

A Text-Mining-Based Approach for Conducting Literature Review of Selected Meshfree Methods

SINDHUSUTA, Sheng-Wei CHI*, Sybil DERRIBLE

*Department of Civil and Materials Engineering
University of Illinois*

Chicago, USA; e-mail: sindh2@uic.edu, derrible@uic.edu

*Corresponding Author e-mail: swchi@uic.edu

The goal of this study is to review the literature in the field of *meshfree methods* using text mining. For this study, the abstracts of around 17 330 relevant articles published from 1990 to 2020 were collected from Scopus. Text mining techniques such as the latent Dirichlet allocation (LDA), along with the calculation of term frequencies and co-occurrence coefficients were used to analyze the text. The study identified a few key topics in the field of meshfree methods and helped to see the evolution of the field over the past three decades. Furthermore, the trend in the number of publications and frequency map highlighted research trends and lack of focus in certain areas. The co-author network visualization provided interesting insights about collaboration between different researchers around the world. Overall, this study facilitates a systematic literature review in the field of meshfree methods and provides a broader perspective of the field to the research community.

Keywords: meshfree methods, text-mining, latent Dirichlet allocation, topic modeling, literature review.

1. INTRODUCTION

Meshfree methods have emerged as a successful computational method in the past 30 years [1–9]. They are the numerical methods for solving partial differential equations (PDEs) that govern various physical systems in which the system’s approximation is established without using a predefined structured mesh. In contrast to conventional mesh-based methods, the entire problem domain, including its boundary, is represented by a set of scattered nodes within the domain without specific connectivity among them [9]. The motivations behind developing this new class of numerical methods were to eliminate some of the major drawbacks or issues of mesh-based numerical methods such as the finite element method (FEM) [10] and finite difference method (FDM) [11]. These methods are ineffective in dealing with problems where the domain has signifi-

cantly large deformation causing mesh entanglements. This type of issue leads to a need for remeshing during simulation, which is not only time-consuming but also introduces data transport errors. Meshfree methods, by removing the strong tie between the quality of mesh and numerical accuracy, alleviate such issues. During their nearly three-decade long development, meshfree methods have found niche applications in problems such as:

- simulation of a problem domain undergoing large deformation that can cause mesh entanglement [5, 12–15],
- simulation of problems involving the creation of mesh of very complex 3D geometry, which is difficult as well as time-consuming [16, 17],
- simulation of fracture and crack growth problems [4, 15, 18, 19],
- penetration and impact problems [20–23],
- problems with strain localization and damage of solids [24–27],
- thin plate and shell problems [28–32].

Meshfree methods can be in general categorized as the Galerkin meshfree methods that use a weak form of the PDE and the collocation meshfree methods that directly use the strong form of the PDE. The initial development for solving PDEs with scattered points traces back to 1972 with the development of generalized finite difference methods [33–37]. The smoothed particle hydrodynamics (SPH) was subsequently developed by kernel estimation of conservation equations for astrophysics modeling [1, 38]. The method later gained popularity in mechanics as means to resolve mesh-related issues due to large deformations.

Many more methods were proposed to improve the shortcomings of SPH in terms of accuracy, stability, and consistency. However, it was not until 1990 when this research field started developing rapidly. The development of many meshfree methods that are still popular today, including the element-free Galerkin method (EFG) [2–4], began in 1994. The reproducing kernel particle method (RKPM) [5, 39] was also introduced around the same time. These are examples of some of the Galerkin meshfree methods. Similarly, the radial basis collocation method [40, 41] is an example of strong-form-based collocation meshfree method developed around 1990. Later, a new method based on an integral form of the equation of motion-peridynamics [8, 42], was introduced around 2000 and it has gained popularity recently.

In the Galerkin weak form meshfree methods, the two most discussed or key issues are domain integration and implementation of boundary conditions. The integration scheme used affects the solution accuracy, convergence, and stability. Various integration methods have been proposed and developed over time to achieve higher stability, accuracy, and convergence in meshfree solutions. These integration methods include stabilized conforming nodal integration [6, 43], variationally consistent integration [44], gradient stabilized conforming

nodal integration, and naturally stabilized nodal integration [45]. The second issue arises from the fact that the Kronecker delta properties are not satisfied by the shape functions in Galerkin methods, and hence the implementation of the essential (Dirichlet) boundary conditions needs special techniques. These techniques can be categorized into two types: (a) methods of strong enforcement of essential boundary condition, which include transformation method [5, 46], and (b) methods of weak enforcement of essential boundary condition which include methods like Nitsche’s [47] method, the implementation of Lagrange multipliers method, and the penalty method in meshfree methods.

It should be acknowledged that many other methods that can be generally classified as meshfree or meshless were developed based on various approximation techniques and were popular in some periods, for more references, see [7, 48–51]. The above overview of meshfree methods is by no means, nor is it intended to be, a complete literature review for readers who like to learn these methods. It merely serves as a terminology introduction and provides the background to read the text mining results, as this study aims to facilitate a systematic literature review of the field of meshfree methods using data mining techniques. To better illustrate the analysis results from diverse data pools using the proposed text-mining approach, in this study, commonly used meshfree methods are selected as keywords, listed in Table 1.

TABLE 1. Keywords and abbreviations used for data extraction from Scopus.

Search no.	Search keywords	Abbreviations
1	“Element Free Galerkin” OR “Element Free Galerkin Method”	EFG
2	“Reproducing Kernel Particle Method” OR (“RKPM” and “mesh-free”) OR (“reproducing kernel approximation” and “meshfree”) OR (“reproducing kernel approximation” and “mesh-free”) OR (“reproducing kernel approximation”)	RKPM
3	“Peridynamics”	–
4	“Weak form meshfree method” OR “Weak form meshless method” OR “Galerkin meshfree method” OR “Galerkin based meshfree method” OR “Galerkin meshless method” OR “Galerkin based meshless method” OR “weak form meshfree” OR “weak-form mesh-free” OR “weak form mesh-free” OR “weak form meshless” OR “meshfree Galerkin” OR “meshless Galerkin”	Weak form
5	“Strong form meshfree method” OR “Collocation meshfree methods” OR “strong form meshfree” OR “strong form mesh-free” OR “strong-form mesh-free” OR “strong form meshless” OR (“strong form” and “meshfree”) OR “collocation meshfree” OR (“collocation” and “meshfree”) OR (“collocation” and “mesh-free”)	Strong form
6	“Meshfree” OR “Meshless methods” OR “Mesh-free”	–
7	“Smoothed particle hydrodynamics” OR (“meshfree” and “SPH”)	SPH
8	“Material point method”	MPM

Literature reviews are essential parts of research in any research field. It is arguably even more essential for someone who is new to the field and wants to get an overview and broader perspective of a particular field of research. However, with a significant number of publications in the field coming out every year, it can become difficult for someone to find the right articles matching their interest. Text mining is a tool that can help ease the literature review process. It helps in the automatic extraction of useful information from a large volume of text. As it is an automated process, it saves a significant amount of time and covers a much larger scope of literature. Different text mining techniques can help highlight important information and patterns from the text. For example, the frequency of different terms in the text can highlight important keywords in the text. Topic modeling is another method that can highlight the theme or summarize the essence of documents in terms of a few topics. Therefore, these methods can be used to visualize the development of any research field over time. Furthermore, they can help identify research areas that have been focused more and the areas that lack focus in terms of research. This can also help draw attention towards the issues or areas where more work or development is needed. Overall, text mining techniques can be used to analyze a large body of literature and can provide a bigger picture of a research area in a short period. Text mining techniques have been used to conduct literature reviews in various fields such as transportation engineering [52, 53], materials engineering [54, 55], industrial ecology [56], biomedical engineering [57, 58], and various other fields. This study has used several text mining techniques to analyze the literature on meshfree methods.

