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Mass optimisation of turbofan engine casing made of sandwich structure

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Materials of a high specific strength and stiffness are used in the aerospace industry to obtain the lowest possible aircraft mass. The object of analysis is the casing of the F124 turbofan engine. The axially compressed cylindrical part of this casing is considered. The aim of the paper is to analyse possible benefits of replacing the original ribbed metal casing with a sandwich structure. The sandwich structure (metal-fibre laminate) of titanium alloy faces and a flax fibre laminate core is proposed. Semi-analytical optimisation of a sandwich structure was performed including a polynomial approximation of the critical load with correction obtained based on numerical analysis. The best mass efficiency was obtained for a core to faces thickness ratio equal to about 4.

Keywords: thin-walled sandwich structure, flax laminate core, buckling, semi-analytical optimisation.

1. Introduction

Thin-walled panels and materials of high specific strength and stiffness are used in the aerospace industry to obtain the lowest possible aircraft mass. High strength titanium (e.g. Ti6Al4V) or aluminium alloys (e.g. 2024T3) as well as composite laminates (e.g. CFRP) are examples of such materials.

Large thin-walled aircraft structures are reinforced with longerons, spars, stringers, ribs and struts in order to increase their stiffness and improve buckling performance [1]. Moreover, various types of thin-walled panels stiffening are used, e.g., corrugation or repetitive equidistant ribs (grid reinforcement) [7]. Composites reinforced with advanced grid structures have been recently analysed in [3]. Simultaneously, composite structural materials such as fibre-metal laminates and sandwich structures are developed. Buckling of thin-walled circular cylinders was analysed in the 1960s and general design criteria for isotropic, orthotropic, corrugated, waffle-stiffened and sandwich cylinders are presented in [6]. Nowadays, the finite element method is commonly used in engineering practice, however, mathematical models of structural materials are developed as well, e.g., comparative bending and buckling analysis of both corrugated and sandwich beams are presented in [4].

The aim of the paper is to analyse possible benefits of replacing the original ribbed metal casing with a sandwich structure. The sandwich structure in the form of a metal-fibre laminate of titanium alloy faces and a flax fibre laminate core is proposed and analysed. Flax fibre reinforced laminates are taken into account due to their advantages (in general advantages of bast fibre laminates), i.e., biodegradability, a relatively small density and a better damping ability compared to synthetic laminates (e.g. glass or carbon). On the other hand, disadvantages of natural fibre laminates, such as their low durability (due to relatively poor moisture resistance and sensitivity to environmental agents/effects) and variability in their mechanical properties, cannot be neglected [13]. The first stage of the aforementioned analysis is presented in [10]. In the paper, analytical optimisation of a sandwich structure including a polynomial approximation of the critical load with correction obtained based on numerical analysis is presented.

2. Object of analysis

The object of analysis is the F124 turbofan engine casing, which is a load carrying element, i.e., transfers load from the fan and other engine elements. The casing stiffness should be sufficient to reduce its deformations during a steady flight, manoeuvres and in the case of a crash landing. Analysis of load conditions indicates that a dimensioning load case is a crash landing, while an axial component of the inertial load is about 30g [MIL-HDBK]. Therefore, the axial load is considered in this paper.

The F124 engine casing is built of thin metal sheets in order to reduce its weight. The casing thickness δ is about 1 mm, whereas its diameter 2R is relatively large and equal to 670 mm. Therefore, buckling is the most important condition that determines the engine casing strength. In the original structure, the ribbing (isogrid stiffening) is used to increase the casing stiffness and, consequently, to increase its critical load with a minimum mass growth. The casing is made of titanium alloy (Ti6Al4V). It consists of one cylindrical and two conical parts (Fig. 1). The cylindrical part is more flexible for buckling, therefore it is considered in the paper.

a) b)

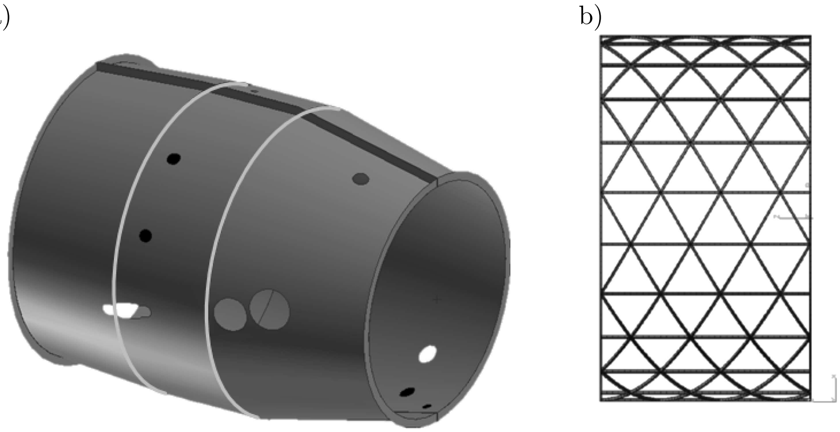

Fig. 1. The F124 turbofan engine casing: a) shell geometry [10], b) ribbing topology of the original structure [11].

