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Optimal design of pressure vessels including the effect of environment

Yuriy M. Pochtman and Mark M. Fridman
Dnepropetrovsk State University, Dnepropetrovsk, Ukraine

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The paper presents the optimization of thin-walled structures such as vertical cylindrical reservoirs subject to pitting corrosion. The function of the structure utility is taken as an optimization criterion. The choice of an optimal thickness distribution of the reservoir shell along its height is determined from the conditions of its uniform reliability.

1. Introduction

The surfaces of reservoirs working under liquid pressure or in a corrosive medium are often subject to pitting corrosion. In this case it is important to estimate the structure strength and reliability because the development of at least one pitting which gives rise to a solid hole causes the loss of its capability and unwanted emergency consequences. On the other hand, as it is mentioned in [1], the reservoirs dominate among metal consuming structures. Due to this, the problems of increasing their strength and decreasing the metal and labour consumption for their manufacture are rather urgent. The construction of a probability model of thin-walled structure breakage due to pitting corrosion has been described in [2]; it is supposed that initial pitting size distribution on the surface of the structure and the emergence time of the first hole are normal variables.

2. Formulation of the problem

The present paper offers a comprehensive approach to designing optimal structures of this class from the utility point of view.

The function of average expected utility [3] is adopted as an optimal criterion of a cylindrical shell (reservoir shell),

$$U(X) = B(X) - H_1(X) - L(X)\,, \tag{1}$$

which includes: an average income $B(X)$ expected from the operation of its designed period of life T with regard to a possible failure at the moment of time $T_{\text{fail}} < T$, the initial cost $H_1(X)$ and a damage due to the structure failure $L(X)$.

The purpose of the optimization task is to find a vector of the optimal structure parameters X maximizing the utility function (1) with a limitation of reliability

$$[U(X) = B(X) - H_1(X) - L(X)] \longrightarrow \max, \qquad P(T) \geq P_*\,, \tag{2}$$

where $P(T)$ denotes the reliability function and P_* is the value of the assumed reliability.

The vector of variable parameters is accepted as follows,

$$X = \{x_1, x_2\}^T = \{n, T\}^T$$

where n is the number of variable thickness belts (sheets) into which the reservoir is divided and T is the designed service live of the reservoir. The reservoir is under the influence of hydrostatic internal pressure of a liquid corrosive medium (such as petroleum products) of density ρ. The general reservoir dimensions, radius R and height H, are known.

3. CALCULATION METHODS

Let us show the determination of the values included in (2). Income and loss due to the structure failure will be determined according to [3],

$$B(X) = \int_0^T B^0(t) p_{\text{fail}}(t)\,\mathrm{d}t\,, \tag{3}$$

where

$$B^0(t) = \frac{b}{r}(1 - e^{-rt})$$

denotes income, b — annual income in the absence of failure, $r = \ln(1 + r')$, r' — interest from the capital and

$$p_{\text{fail}}(t) = P'_{\text{fail}}(t) = [1 - P(t)]' \tag{4}$$

— density of the structure failure probability during the period of its service. Prime $'$ denotes time derivative.

The average value of losses (or failure due to loss) reduced to the present time is determined similarly,

$$L(X) = \int_0^T L^0(t) p_{\text{fail}}(t)\,\mathrm{d}t, \tag{5}$$

where $L^0(t) = L_t e^{-rt}$ denotes losses and L_t — sum of the damage evaluated before the structure operation.

The initial cost of the structure is determined as in [1]:

$$H_1(X) = C_3 + C_{\text{m}}\,, \tag{6}$$

where

$C_3 = C_{\text{om}} + C_{\text{i}} + C_{\text{p}}$ – manufacturing cost,

$C_{\text{om}} = 1.07\left(\sum_{i=1} nC_{npi}k_{npi}G_i + 1.5G\right)$ – cost of main materials,

C_{npi} – wholesale price of sheets for i-cost of main structure part,

$G_i = 2\pi R h_i H\gamma/n$ – weight of the i-th part,

$C_{\text{i}} = C_{\text{ob}} + C_{\text{cb}} + C_{\text{n}} \cong 0.62\sqrt{Gn}$ – cost of the structure manufacture,

$C_{\text{ob}}\,,\ C_{\text{cb}}\,,\ C_{\text{n}}$ – costs of machining, mounting welding and rolling,

$C_{\text{p}} = 1.1406[1.0054(C_{\text{om}} + C_{\text{i}}) + 2.66]$ – structure cost,

$C_{\text{m}} = 0.641\sqrt{Gn}$ – cost of the reservoir assembling.