Specifically, the main objectives of this study are:

- facilitate a systematic literature review of the field of meshfree methods,
- provide a brief overview of the field of meshfree methods, highlighting major topics and key issues discussed in this field,
- show the evolution of this research field over the last 30 years by highlighting research trends and changes in research trends over time,
- detect research areas that have received less attention,
- highlight the collaboration network between co-authors (which also highlights the interdisciplinary nature of a research field as authors from different disciplines collaborate and work together),
- identify leading authors in the field of meshfree methods.

2. DATA AND METHODOLOGY

2.1. Dataset and data preparation methods

The current study used data from 17330 articles downloaded from Scopus that contained the keywords given in Table 1. Since the evolution of many po-

pular meshfree methods traces back to around 1990 and the literature in this research field has also expanded rapidly, this study has included articles published between 1990 and 2020 [1–9, 22, 38, 39, 48, 59].

The keywords used to search the articles on Scopus were selected based on the literature review conducted on the field of meshfree methods. The keywords selected were either name of a popular meshfree method such as the element-free Galerkin method and reproducing kernel particle method or some category of meshfree methods such as weak form meshfree methods and strong form meshfree methods. The keywords were used to search for articles containing them in the title, the abstract, or keywords. The logical “or” operator was used to include similar terms for the same keyword as “meshfree methods” or “meshless methods”. The data for around 17 330 articles containing these keywords were extracted from Scopus in comma-separated values (CSV) file format.

The data included abstract, title, keywords, author names, author ID, author affiliations, number of citations, publication year, and journal. Data from journal publications, books, and conference proceedings were all given the same weight.

The data used for analysis in this study were: abstract, title, author name, author ID, citations, and publication year. Some of the data such as authors’ affiliations and journal names have not been used in this study but are intended to be used in a future study. Further, the data were processed to drop any duplicate data points, and the uniqueness of data points based on their title was ensured. Figure 1a shows the number of articles used in this study for each publication year from 1990 to 2020. Figures 1b and 1c show a comparison of trends in publications for different computational methods. Data were collected for the number of publications in the field of other computational methods such as the finite element method (FEM), isogeometric analysis (IGA) and meshfree methods. The data have been plotted and shown in Figs. 1b and 1c, which compare the trend in publications for all three computational methods. Since the number of publications for FEM was very large compared to meshfree and IGA publications, two separate plots have been generated. Figure 1b corresponds to the FEM publications and Fig. 1c corresponds to the meshfree and IGA publications. As observed in the plots in Figs. 1b and 1c, FEM has a relatively large number of publications compared to the other two methods. As the development of FEM traces back to much earlier times than meshfree methods and IGA, FEM is more robust, simple, easy to implement, and has found applications in various fields, which shows in its high number of publications. Since IGA is relatively new among all these three computational methods, it has the lowest number of publications, but we can observe an increase in its popularity after 2010. We can also observe that meshfree methods started to gain popularity after 2000.

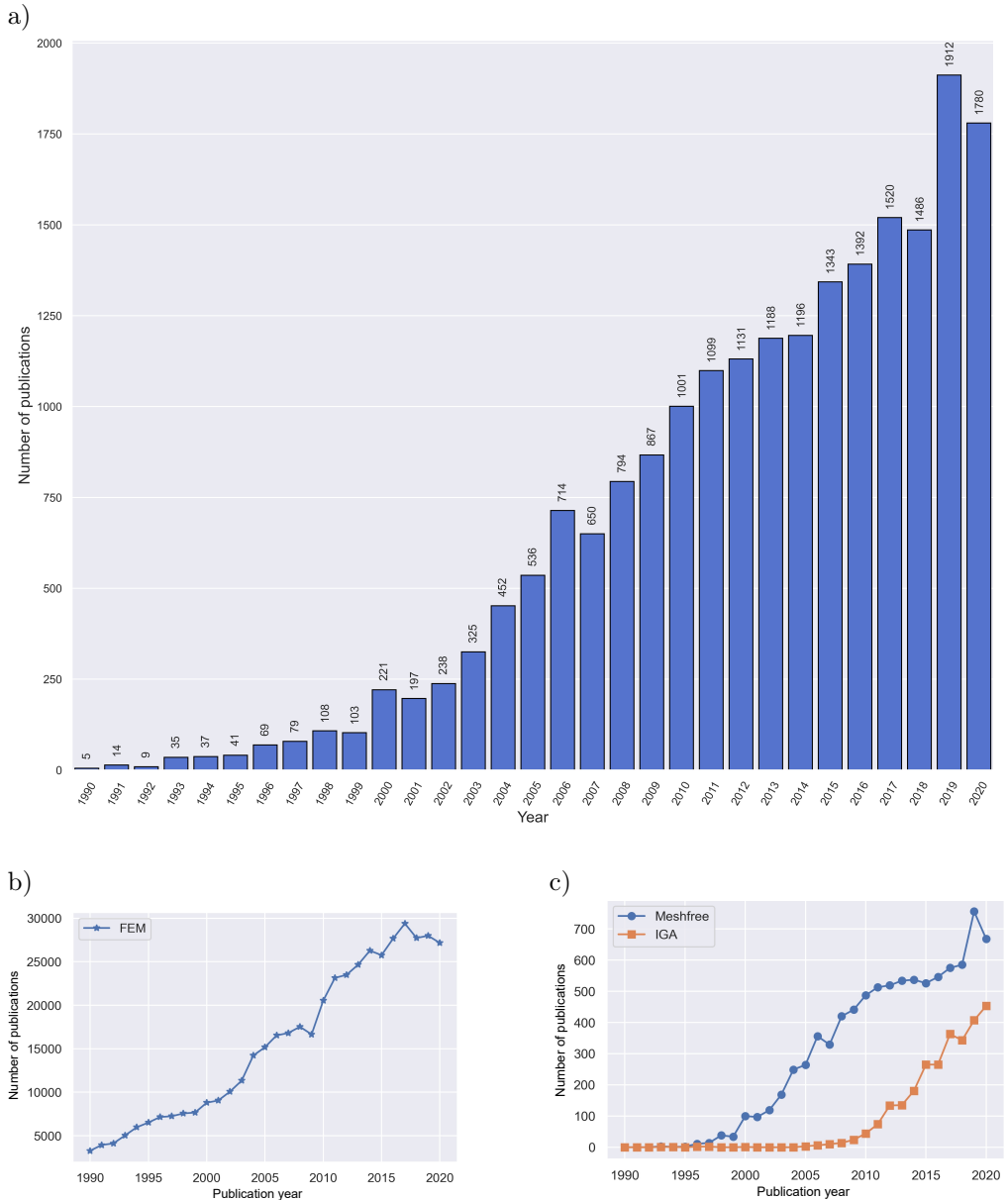


FIG. 1. a) Number of publications on meshfree methods yearly, b) trend in publications for FEM, c) trend in publications for meshfree methods and IGA.

Further, the abstracts were tokenized using the NLTK library [60] in Python, and all the punctuations and stop words (e.g., “a”, “an”, “the”) were removed. The tokenized data were also lemmatized to only keep the root of each term so that terms such as [node] and [nodes] are grouped together. Some other terms, such as

“results”, “methods”, and “copyright”, that frequently occurred in abstracts but were generic and did not provide any useful information were also removed. The tokenized data were then used for analysis. With some modifications, the LiT-CoF (literature topic co-occurrence and frequency) Python-based tool developed by Dayeen, Sharma [56] was used to perform the frequency and co-occurrence analyses and for topic modeling in this study.

The co-authors’ data containing all co-authors’ names and IDs were used to create and visualize a co-author network. For each article, the row with the co-authors’ name and ID were split to get the individual author’s name and ID. The uniqueness of the result was ensured in Python and the data were used as nodes to create the networks. To create data for the links, the combination of the co-authors’ IDs was taken for each article, two at a time. These data were used to create a link between the two nodes in the network. The data were then imported in Gephi [61] for visualization of the entire co-author network.

