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Interval response data based system identification of multi storey shear buildings using interval neural network modelling

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This paper uses artificial neural network (ANN) technique for the identification of structural parameters of multistorey shear buildings. First, the identification has been done using response of the structure subject to ambient vibration with interval initial condition. Then, forced vibration with horizontal displacement in interval form has been used to investigate the identification procedure. The neural network has been trained by a methodology so as to handle interval data. This is because, in general we may not get the corresponding input and output values exactly (in crisp form) but we may only have the uncertain information of the data. These uncertain data are assumed in term of interval and the corresponding problem of system identification is investigated. The model has been developed for multistorey shear structure and the procedure is tested for the identification of the stiffness parameters of simple example problem using the prior values of the design parameters.

Keywords: identification, inverse vibration, modeling, interval neural network, shear buildings, structures.

1. Introduction

Dynamic behaviour of complicated systems often needs to be investigated by system identification, since it usually has to meet certain requirements. System identification methods in structural dynamics, in general solve inverse vibration problems to identify properties of a structure from measured data. Rapid progress in the field of computer science and the use of efficient mathematical tools allow for identification of the process dynamics by evaluating the input and output signals of the system. System identification (SI) techniques play an important role in investigating and reducing gaps between the structural systems and their structural design models. This is also true in structural health monitoring for damage detection. A great amount of research has been conducted in SI. Modal-parameter SI and physical-parameter SI are two major branches in SI. The SI methods are generally used to extract dynamic characteristics of structures, to know current behaviour of complicated systems against dynamic loads such as earthquake, etc. The SI techniques are also applied to determine vibration characteristics, modal shapes and damping ratios of complex structural systems so as to frame knowledge for modelling and assessing current design procedures. The result of such process identification is usually a mathematical model by which the dynamic behaviour can be estimated or predicted. As regards [1–6] gave various methodologies for different type of problems in system identification. Various techniques for improving structural dynamic models were reviewed in [7, 8]. Also studies related to system identification of buildings using different methods have been done by [9, 10]. Some of the related publications may be mentioned as those of [11–13]. Recent publications on system identifications were given by [14, 15]. A unique research program that investigates the dynamic behaviour of a full scale 13-storey reinforced concrete building under forced vibration, ambient vibration and distal earthquake excitation is described in [16].

When the systems are generally modeled as linear identification problem, it often turns out to be a non-linear optimization problem and difficulties are faced while calculating for a large number of parameters. So as to overcome these difficulties, researchers have developed various identification methodologies for the said problem by using powerful technique of artificial neural network (ANN). A number of studies [17, 18] and the references mentioned below have used ANN for solving structural identification problems. An application of neural networks for detection of changes in nonlinear systems has been given by [19]. Identification of substructures using neural networks has been proposed by [20]. A decentralized stiffness identification method with neural networks for a multi-degree- of- freedom structure has been developed in [21]. A localized identification strategy with neural networks and its application to structural health monitoring was proposed in [22]. In particular, [17] presented a novel procedure for identifying the dynamic characteristics of a building using a back propagation neural network technique. A new soft-computing technique to identify dynamic systems was proposed in [23]. System identification of linear structures based on Hilbert-Huang spectral analysis by using normal modes has been given in [12]. Another novel neural network based approach has been presented in [24] for detecting structural damage. Neural network based strategy was also developed in [25] for direct identification of structural parameters from the time domain dynamic responses of a structure without any eigen-value analysis. A novel procedure for identification of structural parameters of two storey shear buildings by an iterative training of neural networks was presented in [26]. A multistage identification scheme for structural damage detection with the use of modal data using a hybrid neural network strategy has been proposed in [27]. Parameter identification of torsionally coupled shear buildings from earthquake response records was given in [14].

