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Multicriterion optimization of space trusses with stiffness variation

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The paper discusses the discrete optimization problem in structural space truss design. The optimal structure should satisfy limit state capacity and serviceability conditions. If the serviceability conditions are violated, the structure is not eliminated from considerations but it is modified by increasing structural stiffness. Many different ways to increase the stiffness of the structure are considered. The cross-sectional areas of truss bars A_k picked from a catalogue of circular hollow sections T and a number of elements in the catalogue t are taken as discrete design variables. The stress, local stability, displacement (design) constraints as well as technological and computational constraints are taken into account. The mass of truss bars including that of joints as well as exploitation and maintenance costs are chosen as optimization criteria. A labour consumption corresponding with a number of elements in the catalogue t is minimized, i.e., it is also regarded as an optimization criterion. Sets of non-dominated (efficient) and compromise (Pareto optimal) solutions, and the preferable solution for space truss are found. The results are presented in the form of diagrams and tables.

1. Introduction

Double-layer space trusses are used for covering exhibition, sports, shopping centre and industrial halls [12]. Design of such metallic structures in the elastic range is based on the selection of appropriate cross-sectional areas of bars to satisfy stress and local stability constraints. Next, the displacements of nodes under combined characteristic loads are calculated and compared with the allowable displacements occurring in codes [15]. If they are violated it is necessary to modify the structure by increasing its stiffness.

The optimal structure should satisfy limit state capacity and serviceability conditions [15]. Many papers in this area of research are only concerned with the limit state capacity of the structure. If the serviceability conditions are violated the structure is eliminated from considerations.

In this paper, a new method is presented that allows to tread concurrently limit state capacity and serviceability conditions. It means that in the first stage of design, the minimum cross-sectional areas of truss bars satisfying the limit state capacity conditions are found. In the second stage, the serviceability conditions are checked. If they are not satisfied the structure is modified. This modification can be done in a few ways.

Displacements decrease or stiffness increase of double-layer space truss can be done by the following means:

i) increasing the depth of the truss h [7, 8];

ii) increasing the distance between truss nodes a [7, 8];

iii) changing the supporting points from the upper layer to the lower one [1];

iv) increasing the number of vertical supports [1, 12];

v) using a material with a lower yield stress [1, 8];

vi) decreasing the number of elements in the catalogue of cross-sections for truss bars t [14];

vii) decreasing the number of different stiffness zones [7, 13];

viii) increasing the cross-sectional areas of truss bars with maximum stresses [8];

ix) changing the truss topology (a type of bar net) [13];

x) introducing an initial truss camber [1, 9];

xi) varying the initial curvature of truss layers [9];

xii) varying the roof layout [1, 9, 12];

xiii) applying truss joints with larger stiffness [5].

In the truss optimization problem, each of the above mentioned means of increasing the truss stiffness can be treated as an independent decision variable.

2. Optimal truss design with stiffness variation

This section deals with the optimal design of double-layer steel space trusses which can be used for covering large span halls. The design codes require the displacements of truss nodes under combined characteristic loads not to exceed some specified permissible values as follows (Fig. 1), [15],

$$\Delta_1 \leq \frac{L_1}{250}, \qquad \Delta_2 \leq \frac{L}{250}, \qquad \Delta_3 \leq \frac{H}{150}. \tag{1}$$

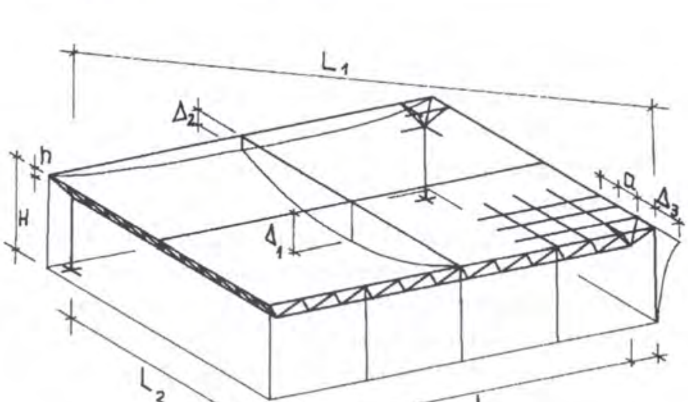

Fig. 1. Layout of double-layer space truss

For a small truss depth h, it is usually easy to find the appropriate cross-sectional areas of truss bars to satisfy the stress constraints but the displacement constraint $(1)_1$ is often violated under given loads. But, increasing the stiffness of the structure, one can satisfy the displacement constraints. In the following, a few means of increasing the truss stiffness are considered separately.

i) Increasing the depth of the truss h [7, 8]

The most effective way to increase the truss stiffness D is to increase the truss depth h, i.e. the truss stiffness can be calculated as follows,

$$D = \frac{h^2 E A_u}{a\left(1 + \frac{A_u}{A_l}\right)} \tag{2}$$

where A_u and A_l are the cross-sectional areas of upper and lower layer bars, E is the Young modulus for steel and a is the distance between truss nodes. However, assuming a constant utility space and increasing the truss depth h cause an increase in the total hall height H. Therefore, the exploitation and maintenance costs are also increased. Some structural systems, e.g. the Mero system [12], have a fixed ratio between truss bar lengths, i.e. it is not possible to increase the truss depth h without increasing the distance between truss nodes a in the upper and lower layers (Fig. 1).

The results of mass, cost and displacement analysis with respect to the truss depth h for the truss with span $L \times L = 24 \times 24$ m are shown in Fig. 2. The results of changing the truss geometry are shown in Figs. 2a and 2b. The diagrams of the displacement $\Delta_1(h)$ as well as of the mass of truss bars $M_b(h)$ with respect to the truss depth h are presented in Fig. 2c. The line $\Delta_1 = \frac{L_1}{250} = 13.58$ cm separates the structures which satisfy and do not satisfy the serviceability constraints. The continuous $\Delta_1(h)$ and dash $M_b(h)$ broken lines in Fig. 2c are found for two different numbers of elements in the catalogue, i.e. for $t = 10$ and $t = 34$. It can be noticed that by increasing h up to $h^* = \frac{L}{10} = 2.4$ m the displacement Δ_1 and the mass of truss bars M_b can be decreased. However, for $h \geq h^*$ the displacement Δ_1 can be decreased but the mass of truss bars M_b is almost constant. It can also be noticed that the structures designed using the catalogue with $t = 34$ are lighter than those with $t = 10$. The broken lines occur because of discrete variables.

The costs of the structure are presented in Fig. 2c with dotted lines. C_1 represents only the cost of the space truss including labour, material and equipment costs; C_2 is the total cost of the

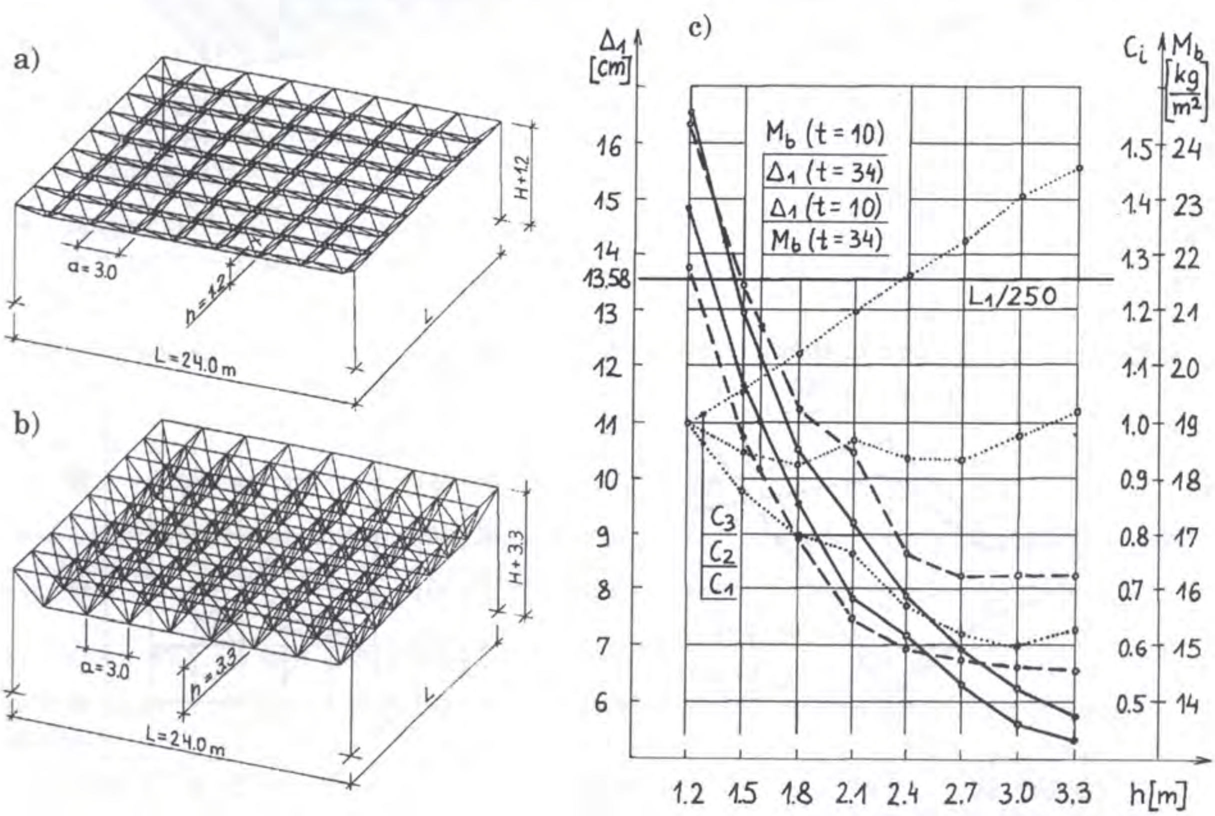

Fig. 2. Variation of truss depth h

object realization including the costs of the space truss, the columns, roof covering, the walls and the foundations; C_3 is the exploitation cost, i.e. the cost of heating and maintenance. The costs C_i $(i = 1, 2, 3)$ are expressed with respect to a reference solution, i.e. for $h_0 = 1.2\,\mathrm{m}$ and $t = 10$, as follows,

$$C_i = \frac{C_i(h)}{C_i(h_0)}, \qquad h \in \langle 1.2,\ 3.3\rangle.$$

The truss depth h should be determined as a compromise between the total cost of the object realization C_2 and the exploitation cost of the object C_3. It can be noticed that taking the truss depth h from the range $\langle 1.2, 1.5\rangle$ and increasing the truss stiffness by another means listed below it is possible to satisfy the serviceability constraints.