2.2. Text-mining methods

2.2.1. Term frequency. Term frequency is a measure of how frequently a term appears in the text. It is defined as:

$$\text{Term frequency} = \frac{\text{Number of times a term appears in text}}{\text{Total number of terms in the text}}. \quad (1)$$

Although term frequency can be used to identify the most important terms from a corpus of text, higher frequencies do not suggestively indicate that a term is important. As mentioned previously, frequent but generic terms such as “results” and “methods” were removed from the text used. Once identified, they were appended to the list of stop words taken from the NLTK library. Then, term frequency values were used to identify some of the most important terms in the text. These terms were then used for topic modeling (described next) to identify the naturally emerging topics from the text.

2.2.2. LDA topic modeling. Topic modeling is an efficient way to analyze a large volume of text as it can capture and represent the information from the text of documents through several topics. Although there are several topic modeling techniques, some of the commonly used methods are latent semantic analysis (LSA), probabilistic latent semantic analysis (PLSA), latent Dirichlet allocation (LDA), and correlated topic model (CTM) [62]. Each of these methods has its benefits and limitations. LSA is a technique in distributional semantics that uses the concept of singular value decomposition and is easy to use and implement. However, it faces multiple issues such as inefficient representation, lack of interpretability, and difficulty in dealing with polysemy (different meanings of the same word in different contexts). PLSA is an advancement over this

method, and it resolves the problem of representation, interpretability, and polysemy. However, it suffers from the problem of complicated parameter space. The number of parameters increases with an increase in the number of documents leading to the problem of overfitting in this method. On the other hand, LDA is a more robust method and allows a better mixture of words and topics. LDA also has a relatively simpler parameter space and hence avoids the overfitting problem. The representation with LDA is also efficient and semantic interpretation is easy. One limitation of LDA is its inability to create relations among topics. This shortcoming is overcome in the CMT method, but this method involves much complicated and larger calculations. Looking at the advantages and disadvantages of each of these methods, LDA topic modeling was chosen for this analysis as it is a more robust method, has simpler parameter space, and is easily interpretable.

Latent Dirichlet allocation (LDA) [63] is a probabilistic model of a corpus (collection of documents) to identify naturally emerging topics from the text of various documents that can best represent the information underlying in the text. The word latent is used since the topics to be identified are not known.

LDA is based on Bayesian probability distribution. It considers documents as probability distributions of topics and topics as probability distributions of terms. It works by creating a probability distribution matrix that shows the probability distribution of terms in each document. This matrix of a term-document probability distribution is created by taking the product of two lower dimension probability distribution matrices. These two matrices contain probability distribution values for each term in topics and the probability distribution of each topic in documents. Once the number of topics is specified, LDA randomly assigns topics to documents and then assigns each term to a topic. This means that it randomly assigns a probability distribution for each term in the term-topic matrix and the topic document matrix, respectively. In the next step, LDA corrects the probability distribution for each term, one at a time, by considering that the probability distribution value assigned to all other terms is correct except for the current term. It then calculates both the probability distribution values for the distribution of terms in topics and the distribution of topics in documents. These values are multiplied to obtain the probability distribution of terms in documents. Based on this new distribution value, new topics are assigned to terms. This process is iterated until satisfactory convergence is reached. Therefore, as a product, LDA gives two matrices containing probability distribution values for the distribution of terms in the topics and the distribution of topics in the documents.

LDA has two main hyper-parameters. One is α denoting the topic distribution in documents or document topic density. A higher value of α indicates that the documents consist of a mixture of many topics and a lower α denotes documents

consisting of fewer topics. The second hyperparameter is β . It represents the term distribution per topic or topic term density. A higher value of β denotes more terms per topic, and a lower β signifies fewer terms per topic. Another important parameter is the right/optimum number of topics. To get the right number of topics for text, coherence and perplexity scores can be used. A high coherence score shows a similarity between high-scoring terms in topics having similar meanings. Therefore, the higher the coherence score the better. Perplexity score is also known as log-likelihood and is interpreted as the model's surprise at the data, so the lower the perplexity score the better.

2.2.3. Co-occurrence coefficient. The co-occurrence of certain keywords and terms indicates how many times they have co-occurred together in one publication. The co-occurrence coefficient c_{ij} for two terms i and j is defined as [56, 64]:

$$c_{ij} = \frac{e_{ij}^2}{n_i n_j}, \quad (2)$$

where e_{ij} is the number of publications in which the terms i and j co-occur, n_i is the number of publications in which term i appears, and n_j is the number of publications in which term j appears. The co-occurrence coefficient gives a measure of the correlation between the two terms.

In this study, co-occurrence or the number of times certain terms appeared together in a publication were calculated first. Then, the number of publications in which each of those terms appeared was taken using Python libraries. Using these values, a co-occurrence matrix was created for the considered terms. The values in the matrix were used to calculate co-occurrence coefficients. These coefficients were then used to plot the co-occurrence between the selected terms.

2.2.4. Relative term frequency. To see the trends in the usage of certain terms over the years, relative term frequency can be used. It can help visualize how a certain research field has evolved. Moreover, it can show the gain or loss of popularity of terms over time. Relative frequency $R_{i,t}$ of term i at time t is defined as [56]:

$$R_{i,t} = \frac{n_{i,t}}{N_t}, \quad (3)$$

where $n_{i,t}$ is the number of times term i occurs in publications at year t , and N_t is the total number of terms in all abstracts combined for year t .

2.2.5. Co-author network. A co-author network gives an idea of the collaboration network between authors. It also highlights the interdisciplinary nature

of a research field as it shows the collaboration between authors who are in different research fields. Network science techniques have been used here for creating the co-author network. To create a network, data about the nodes and links in the network are required. The nodes are individual entities; in this study, they are unique authors. The links show the connections between each node. Gephi was used in this study for network visualization. The node and link data were prepared as CSV files and imported into Gephi. In this study, the network is undirected, meaning that links do not have directions, i.e., a link from author A to B is also a link from author B to A. In addition, measures such as degree and centrality were used to improve the visualization. The degree of a node denotes the number of links that are connected to it. Nodes with a higher degree were assigned darker colors in this study. Further, modularity was used to partition the nodes into groups. Modularity denotes the strength or density of divisions or groups or clusters within the network. Based on modularity, different groups were assigned different colors. All nodes belonging to the same class were given the same color. Further, betweenness centrality was calculated; it is a measure of the number of times a node acts as a link or bridge between two other nodes. Node with a high betweenness centrality was given a larger size. The names of authors were also kept proportional to the node size so that the names of leading authors appear more clearly.

3. RESULTS AND DISCUSSION

3.1. Trend of number of publications for keywords

Figure 2a shows the number of publications for each keyword (see Table 1) between 1990 and 2020. The figure shows an increase in the number of publications for meshfree methods such as SPH, weak form meshfree methods, strong form meshfree methods, EFG, RKPM, MPM, and peridynamics over the years.

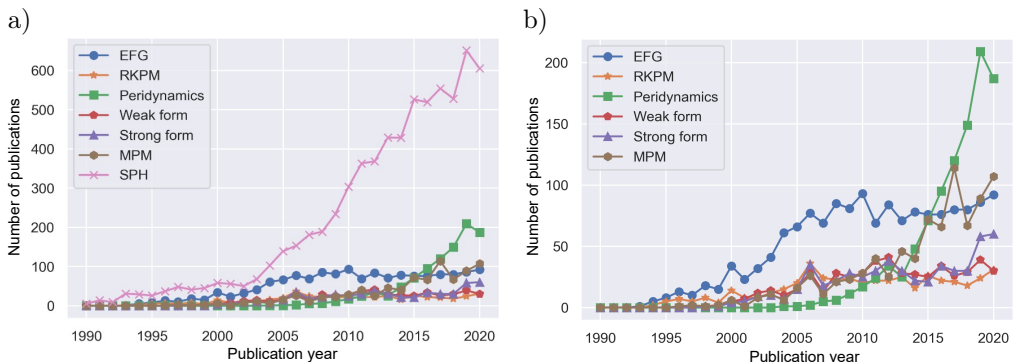


FIG. 2. a) Number of publications for keywords between 1990 and 2020, b) number of publications for keywords except SPH between 1990 and 2020.