Identification of dynamic models of a building structure using multiple earthquake records has been developed in [28]. An approach to detect structural damage using ANN method with progressive substructure zooming was presented in [29]. This method also uses the substructure technique together with a multi-stage ANN models to detect the location and extent of the damage. The application of neural networks to damage detection in structures is studied in [30]. Model updating of multistorey shear buildings for simultaneous identification of mass, stiffness and damping matrices using two different soft-computing methods have been developed in [15]. System identification using frequency response functions with the help of artificial neural networks (ANN) has been studied in [31] for single-input, single-output and multiple-input single-output (MISO) system. The application of artificial neural networks and wavelet analysis to develop the intelligent and adaptive structural damage detection system has been investigated in [32]. A structural parameter identification and damage detection approach using displacement measurement time series has been proposed in [33] and the performance of this approach has been validated experimentally with a frame structure model in a healthy condition and with joint connection damages. The approach also provides an alternative way for damage detection of engineering structures by direct use of structural dynamic displacement measurements. A process for predicting the recyclable amount of concrete and reinforcement residential buildings based on artificial neural network has been provided in [34]. It may be seen from the above that artificial neural networks (ANNs) provide a fundamentally different approach to system identification problems. They have been successfully applied for identification and control of dynamic systems in various field of engineering because of its excellent learning capacity and high tolerance to partially inaccurate data.

It is revealed from the above literature review that various authors developed different identification methodologies using ANN. They supposed that the data obtained are in exact or crisp form. But in actual practice the experimental data obtained from equipments are with errors that may be due to human or equipment errors thereby giving uncertain form of the data. Although one may also use probabilistic methods to handle such problems, but the probabilistic method requires huge quantity of data which may not be easy or feasible. As such [35] presented a robust converging mathematical procedure and applied selective sensitive excitations to identify essential structural parameters. As regards, few research works have been done using interval neural networks in different fields. [36] defined Interval neural network and categorized general three-layer neural network training problems into two types, i.e., type1 and type2 according to their mathematical model. Using these general algorithms one can develop specific software which can efficiently solve interval weighted neural network problems. These techniques can be applied to traditional non-interval neural networks as well. In this respect an algorithm for interval neural networks was presented in [37]. An application of interval valued neural networks to a regression problem has been presented in [38]. The work was concerned with exploiting uncertainty in order to develop a robust regression algorithm for a pre-sliding friction process based on a non-linear auto-regressive with exogenous inputs neural network. In addition to this, it has also shown that an interval-valued neural network allows a trade-off between the model error and the interval width of the network weights or a 'degree of uncertainty' parameter. An interval GA (Genetic Algorithm) for evolving neural networks with interval weights and biases was developed in [39], where an extension of genetic algorithm for neuro evolution of interval-valued neural networks was proposed. In order to handle the interval-valued genotypes, interval-valued GA (IvGA) extends its processes of initialization of populations, fitness evaluation, crossover and mutation. The IvGA was applied to approximate modeling of interval functions with interval-valued neural networks.

As such, the uncertainty may sometimes be modeled by considering the data in term of interval. In this paper, minimum numbers of data are taken in interval form to have the essence of the uncertainty. Although the interval data requires complex interval arithmetic to handle the problem, here a simple back propagation neural network has been used. Identification methodologies for multi-storey shear buildings have been proposed using the powerful technique of artificial neural network (ANN) models which can handle interval data. It was already mentioned that identification with crisp data is known and also neural network method has already been used by various researchers for this case. Here the input and output data may be in interval form.

In this paper, forward problem for each time step is solved for a given input to the system, rather than solving the inverse vibration problem. Thus, the solution vector is generated. The initial design parameters viz. stiffness, mass and so the responses of the said problem are known. The initial values of the physical parameters of the system are used to obtain the interval responses. Responses and the corresponding parameters are used as the input/output in the neural net. Next, the interval artificial neural network (IANN) model is trained by the proposed interval error back propagation training algorithm (IEBPTA) scheme. After training of the model, physical parameters may be identified in interval form if new maximum response data is supplied as input to the net which are also in interval form. The procedure has been demonstrated for multi-storey structures and the structural parameters are identified in interval form using the response of the structure subject to initial condition and horizontal displacement all are in interval form. Corresponding methodology is demonstrated for multistorey structure and example problem of two-storey shear structures are solved. Results are reported to show the reliability and powerfulness of the model.