ii) Increasing the distance between truss nodes a [7, 8]

Decreasing the distance between truss nodes a causes an increase in the mass of truss bars M_b and some fluctuation of the vertical displacement of truss nodes (Fig. 3a, 3b and 3c). A concurrent decrease in the truss mass and increase in the truss stiffness is useful from the economical and structural viewpoints (see the range $2.0 \leq a \leq 3.0\,\mathrm{m}$ in Fig. 3c). It can be noticed that the mass of roof covering elements rapidly increases for $a \geq 4.0\,\mathrm{m}$ [7]. In Fig. 3c two ways of modelling of a dead load are distinguished. In the first case it can be assumed that a dead load $q = 0.2\,\mathrm{kN/m^2}$ is

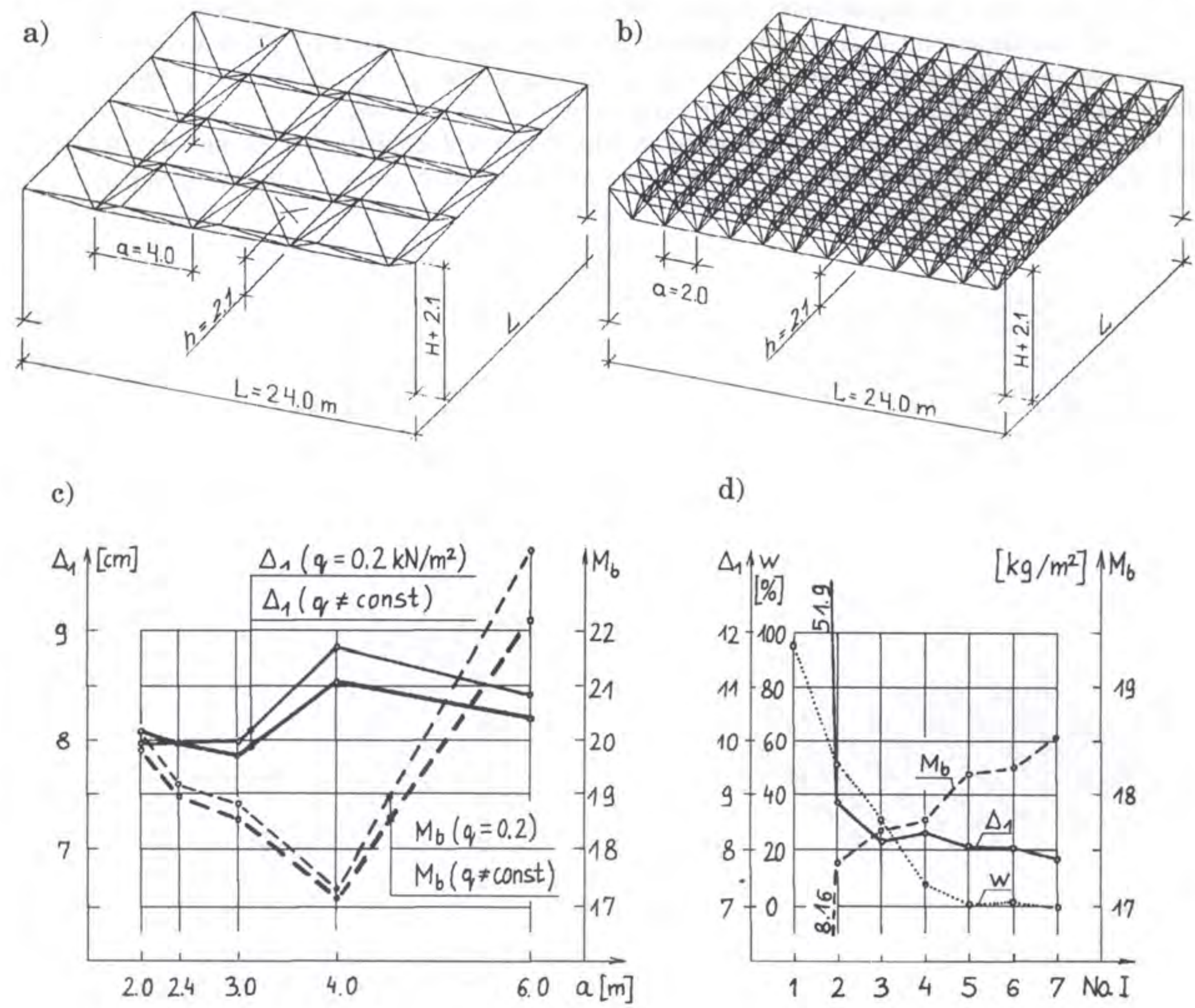

Fig. 3. Variation of distance between truss nodes a

constant during the iteration process; it is also uniformly distributed and acts at truss nodes. In the second case it can be assumed that a dead load q is not constant; it varies during the iteration process according to the real weight of truss elements calculated in each iteration (Fig. 3d). It can be noticed that in the first case, the displacement $\Delta_1(a) = \text{const}$ can be obtained for $q = \text{const}$ and for a catalogue with a large number of elements. The way of modelling of a dead load may have a significant influence on the results of truss structure optimization. The optimization process is presented in Fig. 3d; real mass distribution is taken into account in each iteration. The index w describes the percentage of truss bars changing their cross-sections in the I-th iteration.

iii) Changing the supporting points from the upper layer to the lower one [1]

Figures 4a, 4b and 4c show different types of support for a truss 27×36 m in dimension, i.e. a truss supported at the upper and lower layers as well as with V-shaped columns. Figure 4c (cases $c = 9$ and $c = 11$) shows a horizontal projection of a quarter of the truss with marked supported nodes. Figure 4a shows different types of technological solutions of particular supports. The schemes of supports are ordered arbitrarily. There is no way to order these schemes according to monotonically increasing values of the objective function. Therefore, the results $\Delta_1(c)$ and $M_b(c)$ are presented in column diagrams (Fig. 4d). The following conclusions can be drawn on the basis of the above analysis:

- truss support at the lower layer is more effective (it requires a smaller mass of truss bars and a smaller vertical displacement of nodes than at the upper layer), e.g. the solutions $c = 1$ and $c = 4$,
- truss support at the lower layer may increase the volume of the object because the space between the walls and columns is dead (see the cases $c = 5, 6, 9, 10, 11, 12, 13$),
- using V-shape columns makes it possible to replace concentrated forces (case $c = 4$) by distributed forces (case $c = 9$).

iv) Increasing the number of vertical supports [1,12]

Increasing the number of truss nodes supported in the vertical direction causes a significant decrease in vertical displacements of internal truss nodes. The following conclusions can be drawn on the basis of the analysis:

- changing the number of supports from 2 to 3 on the span L or L_2 is more effective than changing the number of supports from 3 to 5 (see the solutions in Figs. 4c and 4d, $c = 14$, $c = 2$, $c = 3$ as well as $c = 4$, $c = 5$, $c = 6$),
- by increasing the number of supports one can decrease the truss mass but it is necessary to increase the mass of columns,
- additional columns can be used for the sway braces of the object or as supports of stud walls (see $c = 6$ and $c = 12$),
- a compromise between the cost of the space truss and the cost of the supports and walls of the object should be found before choosing the type of support.

v) Using a material with a lower yield stress [1, 8]

In steel space truss design, the mechanical properties of the material of bars and joints should be established. The strength of the material for truss bars (f_d) is particularly important to satisfy the serviceability constraints. Smaller cross-sectional areas of truss bars can be obtained with high-strength steel but this leads to smaller stiffness of the structure. Varying the truss bars material may concern the whole structure or some specified zones or layers of the structure. Figure 5 shows