Overall, we see that the SPH method has a higher number of publications, and hence we can say that this method has received more focus or is more developed over the years. SPH is the earliest developed meshfree method, being introduced in 1977, compared to most other methods that were developed in the 1990s. Since it was introduced early, it has been used more and has developed into a more robust method and found applications in various fields, including astrophysics, fluid mechanics, and solid mechanics. Since the number of publications of SPH is high compared to others, Fig. 2b shows the same data as Fig. 2a without SPH. The first method that catches attention in Fig. 2b is peridynamics. We can see that peridynamics has evolved after 2000, and there has been a significant increase in the number of publications using peridynamics after 2010, indicating an increase in popularity. Peridynamics works by reformulating the governing equations of solid mechanics into non-local integral equations. Since this method does not contain derivatives, the accommodation of discontinuities does not require any modifications. Therefore, this method has emerged as a simple but effective method for dealing with problems involving modeling and simulation of discontinuities such as fracture and fragmentations. The simplicity and effectiveness of the method for such complex problems seem to be increasing its popularity. The other method that captures attention is MPM. A significant increase in publications for this method is observed after 2015. It has evolved from the particle-in-cell method, which was originally developed for fluid dynamics problems. The MPM method has recently found its applications in geotechnical and geoenvironmental problems [65] apart from the solid mechanics' problems. It has also gained attention in computer graphics and has been used in the movie industry [66]. These new applications explain the increase in its publications in recent years. The figure shows a similar trend for both weak form and strong form meshfree methods. We can also see that EFG method publications have been consistently higher than RKPM, indicating that this method has received more attention than some of the other weak form meshfree methods. These results may be indicative of more areas of opportunity or development in other weak form meshfree methods such as RKPM. However, it is important to note that these results by no means show importance of one method over another as the number of publications cannot be the sole criterion to determine or judge the importance of any method.

3.2. List of top-cited articles

Table 2 shows the list of the 24 most cited articles between 1990 and 2020 in the field of meshfree methods. The list was obtained by sorting the data based on the number of citations. The purpose of this list is to identify the most popular articles in the field. As a side note, the table does not identify or list them as

TABLE 2. Top-cited articles.

Publication year	Title	Citations
1994	Element free Galerkin methods	4650
2005	The cosmological simulation code GADGET-2	4118
1996	Meshless methods: An overview and recent developments	2740
1994	Simulating free surface flows with SPH	2256
1995	Reproducing kernel particle methods	2178
1998	A new Meshless Local Petrov-Galerkin (MLPG) approach in computational mechanics	2064
2005	Smoothed particle hydrodynamics	1810
2002	Meshfree methods: Moving beyond the finite element method	1601
2005	How do galaxies get their gas?	1462
1997	Modeling low Reynolds number incompressible flows using SPH	1302
2003	Cosmological smoothed particle hydrodynamics simulations: A hybrid multiphase model for star formation	1297
2001	GADGET: A code for collisionless and gasdynamical cosmological simulations	1184
2003	Numerical simulation of interfacial flows by smoothed particle hydrodynamics	1036
2005	An introduction to meshfree methods and their programming	999
2010	Smoothed particle hydrodynamics (SPH): An overview and recent developments	965
2001	Stabilized conforming nodal integration for Galerkin mesh-free methods	944
2005	A meshfree method based on the peridynamic model of solid mechanics	941
1996	Smoothed particle hydrodynamics: Some recent improvements and applications	875
2003	Particle-based fluid simulation for interactive applications	874
2010	E pur si muove: Galilean-invariant cosmological hydrodynamical simulations on a moving mesh	849
2004	Cracking particles: A simplified meshfree method for arbitrary evolving cracks	845
2008	Meshless methods: A review and computer implementation aspects	802
1994	Dynamics of binary-disk interaction. I. Resonances and disk gap sizes	792
2007	Peridynamic states and constitutive modeling	791

the most important papers, as citations alone cannot be a governing criterion to identify important papers in any field of research.

3.3. Results from LDA topic modeling and word-cloud

The LDA results are shown in Table 3. The table shows 8 topics containing 12 terms each. As LDA is an unsupervised method, the tag or name of the topics or the meaning of the topics are not known as suggested by the name latent and needs interpretation based on the terms that constitute them. In other words, LDA produces a specified number of latent topics, which consist of terms that maximize the probability of word assignment to the topics, as explained in the previous section. For each topic in Table 3, the terms highlight some of the important keywords in this field of research. Terms like “mpm” and “hydrodynamics” highlight some of the popular meshfree methods like SPH and MPM (which has recently gained more popularity). Terms such as “integration”, “convergence”, “accuracy”, “boundary_conditions”, “discretization”, and “stability” highlight some of the major or key issues discussed in the field. As discussed previously, domain integration is one of the most discussed issues in this field and has developed over the years for better convergence, stability, and accuracy. The imposition of essential boundary conditions is another major issue discussed in the field that is also identified here. Some other terms such as “large_deformation”, “deformation”, “fracture”, “fluid”, “flow”, “solid”, “soil”, “plate”, “impact”, and “damage” highlight the problems where meshfree methods have found their application. Other than these, terms such as “shape functions” and “radial_basis” have also emerged. Shape functions are used to find the values of the main variables at any point in a domain using their values at nodes that have been used for discretization of the domain. The appearance of terms such as “simulation”, “procedure”, “scheme”, “computation”, “construct”, “design”, “numerical”, “problem”, “solution”, “framework” is self-explanatory.

Figure 3 shows a word cloud generated with the top 1000 terms based on their frequency. The figure reiterates the importance of some of the keywords that were also identified by LDA and highlights some other important terms. In the word cloud, we can see that smooth particle hydrodynamics is the term that has appeared most frequently in the text, which is also evident from the trend in the number of publications. Some of the obvious terms such as “numerical”, “model”, “simulation”, “algorithm”, and “partial differential equations” have been removed from the word cloud. We can also see terms such as “boundary condition”, “essential boundary”, “integral”, “instability”, “discretization”, and “accurate”, reiterating that these are some important issues discussed in the field. Terms such as “large deformation”, “damage”, “collision”, “impact”, “crack propagation”, “fluid flow”, “heat transfer”, “fracture”, “failure”, “star formation”, “shock”, and “contact” describe the wide variety of fields where meshfree methods have been implemented to solve problems. Furthermore, we can observe terms such as “weak form”, “reproduce kernel”, “element free”, “free galerkin”, “material point”,

3.4. Co-occurrence plot

The co-occurrence of terms such as “integration”, “essential boundary”, “convergence”, “accuracy”, “stability”, and “discretization” that appeared in the LDA analysis are shown in Fig. 4. These terms were selected as they highlight the key issues in the field and co-occurrence can help visualize how correlated these key issues are with one another.

