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Parametric Study on the Connection Between Poisson's Ratio, GSI and Environmental Stress

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According to its definition, Poisson's ratio is a measure of the Poisson effect, the phenomenon in which a material tends to expand in directions perpendicular to the direction of compression or, in other words, Poisson's ratio is the ratio of relative contraction to relative expansion. For the sake of simplicity, it is handled as a constant. However, in reality, it is a variable. One can find several studies on this topic. This paper aims to study the impact of the environmental (confining) pressure and rock quality (namely the geological strength index, GSI) of rock mass on Poisson's ratio. By using parametric investigation, several cases were calculated to present the functional dependency between the quantities mentioned above.

Keywords: Poisson's ratio, geological strength index (GSI), stress dependence, variable environmental stress.

1. Introduction

Poisson's ratio is a measure of the Poisson effect [1]. According to the original definition of the Poisson's ratio, one can define it by use of the deformations of a differential volume element loaded with uniaxial stress. Specifically, it is the ratio between the transverse deformation and the deformation in the direction of stress. This is a dimensionless number and can be defined by the Lame's theory as well, namely in case of isotropic material it is proportional by the Young's and the bulk modulus. For the sake of simplicity, it is handled as a constant; however, in reality, it is a variable. One of the first researches of the variable Poisson's ratio was Nadai's [2] work in 1963. The value of the Poisson's ratio can be in principle between $+0.5$ and -1. It is noted that this is easily verify by the Lame's definition. Approximately, it is between $+0.1$ and $+0.3$ for most civil engineering materials. (A negative Poisson's ratio means that when a body of such a material is pulled in some direction, it does not contract transversely, but expands. When compressed, its size also decreases transversely. By creating a special microstructure, such materials can actually be created.) As it was noted earlier, the Poisson's ratio of a stable, isotropic, linear elastic material must be between -1.0 and $+0.5$ because of the positivity of Young's modulus, the shear modulus and bulk modulus. According to the Lame's equation, the Poisson's ratio is 0.5 minus the ratio of the Young's modulus and twice of the bulk modulus. Lógó and Vásárhelyi [3] investigated the influence of the intact rock rigidity on the Poisson's ratio. Vásárhelyi [4] presented a linear equation in which Poisson's rate was increasing as the quality of the rock mass was decreasing.

The Poisson's ratio of natural materials covers the entire range, as in the case of rubber it is nearly 0.5 (i.e., it is capable of large lateral deformation), while in the case of cork it is close to 0, showing very little lateral expansion when compressed. Poisson's ratio of rock is usually between 0.1–0.4 [5].

Poisson's ratio as a material constant, even though it is found in most formulas in rock mechanics, has not been studied frequently. According to Bieniawski [6], Poisson's ratio of rocks is constant under linear elastic deformation but begins to increase due to the appearance of new micro-cracks or expansion of existing ones. Years later, Kumar [7] investigated the effect of the Poisson's ratio on the intact rock properties.

The role of the Poisson's ratio, its fields of application, and methods of calculation are briefly presented in this article. Then, depending on the changes in the GSI and the environmental pressure, we examine the differences in the Poisson's ratio. The purpose of this article is to study the impact of the environmental pressure and the GSI of rock mass on Poisson's ratio.

2. Theoretical background

The Poisson's ratio (ν) is a unit of measure defined as the ratio of transverse deformation to longitudinal deformation. However, this value can be calculated in many ways, not just as a ratio of the deformations. One such calculation method is when the internal friction angle is used to obtain the Poisson's ratio. This method of obtaining the internal friction angle of each rock requires knowledge of the Hoek–Brown failure criterion.