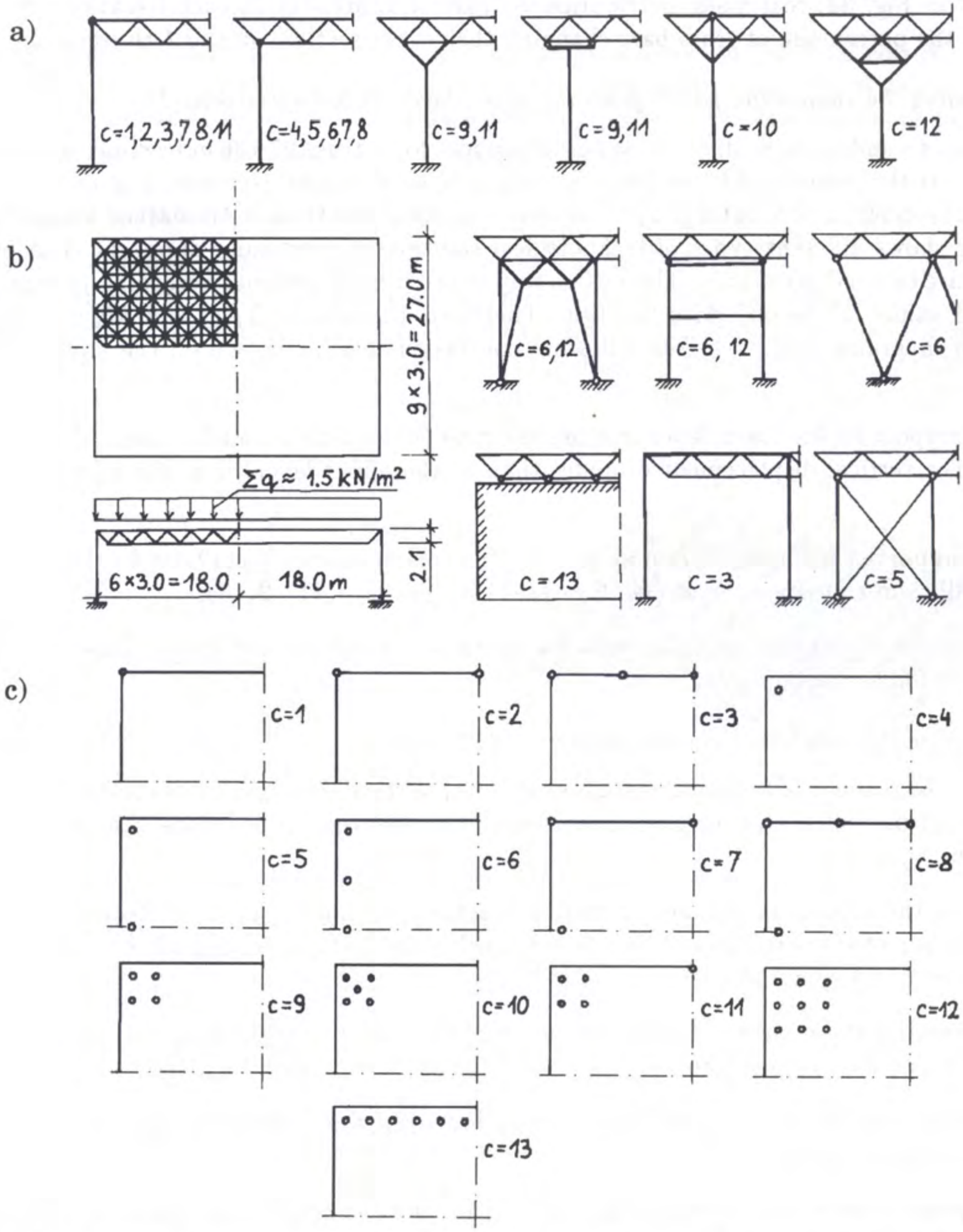

Fig. 4. Analysis of different types of supports for space truss with span 27×36 m *(continued in the next page)*

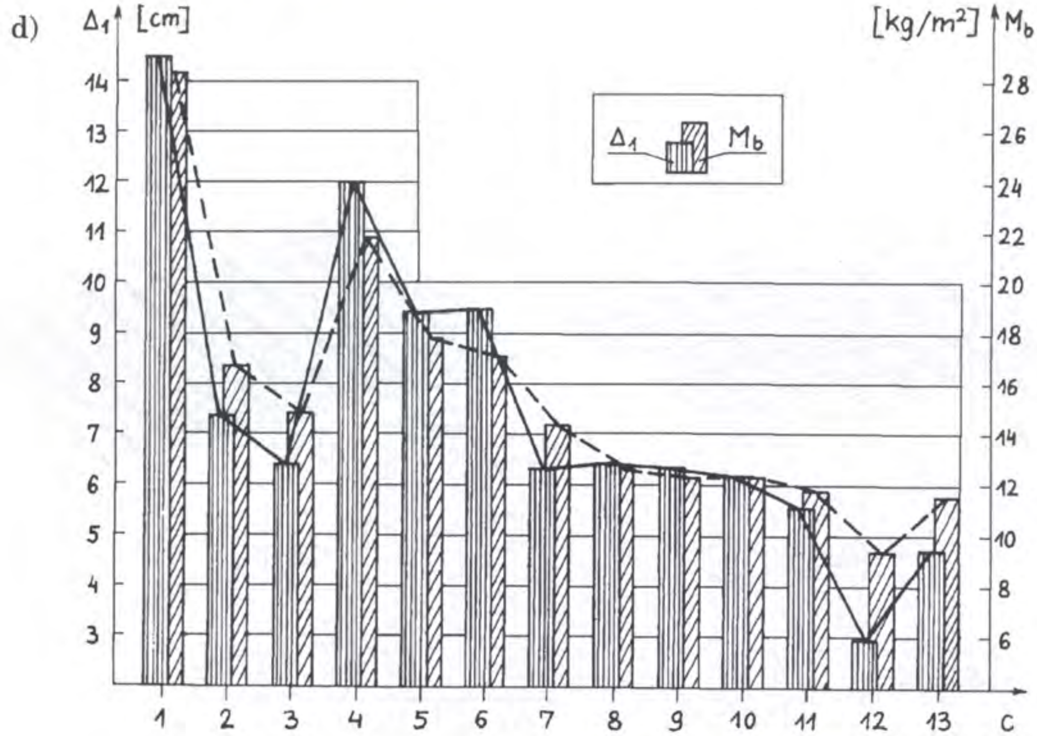

Fig. 4. *(continued)*

the results of varying the material of truss bars in the upper, middle and lower layers for the following truss parameters: $h = 2.1\,\mathrm{m}$, $a = 3.0\,\mathrm{m}$ and $t = 9$ and for the dead load varying in the iteration process. Two trusses are considered, one with $24\times24\,\mathrm{m}$ span (Fig. 5a) and the other with $27\times36\,\mathrm{m}$ span (Fig. 5b). The smallest vertical displacements occur for the trusses with $S_t = 1$ and for the steel with the smallest strength of material $f_d = 210\,\mathrm{MPa}$. The smallest mass of trusses occurs for $S_t = 7$ and for the steel with the largest strength of material $f_d = 305\,\mathrm{MPa}$. It is rational to make tensile elements from high-strength steel (see e.g. the solutions $S_t = 1$, 5 and 8). By increasing the strength of material in the lower layer from 210 MPa to 305 MPa, about 10% decrease in the mass of the truss and about 22% increase in the vertical displacement of the central truss node can be obtained (see the solutions $S_t = 1$ and 5). Changing the high-strength material from tensile layer ($S_t = 5$) to compression layer ($S_t = 8$) results in the increased truss mass M_b (about 15% in the case of the $24\times24\,\mathrm{m}$ truss and about 3% in the case of the $27\times36\,\mathrm{m}$ truss). The corresponding displacement Δ_1 can be decreased by about 17% and 7%, respectively. The decision concerning an appropriate distribution of high-strength steel in the structure should be a compromise between the material cost and the structure stiffness. It can be noticed that the total mass of truss bars is not proportional to the material cost.

vi) Decreasing the number of elements in the catalogue t [14]

Variation of the number of the elements t in the catalogue T is presented in Section 3.

vii) Decreasing the number of different stiffness zones [7, 13]

Variation of the number of different stiffness zones for a space truss is presented in Fig. 6. Designing a space truss with serviceability constraints can be done in two ways. In the first one, each cross-section area of truss bar can be designed individually. In the second one, a few stiffness zones can be established. Each element in a particular zone has the same cross-sectional area. Too many different stiffness zones result in too many different cross-sections of elements. Such a solution is not recommended because of increased labour and erection costs. On the other hand, too small a number of stiffness zones leads to an increased dead load of the truss and consequently to decreased