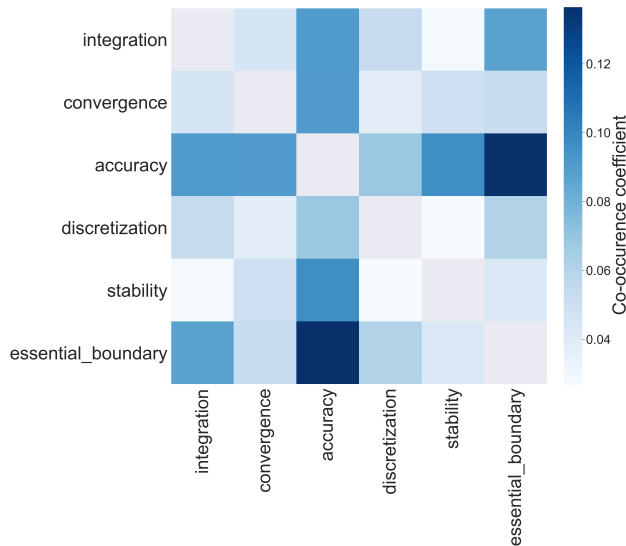


FIG. 4. Co-occurrence matrix.

The co-occurrence of terms shows how frequently they have appeared together in the same abstract. The figure shows that “essential boundary” is highly correlated with “accuracy”. This result re-emphasizes the previous finding from the literature that the imposition of essential boundary conditions is a major issue in the field, and the method used to impose them affects the accuracy of the solution. We can also see that there is a relatively high co-occurrence between “accuracy” and “integration”. We also observe that the terms “integration” and “convergence” co-occur frequently. Further, we can see that the terms “accuracy”, “stability”, and “convergence” also co-occur relatively frequently. These results are consistent with our findings from the literature review that the integration method selected for domain integration strongly affects the solution’s convergence, stability, and accuracy. Various integration methods have been proposed and developed over time to achieve better stability, accuracy, and convergence in meshfree solutions. Further, we see that “discretization” also co-occurs with “integration”, “accuracy”, and “convergence”. It brings up the fact that the type of discretization such as uniform and non-uniform discretization of a domain is

correlated with integration and affects the accuracy and convergence of the final solution.

3.5. Term-relative frequency map

Two different term-relative frequency maps were created and are shown in Figs. 5 and 6. These frequency maps show the trend of using certain keywords/terms over the past 30 years in the field of meshfree methods.

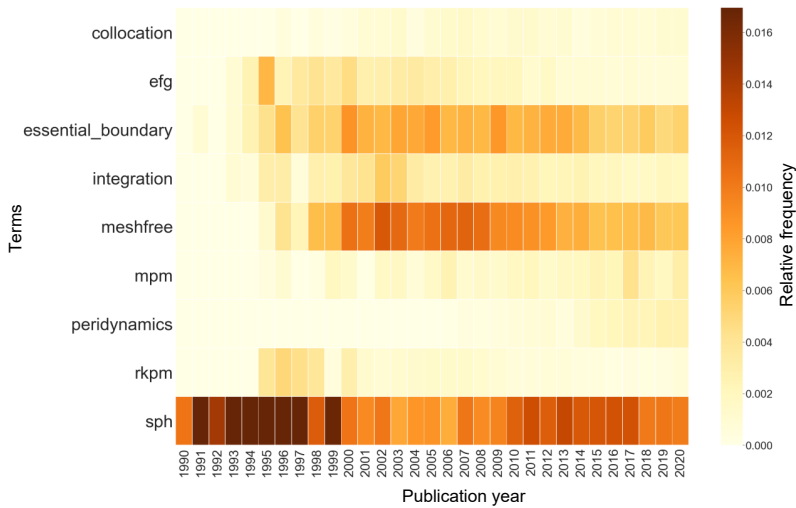


FIG. 5. Keyword-relative frequency map with time for different methods and issues.

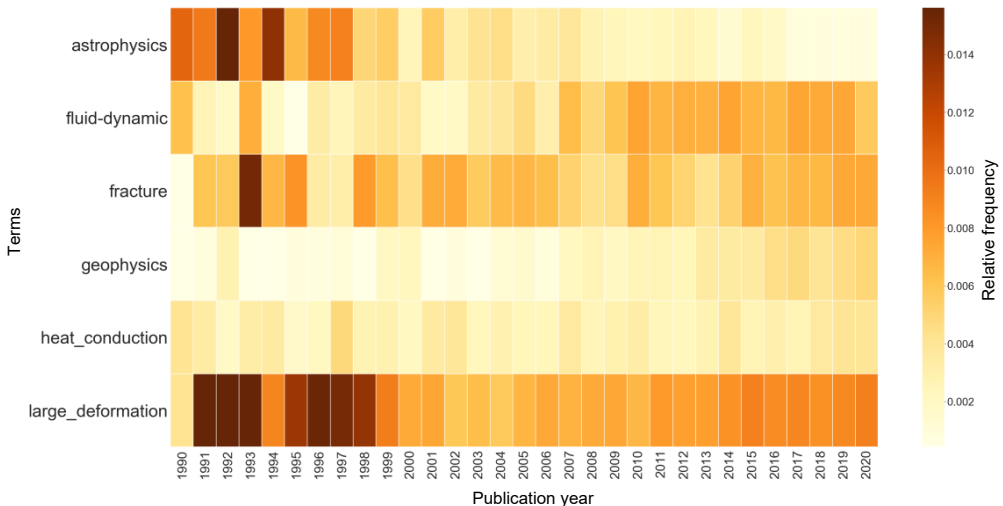


FIG. 6. Keywords relative frequency map with time for the field of application.

To calculate relative frequency, the frequency of some of the terms was combined since they had the same meaning or represented the same term. For example, the frequency of the terms “meshless”, “meshfree”, and “mesh-less” has been combined here and shown as the frequency for “meshfree” in Fig. 5. Similarly, the frequency for “mpm”, shown in Fig. 5, is the sum of the frequency of the terms “mpm”, “material point”, and “material point method”. Table 4 below shows the different terms that were combined for frequency map generation. The frequency map in Fig. 5 shows the increasing frequency of the term “meshfree” over time and shows that the field has been evolving continuously. A high relative frequency of the term “sph” between 1990 and 2000 can also be observed. It shows that the SPH method has been used more than other methods consistent with our findings from the publication trends. The frequency map also highlights the fact that peridynamics has evolved after 2000 and has gained more popularity

TABLE 4. The terms combined and their representative term for the relative frequency map.

Term in frequency map	Combined terms
meshfree	meshfree, meshless, mesh-free, mesh-less
sph	sph, smooth_particle, particle_hydrodynamics, hydrodynamics_sph
rkpm	rkpm, reproduce_kernel, reproducing_kernel, kernel_particle
efg	efg, efgm, galerkin_efg, galerkin_efgm
peridynamics	peridynamics, peridynamic, peridynam, peridynamics_meshfree
mpm	mpm, material_point, material_point_method, material-point
integration	integration, integrate, integral, quadrature
essential_boundary	essential_boundary, boundary, boundary_condition, boundary_value
heat_conduction	thermal, heat, heating, heat_flow, temperature, heat_transfer, heat-flow, heat-transfer, heat_flux, heat-flux, heat_conduction, transient_heat
fluid_dynamic	water, fluid_flow, fluid, liquid, fluid_dynamic, tsunami, fluid_dynamics, fluid-dynamic, fluid-dynamics, fluid-flow, surface_tension, flood
fracture	crack, crack_propagation, crack_growth, cracks, crack_tip, cracks, discontinuity, fracture, fracture_growth, fracture_mechanic, fragmentation, fragment
large_deformation	impact, penetration, high_velocity, collision, large_deformation, impact_velocity, shock, vibration, high_speed, damage, collapse, explosion
astrophysics	cosmological, planet, star, star_formation, black_hole, orbit, astrophysics, black-hole, star-formation, dust, satellite, astronomical
geophysics	soil, rock, porosity, landslide, erosion, sediment, debris_flow, erosive, geophysics, porous_medium, grain, sand

after 2015. An increase in the popularity of the MPM method is also evident in Fig. 5. The terms “EFG” and “RKPM” were more popular between 1993–2000, but their frequency has decreased in recent years in contrast to SPH, MPM, and peridynamics. Similarly, the popularity of collocation or strong form meshfree methods can also be seen to have increased gradually, but it is consistently less than methods such as MPM, peridynamics, and SPH. These results highlight a shift in trend or usage of methods in this field. A similar trend was seen from the publication trends as well. These results may suggest the need to focus more on these methods or show that they still have more development opportunities. Further, the usage of terms “integration” and “essential boundary” is almost consistent throughout the last 30 years, indicating that these have been key issues in the field and have been consistently discussed. This finding also highlights the fact that there has been a significant development in this area over the past three decades, but it still has more scope of research as this remains a topic of discussion.

