simulated the behaviour of the material used for the plate. The screws connecting the stabilizer plate with the bone were modelled using BEAM3D beam elements (Fig. 6).

The strength analysis was based on three general patterns of the tibia:

- without fracture,
- with transverse fracture (Fig. 6),
- with oblique fracture.

The latter two patterns were taken together with the ZESPOL stabilizer.

The research of Pauwels indicated that the unilateral loading of the total value of the force acting on the tibia was equal to the sum of the body weight and the force of the acting muscles. During the load transfer, with each step the tibia bears almost the same body weight as in the unilateral standing position (approximately equal to 95% of the body weight). The extent of the forces during walking, besides the body weight and forces of balancing muscles, depends on the gravity centre position and inertia, varying at each stage of walking. This induces a load, acting on the tibia, varying at each stage of walking up to six times the body weight. Moreover, during the walk, an asymmetric load of the different knee compartments caused by a medial shift of the resultant force onto the proximate tibia epiphysis in the horizontal plane increases.

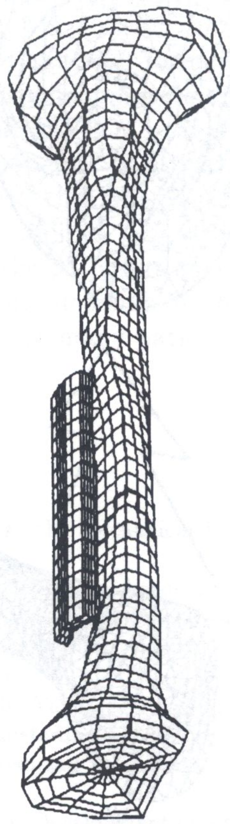


Fig. 6. Elementary meshes of a fractured tibia with the ZESPOL stabilizer — transverse fracture

4. THE ANALYSIS AND FINAL CONCLUSIONS

The strength analysis, both for the normal and the fractured bones with the ZESPOL stabilizer was performed using the COSMOS/M System of the Structural Research and Analysis Corporation. Some introductory results of this analysis are shown in Figs. I and II. The results of the calculations demonstrate that the normal tibia has the maximum stress at 1/3 of its height. This is also confirmed by the clinical analysis showing that place as the most frequent location of fractures. The distribution of stress geodesic isolines indicates that the bone deformation at plain loads has a bending character. An exact analysis shows, however, a little asymmetry round the concentration point which, in turn, indicates a small twisting tension present during loading.

The work of the tibia with the stabilizer is much more complicated. Numerous concentrations of stresses in the plate perpendicular to the plate stabilizer indicate that the placing of the stabilizer is critical for the behaviour of the whole system. It is also possible to observe the work of the plate under the stresses — it usually bends. The proximity of the geodesic isolines on one side of the plate