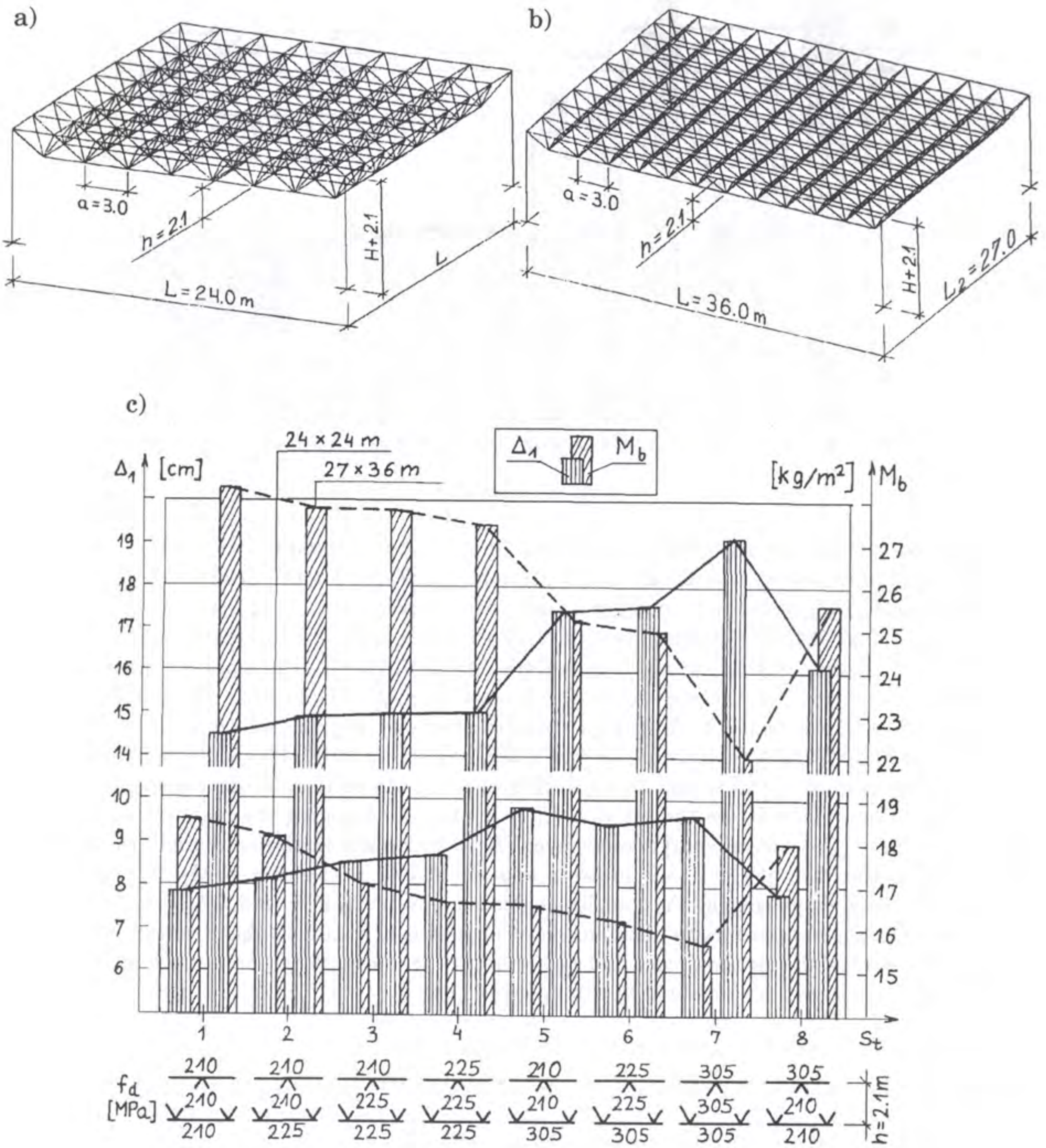

Fig. 5. Variation of material properties for truss bars

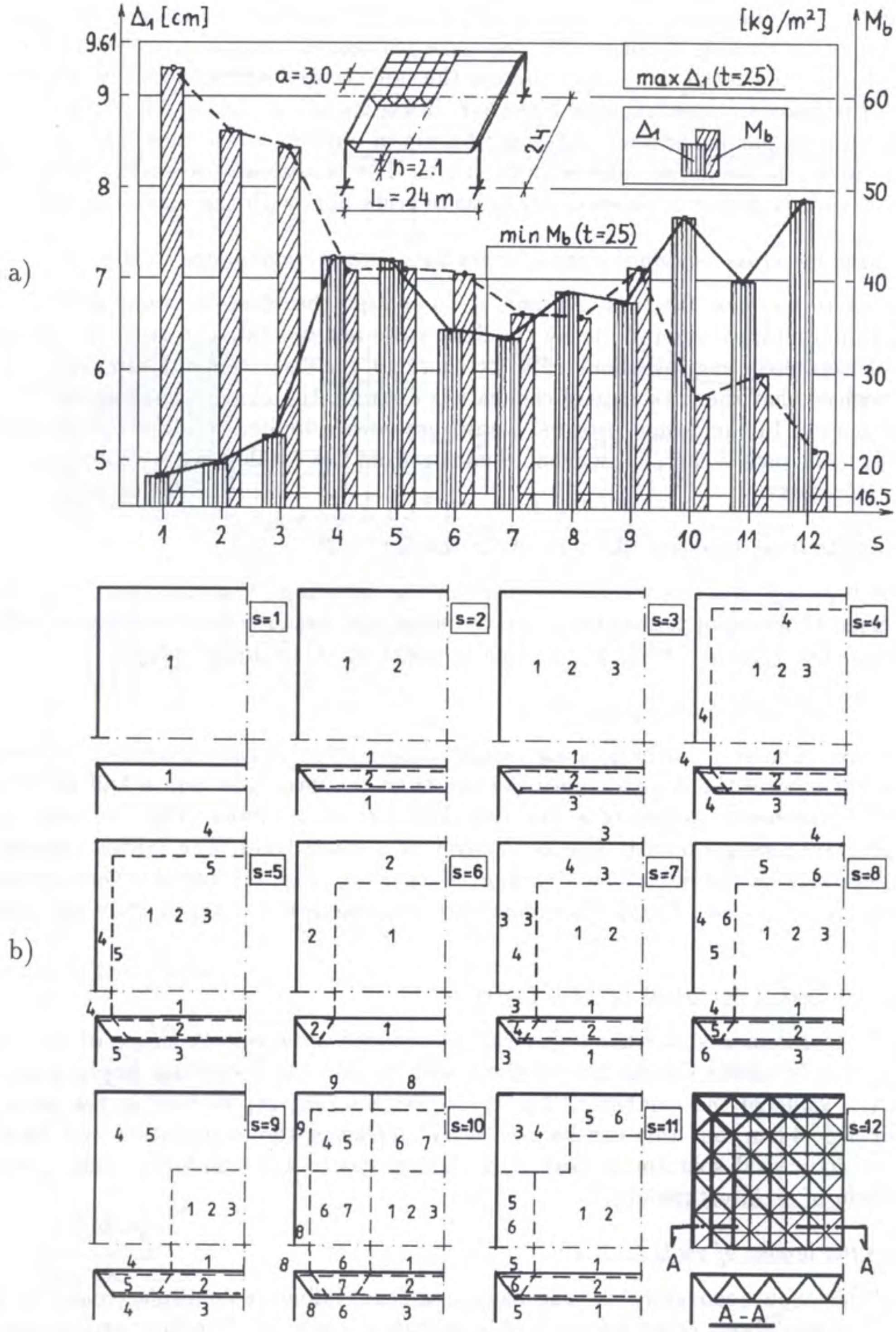

Fig. 6. Variation of a number of different stiffness zones of space truss

displacements of the truss nodes. Figure 6 shows the multicriterion choice of stiffness zones for a space truss 24×24 m in span, supported at the corners. In the case of $s = 13$ all cross-sections of the truss elements are chosen individually from the catalogue with $t = 25$. In this case, the minimum mass criterion gives the following results: $M_b = 16.5\,\mathrm{kg/m^2}$ and $\Delta_1 = 9.61\,\mathrm{cm}$. These values determine the boundaries of domain of the feasible solutions as it is shown in Fig. 6a. The distribution and numbering of zones with the same stiffness of truss bars are separated with dash lines (Fig. 6b). In the case of $s = 1$ the truss has the same cross-sectional areas for all truss bars. In other cases, the cross-sectional areas of truss bars in the upper, middle and lower layers are chosen individually in each particular zone. The distribution of stiffness zones should be determined as a compromise between the labour and material costs. The fastest way to satisfy the serviceability constraints is to enlarge the cross-sectional areas of truss bars with the maximum stresses.

viii) Increasing the cross-sectional areas of truss bars with the maximum stresses [8]

The next way to increase the truss stiffness is to enlarge the cross-sectional areas of truss bars with the maximum stresses found during the limit state analysis [8]. The cross-sectional areas can be increased iteratively, e.g. by about 10%, by checking the displacements after each iteration. It should be noticed that the catalogue of elements is discrete, therefore, a finite number of elements should be checked. In the design process, catalogues with a limited number of elements are very often used [14]. In such a case, subsequent elements from the catalogue are chosen and checked in subsequent iterations.

ix) Changing the truss topology (the type of the bar net) [13]

Usually, the topology of a space truss is chosen in the first stage of design. For large span trusses (above 40 m), orthogonal or trigonal nets are recommended because they have higher stiffness than diagonal ones. The type of a truss net is often imposed by the framing system.

x) Introducing an initial truss camber [1, 9]

An initial truss camber f for large span trusses (above 30 m) is recommended. Its value should be equal to the sum of the displacements coming from the dead load and a half of the persistent live load. The cambered shapes of a hip-roof (Fig. 7a) or a cupola (Fig. 7b) can be obtained by shortening appropriate bars in the lower layer of a space truss. The initial camber f has no significant influence on the reduction of truss displacements (Fig. 7c), but it is very important from the aesthetic point of view. The displacements are not practically changed; only the reference line is changed.

xi) Varying the initial curvature of truss layers [9]

The vertical displacements of truss nodes can be reduced by a convex shape of the truss layers (Fig. 8a). Figure 8c shows the vertical displacement Δ_1 and the horizontal displacement Δ_3 with respect to the value of the camber z. The truss displacement Δ_1 as well as the truss mass M_b decrease for $f > 4.8\,\mathrm{m}$, but this can be gained by increasing the exploitation and framing costs. This kind of structural solution is used in designing sports and exhibition halls because of an attractive form of architecture.

xii) Varying the layout of roofs [1, 9, 12]

Variation of the layout (convexity) of roofs has a significant influence on displacements and the truss mass as well as on labour consumption and exploitation costs [9]. The best proportions between the labour consumption cost and the displacements of truss nodes occur for hip-roofs (Fig. 9).

xiii) Applying truss joints with larger stiffness [5]

The pinned-joint connections are assumed in truss design. In reality, truss joints have some stiffness. The elasticity coefficient for particular bar connections depends on framing systems [4]. In order