Parameters of material mass efficiency are required to perform mass-strength optimization [2]. The advantage of a ribbed structure, compared to a plain structure, is an increased global bending stiffness. In the case of the F124 engine casing, waffle ribbing causes the 26 percent increase in material structure efficiency and a buckling mode is characterised with a large number of waves corresponding to ribbing density [10].

3. Mass-strength analysis

A plain cylindrical shell made of titanium alloy was selected as a reference structure for the sandwich structure mass optimisation. The critical load of a plain metal cylindrical shell is given with a well-known formula (1) [12]. In the case of a sandwich structure (a metal-fibre laminate) presented in Fig. 2, the critical load can be estimated with formula (2) [10]

$$P_{cr} = 2\pi k E_\delta h^2, \tag{1}$$

$$P_{cr} = 2\pi k E_\delta \delta^2 \cdot \alpha\sqrt{(1 + \xi_2\eta)(1 + 3\eta + 3\eta^2 + \xi_1\eta^3)}, \tag{2}$$

where E_δ – Young Modulus of metal alloy and face sheet material, E_i – Young Modulus of a laminate core in i direction (1 – axial direction, 2 – hoop direction), k – coefficient dependent on boundary conditions, h – metal shell thickness, δ – overall thickness of face sheets, H – laminate core thickness, $\eta = H/\delta$, $\xi_i = E_i/E_\delta$, $\alpha \le 1$ – coefficient dependent on a laminate configuration and its shear Modulus (for isotropic material $\alpha = 1$).

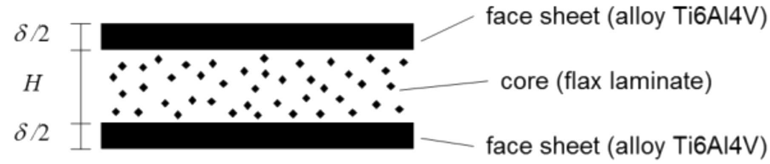

Fig. 2. Sandwich structure, thickness notation [10].

Mass efficiency of materials used in the aircraft industry is studied in [2]. A sandwich to the plain structure mass ratio N, with an assumed fixed value of the critical load (mass efficiency reciprocal), is given with formula (3)

$$N = \frac{m_{\text{sandwich}}}{m_{\text{plain}}} = \frac{(1 + \lambda\eta)}{\sqrt[4]{\alpha^2 (1 + \xi_2\eta)(1 + 3\eta + 3\eta^2 + \xi_1\eta^3)}}, \tag{3}$$

where ρ_δ and ρ_H – density of titanium alloy and flax laminate, respectively, $\lambda = \rho_H/\rho_\delta$.

Analytical and numerical analyses were performed for three material models of cylindrical shell subjected to axial compression:

- model I – a waffle ribbed (stiffened) cylindrical shell made of Ti6Al4V titanium alloy (basic model) with the following rib dimensions: height – 5 mm, width – 1.254 mm and length – 35.08 mm [10],
- model II – a plain cylindrical shell made of Ti6Al4V titanium alloy (reference model in comparative analysis),
- model III – a cylindrical shell built of a sandwich structure of titanium alloy face sheets and a flax fibre (unidirectional laminate) core. A face to core to face thickness ratio was initially assumed as 1:4:1 [10] and it is optimised in the paper.

The algorithm of analysis based on [10] was as follows:

1) Evaluation of the critical load P_{cr} for a basic model (model I).
2) Determination of the thickness δ_i for each structure (model II, III) for the assumed value of critical load P_{cr}.
3) Calculation of a mass ratio for each structure (model I, II, III).
4) Comparison of the obtained results.
5) Sandwich structure optimisation (model III).

Numerical models of a cylindrical shell were subjected to axial compression. Both ends of a shell were fixed: 6 and 5 (without axial) degrees of freedom were fixed at the bottom and the top end, respectively.