The Hoek–Brown failure criterion [8] was used in our calculations. This theory was developed to model the fracture boundary state of intact rock and rock mass. The theory itself, especially in tunnel construction, is used often for calculating the stress state of the rock environment and for modeling the boundary curve of brittle rocks. This method, as proved by practice, can be modeled better than the Mohr–Coulomb theory, and even within certain limits, the internal friction angle and cohesion of the rock mass can be determined [8, 18]. The formula for the fracture interface, in the case of intact rock, looks like this:

$$\sigma_1' = \sigma_3' + \sigma_{ci} \left(m_i \frac{\sigma_3'}{\sigma_{ci}} + 1 \right)^{0.5}, \tag{1}$$

where σ_1', σ_3' are the major and minor effective principal stresses at failure where σ_1 is the biggest principal stress in compression (the paper follows the principles applied in engineering geology), σ_3' is the environmental stress, σ_{ci} is the uniaxial compressive strength, and m_i is the Hoek–Brown constant for intact rock.

This formula was later transformed into a general case so that it could not only be applied to the intact rock. The transformed equation is as follows:

$$\sigma_1' = \sigma_3' + \sigma_{ci} \left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s \right)^{a}, \tag{2}$$

where m_b is a reduced value of the material constant m_i and it is given by:

$$m_b = m_i \exp\left(\frac{\mathrm{GSI} - 100}{28 - 14D} \right). \tag{3}$$

Here s and a are constants for the rock mass given by the following relationships:

$$s = \exp\left(\frac{\mathrm{GSI} - 100}{9 - 3D} \right), \tag{4}$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{-\mathrm{GSI}/15} - e^{-20/3} \right). \tag{5}$$

D is a factor that depends upon the degree of disturbance to which the rock mass has been subjected by blast damage and stress relaxation. It varies from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses [8].

The disadvantage of the method is that the Hoek–Brown constant can only be determined by a large number of triaxial tests. In practice, this is not possible in most cases, and there may be significant discrepancies between the results of each measurement.

The so-called fracture limit state has been developed for modeling the fracture boundary states of rock blocks and rock mass. The Hoek–Brown failure criterion is very widespread in practice, especially in tunnel construction, for modeling the boundary curve of brittle rocks and calculating the stress state of the rock environment. The empirical theory has been proved by practice because it is better to model the real state than the traditional Mohr–Coulomb theory. The Hoek–Brown fracture state also allows, within certain limits, to calculate the friction angle and cohesion of the rock mass.

The Hoek–Brown constants (m_i) of the intact rocks are usually well-known. These values for the most important rock types are collected in Table 1 (using the published data of Hoek [9]). According to this table, the minimal value of m_i is 2 (e.g., claystone) and the maximum value is 35 for some granitic rocks.

TABLE 1. Values of m_i for intact rock group [9].

Texture			
Coarse	Medium	Fine	Very fine
Sedimentary rock types			
Conglomerates (21 ± 3)	Sandstone (17 ± 4)	Siltstone (7 ± 2)	Claystone (4 ± 2)
Breccia (19 ± 5)		Greywacke (18 ± 3)	Shales (6 ± 2)
Crystalline limestone (12 ± 3)	Sparitic limestone (10 ± 2)	Micritic limestone (9 ± 2)	Dolomites (9 ± 3)
			Chalk (7 ± 2)
Metamorphic			
Marble (9 ± 3)	Hornfels (19 ± 4)	Quartzite (20 ± 3)	
	Metasandstone (19 ± 3)		
Migmatite (29 ± 3)	Amphibolite (26 ± 6)		
Gneiss (28 ± 5)	Schist (12 ± 3)	Phyllite (7 ± 3)	Slate (7 ± 4)
Igneous			
Granite (32 ± 3)	Diorite (25 ± 5)		
Granodiorite (29 ± 3)			
Gabbro (29 ± 3)	Dolerite (16 ± 5)		
Norite (20 ± 5)			
Porohyrite (20 ± 5)		Diabase (15 ± 5)	Peridotite (25 ± 5)
	Rhyolite (25 ± 5)	Dacite (25 ± 3)	Obsidian (19 ± 3)
	Andesite (25 ± 5)	Basalt (25 ± 5)	
Agglomerate (19 ± 3)	Breccia (19 ± 5)	Tuff (13 ± 5)	

However, this constant depends on many things, such as water content [10] or heat cycles [11].