Figure 6 is a frequency map of terms highlighting various fields and problems where meshfree methods have been implemented. We can see that the problems in solid mechanics such as large deformation and fracture have found more application of meshfree methods consistently over the years [4, 5, 12, 18, 24, 70]. Meshfree methods such as SPH, EFG, and RKPM have been used extensively to solve problems including large deformation, impact and penetration, fracture mechanics, and explosion. Peridynamics has also been used for modeling and simulating discontinuities such as fracture and fragmentations more recently. Fluid dynamics is another field that has benefited from applying meshfree methods. Meshfree methods such as SPH, particle-in-cell method, RKPM, and finite point method have been used to solve fluid flow problems. Further, we observe a high frequency of problems related to astrophysics between 1990 and 2000. This is because the SPH method was initially used more in astrophysics to solve problems such as star formation, galaxy formation, cloud dust, and stellar collisions. We can see that meshfree methods have also been applied in studying geophysics problems such as landslide and erosion, mainly because of the increasing popularity of methods such as the MPM method in geophysics and geoengineering problems in recent years, as reflected in the map.

3.6. Co-authorship network

Figure 7 shows the co-authorship network. The network highlights the major collaboration groups in the field of meshfree methods. The collaboration network not only shows collaboration between different authors but also highlights the inter-disciplinary nature of this research field as authors from many different disciplines have been seen to collaborate closely.

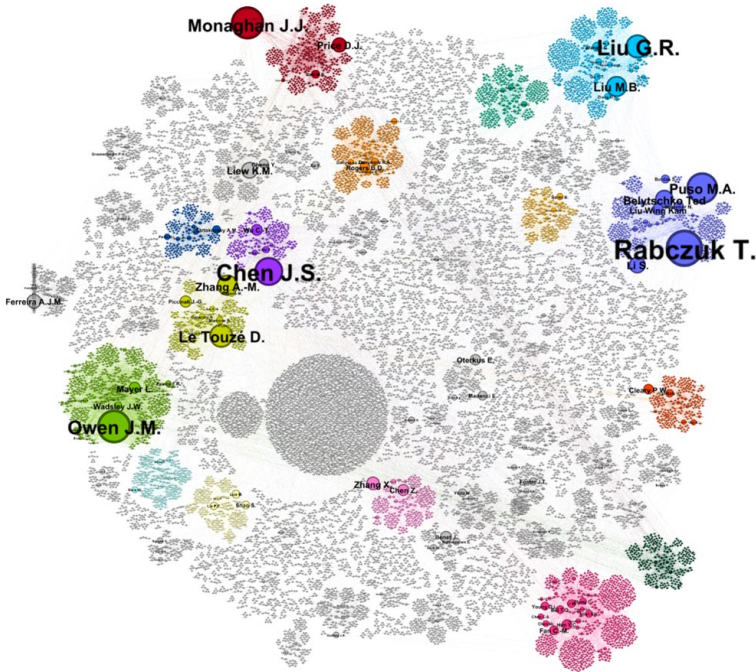


FIG. 7. Co-authorship network [https://sindh2.github.io/meshfree_network/].

The figure highlights the wide range of applications of this category of computational methods into different disciplines. The node size in the network is proportional to the betweenness centrality of the node, which is a measure of the number of times a node acts as a link or bridge between two nodes. The authors' name was kept proportional to the node's size. It helps to highlight some of the leading authors in the field, such as J.J. Monaghan, T. Rabczuk, G.R. Liu, J.S. Chen, T. Belytschko, M.A. Puso, and W.K. Liu. The different colors were provided to nodes by modularity that partitions networks based on the strength or density of different divisions or groups or clusters. All nodes belonging to the same class were given the same color. This network can be helpful for new researchers to identify leading authors and their network in the field. Further, the network could also be used to highlight collaboration between different universities and countries. A link has also been provided in the caption of Fig. 7 to access the network.

4. SUMMARY AND CONCLUSIONS

Text mining techniques and measures such as LDA, term frequencies, and co-occurrence coefficients can be used to analyze a large corpus of articles from the

literature to facilitate the review of a research field. The text mining technique used in this study identified the correlation between search keywords and output relevant words with higher frequency, and thus helped reduce bias in keyword selection. It identified the trending topics in the field of meshfree methods and helped to see how the field has evolved over the past 30 years. Popular meshfree methods such as EFG, RKPM, SPH, MPM, peridynamics, and collocation mesh-free methods were identified. The major subtopics or key issues of the field also emerged as topics through LDA. These topics are domain integration, imposition of essential boundary conditions, and their effect on convergence, stability, and accuracy of meshfree solutions. We also observed that SPH method, in general, has received more focus than other meshfree methods. Nonetheless, an increase in popularity has been observed for collocation methods, MPM, and peridynamics in the last decade through the number of publications and relative frequency mapping. The co-author network visualization provided some interesting insights about collaboration between different scientists worldwide. Some of the leading authors were also identified from the co-authorship network. The work on the network can further be extended to visualize collaboration between universities or countries all over the world based on the data of collaboration between authors. The co-occurrence matrix also highlighted the correlation between integration and boundary conditions with convergence, stability, and accuracy of solutions in meshfree methods.

REFERENCES

1. L.B. Lucy, A numerical approach to the testing of the fission hypothesis, *The Astronomical Journal*, **82**(12): 1013–1024, 1977.
2. T. Belytschko, Y.Y. Lu, L. Gu, Element-free Galerkin methods, *International Journal for Numerical Methods in Engineering*, **37**(2): 229–256, 1994, doi: 10.1002/nme.1620370205.
3. Y.Y. Lu, T. Belytschko, L. Gu, A new implementation of the element free Galerkin method, *Computer Methods in Applied Mechanics and Engineering*, **113**(3–4): 397–414, 1994 doi: 10.1016/0045-7825(94)90056-6.
4. T. Belytschko, Y.Y. Lu, L. Gu, Crack propagation by element-free Galerkin methods, *Engineering Fracture Mechanics*, **51**(2): 295–315, 1995, doi: 10.1016/0013-7944(94)00153-9.
5. J.-S. Chen, C. Pan, C.-T. Wu, W.K. Liu, Reproducing kernel particle methods for large deformation analysis of non-linear structures, *Computer Methods in Applied Mechanics and Engineering*, **139**(1–4): 195–227, 1996, doi: 10.1016/S0045-7825(96)01083-3.
6. J.S. Chen, C.-T. Wu, S. Yoon, Y. You, A stabilized conforming nodal integration for Galerkin mesh-free methods, *International Journal for Numerical Methods in Engineering*, **50**(2): 435–466, 2001, doi: 10.1002/1097-0207(20010120)50:2<435::AID-NME32>3.0.CO;2-A.