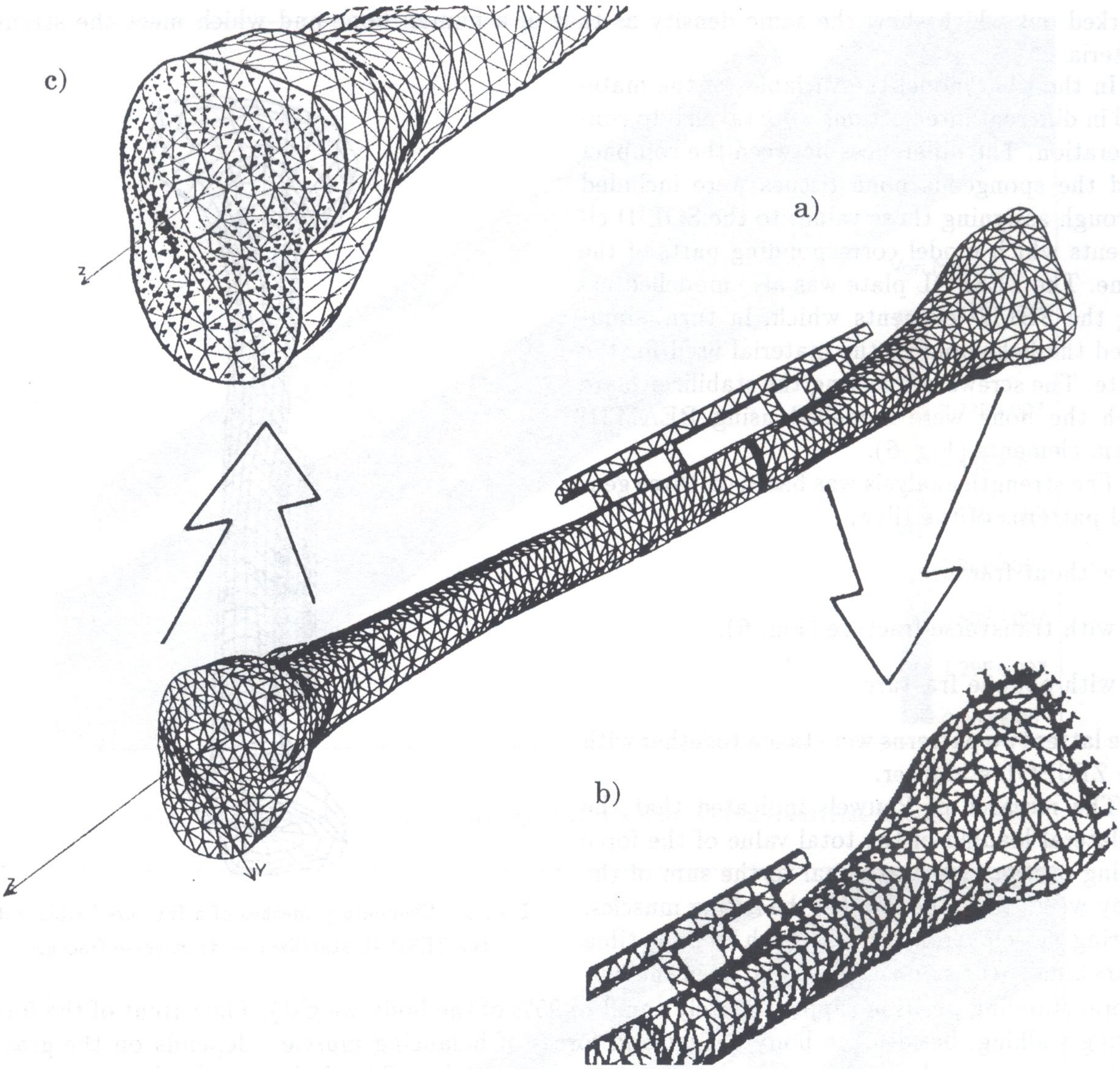


Fig. 7. A tibia model with the ZESPOL stabilizer; a virtual connection of the stabilizer screws to the bone: (a) general view; (b) received degrees of freedom; (c) external load

suggests that an extra bending stress is present there. The shape of the geodesic isolines enables to locate the place of the fracture by the discontinuity of the geodesic isolines.

The stabilizer model (Fig. 6) worked out, in which the screws were digitized by means of BEAM3D beam elements, turned out to be too simplified. The results obtained from the FEM calculations at the place of connecting the screws to the bone (Fig. II) are inadequate since a spot connection of the screw to the bone appears here. In reality, the stabilizer screws are connected through the thread over a certain area and such a connection is provided by a model shown in Fig. 7. The results obtained from the calculations for the model (Fig. 7) in which the stabilizer screws are digitized with SOLID type volumetric elements are shown in a view in Fig. III as well as in a cross-section through the stabilizer and the screws in Fig. IV. This model allows to consider the local change in the bone geometry under the influence of a concentration of stresses in the vicinity of metal elements (the so-called 'bone remodelling').

The distribution of stresses as shown in Fig. III illustrates the first stage of treating a fractured bone. One may see exactly herein (Fig. III) how the flux of effort flows from one part of the bone via the stabilizer and the screws onto the other part of the bone. The state of stresses is, however, equal to zero at the place of a fracture. The nature of mating and transferring the loads from the screws onto the bone tissue is presented in Fig. IV. In this case the bone tissue on the other side of the stabilizer plate at the junction place with the bolts is very strained.

Using this computational model (Fig. 7) one may perform a numerical simulation of the state of effort of a fractured bone not only at the first stage of treatment but also for the next treatment stages when the bone knits and at the place of a fracture the calls has been formed which, during different treatment periods, has got from 0% up to 100% of the sound bone rigidity. The aim of the numerical simulation of the effort of a bone treated using the ZESPOL stabilizer is to select its rigidity for individual treatment stages so as to allow the orthopedist to adjust (to select) the stabilizer rigidity in a continuous way. To sum up, we may state that the FEM is well suited for simulating the biomechanical systems such as the ZESPOL stabilizer. The whole information related to the connection allows to assess not only the rigidity of the plate but also to optimize its mechanical properties. An appropriate selection of the plate variables depends on the evaluation of the bone system of the patient. To do that it is necessary to study as many patients as possible in order to further optimize the performances of the stabilizer.

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