Now, let us determine the function of the shell reliability introduced in [2]. The reliability of the shell structure means the probability of a random event consisting in the fact that no pitting will go outside the permissible level, in this case, the thickness of the sheet during the preset period of its operation $0 \le t \le T$. We assume the equation of corrosion in the form

$$\frac{\mathrm{d}l_i}{\mathrm{d}t} = \alpha + \beta\sigma_i \tag{7}$$

where l_i denotes current depth of pitting, α, β are constant coefficients and σ_i is effective stress in the i-th sheet. The solution of Eq. (7) for $t = 0$, $l_i = l_{0i}$, is as follows,

$$l_i = l_{0i} + (\alpha + \beta\sigma_i)t = l_{0i} + b_i t\,.$$

If we take the initial depression l_{0i} as a random value with a normal density

$$q(l_{0i}) = \frac{1}{\sqrt{2\pi}\sigma_{l_{0i}}} \exp\left[-\frac{(l_{0i} - \bar{l}_{0i})^2}{2\sigma_{l_{0i}}^2}\right]$$

and take into account the fact that b_i is a constant value, then l_i is a fixed random value. In this case, the probability that no pitting will go outside the thickness of the i-th sheet h_i (taken as a constant value) is determined as follows, [4],

$$P_i^* = \frac{1}{\sqrt{2\pi}\sigma_{l_{0i}}} \int_{-\infty}^{h_i - b_i t} \exp\left[\frac{(l_{0i} - \bar{l}_{0i})^2}{2\sigma_{l_{0i}}^2}\right] \mathrm{d}l_{0i} = 0.5 + \Phi(a_i)\,,$$

where

$$a_i = \frac{h_i - b_i t - \bar{l}_{0i}}{\sigma_{l_{0i}}}\,, \qquad \Phi(a_i) = \frac{1}{\sqrt{2\pi}} \int_0^{a_i} e^{-\frac{z^2}{2}}\,\mathrm{d}z \text{ — Laplace integral.}$$

If we denote the total number of pitting formations on the surface of the whole reservoir by N and assume the thickness distribution along the height of the shell in such a way that the reliability of each sector is constant (equally reliable), then the reliability of the whole reservoir is determined by the expression

$$P = (P_i^*)^N\,.$$

Assuming a permissible level of reliability P_* and the total number of pitting formations, it is easy to calculate the required values of thickness h_i according to the principle of equal reliability of the reservoir. In this case

$$P_i^* = (P_*)^{\frac{1}{N}}$$

and

$$a = \frac{h_i - b_i t - \bar{l}_{0i}}{\sigma_{l_{0i}}} = \text{const},$$

where a is found from the conditions

$$(P_*)^{\frac{1}{N}} = \Phi(a)\,.$$

Assuming $\bar{l}_{0i} = c_1 h_i$ and $\sigma_{l_{0i}} = c_2 h_i$, we obtain

$$a = \frac{h_i - c_1 h_i - b_i t}{c_2 h_i}\,, \qquad b_i = \alpha + \beta\sigma_i\,. \tag{8}$$

As the effective stress in the i-th sheet is $\sigma_i = \rho H_i R / n h_i$, the unknown quantities h_i are determined as follows,

$$h_i = k + \sqrt{k^2 + p}\,, \qquad i = 1, \ldots, n\,,$$

where

$$k = \frac{\alpha t}{2(1 - c_1 - c_2 a)}\,, \qquad p = \frac{\beta H_i \rho R t}{n(1 - c_1 - c_2 a)}\,.$$

Having determined the function of reliability and substituted the value p_{fail} in (3) and (5) found according to (4), we can find the expressions of the income and damage due to the failure of the structure:

$$B(X) = \frac{bN}{r}\left\{[0.5 - \Phi(a)] - \exp\left(-b_1 + \frac{a_1^2}{2}\right)[0.5 - \Phi(a - a_1)]\right\}, \tag{9}$$

$$L(X) = L_T N \exp\left(-b_1 + \frac{a_1^2}{2}\right)[0.5 - \Phi(a - a_1)], \tag{10}$$

where

$$b_1 = \frac{rh_i(1-c_1)}{b_i}, \quad a_1 = \frac{rc_2h_i}{b_i}.$$

It is necessary to note that $a_1 = \text{const}$ and $b_1 = \text{const}$ at any values of is resulting from Eq. (8).