2. Analysis and modeling

System identification refers to the branch of numerical analysis which uses the experimental input and output data to develop mathematical models of systems which finally identify the parameters. Let us consider a multi- degree- of- freedom system (shear building) with n storey as shown in Fig. 1. The floor masses for this application problem are assumed to be $[\underline{m}_1, \overline{m}_1]$, $[\underline{m}_2, \overline{m}_2]$, ..., $[\underline{m}_n, \overline{m}_n]$ and the stiffness parameters $[\underline{k}_1, \overline{k}_1]$, $[\underline{k}_2, \overline{k}_2]$, ..., $[\underline{k}_n, \overline{k}_n]$ are the structural parameters which are to be identified. It may be seen that all the mass and stiffness parameters are taken in interval form. As such for each mass m_i we have $\underline{m}_i$ as the lower value and $\overline{m}_i$ as the upper value of the

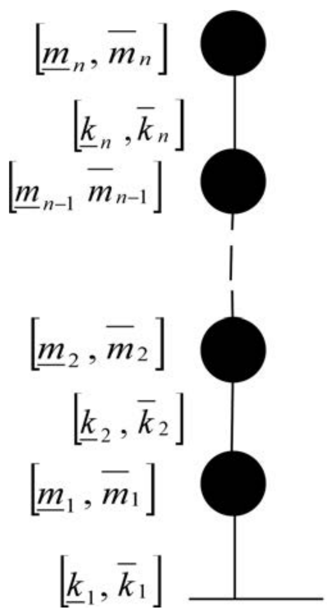


Fig. 1. Multi-storey shear structure with n-levels having interval structural parameters.

interval. Similarly for the stiffness parameter for each k_i we have $\underline{k}_i$ as the left value and $\overline{k}_i$ as the right value of the interval. Corresponding dynamic equation of motion for n-storey (supposed as n degrees of freedom) shear structure without damping [40] may be written as

$$[\widetilde{M}]\left\{\ddot{\widetilde{X}}\right\} + [\widetilde{K}]\left\{\widetilde{X}\right\} = \left\{\widetilde{F}(t)\right\}, \tag{1}$$

where $\left\{\widetilde{M}\right\} = \left[\underline{M}, \overline{M}\right]$ is $n \times n$ mass matrix of the structure, and is given by

$$\left\{\widetilde{M}\right\} = \begin{bmatrix} [\underline{m}_1, \overline{m}_1] & 0 & \dots & \dots & 0 \\ 0 & [\underline{m}_2, \overline{m}_2] & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & 0 & [\underline{m}_{n-1}, \overline{m}_{n-1}] & 0 \\ 0 & \dots & \dots & 0 & [\underline{m}_n, \overline{m}_n] \end{bmatrix},$$

$\left\{\widetilde{K}\right\} = \left[\underline{K}, \overline{K}\right]$ is $n \times n$ stiffness matrix of the structure and may be written as

$$\left\{\widetilde{K}\right\} = \begin{bmatrix} [\underline{k}_1, \overline{k}_1] + [\underline{k}_2, \overline{k}_2] & -[\underline{k}_2, \overline{k}_2] & 0 & \dots & 0 \\ -[\underline{k}_2, \overline{k}_2] & [\underline{k}_2, \overline{k}_2] + [\underline{k}_3, \overline{k}_3] & -[\underline{k}_3, \overline{k}_3] & \dots & 0 \\ \dots & \dots & & & \dots \\ 0 & \dots & -[\underline{k}_{n-1}, \overline{k}_{n-1}] & [\underline{k}_{n-1}, \overline{k}_{n-1}] + [\underline{k}_n, \overline{k}_n] & [\underline{k}_n, \overline{k}_n] \\ 0 & \dots & -[\underline{k}_n, \overline{k}_n] & [\underline{k}_n, \overline{k}_n] & \end{bmatrix},$$

$\left\{\widetilde{F}(t)\right\} = \left\{\underline{F}(t), \overline{F}(t)\right\}$ is assumed as $n \times 1$ interval horizontal displacement forcing vector

$$\left\{\widetilde{F}(t)\right\} = \begin{bmatrix} \left[\underline{f}_1(t), \overline{f}_1(t)\right] \\ \left[\underline{f}_2(t), \overline{f}_2(t)\right] \\ \cdots \\ \left[\underline{f}_n(t), \overline{f}_n(t)\right] \end{bmatrix}.$$