If it is not possible to perform a large number of triaxial tests, then the constant value of m_i can be determined by both uniaxial compressive strength (σ_c) and tensile strength (σ_t):

$$m_i = \left(\left(\frac{\sigma_t}{\sigma_c}\right)^2 - 1\right)\left(\frac{\sigma_c}{\sigma_t}\right). \tag{6}$$

In such a case, if this ratio is quite small, one can determine the m_i constant with a relatively high error rate. According to the theory of Cai [12], if $m_i > 8$, the following equation can also be used (the estimated error is plotted in Fig. 1, according to [12]):

$$m_i = \frac{\sigma_c}{\sigma_t}. \tag{7}$$

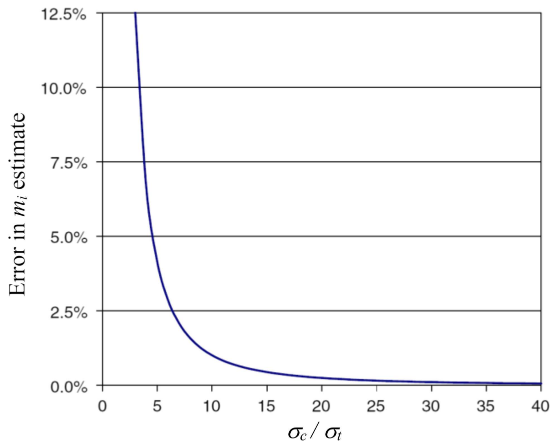

Fig. 1. Relationship between error in m_i (the Hoek–Brown material constant) estimate and the strength ratio R, according to [12].

As can be seen from Eqs (2)–(5), the GSI plays an important role in the Hoek–Brown failure criterion being one of the most significant failure-criteria for rock formations. It enables in a fast and easy way, together with the Hoek–Brown failure criterion, to obtain indirect quantifiable data that can be used to infer internal friction angle, cohesion, unidirectional compressive strength or deformation modulus, for example. The method was originally developed for high-strength homogeneous rocks but soon extended to heterogeneous rocks. The GSI can be used to indicate the structure of rock and the state of the subdivision by specifying a single index (Fig. 2) [13].