Having calculated the current values $B(X)$, $H_1(X)$ and $L(X)$ we turn our attention directly to the reservoir optimization. In the case of complex, multiextreme tasks of non-linear programming such as (2) it is advisable to use one of the effective algorithms of the random search method [5].

4. Design Example

As an illustration, let us consider the problem of reservoir optimization with the following initial data: $R = 2\,\text{m}$, $H = 4\,\text{m}$, $N = 48$, $\alpha = 0.6\,\text{cm/year}$, $\rho = 0.8 \times 10^3\,\text{kg/m}^3$, $c_1 = 0.2$, $c_2 = 0.01$, $P_* = 0.99$, $b = 10^4$, $L_T = 10^4$, $r' = 10\%$.

Four variants of different values of relationship coefficient between the corrosion and stress β have been considered: $\beta_1 = 0.085\,\text{cm/Ton·year}$, $\beta_1 = 0.17\,\text{cm/Ton·year}$, $\beta_1 = 0.25\,\text{cm/Ton·year}$ and $\beta_1 = 0.34\,\text{cm/Ton·year}$. The ranges of changes in variable parameters are $2 \le n \le 10$, $5\,\text{years} \le T \le 10\,\text{years}$.

The results of the numerical experiment on the optimization of the reservoir shell are given in Table 1.

Table 1

β [cm^3/Ton·year]	n	T [year]	h_1 [cm]	h_n [cm]	$B(X)$	$H_1(X)$	$L(X)$	$U(X)$
0.085	10	27.88	2.188	2.19	969.42	217.24	7.11	745.07
0.17	10	28.03	2.2	2.21	969.73	218.58	7.08	744.07
0.25	10	28.06	2.2	2.21	969.19	218.92	7.13	743.14
0.34	10	28.25	2.22	2.23	969.73	220.54	7.08	742.11

5. Conclusion

It is evident that the increase of the coefficient β has only a little influence on the optimal design of the reservoir (this is due to the fact that the effective stresses in it are small). Thus, the designed service life of the reservoir is $T = 28\,\text{years}$ and the number of variable thickness belts n in all variants reaches the maximum. Due to this, the change of thickness h_i along the height found from the principle of equal reliability of the reservoir shell is practically insignificant.

A similar problem has been solved with the use of the traditional statement (Table 2):

We find X at which

$$G \longrightarrow \min, \qquad P(T) \ge P_*, \tag{11}$$

where G is the reservoir weight.

Table 2

β	n	T	h_i	G
[cm^3/Ton·year]		[year]	[cm]	[kg]
0.085	2	3.7	0.3	120.1
0.17	2	3.58	0.301	121.6
0.25	2	3.48	0.302	123.2
0.34	2	3.36	0.304	124.1

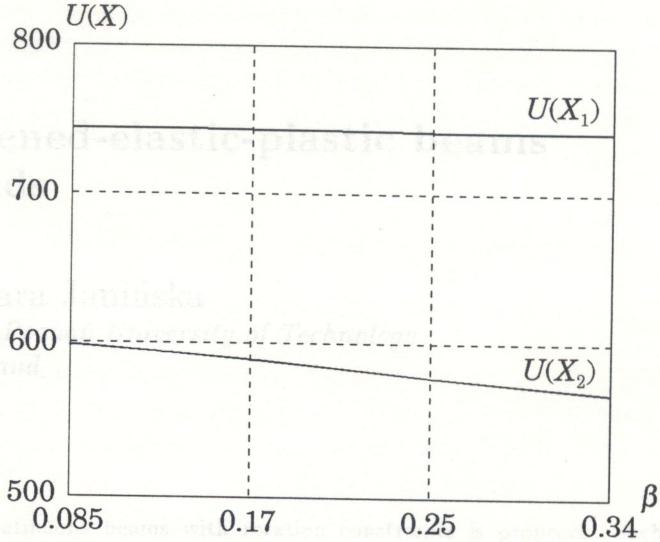

Fig. 1

When comparing he results of Tables 1 and 2 it can be concluded that the thickness of the reservoir in the second case decreases sharply (as would be expected) and reaches the minimum. Naturally, this results in a great reduction of the designed service life of the reservoir.

The values of the structure utility function $U(X_2)$ in the optimal points X_2 obtained according to the statement (11) have been determined. As indicated in Fig. 1, the utility of the structure in this case decreases by 20–24%.

Based on this, the use of the statement (2) allows to expand the usual range of problems considered in optimization and is thus more preferable.

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