Let us consider that the initial conditions in interval form are given by Eqs. (2) and (3) as

$$\{\widetilde{x}(0)\} = \{\underline{x}(0), \overline{x}(0)\} = \{\widetilde{x}_1(0) \quad \widetilde{x}_2(0) \quad \cdots \quad \widetilde{x}_n(0)\}^T, \tag{2}$$

$$\left\{\dot{\widetilde{x}}(0)\right\} = \{\underline{\dot{x}}(0)\ \overline{\dot{x}}(0)\} = \left\{\dot{\widetilde{x}}_1(0) \quad \dot{\widetilde{x}}_2(0) \quad \cdots \quad \dot{\widetilde{x}}_n(0)\right\}^T. \tag{3}$$

Solution of Eq. (1) for free vibration, with given interval values of mass and stiffness, gives the corresponding interval eigenvalues and eigenvectors. These are denoted respectively by $\widetilde{\lambda}_i$ and $\left\{\widetilde{A}\right\}_i = \{\underline{A}, \overline{A}\}_i$, $i = 1, \ldots, n$ where $\widetilde{\omega}_i^2 \left(= \widetilde{\lambda}_i\right)$ are the system's interval natural frequency. It may be noted that the free vibration equation will be an interval eigenvalue problem. The interval eigenvalue and vector are obtained then by considering different sets of lower and upper stiffness and mass values. Although there exist different methods to handle interval eigenvalue problems but here the above procedure has been used so that we may handle the inverse of the matrices in crisp form separately as lower and upper value. And that is why now we will replace the '$\sim$' from all notations and will consider the case for lower form first and simultaneously for upper form. Hence, the modal matrix for lower form $\{\underline{A}\}$ may be written as

$$[\underline{A}] = [\{\underline{A}\}_1 \ \{\underline{A}\}_2 \ \cdots \ \{\underline{A}\}_n]. \tag{4}$$

Denoting the diagonal matrix made up of the eigenvalues in lower form as $\underline{\lambda}_i$, as $[\underline{\lambda}]_{n\times n}$, a new set of coordinates in lower form $\{\underline{y}\}$ related to the coordinates $\{\underline{X}\}$ is introduced by the well-known transformation

$$\{\underline{X}\} = [\underline{A}] \ \{\underline{y}\}. \tag{5}$$

If the system (1) is subjected to an initial velocity only then substituting Eq. (5) into Eq. (1) for ambient vibration, the following equation is obtained for the response in lower form as:

$$\{\underline{X}\} = [\underline{A}]\,[\underline{D}]\,[\underline{\omega}]^{-1}\,[\underline{A}]^{-1}\,\{\underline{\dot{x}}(0)\}, \tag{6}$$

whereas for the horizontal displacement in lower form we have the equation

$$\{\underline{\ddot{y}}\} + [\underline{\lambda}]\,\{\underline{y}\} = [\underline{P}]^{-1}\,[\underline{A}]^T\,\{\underline{F}(t)\}, \tag{7}$$

where

$$[\underline{P}] = [\underline{A}]^T\,[\underline{M}]\,[\underline{A}]. \tag{8}$$

In order to obtain the final response in term of the original coordinates $\{\underline{X}\}$, we solve Eq. (7) for$\underline{y}$and then it is substituted in Eq. (5). In the similar manner we can compute the upper form. The patterns are now trained using interval error back propagation training algorithm (IEBPTA) of generalized delta learning rule.

2.1. Interval computation

An interval defined on real line $\mathcal{R}$ is said to be a subset of $\mathcal{R}$. For instance, if an interval is denoted as $\mathrm{A} = [\underline{a}, \overline{a}]$ where $\underline{a}, \overline{a} \in \mathcal{R}$, $\underline{a} < \overline{a}$ we may regard this as one kind of set. Expressing the interval as membership function we may get

$$\mu_A(x) = \begin{cases} 0, & x < \underline{a}, \\ 1, & \underline{a} \le x \le \overline{a}, \\ 0, & x > \overline{a}. \end{cases}$$

If $\underline{a} = \overline{a}$, this interval indicates a point [41].