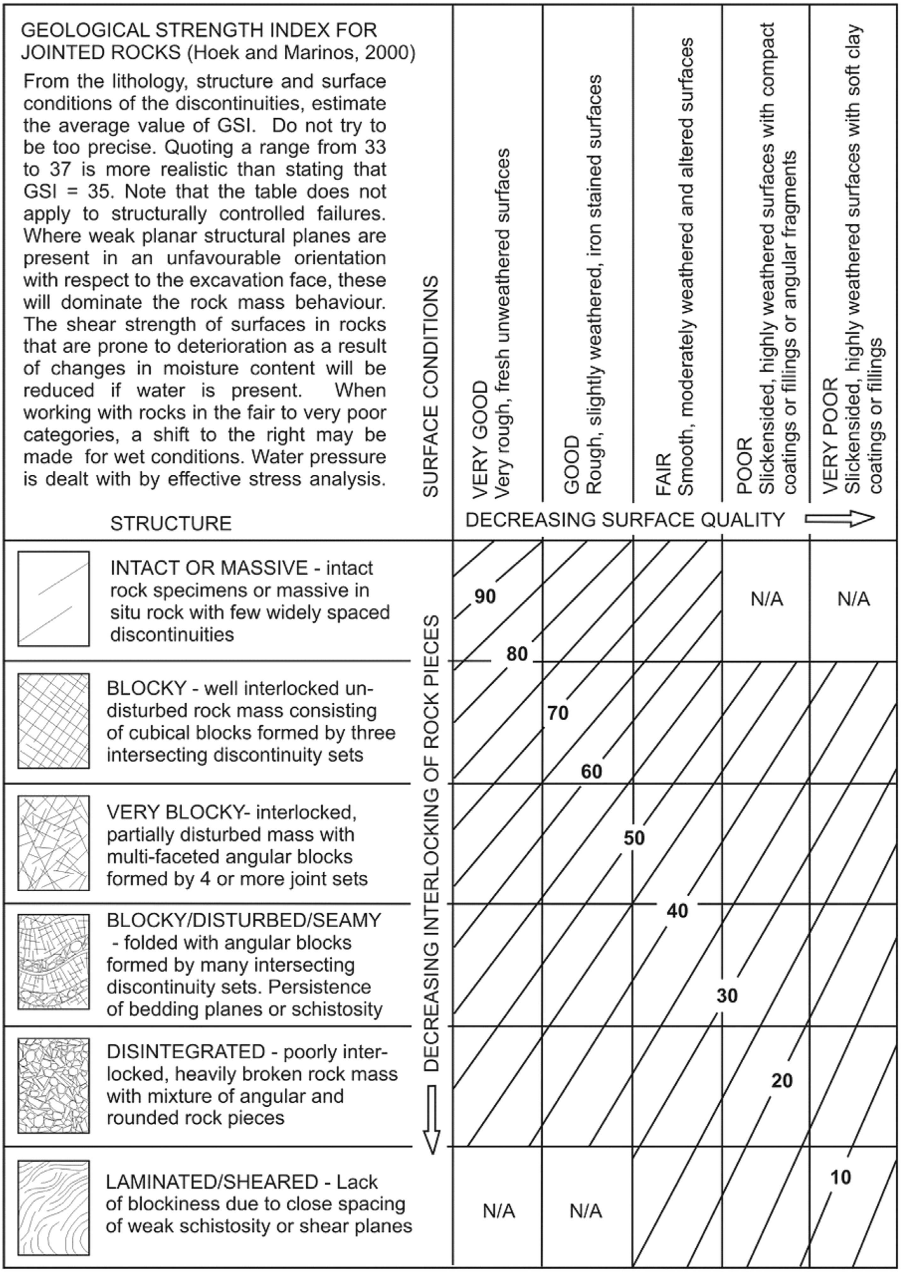

Fig. 2. The GSI for jointed rocks [8].

3. Fundamental relationships

Poisson's ratio is determined in several ways during the calculations. These equations establish the relationship between the internal friction angle and Poisson's ratio [14]. It is important to mention Jaky's [15] and Terzhagi's [16] equations,

$$K_0 = \frac{P_h}{P_v} = 1 - \sin\varphi, \tag{8}$$

$$K_0 = \frac{P_h}{P_v} = \frac{\nu}{1-\nu}, \tag{9}$$

since these equations serve as the basis for Eq. (10). Because Eqs (8) and (9) are used for the same quantity, one can define the following formulation [5]:

$$\nu = \frac{1-\sin\varphi}{2 - sin\varphi}. \tag{10}$$

Zhang *et al.* [14] summarized the most important relationships between the internal friction angle (φ) and Poisson's rate (ν) of the intact solid material. All of these equations are based on the Mohr–Coulomb theory, and they use different equilibrium methods presented by Stagg and Zienkiewicz [17]. The following correlations were collected:

$$\nu = \frac{1}{2}(1-\sin\varphi), \tag{11}$$

$$\nu = \frac{\arctan[\cos\varphi - (1-\sin\varphi)\tan\varphi]}{90^\circ}, \tag{12}$$

$$\nu = \frac{\cos\varphi - (1-\sin\varphi)\tan\varphi}{2}, \tag{13}$$

$$\nu = \frac{\tan(45^\circ - \frac{\varphi}{2})}{2}, \tag{14}$$

$$\nu = \frac{\tan(45^\circ - \frac{\varphi}{2})}{1+\tan(45^\circ - \frac{\varphi}{2})}. \tag{15}$$