Thin-walled structure stability is mostly influenced by such imperfections as material, geometrical and technological (manufacturing) [9]. In the paper, geometrical imperfections, i.e., a global imperfection corresponding to boundary conditions at the ends of a cylindrical shell and a local imperfection (local variation of a shell surface), are considered. A local variation of a shell mid-surface was obtained by means of shifting the selected nodes (placed on area of about 14×14 mm^2) in the radial direction by 1 mm (which is equal to the thickness of the ribbed casing cylindrical part). This imperfection was located in the middle of the cylindrical shell length/height [10].

The titanium alloy was defined using a bilinear elasto-plastic material model with Young Modulus 114 GPa, Poisson coefficient 0.34, Yield stress 950 MPa, Ultimate stress 1050 MPa and density 4500 kg/m^3. Flax lamina was defined using an orthotropic material model with Young Modulus E_{11} 49500 MPa, E_{22} is 8000 MPa, shear Modulus G_{12} is 3000 MPa, the Poisson coefficient 0.32 and density 1330 kg/m^3. Ultimate stress for a quasi-isotropic flax laminate is 150 MPa [8, 13]. The metal-fibre laminate (model III) was defined using a composite material model [5].

Numerical models of cylindrical shells were developed using 4-node shell finite elements (Quad4). A plain cylinder model (cylindrical part) was built from 36000 finite elements and the ribbing model from 7800 elements.

Nonlinear analysis using the Newton-Raphson method was performed with the Marc code. The critical load estimated based on nonlinear analysis of a ribbed cylindrical shell with local imperfection is 1306 kN [10].

4. Results and discusion

In order to find the mass ratio minimum for the initially assumed $\alpha = 1$, formula (3) was differentiated with respect to η, compared to zero and, afterwards, transformed and simplified to the third order polynomial (4) with the assumption that only positive solutions ($\eta > 1$) are sought

$$e_3\eta^3 + e_2\eta^2 + e_1\eta + e_0 = 0, \tag{4}$$

where

$$e_3 = \lambda\left(\xi_1 + 3\xi_2\right) - 4\xi_1\xi_2,$$

$$e_2 = 6\lambda\left(1 + \xi_2\right) - 3\left(\xi_1 + 3\xi_2\right),$$

$$e_1 = 3\lambda\left(3 + \xi_2\right) - 6\left(1 + \xi_2\right),$$

$$e_0 = 4\lambda - \left(3 + \xi_2\right).$$

In general, coefficients of this polynomial depend on parameters λ, ξ_1 and ξ_2. However, for selected material properties of titanium alloy and flax laminate λ is equal to 0.2955, ξ_1 – 0.4342 and ξ_2 – 0.07. Equation (5) was obtained by substituting those values into (4)

$$\eta^3 - 0.515956\eta^2 - 53.7681\eta - 27.4478 = 0. \tag{5}$$

The initial estimation of the function N (for $\alpha = 1$) is presented in Fig. 3. The minimum of N (equal to 0.657) is reached for the argument $\eta = 7.83$ (marked as a circle) obtained as a positive solution of equation (5).

In general, the coefficient α depends on a core to faces thickness ratio, i.e. $\alpha = \alpha(\eta)$. The influence of a core to an overall sandwich thickness ratio $\eta/(1+\eta)$ on the parameter α is presented in Fig. 4a. This graph was obtained from (3) and numerical calculation of the critical load for a cylindrical shell built of a sandwich structure.

Based on Fig. 4a, it can be concluded that if a core to sandwich thickness ratio is less than 0.6, the coefficient α is, in fact, equal to unity. However, if $\eta = 7.83$ ($\eta/(1+\eta) = 0.887$) then $\alpha \approx 0.87$ and it is lower than 1, as it is shown in Fig. 4a. Therefore, in order to improve an optimal solution, an appropriate function $\alpha(\eta)$ is required. The function $\alpha(\eta)$ was approximated with the second order

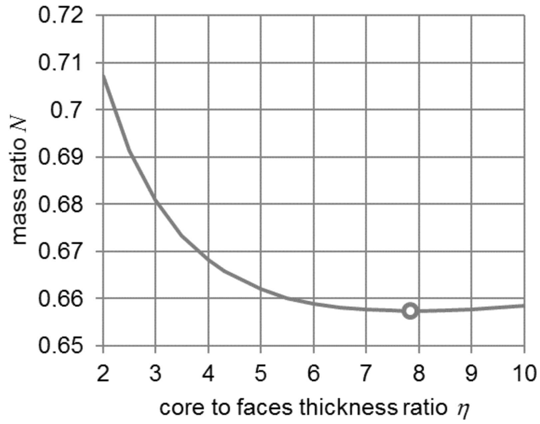

Fig. 3. Mass ratio as a function of core to faces thickness ratio (initial estimation for $\alpha = 1$).