2.2. Interval arithmetic

Let us assume A and B as numbers expressed as interval. For all $\underline{a}, \overline{a}, \underline{b}, \overline{b} \in \mathcal{R}$ where $\mathrm{A} = [\underline{a}, \overline{a}]$, $\mathrm{B} = [\underline{b}, \overline{b}]$, the main operations of intervals may be written as [41],

1. addition

$$[\underline{a}, \overline{a}]\,(+)\,[\underline{b}, \overline{b}] = [\underline{a} + \underline{b}, \overline{a} + \overline{b}],$$

2. subtraction

$$[\underline{a}, \overline{a}]\,(-)\,[\underline{b}, \overline{b}] = [\underline{a} - \overline{b}, \overline{a} - \underline{b}],$$

3. multiplication

$$[\underline{a}, \overline{a}]\,(\times)\,[\underline{b}, \overline{b}] = [\min(\underline{a} \times \underline{b}, \underline{a} \times \overline{b}, \overline{a} \times \underline{b}, \overline{a} \times \overline{b}),\ \max(\underline{a} \times \underline{b}, \underline{a} \times \overline{b}, \overline{a} \times \underline{b}, \overline{a} \times \overline{b})],$$

4. division

$$[\underline{a}, \overline{a}]\,(\div)\,[\underline{b}, \overline{b}] = [\min(\underline{a} \div \underline{b}, \underline{a} \div \overline{b}, \overline{a} \div \underline{b}, \overline{a} \div \overline{b}),\ \max(\underline{a} \div \underline{b}, \underline{a} \div \overline{b}, \overline{a} \div \underline{b}, \overline{a} \div \overline{b})]$$

excluding the case $\underline{b} = 0$ or $\overline{b} = 0$.

3. Interval artificial neural network (IANN) and interval error back propagation training algorithm (IEBPTA)

Traditional ANN and EBPTA are well known but here for the sake of completeness those are developed for interval case. In ANN, the first layer is considered to be input layer and the last layer is the output layer. Between the input and output layer, there may be more than one hidden layer. Each layer will contain number of neurons or nodes (processing elements) depending upon the problem. These processing elements operate in parallel and are arranged in patterns similar to the patterns found in biological neural nets. The processing elements are connected to each other by adjustable weights. The input/output behaviour of the network changes if the weights are changed. So, the weights of the net may be chosen in such a way so as to achieve a desired output. To satisfy this goal, systematic ways of adjusting the weights have to be developed to handle the interval data which are known as training or learning algorithm as in [37]. Neural network basically depends upon the type of processing elements or nodes, the network topology and the learning algorithm. Here, interval error back propagation training algorithm and feed forward neural network has been used to handle the uncertain system. Following IANN is computed based on the interval computation defined above. The interval weights and interval biases are also computed based on above interval computations. A typical network is shown in Fig. 2.

