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# A method of identification of kinematic chains and distinct mechanisms

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A new method is proposed to identify the distinct mechanisms derived from a given kinematic chain in this paper. The kinematic chains and their derived mechanisms are presented in the form of a flow matrix. Two structural invariants, sum of the absolute values of the characteristic polynomial coefficients (SCPC) and maximum absolute value of the characteristic polynomial coefficient (MCPC) are determined using the software MATLAB. These invariants are used as the composite identification number of a kinematic chain and mechanisms and clearly identify the distinct mechanisms derived from the family of 1-F, 8-links and 10-links KC as well as 2-F, 9-links simple joined KC. This study will help the designer to select the best possible mechanism to perform the specified task at the conceptual stage of design. The proposed method does not require any test for isomorphism separately. Some examples are provided to demonstrate the effectiveness of this method.

**Keywords:** kinematic chain (KC), distinct mechanism (DM), flow matrix, SCPC, MCPC

## 1. Introduction

In a mechanism design problem, systematic steps are type of synthesis, structural number synthesis and dimensional synthesis. Structural synthesis of the KC and mechanism has been the subject of a number of studies in recent years. One important aspect of structural synthesis is to develop all the possible arrangements of KC and their mechanisms derived for a given number of links, joints and degrees of freedom, so that the designer has the liberty to select the best or optimum mechanisms according to his requirements. In the course of development of KC and mechanisms, duplication may be possible. For this reason, many methods have been proposed by many researchers to check for duplication or, in other words, to detect the isomorphism among the kinematic chains. Most of these methods are based on the adjacency matrix [1] and the distance matrix [2]. Determining the structurally DM of a KC, the link disposition method [3], the flow matrix method [4] and the row sum of extended distance matrix methods [5] are used. Minimum code [6], characteristic polynomial of matrix [7], identification code [8], link path code [9], summation polynomial [10] etc. are used to characterize the KC. With regard to these methods, either there is a lack of uniqueness or they take too much time. Row sum of the extended adjacency matrix method [5] identifies 10 distinct mechanisms derived from the family of 6-links, 1-F kinematic chains but it distinguishes only 69 DM derived from the family of 8-links, 1-F, KC instead of the 71, reported by other researchers. Hence, there is a need to develop a computationally efficient method for determining the DM of a KC. In the present work, a new method is proposed to determine the DM of a KC and a flow matrix has been defined. Two structural invariants $SCPC$ and $MCPC$ are derived from the flow matrix, based on their characteristic polynomial coefficients, using the software MATLAB. These structural invariants are the same for identical or structurally equivalent mechanisms and different for DM. Hence, in this way, it is possible to identify all DM derived from the given KC. These invariants may also be used to detect isomorphism in the KC having simple joints and even the KC having co-spectral graph. The method is explained with the help of examples of planner KC having all simple joints and the results of all the DM derived from the family of 1-F, 8-links and 10-links KC as well as 2-F, 9-links simple joined KC summarized in Table 3 and 4.

## 2. Definitions of terminology

The following definitions are to be explained clearly before applying this method. Various definitions with their abbreviations are given below.

**Flow Path Value.** It is defined as the minimum number of joints between two links under consideration. For example in Fig. 1, the minimum number of joints between links 1 and 7 are 2, so the flow path value between link 1 and 7 will be 2.

**Link Flow Matrix.** For an $n$-link KC it is defined as an $n \times n$ square matrix,

$$\mathbf{FM} = \{F_{ij}\}_{n\times n},$$

whose element $F_{ij}$ is a minimum number of joints between link $i$ and $j$, and is equal to zero if $i$ is equal to $j$. Of course, all the diagonal elements $F_{ii} = 0$.

$$\mathbf{FM} = \begin{bmatrix} 0 & F_{12} & F_{13} & \dots & F_{1n} \\ F_{21} & 0 & F_{23} & \dots & F_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ F_{n1} & F_{n2} & F_{n3} & \dots & 0 \end{bmatrix}.$$

## 3. New structural invariants

The characteristic polynomial [1] is generally derived from the $(0,1)$ adjacency matrix. The roots of the $n$-th order characteristic polynomial are the set of $n$ eigenvalues called eigenspectrum. Many researchers have reported co-spectral graphs (the non-isomorphic graphs having the same eigenspectrum derived from the $(0,1)$ adjacency matrix). The proposed flow matrix contains additional information about the number of joints existing between two links of a KC. Therefore, it is expected that the characteristic polynomial and its coefficients will be unique to clearly identify the KC and even the KC with co-spectral graphs. The characteristic polynomial of flow matrix is given by $D(\lambda)$. The monic polynomial of degree $n$ is given by Eq. (1).