The relationships between major (σ_1) and minor (σ_3) principal stresses for Hoek–Brown and equivalent Mohr–Coulomb criteria can be determined [8, 18]. The fitting principal (process) involves balancing the areas above and below the Mohr–Coulomb plot. By taking into account the Mohr–Coulomb failure criterion, the relation [9] between the major principal stress σ_1, the environmental (minor principal) stress σ_3 and the friction angle can be written as follows:

$$\sigma_1' = \frac{2c'\cos\varphi'}{1-\sin\varphi'} + \frac{1+\sin\varphi'}{1-\sin\varphi'}\sigma_3'. \tag{16}$$

By the use of the formulation (10)–(15) and (16), one can create graphs for the variation of Poisson's ratio in function of the environmental (minor principal) stress σ_3.

Although some equations were calculated using the same baseline data, the results show significant variation.

4. Parametric study

The results of each equation have been compared in several ways. One of the aims of the study was to determine whether there is any relationship between the Poisson's ratio and the magnitude of the ambient pressure, and how the change in the GSI affects the Poisson's ratio.

Figure 3 shows Poisson's ratio values for intact rock as a function of σ_3. It can be observed that the results given by each equation, although different in value, behave the same. It can be seen that although there is no constraint in Eq. (15), it cannot be used in all ranges as it does not give a relevant result.

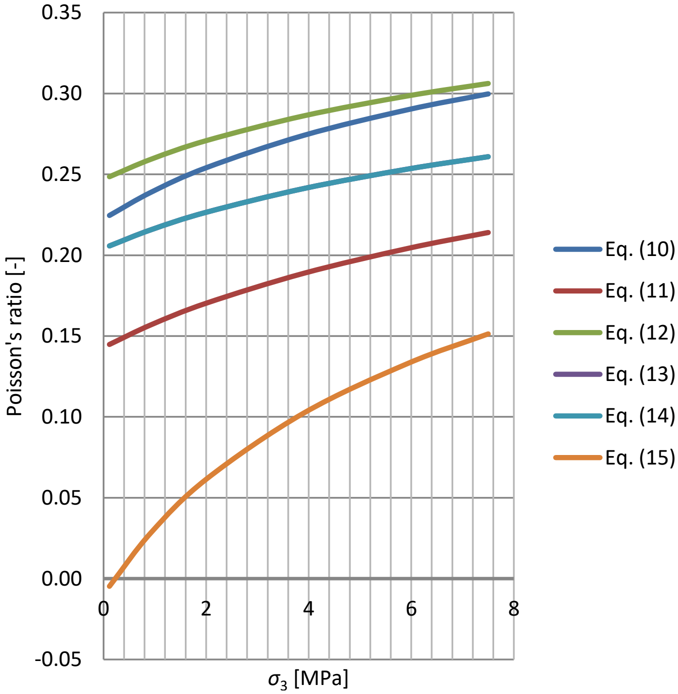

Fig. 3. The results for the intact rock (100 GSI).

This trend is also shown in Fig. 4. Here, too, behavior similar to that of intact rock can be observed, whereby the Poisson's ratio's value changes suddenly and rapidly as the environmental pressure increases, then slows down but does not disappear. In this case, too, it can be observed that Eq. (15) gives meaningful results only within certain limits.

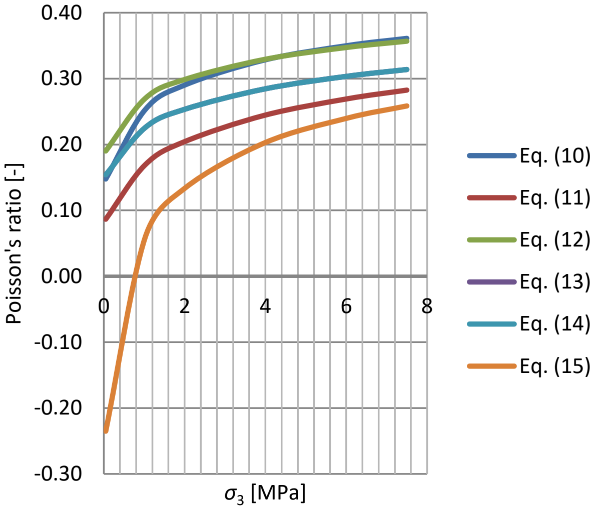

Fig. 4. The results for 60 GSI.