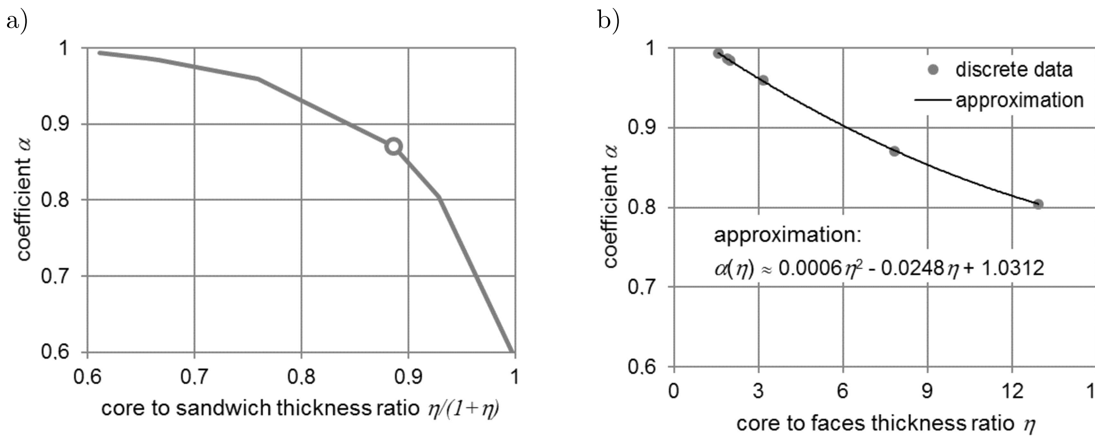

Fig. 4. The influence of a core thickness ratio on the critical load of a metal-fibre cylindrical shell: a) core to sandwich ratio $\eta/(1+\eta)$, b) core to faces ratio η.

polynomial based on the discrete data obtained from numerical calculations (Fig. 4b). Formula (6) is obtained from (3) by substituting α with a general form of the second order polynomial

$$N = \frac{m_{\text{sandwich}}}{m_{\text{plain}}} = \frac{(1+\lambda\eta)}{\sqrt[4]{(c+b\eta+a\eta^2)^2\,(1+\xi_2\eta)\,(1+3\eta+3\eta^2+\xi_1\eta^3)}}. \tag{6}$$

Coefficients of a polynomial $\alpha(\eta)$ for the analysed material are as follows: $c = 1.0312$, $b = -0.0248$, $a = 0.0006$ (Fig. 4b). In this case, a linear approximation of $\alpha(\eta)$ would be sufficient since a is close to zero. However, if a sequence of arbitrary assumptions and transformations (including numerical calculations) are necessary to solve the problem, it is more perspective to begin from analysis of a more general problem and round the final solution only.

Taking into account the aforementioned assumptions, the minimum of a mass ratio N was calculated anew. The derivative of (6) with respect to η was compared to zero and the obtained equation was simplified to the sixth order polynomial (7), similarly as in the case explained above

$$f_6\eta^6 + f_5\eta^5 + f_4\eta^4 + f_3\eta^3 + f_2\eta^2 + f_1\eta + f_0 = 0, \tag{7}$$

where

$$f_6 = -4\lambda a\xi_1\xi_2,$$

$$f_5 = -3\lambda a\,(\xi_1 + 3\xi_2) - 2\,(\lambda b + 4a)\,\xi_1\xi_2,$$

$$f_4 = -6\lambda a\,(1 + \xi_2) - (\lambda b + 7a)\,(\xi_1 + 3\xi_2) - 6b\xi_1\xi_2,$$

$$f_3 = ce_3 - \lambda a\,(3 + \xi_2) - 18a\,(1 + \xi_2) - 5b\,(\xi_1 + 3\xi_2),$$

$$f_2 = ce_2 + (\lambda b - 5a)\,(3 + \xi_2) - 12b\,(1 + \xi_2),$$

$$f_1 = ce_1 - 3b\,(3 + \xi_2) + 2\lambda b - 4a,$$

$$f_0 = ce_0 - 2b.$$

The function N reaches its minimum for the argument η obtained as the first positive solution of Eq. (8)

$$\eta^6 + 1.99775\eta^5 - 250.502\eta^4 - 6435,12\eta^3 - 11603.9\eta^2 + 167126.6\eta + 88019.87 = 0. \tag{8}$$

The results of the aforementioned mass optimisation are presented in Fig. 5. The consideration of the function $\alpha(\eta)$ caused a significant change in an optimal solution, i.e., the mass ratio N extremum equal to 0.688 was obtained for the argument $\eta = 4.31$ (marked as a diamond in Fig. 5). A slight (below 5%) increase in the mass ratio minimum, compared to the results of initial analysis with $\alpha = 1$ (marked as a circle), was obtained for a significantly (about 45%) lower core to faces thickness ratio.