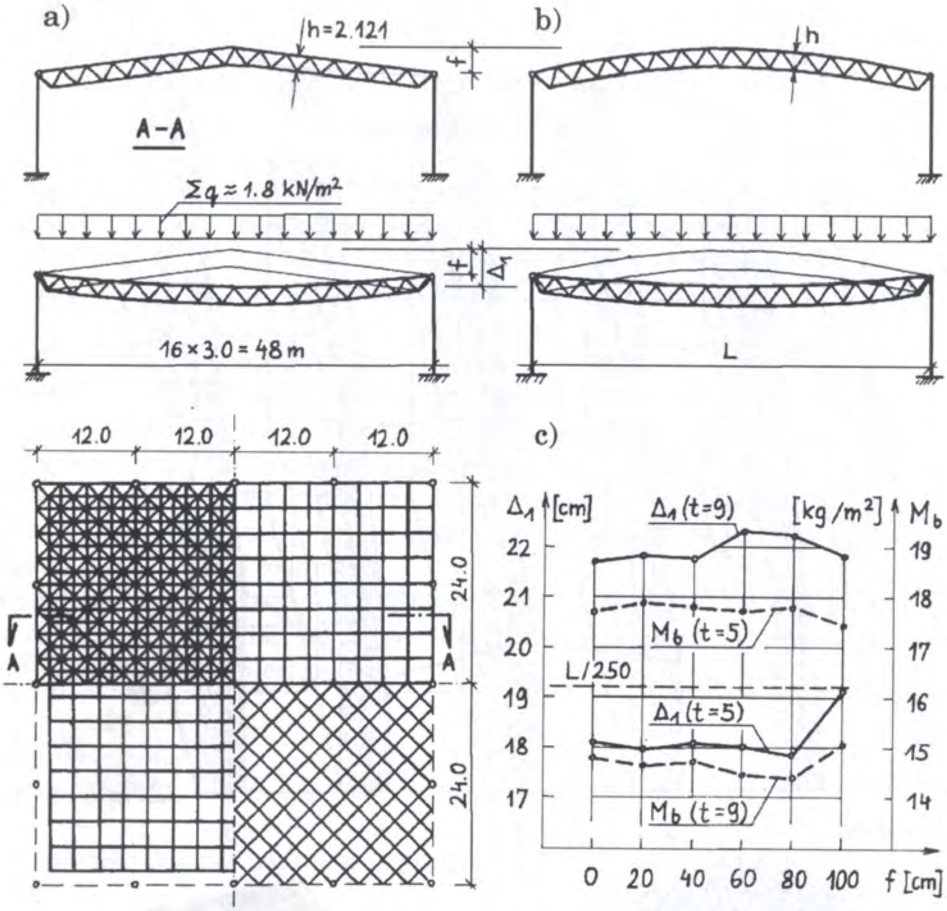

Fig. 7. Variation of initial truss camber

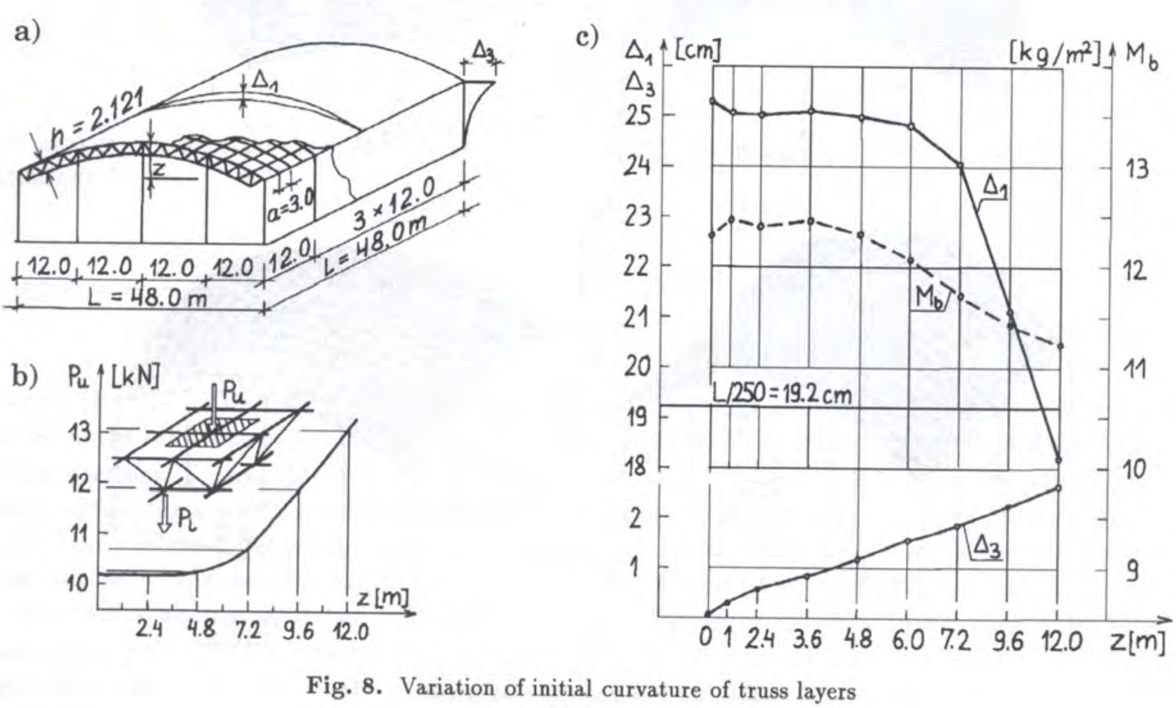

Fig. 8. Variation of initial curvature of truss layers

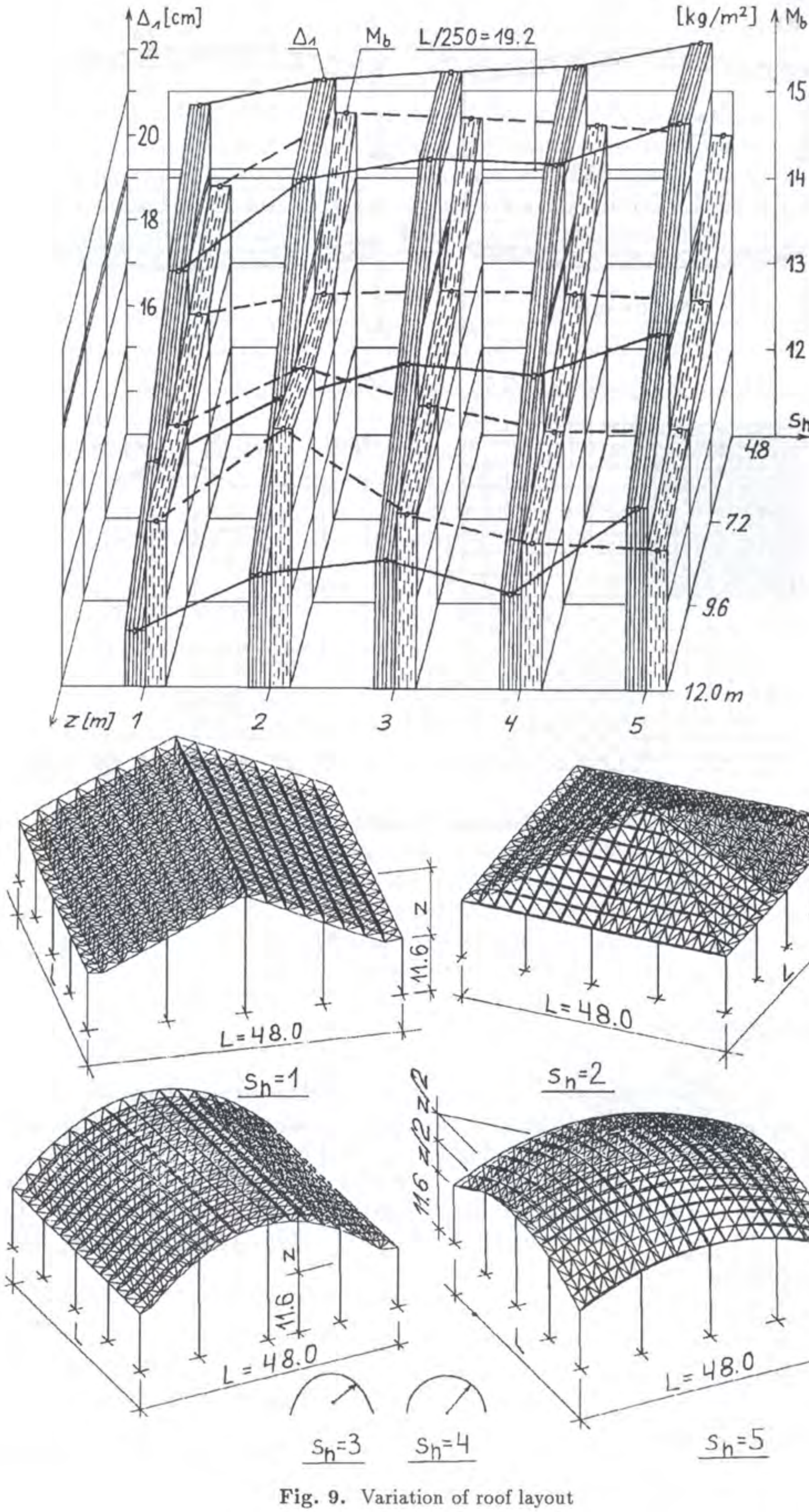

Fig. 9. Variation of roof layout

to consider the elastic properties of truss joints, it is necessary not only to change the number of degrees of freedom at nodes from 3 to 6 but also to investigate the identification of real joints.

3. Multicriterion optimal choice of truss bar catalogue

This section deals with optimization of a steel space truss used for covering exhibition halls (Fig. 10). The orthogonal double-layer space truss is supported at the corners of the upper layer. Circular hollow sections of truss bars are made from hot-rolled steel with the strength $f_d = 210\,\mathrm{MPa}$. The five single-layer parabolic lattice-work domes are used as abatjours and they are placed on the roof (Fig. 10).

The maximum loading condition which causes the maximum displacements Δ_1 and Δ_2 (see Fig. 10) consists of the following loadings: the weight of the roof covering $0.5\,\mathrm{kN/m^2}$, the snow loading $0.72\,\mathrm{kN/m^2}$, the technological loading $0.2\,\mathrm{kN/m^2}$ and the concentrated loadings transmitted from abatjours to truss nodes, shown in Fig. 11. The supporting structure has a crucial influence on the horizontal displacement Δ_3. But Δ_3 is not considered in this paper and it is not checked in the static analysis of the space truss. A quarter of the space truss is only considered because of the symmetry of structure and loadings.

The multicriterion space truss optimization problem can be formulated as follows [2, 6, 7, 10, 11]: Find such a catalogue of circular hollow sections for truss bars which minimizes the mass of the truss for $1\,\mathrm{m^2}$ of horizontal projection

$$f_1(\mathbf{x}) = \frac{M_t^{\frac{t}{t+0.1}}}{\max\left[M_t^{\frac{t}{t+0.1}}\right]} \tag{3}$$

where: M_t is the total mass of bars and joints in one quarter of the truss, i.e.

$$M_t = \frac{1.1\gamma}{(0.5L)^2}\sum_{k=1}^{k_b} A_k l_k\,,$$

and

$$\sum_{i=1}^{t} b_i = k_b$$

and also minimizes labour consumption corresponding with the number of the elements t in the catalogue T, i.e.

$$f_2(\mathbf{x}) = \frac{\sum\limits_{i=1}^{t} b_i^{C(t)}}{\max\left[\sum\limits_{i=1}^{t} b_i^{C(t)}\right]}$$

where $C(t) = (1-0.4e^{-0.35t})$ is labour consumption on the truss realization shown later in Fig. 18; it describes the influence of the labour cost due to the number of elements t in the catalogue of circular hollow sections T; $\max[M^{\frac{t}{t+0.1}}] = 75.14\,\mathrm{kg/m^2}$ and $\max[\sum_{i=1}^{t} b_i^{C(t)}] = 111.04$ are the maximum values of functions which can be reached in the domain of feasible solutions, $\gamma = 7850\,\mathrm{kg/m^3}$ is the bulk density of steel, A_k is the cross-sectional area of the k-th truss bar, l_k is the length of the k-th truss bar, $k_b = 168$ denotes the number of truss bars in one quarter of the space truss, $L = 30\,\mathrm{m}$ is the truss span and b_i is the number of bars with the same cross-sectional area. The mass of truss joints for $1\,\mathrm{m^2}$ of horizontal projection is estimated by the empirical coefficient $1.1^{\frac{t}{t+0.1}}$ which equals approximately 10% of the mass of the truss bars, i.e. $M_t = 1.1M_b$ and $\frac{t}{t+0.1}$ is the rectifying exponent for the truss mass dependent on the number of elements in the catalogue.