In the case of fragmented rocks, the results show similar behavior, as shown in Fig. 5.

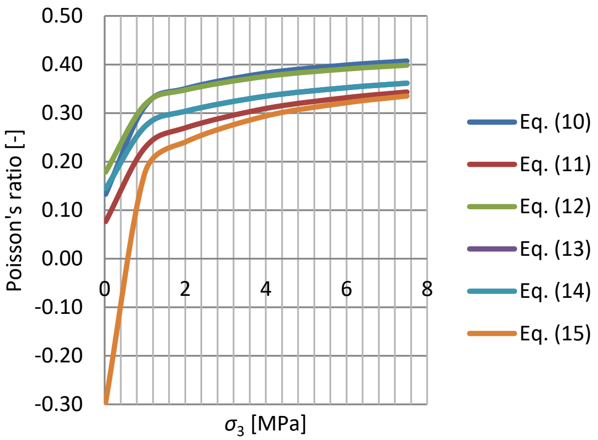

Fig. 5. The results for 30 GSI.

The following figures (Figs 6–11) show that the value of the Poisson coefficient is not only dependent on the magnitude of the environmental pressure but is significantly influenced by the fragmentation of the rock. It can be seen that the higher the GSI value of a given sample, the higher the value of the Poisson's ratio will be. In Figs 6–11, the red dots represent the values calculated using each equation. A surface fitted with these results can give a good approximation of the areas between the calculated results.

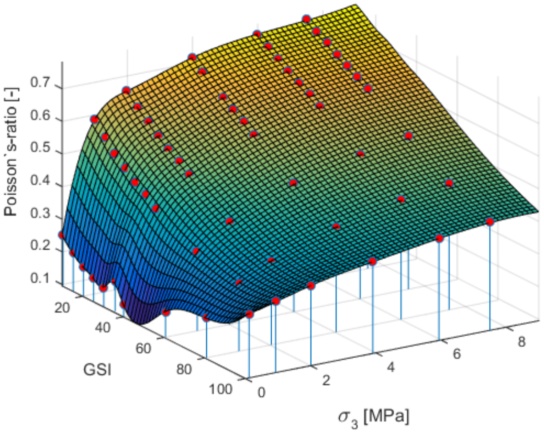

FIG. 6. The results from Eq. (10).

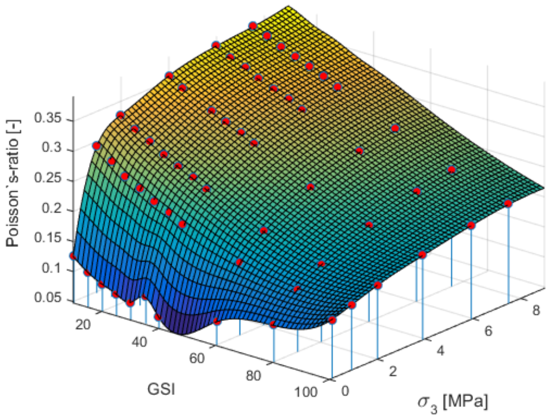

FIG. 7. The results from Eq. (11).

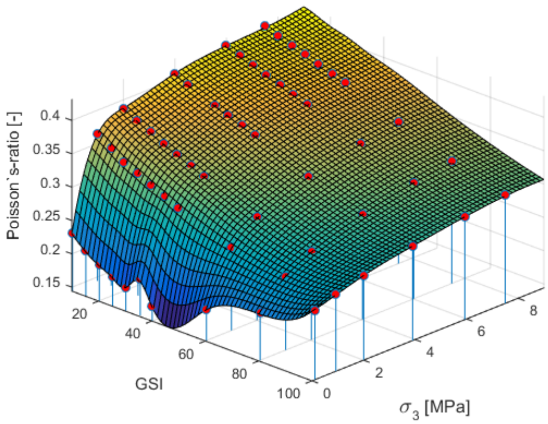

FIG. 8. The results from Eq. (12).