$$|\mathbf{FM} - \lambda\mathbf{I}| = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_{n-1}\lambda + a_n, \tag{1}$$

where $n$ is the number of joints and 1, $a_1$, $a_2$, $a_{n-1}$, $a_n$ are the characteristic polynomial coefficients. The two important properties of the characteristic polynomials are:

1. The sum of the absolute values of the characteristic polynomial coefficients ($SCPC$) is constant for a **FM** matrix. i. e. $|1| + |a_1| + |a_2| + \dots + |a_{n-1}| + |a_n| = const.$
2. The maximum absolute value of the characteristic polynomial coefficient ($MCPC$) is also constant for a **FM** matrix.

Therefore, the proposed structural invariants $SCPC$ and $MCPC$ are unique and used as identification numbers to identify the distinct mechanisms of a kinematic chain.

### 3.1. Test for isomorphism of kinematic chains

**Theorem.** *Two similar square symmetric matrices have the same characteristic polynomial [12].*

**Proof.** *Let the two KC be represented by the two similar matrices* $\mathbf{A}$ *and* $\mathbf{B}$ *such that* $\mathbf{B} = \mathbf{P^{-1}}\,\mathbf{A}\,\mathbf{P}$, *taking into account that the matrix* $\lambda\mathbf{I}$ *commutes with the matrix* $\mathbf{P}$ *and* $\left|\mathbf{P}^{-1}\right| = |\mathbf{P}|^{-1}$. *Since the determinant of the product of two square matrices equals the product of their determinants, we have*

$$\begin{aligned}|\mathbf{B}-\lambda\mathbf{I}| &= \left|\mathbf{P}^{-1}\mathbf{A}\mathbf{P}-\lambda\mathbf{I}\right| \\ &= \left|\mathbf{P}^{-1}\left(\mathbf{A}-\lambda\mathbf{I}\right)\mathbf{P}\right| \\ &= \left|\mathbf{P}^{-1}\right|\left|\mathbf{A}-\lambda\mathbf{I}\right|\left|\mathbf{P}\right| = |\mathbf{A}-\lambda\mathbf{I}|.\end{aligned}$$

*Hence, if* $D(\lambda)$ *is the characteristic polynomial of a matrix,* $D(\lambda)$ *of matrix* $\mathbf{A}$ *is equal to* $D(\lambda)$ *of matrix* $\mathbf{B}$.

*It means that if* $D(\lambda)$ *of two matrices* **FM** *representing two KC are the same, their structural invariants* $SCPC$ *and* $MCPC$ *will also be the same and the two KC are isomorphic otherwise non-isomorphic chain.*

### 3.2. Identification of structurally equivalent links and distinct mechanisms

A KC is represented by the matrix **FM**. When any link of a KC is fixed, a mechanism results. It means that the corresponding joints of the fixed link work as pivots. If in the matrix **FM** the diagonal elements of the corresponding fixed link '1' are changed from 0 to 1 (zero to one), it will be the representation of the first mechanism with fixed link '1'. The structural invariants $SCPC$-1 and $MCPC$-1 of this matrix **FM**-1 are then calculated using software MATLAB. This process is repeated for the second link '2' and so on. In this way, a set of invariants equal to the number of the links are obtained. Some of them may be same and others are different. The same structural invariants represent the corresponding structurally equivalent links that constitute one DM.

## 4. Application to kinematic chain

**Example 1.** The first example concerns 8-bars, 10-joints, single-degree-of-freedom kinematic chain shown in Fig. 1.

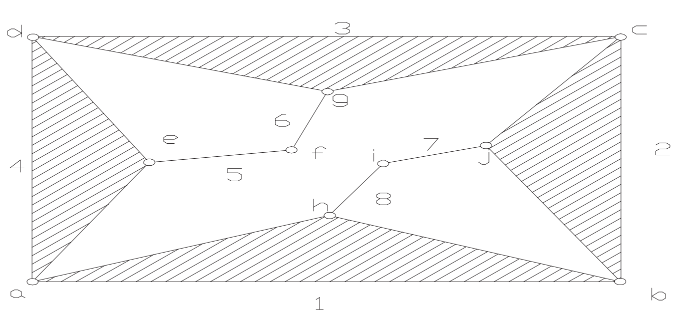


**Fig. 1.** Eight-links single-degree-of-freedom kinematic chain

*Link Flow Matrix* **FM**

The link flow matrix **FM** representing the kinematic chain shown in Fig. 1 is as follows:

$$\begin{array}{c c}
\text{Link} & \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} &
\begin{bmatrix}
0 & 1 & 2 & 1 & 2 & 3 & 2 & 1 \\
1 & 0 & 1 & 2 & 3 & 2 & 1 & 2 \\
2 & 1 & 0 & 1 & 2 & 1 & 2 & 3 \\
1 & 2 & 1 & 0 & 1 & 2 & 3 & 2 \\
0 & 1 & 2 & 1 & 2 & 3 & 2 & 1 \\
1 & 0 & 1 & 2 & 3 & 2 & 1 & 2 \\
2 & 1 & 0 & 1 & 2 & 1 & 2 & 3 \\
1 & 2 & 1 & 0 & 1 & 2 & 3 & 2
\end{bmatrix}
\end{array}.$$

*Structural invariants of the mechanisms*

The structural invariants of the mechanisms derived from the above kinematic chain are listed below in Table 1.

**Table 1.** Structural invariants of various links

| Link | *SCPC* | *MCPC* |
|---|---|---|
| 1 | 28.6459 | 14.3467 |
| 2 | 28.6459 | 14.3467 |
| 3 | 28.6459 | 14.3467 |
| 4 | 28.6459 | 14.3467 |
| 5 | 28.6577 | 14.4099 |
| 6 | 28.6577 | 14.4099 |
| 7 | 28.6577 | 14.4099 |
| 8 | 28.6577 | 14.4099 |

*Identification of the distinct mechanisms*

Observing the structural invariants for the above eight mechanisms, it is found that the structural invariants of links $(1, 2, 3, 4)$ are the same. Hence, they are treated as equivalent links and form only one distinct mechanism. Similarly, the structural invariants of links $(5, 6, 7, 8)$ are the same, hence form second distinct mechanism. Therefore, two distinct mechanisms are obtained from kinematic chain shown in Fig. 1.

**Example 2.** The second example concerns 9-bars, 11-joints, and the two-degrees-of-freedom kinematic chain shown in Fig. 2.

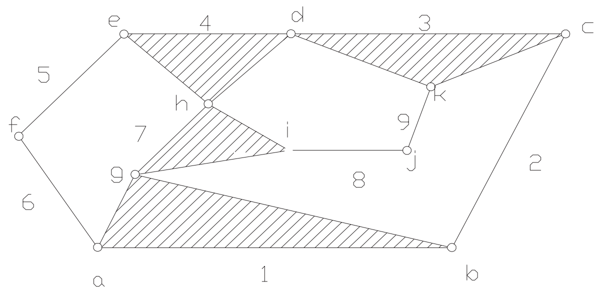


**Fig. 2.** Nine-links two-degrees-of-freedom kinematic chain

*Link Flow Matrix* **FM**

The link flow matrix **FM** representing the kinematic chain shown in Fig. 2 is as follows:

$$
\begin{array}{cc}
\text{Link} & \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array} &
\begin{bmatrix}
0 & 1 & 2 & 2 & 2 & 1 & 1 & 2 & 3 \\
1 & 0 & 1 & 2 & 3 & 2 & 2 & 3 & 2 \\
2 & 1 & 0 & 1 & 2 & 3 & 2 & 2 & 1 \\
2 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 2 \\
2 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 3 \\
1 & 2 & 3 & 2 & 1 & 0 & 2 & 3 & 4 \\
1 & 2 & 2 & 1 & 2 & 2 & 0 & 1 & 2 \\
2 & 3 & 2 & 2 & 3 & 3 & 1 & 0 & 1 \\
3 & 2 & 1 & 2 & 3 & 4 & 2 & 1 & 0
\end{bmatrix}
\end{array}.
$$

*Structural invariants of the mechanisms*

The structural invariants of the mechanisms derived from the above kinematic chain are listed below in Table 2.

**Table 2.** Structural invariants of various links

| Link | $SCPC$ | $MCPC$ |
|---|---|---|
| 1 | 32.1521 | 15.8717 |
| 2 | 32.0081 | 15.8996 |
| 3 | 32.1521 | 15.8717 |
| 4 | 32.0313 | 15.8608 |
| 5 | 31.8374 | 15.9119 |
| 6 | 31.9491 | 15.9264 |
| 7 | 32.0313 | 15.8608 |
| 8 | 31.8374 | 15.9119 |
| 9 | 31.9491 | 15.9264 |

*Identification of the distinct mechanisms*

Observing the structural invariants for the above nine mechanisms, it is found that the links $(1,3)$, $(4,7)$, $(5,8)$ and $(6,9)$ are equivalent links since their structural invariants are the same and form four distinct mechanisms. Link 2 has the distinct invariants, forms the fifth distinct mechanism. Therefore, five distinct mechanisms are obtained from the kinematic chain shown in Fig. 2.