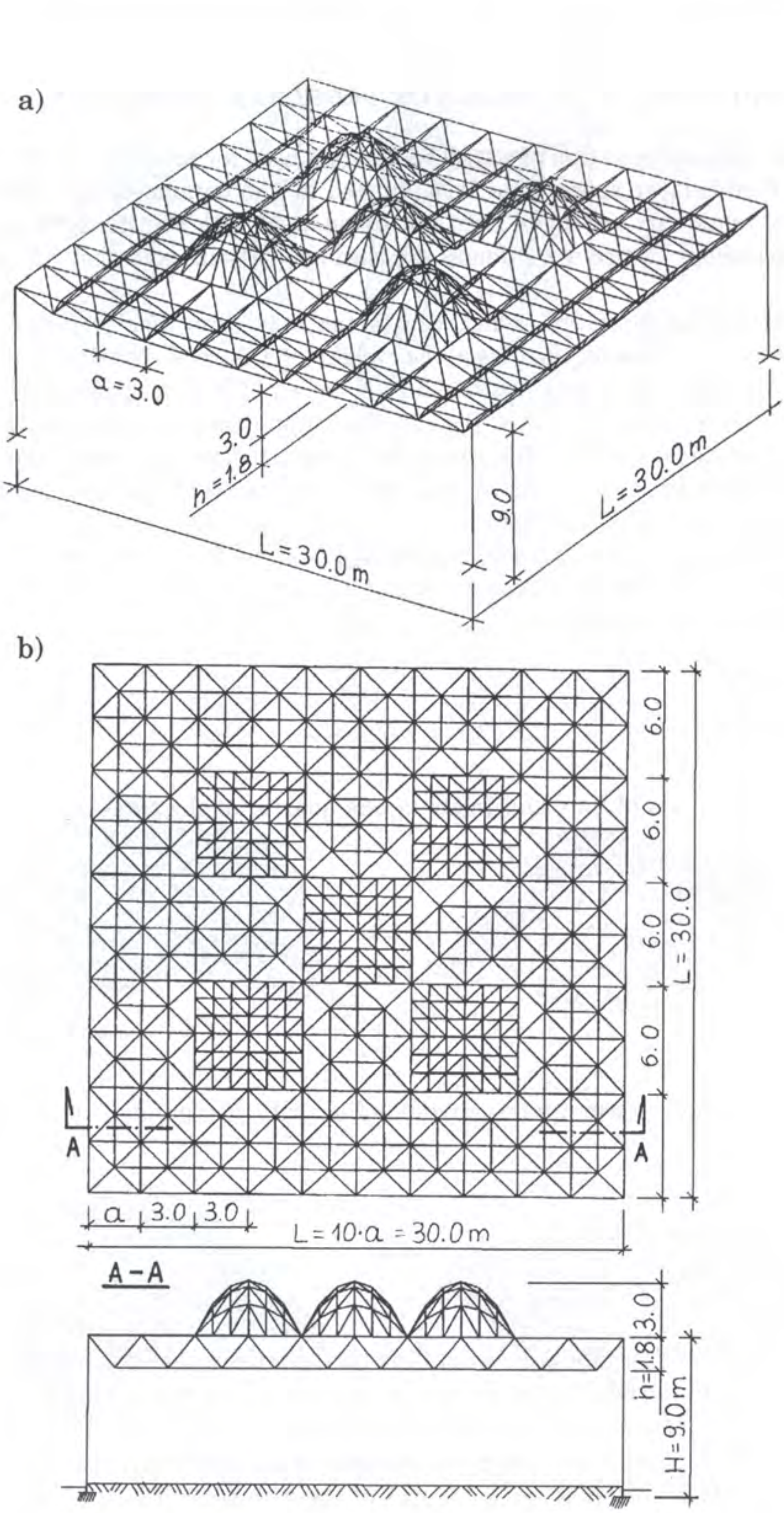

Fig. 10. Layout of exhibition hall covering

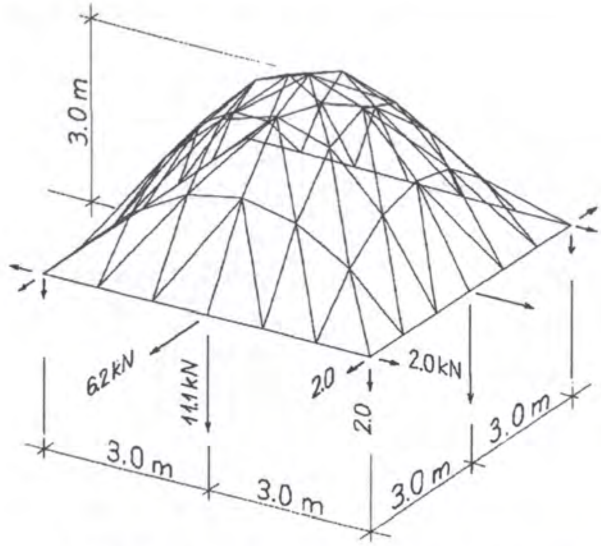

Fig. 11. Concentrated loadings transmitted from abatjours to double-layer truss nodes

The vector of the objective function has the following form

$$\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x})]^T. \tag{4}$$

The vector of decision variables has the form

$$\mathbf{x} = [t, T]^T \in X\,, \tag{5}$$

where T is a catalogue of circular hollow sections for truss bars with the number of elements t (see Figs. 12 and 16).

The domain of feasible solutions is determined by a set of design, computational and technological constraints (Table 1). The following notation is used: σ_k^t and σ_k^c are the tensile and compressive stresses in the k-th truss bar, respectively, m_{bk} is the buckling coefficient for the k-th compressed truss bar, $f_d = 210\,\text{MPa}$ is the strength of material, λ_k is a slenderness ratio for the k-th compressed truss bar, i_k is the radius of inertia of the cross-sectional area of the k-th truss bar, $\{P\}_n$ is the loading vector, n is the number of iteration, T is the catalogue of circular hollow sections for truss bars, T_B is the basic catalogue of cross-sectional areas of truss bars and T_M is the metallurgical catalogue of hot-rolled steel circular hollow sections, g and D are wall thickness and diameter of the circular hollow section, respectively (Fig. 12).

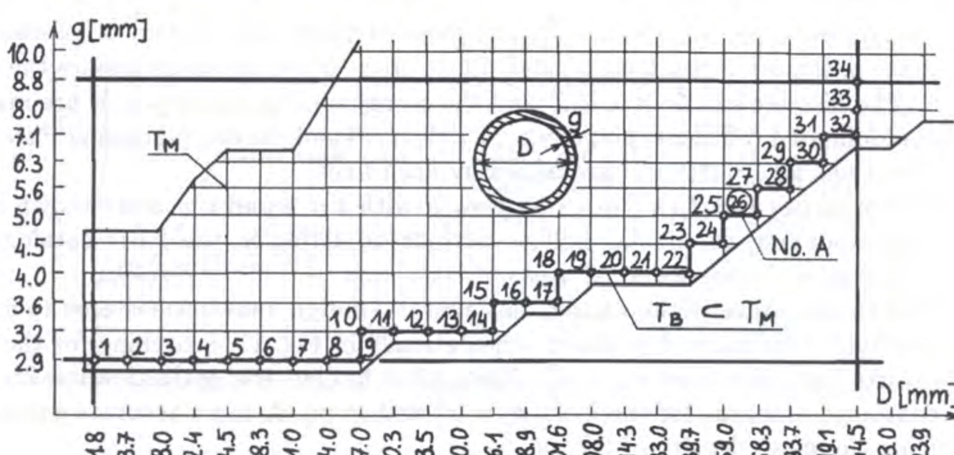

Fig. 12. Segment of the metallurgical catalogue T_M and the basic catalogue T_B

Table 1. Optimization constraints for space truss design

Design constraints	(6)	Technological constraints	(7)
$\sigma_k^c = \frac{N_k m_{bk}}{A_k} \le f_d = 210\,\mathrm{MPa}$	$(6)_1$	$A_k \in T \subset T_B \subset T_M$	$(7)_1$
$\sigma_k^t = \frac{N_k}{A_k} \le f_d = 210\,\mathrm{MPa}$	$(6)_2$	$2.9 \le g \le 8.8\,\mathrm{mm}$	$(7)_2$
$\lambda_k = \frac{l_k}{i_k} \le 250$	$(6)_3$	$31.8 \le D \le 244.5\,\mathrm{mm}$	$(7)_3$
$[K]_n\{\delta\}_n = \{P\}_n$	$(6)_4$	$1 \le t \le 34$	$(7)_4$
$\Delta_1 \le \frac{\sqrt{2}L}{250} = 16.97\,\mathrm{cm}$	$(6)_5$	Computational constraints	(8)
$\Delta_2 \le \frac{L}{250} = 12.0\,\mathrm{cm}$	$(6)_6$	$n \in \{1,2,\ldots,N\}$	$(8)_1$
		$1 \le N \le 10$	$(8)_2$
		$1 \le T \le 14$	$(8)_3$

The static analysis computer program is based on the displacement method. The cross-sectional areas of truss bars are chosen automatically from a given catalogue of circular hollow sections. The local stability constraint $(6)_3$ is taken into account. Internal forces in truss bars are determined. The appropriate cross-sectional areas of truss bars satisfying the constraints $(6)_1$–$(6)_4$ and $(7)_1$ are chosen in the n-th iteration for the known stiffness matrix $[\mathbf{K}]_n$ which has been determined in the previous iteration. The optimization process is terminated when the results of two sequential iterations are identical or the maximum number of iterations N is reached. Next, the calculated loadings are replaced by characteristic loadings to find the actual displacements of truss nodes. If the displacement constraints $(6)_5$ and $(6)_6$ are not satisfied, the next catalogue is chosen and the iteration process is repeated. It is assumed that the displacement constraints can be satisfied by choosing the appropriate catalogue of cross-sectional areas of truss bars.

Based on [14], the basic catalogue of cross-sectional areas of truss bars T_B is determined. The basic catalogue T_B has the following features: the ratio of cross-sectional areas A_{n+1}/A_n is constant and the corresponding critical N^c and limit N^t forces increase simultaneously. The maximum cross-sectional area is determined on the basis of the maximum force which occurs in the truss with the same cross-sectional areas for each bar, i.e. $t = 1$. The dimensions of cross-sections in the basic catalogue T_B and a segment of a metallurgical catalogue are shown in Fig. 12.