7. J.-S. Chen, M. Hillman, S.-W. Chi, Meshfree methods: progress made after 20 years, *Journal of Engineering Mechanics*, **143**(4): 04017001, 2017, doi: 10.1061/(ASCE)EM.1943-7889.0001176.
8. S.A. Silling, E. Askari, A meshfree method based on the peridynamic model of solid mechanics, *Computers & Structures*, **83**(17–18): 1526–1535, 2005, doi: 10.1016/j.compstruc.2004.11.026.
9. G.-R. Liu, Y.-T. Gu, *An Introduction to Meshfree Methods and Their Programming*, Springer Science & Business Media, Dordrecht, 2005.
10. O. Zienkiewicz, R. Taylor, *The Finite Element Method*, McGraw-Hill, New York, 1977.
11. G.E. Forsythe, W.R. Wasow, *Finite Difference Methods for Partial Differential Equations*, Wiley&Sons, New York, 1960.
12. W.K. Liu, S. Jun, Multiple-scale reproducing kernel particle methods for large deformation problems, *International Journal for Numerical Methods in Engineering*, **41**(7): 1339–1362, 1998, doi: 10.1002/(SICI)1097-0207(19980415)41:7<1339::AID-NME343>3.0.CO;2-9.
13. S. Yoon, J.-S. Chen, *Accelerated meshfree method for metal forming simulation*, *Finite Elements in Analysis and Design*, **38**(10): 937–948, 2002, doi: 10.1016/S0168-874X(02)00086-0.
14. J.-S. Chen, C. Pan, C.-T. Wu, Large deformation analysis of rubber based on a reproducing kernel particle method, *Computational Mechanics*, **19**(3): 211–227, 1997, doi: 10.1007/s004660050170.
15. T. Rabczuk, T. Belytschko, A three-dimensional large deformation meshfree method for arbitrary evolving cracks, *Computer Methods in Applied Mechanics and Engineering*, **196**(29–30): 2777–2799, 2007, doi: 10.1016/j.cma.2006.06.020.
16. S.-W. Chi, *Image-Based Computational Mechanics Frameworks for Skeletal Muscles*, University of California, PhD Dissertation, ProQuest Dissertations Publishing, Los Angeles, 2009.
17. J.-S. Chen *et al.*, Pixel-based meshfree modelling of skeletal muscles, *Computer Methods in Biomechanics and Biomedical Engineering: Imaging & Visualization*, **4**(2): 73–85, 2016, doi: 10.1080/21681163.2015.1049712.
18. T. Belytschko, L. Gu, Y. Lu, Fracture and crack growth by element free Galerkin methods, *Modelling and Simulation in Materials Science and Engineering*, **2**(3A): 519–534, 1994, doi: 10.1088/0965-0393/2/3a/007.
19. X. Zhuang, C. Augarde, S. Bordas, Accurate fracture modelling using meshless methods, the visibility criterion and level sets: formulation and 2D modelling, *International Journal for Numerical Methods in Engineering*, **86**(3): 249–268, 2011, doi: 10.1002/nme.3063.
20. S.W. Chi, C.-H. Lee, J.-S. Chen, P.-C. Guan, A level set enhanced natural kernel contact algorithm for impact and penetration modeling, *International Journal for Numerical Methods in Engineering*, **102**(3–4): 839–866, 2015, doi: 10.1002/nme.4728.
21. J. Chen *et al.*, *A Multiscale Meshfree Approach for Modeling Fragment Penetration into Ultra High-Strength Concrete*, California University, Los Angeles Department of Civil and Environmental Engineering, 2011.
22. P.W. Randles, L.D. Libersky, Smoothed particle hydrodynamics: some recent improvements and applications, *Computer Methods in Applied Mechanics and Engineering*, **139**(1–4): 375–408, 1996, doi: 10.1016/S0045-7825(96)01090-0.

23. B. Li, A. Kidane, G. Ravichandran, M. Ortiz, Verification and validation of the optimal transportation meshfree (OTM) simulation of terminal ballistics, *International Journal of Impact Engineering*, **42**: 25–36, 2012, doi: 10.1016/j.ijimpeng.2011.11.003.
24. J.-S. Chen, C.-T. Wu, T. Belytschko, Regularization of material instabilities by mesh-free approximations with intrinsic length scales, *International Journal for Numerical Methods in Engineering*, **47**(7): 1303–1322, 2000, doi: 10.1002/(SICI)1097-0207(20000310)47:7<1303::AID-NME826>3.0.CO;2-5.
25. J.-S. Chen, X. Zhang, T. Belytschko, An implicit gradient model by a reproducing kernel strain regularization in strain localization problems, *Computer Methods in Applied Mechanics and Engineering*, **193**(27–29): 2827–2844, 2004, doi: 10.1016/j.cma.2003.12.057.
26. J. Chen, W. Hu, M.A. Puso, Y. Wu, X. Zhang, Strain smoothing for stabilization and regularization of Galerkin meshfree methods, [in:] M. Griebel, M.A. Schweitzer [Eds.], *Meshfree Methods for Partial Differential Equations III*, Vol. 57, pp. 57–75, Springer, Berlin, Heidelberg, 2007.
27. T. Rabczuk, E. Samaniego, Discontinuous modelling of shear bands using adaptive mesh-free methods, *Computer Methods in Applied Mechanics and Engineering*, **197**(6–8): 641–658, 2008, doi: 10.1016/j.cma.2007.08.027.
28. J.-S. Chen, D. Wang, A constrained reproducing kernel particle formulation for shear deformable shell in Cartesian coordinates, *International Journal for Numerical Methods in Engineering*, **68**(2): 151–172, 2006, doi: 10.1002/nme.1701.
29. D. Wang, H. Peng, A Hermite reproducing kernel Galerkin meshfree approach for buckling analysis of thin plates, *Computational Mechanics*, **51**(6): 1013–1029, 2013, doi: 10.1007/s00466-012-0784-9.
30. G.R. Liu, X. L. Chen, A mesh-free method for static and free vibration analyses of thin plates of complicated shape, *Journal of Sound and Vibration*, **241**(5): 839–855, 2001, doi: 10.1006/jsvi.2000.3330.
31. P. Krysl, T. Belytschko, Analysis of thin plates by the element-free Galerkin method, *Computational Mechanics*, **17**(1): 26–35, 1995, doi: 10.1007/BF00356476.
32. P. Krysl, T. Belytschko, Analysis of thin shells by the element-free Galerkin method, *International Journal of Solids and Structures*, **33**(20–22): 3057–3080, 1996, doi: 10.1016/0020-7683(95)00265-0.
33. P.S. Jensen, Finite difference techniques for variable grids, *Computers & Structures*, **2**(1–2): 7–29, 1972, doi: 10.1016/0045-7949(72)90020-X.
34. T. Liszka, J. Orkisz, The finite difference method at arbitrary irregular grids and its application in applied mechanics, *Computers & Structures*, **11**(1–2): 83–95, 1980, doi: 10.1016/0045-7949(80)90149-2.
35. L. Gavete, M.L. Gavete, J.J. Benito, Improvements of generalized finite difference method and comparison with other meshless method, *Applied Mathematical Modelling*, **27**(10): 831–847, 2003, doi: 10.1016/S0307-904X(03)00091-X.
36. T. Liszka, An interpolation method for an irregular net of nodes, *International Journal for Numerical Methods in Engineering*, **20**(9): 1599–1612, 1984, doi: 10.1002/nme.1620200905.
37. N. Perrone, R. Kao, A general finite difference method for arbitrary meshes, *Computers & Structures*, **5**(1): 45–57, 1975, doi: 10.1016/0045-7949(75)90018-8.