**Example 3.** The third example concerns the case with 10-links, 12-joints, three-degrees-of-freedom, co-spectral kinematic chains shown in Figs. 3 and 4. The graphs having the same characteristic polynomials derived from the $(0, 1)$ adjacency matrix are called the co-spectral graphs.

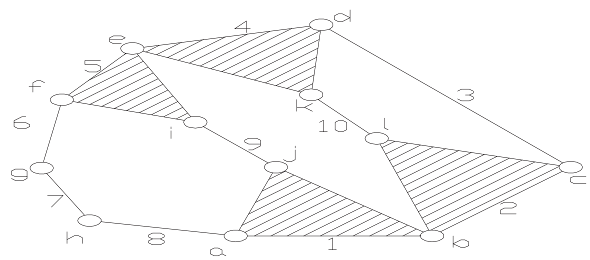


**Fig. 3.** Ten-links three-degrees-of-freedom kinematic chain

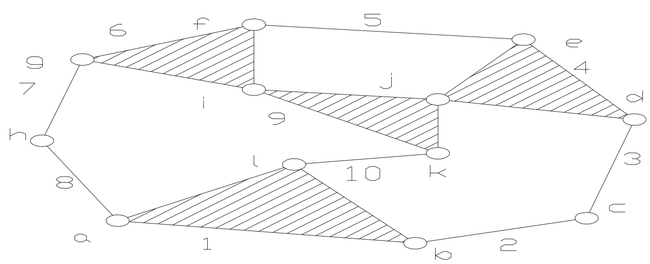


**Fig. 4.** Ten-links three-degrees-of-freedom kinematic chain

*Link Flow Matrix* **FM**

The link flow matrix **FM** representing the kinematic chain shown in Fig. 3 is as follows:

$$
\begin{array}{c c}
\text{Link} & \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} &
\begin{bmatrix}
0 & 1 & 2 & 3 & 2 & 3 & 2 & 1 & 1 & 2 \\
1 & 0 & 1 & 2 & 3 & 4 & 3 & 2 & 2 & 1 \\
2 & 1 & 0 & 1 & 2 & 3 & 4 & 3 & 3 & 2 \\
3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 2 & 1 \\
2 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 1 & 2 \\
3 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 2 & 3 \\
2 & 3 & 4 & 3 & 2 & 1 & 0 & 1 & 3 & 4 \\
1 & 2 & 3 & 4 & 3 & 2 & 1 & 0 & 2 & 3 \\
1 & 2 & 3 & 2 & 1 & 2 & 3 & 2 & 0 & 3 \\
2 & 1 & 2 & 1 & 2 & 3 & 4 & 3 & 3 & 0
\end{bmatrix}
\end{array},
$$

and the link flow matrix **FM** representing the kinematic chain shown in Fig. 4 takes the following form:

$$\begin{array}{c c}
\text{Link} & \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} &
\begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 3 & 2 & 1 & 2 & 1 \\
1 & 0 & 1 & 2 & 3 & 4 & 3 & 2 & 3 & 2 \\
2 & 1 & 0 & 1 & 2 & 3 & 4 & 3 & 2 & 3 \\
3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 1 & 2 \\
4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 2 & 3 \\
3 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 1 & 2 \\
2 & 3 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\
1 & 2 & 3 & 4 & 3 & 2 & 1 & 0 & 3 & 2 \\
2 & 3 & 2 & 1 & 2 & 1 & 2 & 3 & 0 & 1 \\
1 & 2 & 3 & 2 & 3 & 2 & 3 & 2 & 1 & 0
\end{bmatrix}.
\end{array}$$

*Structural invariants of the mechanisms*

The characteristic polynomial coefficients are the following. For the kinematic chain shown in Fig. 3: 7.8576, −6.0000, −3.3157, −2.0000, −1.0665, −0.8290, 0.0000, 0.0000, 1.1139, 19.9549. For the kinematic chain shown in Fig. 4: 7.3117, −6.8284, −3.9025, −1.3372, −1.1716, −0.0000, 0.0000, 0.0000, 0.6751, 19.8762. For the kinematic chain shown in Fig. 3 $SCPC = 42.1376$ and $MCPC = 19.9549$. For the kinematic chain shown in Fig. 4 $SCPC = 41.1027$ and $MCPC = 19.8762$.

The structural invariants of the two KC having co-spectral graphs shown in Fig. 3 and Fig. 4 are different. Therefore, both the KC are non-isomorphic. Note that by using other method of summation polynomials [10], the same conclusion is obtained.