The optimization process starts from the basic catalogue T_B. The displacement constraints for the assumed depth of a double-layer space truss, $h = 1.8\,\mathrm{m}$, are violated (Fig. 13). The subsequent catalogues are created from the previous one by erasing every second cross-section (Fig. 14). The displacement constraints are only satisfied by catalogues with $t \le 4$ (Fig. 13). Four non-dominated catalogues are chosen from among these catalogues. The space of decision variables $\mathcal{A}$ and the objective space $\mathcal{B}$ are shown in Fig. 15.

The active constraints $(6)_2$, $(6)_5$, $(7)_1$, $(7)_4$ and $(8)_3$ determine the domain of feasible solutions $X \subset \mathcal{A}$. The image of the set of constraints listed above in the objective space creates the attainable domain $Y \subset \mathcal{B}$. The discrete solutions $\mathbf{x} \in X$ and the corresponding points $\mathbf{y} \in Y$ are marked with circles. The non-dominated (efficient) points $\mathbf{y}^i_{\mathrm{ND}}$, $i = 1,\ldots,4$ and the corresponding Pareto optimal (compromise) solutions $\mathbf{x}^i_{\mathrm{ND}} = [\mathbf{f}(\mathbf{x})]^{-1}$ are linked by dash lines.

The broken lines $\Delta_1(\mathbf{x})$ and $f_1(\mathbf{x})$ for catalogues T with $t = 2$ and $t = 3$ are shown in Fig. 16. The numbers corresponding to circular hollow sections occurring in the basic catalogue T_B are given for each catalogue of cross-sectional areas of truss bars.

The controlled enumeration method based on structural design knowledge is used to find a set of compromise solutions. This method is shown schematically in Fig. 17. Searching for the minimum value of $f(\mathbf{x})$ starts from the fixed $x_1 = x_1^0$. Next, after finding the optimal value for $x_2 = x_2^1$, the search is continued along x_1 for the fixed $x_2 = x_2^1$ and so on. A more accurate solution can be found by reducing the mesh density of the net.

The preferable solution $\mathbf{x}_p$ can be chosen from the set of compromise solutions X_{ND} by means of e.g. the metric function or utility function methods [2, 6, 9]. In this paper, the latter method

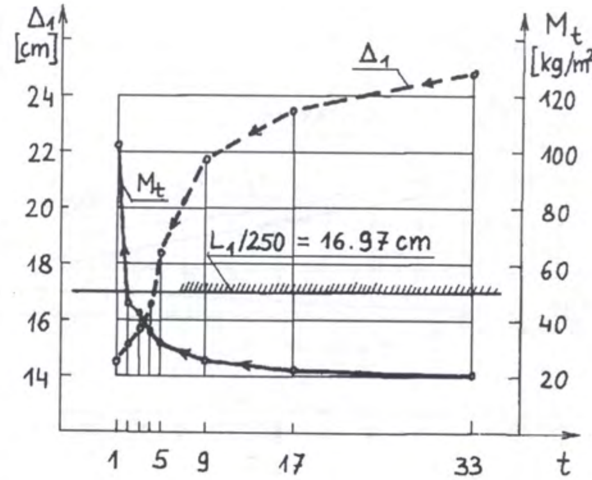

Fig. 13. Diagrams of the displacement Δ_1 and truss mass $f_1(\mathbf{x})$ versus a number of elements t in the catalogue T

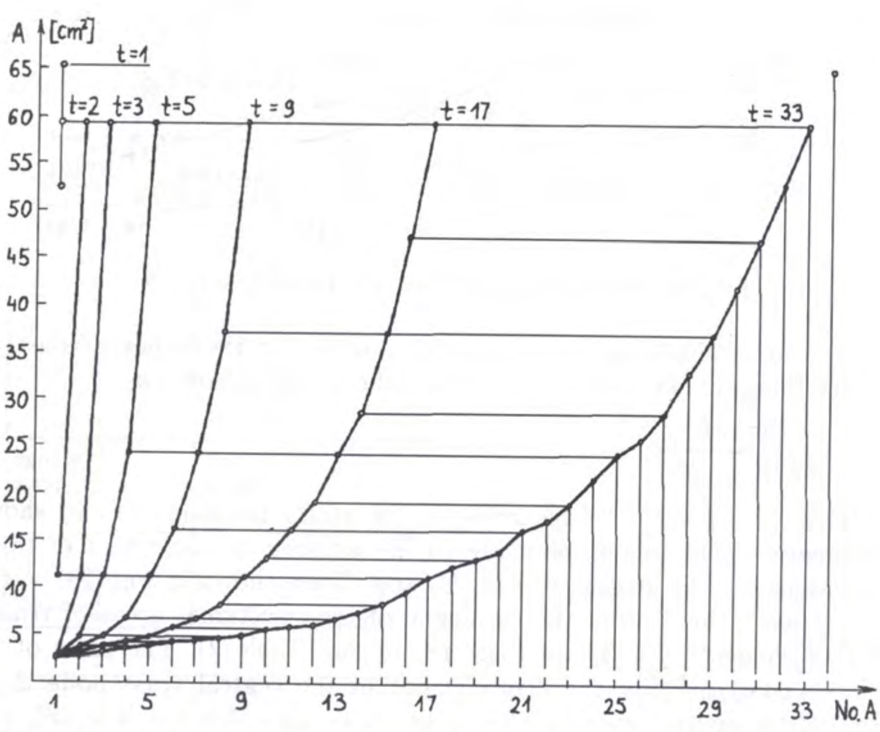

Fig. 14. Creation of catalogues by erasing every second cross-section (No. A is the number of cross-section in the catalogue T with the number t)

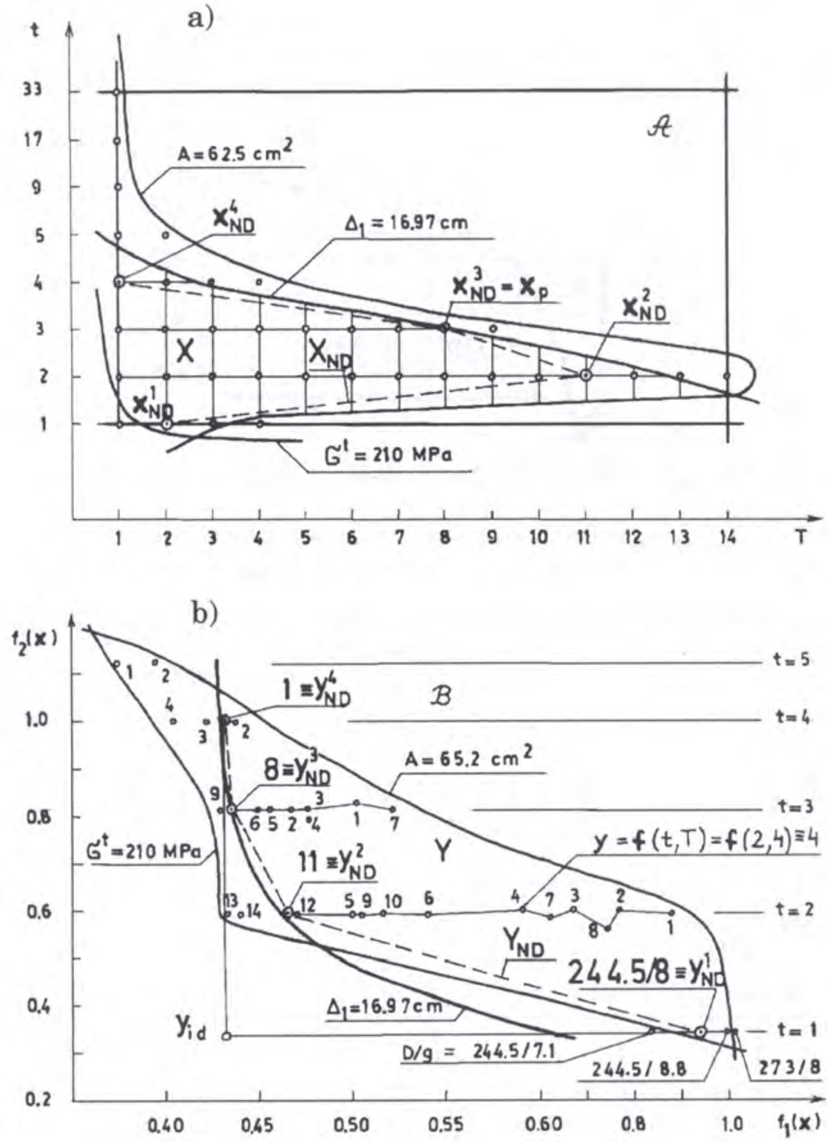

Fig. 15. Spaces of decision variables $\mathcal{A}$ and objectives $\mathcal{B}$

is chosen to find the preferable solution, i.e. $\min(U)$, subject to the imposed constraints listed in Table 1. The utility function takes into account the labour cost as follows,

$$U(\mathbf{x}_p) = \min_{\mathbf{x}\in X_{\mathrm{ND}}} \frac{C(t)\, M_t}{\max[C(t)\, M_t]} \,. \tag{9}$$

The function $C(t) = (1 - 0.4e^{-0.35t})$ occurring in the utility function (9) and shown in Fig. 18 describes the influence of the labour cost due to the number of elements t in the catalogue of circular hollow sections T. The diagrams of U, M_t and C_t are shown in Fig. 18.

The preferable truss is built from the catalogue of cross-sectional areas of truss bars $T = 8$ with the number of elements $t = 3$ (see Figs. 18, 19 and Table 2). The mass of the preferable truss is $f_1(\mathbf{x}) = 36.64\,\mathrm{kg/m}^2$ and the displacement of the central truss node $\Delta_1 = 17.02\,\mathrm{cm}$. The displacement of the central node is 0.3% larger than allowable but it is still lesser than the permissible violation which is 2%.

The Pareto optimal and preferable solutions are shown in Fig. 19. The structure is shown as four independent quarters corresponding with the different non-dominated (compromise) solutions. One of them minimizes the utility function (9) and it is chosen as the preferable solution. Different thickness of lines corresponds with different cross-sectional areas of truss bars.