38. R.A. Gingold, J.J. Monaghan, Smoothed particle hydrodynamics: theory and application to non-spherical stars, *Monthly Notices of the Royal Astronomical Society*, **181**(3): 375–389, 1977, doi: 10.1093/mnras/181.3.375.
39. W.-K. Liu, S. Jun, Y.F. Zhang, Reproducing kernel particle methods, *International Journal for Numerical Methods in Fluids*, **20**(8–9): 1081–1106, 1995, doi: 10.1002/flid.1650200824.
40. E.J. Kansa, Multiquadrics – A scattered data approximation scheme with applications to computational fluid-dynamics – I surface approximations and partial derivative estimates, *Computers & Mathematics with Applications*, **19**(8–9): 127–145, 1990, doi: 10.1016/0898-1221(90)90270-T.
41. E.J. Kansa, Multiquadrics – A scattered data approximation scheme with applications to computational fluid-dynamics – II solutions to parabolic, hyperbolic and elliptic partial differential equations, *Computers & Mathematics with Applications*, **19**(8–9): 147–161, 1990, doi: 10.1016/0898-1221(90)90271-K.
42. S.A. Silling, M. Epton, O. Weckner, J. Xu, E. Askari, Peridynamic states and constitutive modeling, *Journal of Elasticity*, **88**(2): 151–184, 2007, doi: 10.1007/s10659-007-9125-1.
43. J.-S. Chen, S. Yoon, C.-T. Wu, Non-linear version of stabilized conforming nodal integration for Galerkin mesh-free methods, *International Journal for Numerical Methods in Engineering*, **53**(12): 2587–2615, 2002, doi: 10.1002/nme.338.
44. J.-S. Chen, M. Hillman, M. Rüter, An arbitrary order variationally consistent integration for Galerkin meshfree methods, *International Journal for Numerical Methods in Engineering*, **95**(5): 387–418, 2013, doi: 10.1002/nme.4512.
45. M. Hillman, J.-S. Chen, Nodally integrated implicit gradient reproducing kernel particle method for convection dominated problems, *Computer Methods in Applied Mechanics and Engineering*, **299**: 381–400, 2016, doi: 10.1016/j.cma.2015.11.004.
46. J.-S. Chen, H.-P. Wang, New boundary condition treatments in meshfree computation of contact problems, *Computer Methods in Applied Mechanics and Engineering*, **187**(3-4): 441–468, 2000, doi: 10.1016/S0045-7825(00)80004-3.
47. J. Nitsche, Über ein Variationsprinzip zur Lösung von Dirichlet-Problemen bei Verwendung von Teilräumen, die keinen Randbedingungen unterworfen sind [in German], [in:] *Abhandlungen aus dem mathematischen Seminar der Universität Hamburg*, **36**: 9–15, Springer, 1971, doi: 10.1007/BF02995904.
48. T. Belytschko, Y. Krongauz, D. Organ, M. Fleming, P. Krysl, Meshless methods: an overview and recent developments, *Computer Methods in Applied Mechanics and Engineering*, **139**(1–4): 3–47, 1996, doi: 10.1016/S0045-7825(96)01078-X.
49. S. Li, W.K. Liu, Meshfree and particle methods and their applications, *Applied Mechanics Review*, **55**(1): 1–34, 2002, doi: 10.1115/1.1431547.
50. V.P. Nguyen, T. Rabczuk, S. Bordas, M. Duflot, Meshless methods: a review and computer implementation aspects, *Mathematics and Computers in Simulation*, **79**(3): 763–813, 2008, doi: 10.1016/j.matcom.2008.01.003.
51. T. Belytschko, Y. Krongauz, J. Dolbow, C. Gerlach, On the completeness of meshfree particle methods, *International Journal for Numerical Methods in Engineering*, **43**(5): 785–819, 1998, doi: 10.1002/(SICI)1097-0207(19981115)43:5<785::AID-NME420>3.0.CO;2-9.

52. N.M. Modak, J.M. Merigó, R. Weber, F. Manzor, J. de Dios Ortúzar, Fifty years of Transportation Research journals: A bibliometric overview, *Transportation Research Part A: Policy and Practice*, **120**: 188–223, 2019, doi: 10.1016/j.tra.2018.11.015.
53. S. Das, A. Dutta, M.A. Brewer, Case study of trend mining in Transportation Research Record articles, *Transportation Research Record*, **2674**(10): 1–14, 2020, doi: 10.1177/0361198120936254.
54. A. Torayev, P.C.M.M. Magusin, C.P. Grey, C. Merlet, A.A. Franco, Text mining assisted review of the literature on Li-O₂ batteries, *Journal of Physics: Materials*, **2**(4): 044004, 2019, doi: 10.1088/2515-7639/ab3611.
55. Y. Kajikawa, J. Yoshikawa, Y. Takeda, K. Matsushima, Tracking emerging technologies in energy research: Toward a roadmap for sustainable energy, *Technological Forecasting and Social Change*, **75**(6): 771–782, 2008, doi: 10.1016/j.techfore.2007.05.005.
56. F.R. Dayeen, A.S. Sharma, S. Derrible, A text mining analysis of the climate change literature in industrial ecology, *Journal of Industrial Ecology*, **24**(2): 276–284, 2020, doi: 10.1111/jiec.12998.
57. A. Korhonen, D.Ó. Séaghdha, I. Silins, L. Sun, J. Högberg, U. Stenius, Text mining for literature review and knowledge discovery in cancer risk assessment and research, *PLoS one*, **7**(4): p. e33427, 2012, doi: 10.1371/journal.pone.0033427.
58. V. Renganathan, Text mining in biomedical domain with emphasis on document clustering, *Healthcare Informatics Research*, **23**(3): 141–146, 2017, doi: 10.4258/hir.2017.23.3.141.
59. J.J. Monaghan, Smoothed particle hydrodynamics, *Reports on Progress in Physics*, **68**(8): 1703–1759, 2005, doi: 10.1088/0034-4885/68/8/r01.
60. E. Loper, S. Bird, NLTK: the natural language toolkit, *arXiv preprint cs/0205028 [cs.CL]*, 2002.
61. M. Bastian, S. Heymann, M. Jacomy, Gephi: an open source software for exploring and manipulating networks, [in:] *Proceedings of the International AAAI Conference on Web and Social Media*, **8**: 361–362, 2009.
62. R. Alghamdi, K. Alfalqi, A survey of topic modeling in text mining, *International Journal on Advanced Computer Science Applications*, **6**(1): 147–153, 2015.
63. D.M. Blei, A.Y. Ng, M.I. Jordan, Latent Dirichlet allocation, *Journal of Machine Learning Research*, **3**(Jan): 993–1022, 2003.
64. M.J. Cobo, A.G. López-Herrera, E. Herrera-Viedma, F. Herrera, An approach for detecting, quantifying, and visualizing the evolution of a research field: A practical application to the fuzzy sets theory field, *Journal of Informetrics*, **5**(1): 146–166, 2011, doi: 10.1016/j.joi.2010.10.002.
65. J. Fern, A. Rohe, K. Soga, E. Alonso, *The Material Point Method for Geotechnical Engineering: A Practical Guide*, CRC Press, Boca Raton, 2019.
66. A. de Vaucorbeil, V.P. Nguyen, S. Sinaie, J.Y. Wu, Material point method after 25 years: Theory, implementation and applications, [in:] S.P.A. Bordas, D.S. Balint [Eds.], *Advances in Applied Mechanics*, Vol. 53, p. 185–398, 2020, doi: 10.1016/bs.aams.2019.11.001.
67. H.Y. Hu, J.S. Chen, W. Hu, Weighted radial basis collocation method for boundary value problems, *International Journal for Numerical Methods in Engineering*, **69**(13): 2736–2757, 2007, doi: 10.1002/nme.1877.

68. J.G. Wang, G.R. Liu, A point interpolation meshless method based on radial basis functions, *International Journal for Numerical Methods in Engineering*, **54**(11): 1623–1648, 2002, doi: 10.1002/nme.489.
69. G.R. Liu, G.Y. Zhang, Y.T. Gu, Y.Y. Wang, A meshfree radial point interpolation method (RPIM) for three-dimensional solids, *Computational Mechanics*, **36**(6): 421–430, 2005, doi: 10.1007/s00466-005-0657-6.
70. M. Liu, G.R. Liu, K.Y. Lam, Z. Zong, Smoothed particle hydrodynamics for numerical simulation of underwater explosion, *Computational Mechanics*, **30**(2): 106–118, 2003, doi: 10.1007/s00466-002-0371-6.

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