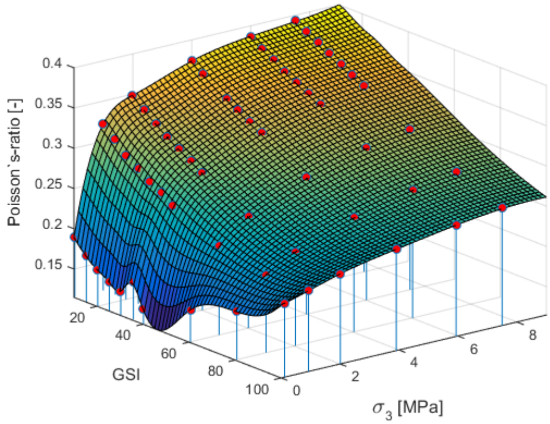

FIG. 9. The results from Eq. (13).

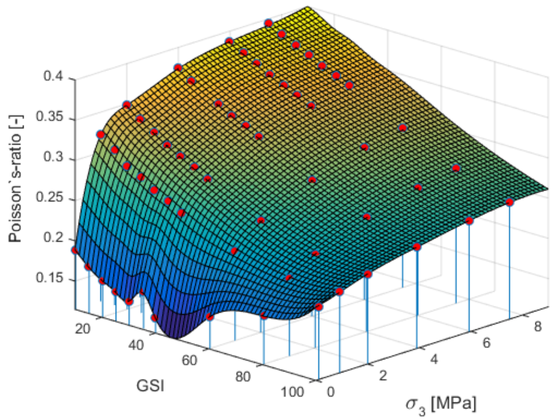

FIG. 10. The results from Eq. (14).

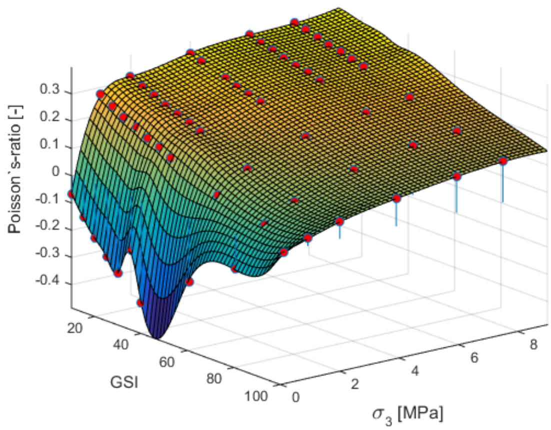

FIG. 11. The results from Eq. (15).

5. Conclusion

On the basis of the results presented above, an interesting correlation can be found between changes in the value of the Poisson's ratio so far considered constant based on the environmental (confining) pressure and the rock mass quality (namely GSI) value. It is clear that decreasing the GSI value increases the value of the Poisson's ratio. If the value of the environmental stress (σ_3) increases, the Poisson's ratio also increases.