**Example 4.** The fourth example concerns the slider crank mechanisms obtained by 6 link 7 joints kinematic chains as shown in Figs. 5 and 6, respectively.

*Structural invariants of the mechanisms*

The characteristic polynomial coefficients are the following. For the kinematic chain shown in Fig. 5: −2.6885, −2.3430, −2.0000, 0.0000, 0.1408, 7.8907. For the kinematic chain shown in Fig. 6: −3.8126, −2.8273, −1.3667, 0.0000, 0.3776, 8.6290. For the kinematic chain shown in Fig. 5 $SCPC = 15.0630$ and $MCPC = 7.8907$. For the kinematic chain shown in Fig. 6 $SCPC = 17.0132$ and $MCPC = 8.6290$. The structural invariants of the both kinematic chain mechanisms are different. Therefore, both the kinematic chain mechanisms are non-isomorphic.

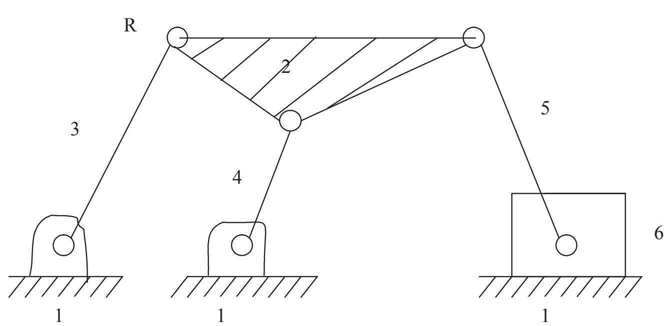


**Fig. 5.** Six-bars slider crank mechanism

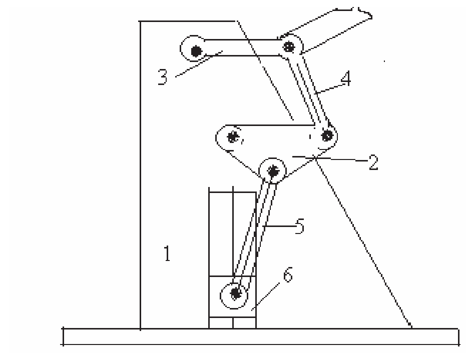


**Fig. 6.** Six bar slider cranck mechanism

## 5. Results

Using the proposed method, the number of mechanisms derived from the family of 1-F; 6-link, 8-link and 2-F 9-link chains are 5, 71 and 254, respectively. These results are in agreement with those reported already in the literature. The distinct mechanisms derived from the family of 1-F; 10-link 1842. A brief summary of these results is presented in Table 3 and 4.

**Table 3.** Total number of distinct mechanisms

| S. No. | Structural synthesis and analysis problem | No. of KC | DM by Ref. [11] | DM by present method |
|---|---|---|---|---|
| 1 | 1-F 8-links simple-joined KC | 16 | 71 | 71 |
| 2 | 2-F 9-links simple-joined KC | 40 | 254 | 254 |
| 3 | 1-F 10-links simple-joined KC | 230 | 1834 | 1842 |

**Table 4.** Description of DM derived from 1-F, KC of 8 and 10 links

| Class | Group | Number of links | KC | DM | Total distinct mechanisms |
|---|---|---|---|---|---|
| III | a | 8 | 9 | 35 | 71 |
| | b | 8 | 5 | 31 | |
| | c | 8 | 2 | 5 | |
| IV | a | 10 | 50 | 342 | 1842 |
| | b | 10 | 95 | 870 | |
| | c | 10 | 15 | 126 | |
| | d | 10 | 57 | 415 | |
| | e | 10 | 8 | 64 | |
| | f | 10 | 3 | 20 | |
| | h | 10 | 2 | 5 | |

## 6. Conclusions

In this paper, a unique method has been developed for identifying distinct mechanisms of a kinematic chain. Two structural invariants $SCPC$ and $MCPC$ are proposed. These invariants are easy to compute, reliable and capable to identifying all distinct mechanisms derived from a given kinematic chain and also able to detect the isomorphism among the KC having simple joints and even the KC with co-spectral graphs. Authors strongly believe that this method is unique and applicable to planar chains of any size and complexity. Unique in the sense that it has taken care of nature and all inherent properties of the mechanisms. It is felt that this paper presents a new concept on which a new identification system can be based. Such a new identification system would be extremely selective and would eliminate the possibility of duplicate identification for structurally different mechanisms. The inherent relation between structural invariants and the mechanisms need further study.

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