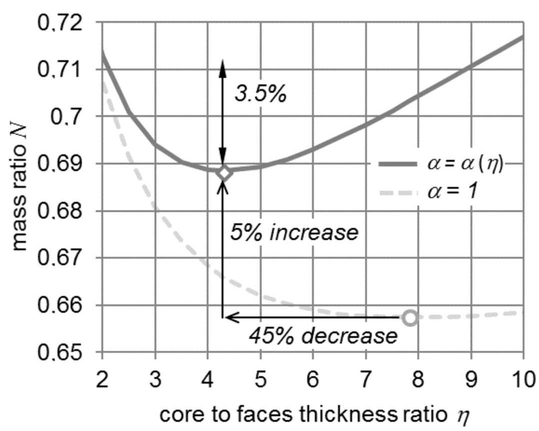

Fig. 5. Mass ratio as a function of core to faces thickness ratio.

5. Summary and conclusions

Three material models of a thin-walled cylindrical shell are analysed in the paper.

In the case of a ribbed structure (model I), the ribbing causes a significant increase in material structure efficiency due to an increase in a global bending stiffness. Additionally, local imperfection occurring within one segment of a ribbed structure causes buckling within this segment only and does not significantly influence the global buckling [10]. This feature is an undoubted advantage of a ribbed structure.

The aim of the paper is to study the possible benefits of replacing the original ribbed metal casing with a sandwich structure. In the case of a sandwich structure, a shift of titanium layers from a neutral surface of the shell is significant since it creates a possibility of a decrease in an overall titanium thickness compared to a reference plain structure (without decreasing its bending stiffness and critical load).

The best mass efficiency was obtained for a sandwich structure (built of titanium alloy faces and a flax fibre core) of a core to faces thickness ratio 4.31, which means that a shift of a titanium layer from a neutral surface of the shell is 4.31 times as large as its thickness. In this case, the shell overall thickness increases about 60% compared to a plain shell, however, at the same time the 70 percent decrease in titanium layers thickness can be obtained (see Table 1).

The conclusion, which can be useful in engineering practice, concerns the change in a mass ratio – in the scope of η from 2 (initial "rational" solution [10]) to 4.31 (optimal solution) only a slight (about 3.5%) decrease in a mass ratio is observed (Fig. 5). The mass efficiency (the mass ratio reciprocal) of a sandwich structure with a face to core to face thickness ratio equals to 1:4:1 ($\eta = 2$) is about 40% higher compared to a plain structure. The mass efficiency increases to 45% for a thickness ratio about 1:8:1 ($\eta = 4.31$) and it is an optimal configuration. Whereas, the mass efficiency of a waffle ribbed cylindrical shell (original solution) is only 26% higher in comparison to a plain structure (analytical and numerical analysis of a cylindrical shell made of a ribbed structure is presented in [10]). A comparison of mass efficiency and shell thicknesses for the assumed critical load is presented in Table 1. The sandwich structure thickness increases with an increase in the thickness ratio η. However, the overall sandwich thickness for the optimal solution is 2.76 mm, whereas, the ribbed structure overall thickness (including a rib height) is 6 mm.

Table 1. Comparison of a shell thickness for the assumed value of the critical load.

Structure	Thickness ratio		Thickness [mm]			Mass efficiency
	ratio	η	overall	face/shell	core/rib	1/N
Ribbed [10]	–	–	1.357*	1.000	5.000	1.26
Plain	–	–	1.717	1.717	–	1.00
Sandwich	1:4:1	2.00	2.308	0.385	1.538	1.40
	~ 1 : 8 : 1	4.31	2.760	0.260	2.240	1.45

* effective thickness δ_{eff} of a ribbed model [10].

In the paper, a homogenised composite material model is used and elastic buckling of cylindrical shells is analysed. In the case of a sandwich structure, material and technological imperfections as well as a material model including delamination in a metal-fibre laminate should be taken into account. This is a direction for further analysis.

Acknowledgments

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