References

1. S.D. Poisson, *Mémoire sur l'équilibre et le mouvement des corps élastiques*, Mém. de l'Acad. Paris, p. 8, 1829.
2. A. Nadai, *Theory of Flow and fracture of solids*, McGraw-Hill Book Company, New York, Toronto, London, 1963.
3. B.A. Lógó, B. Vásárhelyi, Estimation of the Poisson's rate of the intact rock in the function of the rigidity, *Periodica Polytechnica Civil Engineering*, **63**(4): 1030–1037, 2019, doi: 10.3311/PPci.14946.
4. B. Vásárhelyi, A possible method for estimating the Poisson's rate values of the rock masses, *Acta Geodaetica et Geophisica Hungarica*, **44**(3): 313–322, 2009, doi: 10.1556/AGeod.44.2009.3.4.
5. H. Gercek, Poisson's ratio values for rocks, *International Journal of Rock Mechanics and Mining Sciences*, **44**(1): 1–13, 2007, doi: 10.1016/j.ijrmms.2006.04.011.
6. Z.T. Bieniawski, Mechanism of brittle fracture of rock. Part I: Theory of fracture process. Part II: Experimental studies. Part III: Fracture in tension and under long term loading, *International Journal of Rock Mechanics and Mining Sciences*, **4**(4): 395–404, 1967.
7. J. Kumar, The effect of Poisson's ratio on rock properties, SPE-6094-MS, SPE Annual Fall Technical Conference and Exhibition, 3–6 October, New Orleans, pp. 1–12, 1976, doi: 10.2118/6094-MS.
8. E. Hoek, E.T. Brown, The Hoek–Brown failure criterion and GSI – 2018 edition, *Journal of Rock Mechanics and Geotechnical Engineering*, **11**(3): 445–463, 2019, doi: 10.1016/j.jrmge.2018.08.001.
9. E. Hoek, *Practical Rock Engineering*, http://www.rocscience.com 2007.
10. B. Vásárhelyi, M. Davarpanah, Influence of water content on the mechanical parameters of the intact rock and rock mass, *Periodica Polytechnica Civil Engineering*, **62**(4): 1050–1066, 2018, doi: 10.3311/PPci.12173.
11. Á. Török, B. Vásárhelyi, Rigidity of sandstone at elevated temperatures, [in:] R. Ulusay, O. Aydan, H. Gerçek, M.A. Hindistan, E. Tuncay [Eds], *Rock Mechanics and Rock Engineering: From the Past to the Future: Proceedings of the 2016 ISRM International Symposium*, EUROCK 2016, Cappadocia, Turkey, 29–31 August 2016, CRC Press/Belkema, Leiden, Netherlands, pp. 345–348, 2016.
12. M. Cai, Practical estimates of tensile strength and Hoek–Brown strength parameter mi of brittle rocks, *Rock Mechanics and Rock Engineering*, **43**: 167–184, 2010, doi: 10.1007/s00603-009-0053-1.

13. B. Vásárhelyi, G. Somodi, Á. Krupa, L. Kovács, Determining the geological strength index (GSI) using different methods, [in:] R. Ulusay, O. Aydan, H. Gerçek, M.A. Hindistan, E. Tuncay [Eds], *Rock Mechanics and Rock Engineering: From the Past to the Future: Proceedings of the 2016 ISRM International Symposium*, EUROCK 2016, Cappadocia, Turkey, 29–31 August 2016, CRC Press/Belkema, Leiden, Netherlands, pp. 1049–1054, 2016, doi: 10.1201/9781315388502-183.

14. N. Zhang, Z. Sheng, X. Li, S. Li, J. He, Study of relationship between Poisson's ratio and angle of internal friction for rocks [in Chinese], *Chinese Journal of Rock Mechanics and Engineering*, **30**(Supl. 1): 2599–2609, 2011.

15. J. Jaky, The coefficient of earth pressure at rest [in Hungarian: A nyugalmi nyomas tényezője], *Magyar Mérnök és Építész-Egylet Közlönye*, 355–358, 1944.

16. K. Terzaghi, F.E. Richart, Jr, Stresses in rock about cavities, *Geotechnique*, **3**(2): 57–90, Harvard Soil Mechanics Series, 41, 1952.

17. K.G. Stagg, O.C. Zienkiewicz, *Rock mechanics in engineering practice*, Wiley, 1968.

18. E. Hoek, C. Carranza-Torres, C. Brent, Hoek–Brown failure criterion – 2002 Edition, [in:] *Proceedings of the NARMS-TAC Conference*, Toronto, Vol. 1, pp. 267–273, 2002.

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