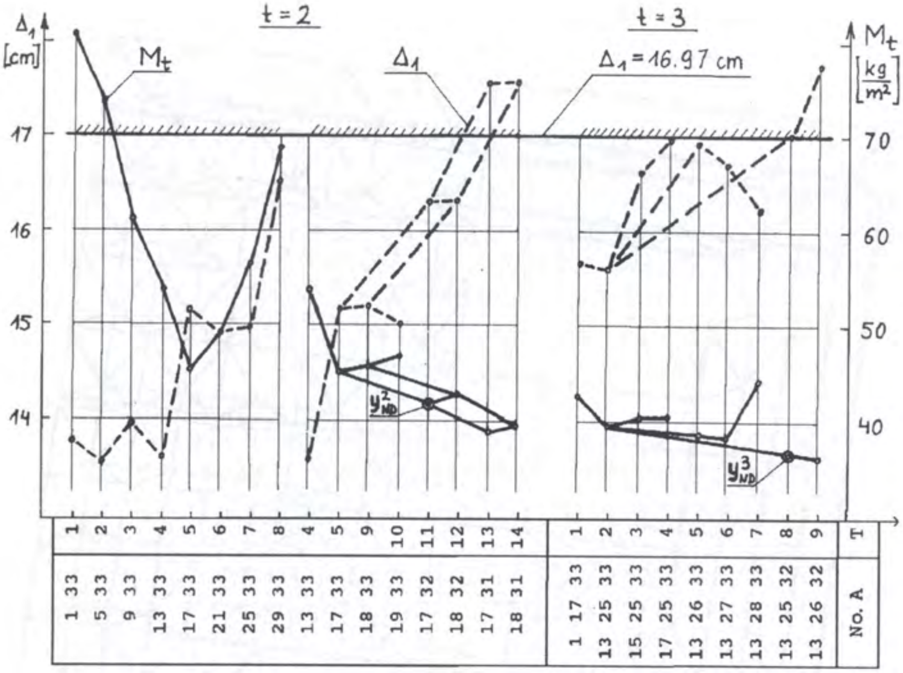

Fig. 16. Broken lines $\Delta_1(\mathbf{x})$ and $f_1(\mathbf{x})$ for the catalogues T with the number of elements $t = 2$ and $t = 3$

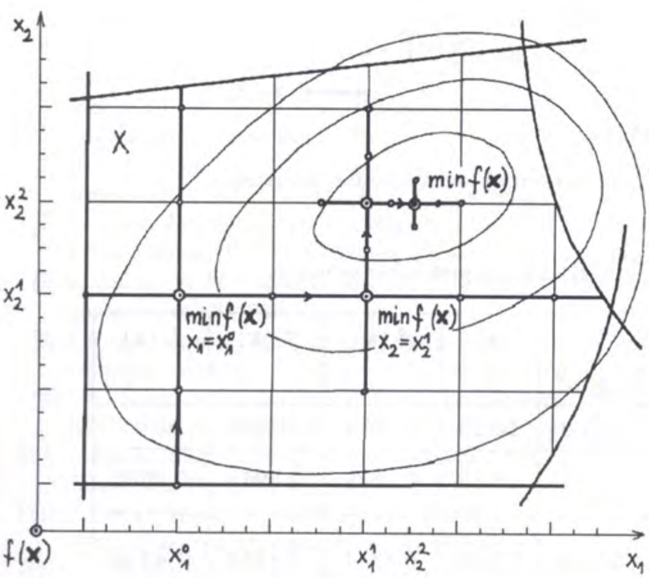

Fig. 17. Performance of the controlled enumeration method

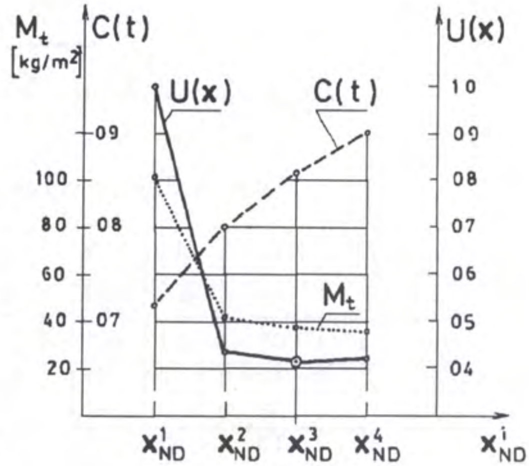

Fig. 18. Diagram of utility function and the preferable solution $x_p = x^3_{\mathrm{ND}}$

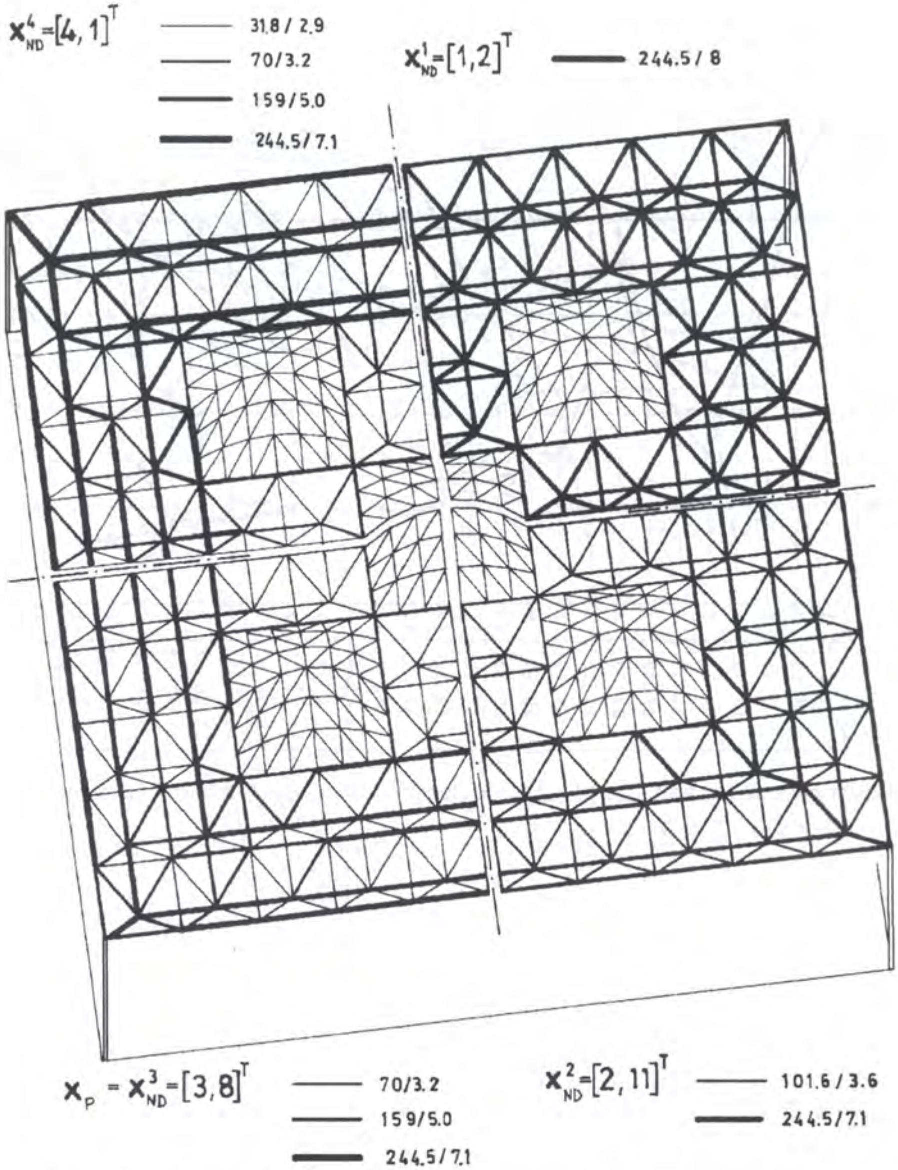

Fig. 19. Layout of Pareto optimal (compromise) and preferable solutions

Table 2. The sets of Pareto optimal and the preferable solutions

$\mathbf{x}=[t,T]^T$		No. A	A	D	g	M_t	Δ_1	$f_1(\mathbf{x})$	$f_2(\mathbf{x})$	$U(\mathbf{x})$
t	T		cm^2	mm	mm	kg/m^2	cm			
1	2	33	59.4	244.5	8.0	106.0	14.03	0.9357	**0.3446**	1.0
2	11	17 32	11.1 53.0	101.6 244.5	3.6 7.1	41.66	16.30	0.4642	0.5945	0.4386
3	**8**	**13** **25** **32**	6.72 24.2 53.0	**70.0** **159.0** **244.5**	**3.2** **5.0** **7.1**	**36.64**	**17.02**	0.4341	0.8146	**0.4140**
4	1	1 13 25 32	2.63 6.72 24.2 53.0	31.8 70.0 159.0 244.5	2.9 3.2 5.0 7.1	35.36	16.92	**0.4313**	1.0	0.4187

4. Conclusions

Rounding a continuous solution to the nearest discrete one can lead to a solution which is far from being the optimal discrete solution. In such cases, which often occur in space truss optimization, the discrete approach is recommended.

The domain of feasible solutions should be determined not only by stress constraints but also by displacement (serviceability) constraints. The optimum design of a truss structure with respect to serviceability constraints requires a different algorithm than that concerned with stress constraints only. The displacement constraints can be satisfied by increasing the stiffness of the structure. The displacement of truss nodes can be decreased by a few independent means presented in Section 2. The optimum design of trusses with respect to serviceability constraints leads to an increased truss mass but decreased labour and exploitation costs of the object.

The sets of non-dominated (efficient) solutions and compromise (Pareto optimal) solutions are found by solving the multicriterion optimization problem , i.e. $\min[f_1, f_2]$, subject to the imposed constraints (see Table 2 and Figs. 15, 16).

The preferable solution can be chosen from a set of non-dominated (efficient) solutions by means of e.g. the metric function or utility function methods. In this paper, the latter method is chosen to find the preferable solution, i.e. $\min(U)$, subject to the imposed constraints. The utility function also takes into account the labour cost.

Minimization of the mass of truss bars including the mass of joints, minimization of labour cost as well as exploitation and maintenance costs are chosen as optimization criteria. The number of elements in the catalogue of cross-sectional areas of truss bars is also minimized to reduce the labour cost.

The particular statements and indications concerned with truss optimization are listed at the end of each subsection so it does not seem necessary to repeat them here.

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