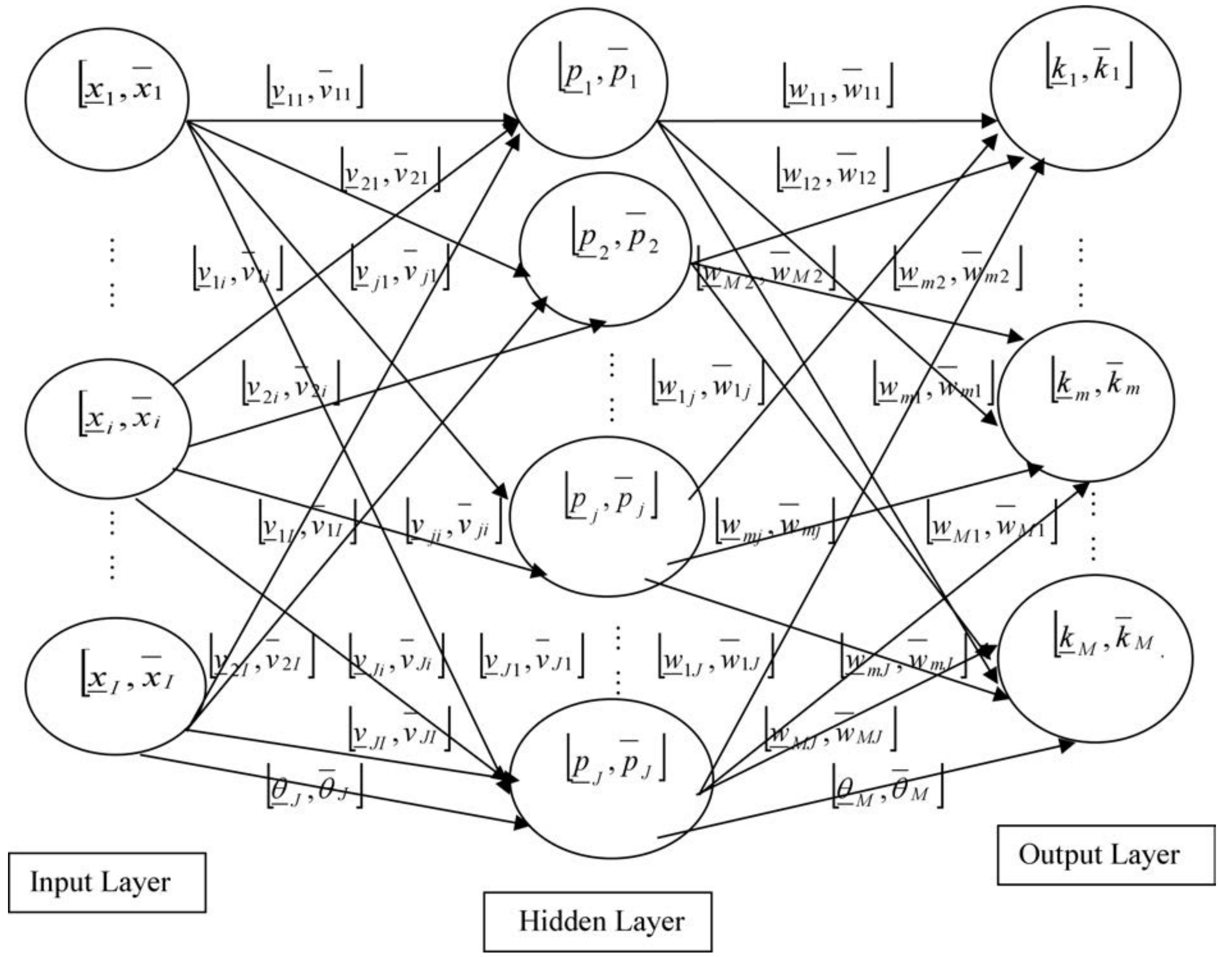


Fig. 2. Generalized feed-forward interval neural network.

In Fig. 2, $\widetilde{Z}_i$, $\widetilde{P}_j$ and $\widetilde{O}_m$ are considered to be the input, hidden and output layer respectively. The weights between input and hidden layers are denoted by $(\widetilde{v}_{ji}) = \left[\underline{v}_{ji}, \overline{v}_{ji}\right]$ and the weights between hidden and output layers are denoted by $(\widetilde{w}_{mj}) = \left[\underline{w}_{mj}, \overline{w}_{mj}\right]$ which are all in intervals. The inputs $\widetilde{Z}_i = \widetilde{X}_i = \left[\underline{X}_i, \overline{X}_i\right]$ are responses in interval and the outputs $\widetilde{O}_m = \widetilde{k}_m = \left[\underline{k}_m, \overline{k}_m\right]$ are stiffness parameters in interval.

Given $\mathcal{R}$ training pairs, $\left\{\widetilde{Z}_1, \widetilde{d}_1; \widetilde{Z}_2, \widetilde{d}_2; ..., \widetilde{Z}_R, \widetilde{d}_R\right\}$ where $\widetilde{Z}_i\,(I \times 1)$ are inputs and $\widetilde{d}_i\,(M \times 1)$ are desired values for the given inputs, the total input to the j-th hidden unit in the second layer can be calculated as

$$\widetilde{P}_j = \left[\underline{P}_j, \overline{P}_j\right] = \left[\underline{v}_{ji}, \overline{v}_{ji}\right] \cdot \left[\underline{Z}_i, \overline{Z}_i\right] + \left[\underline{\theta}_j, \overline{\theta}_j\right],$$

where

$$\underline{P}_j = \sum_{\substack{i=1 \\ \underline{v}_{ji} \geq 0}}^{I} \underline{v}_{ji}\underline{Z}_i + \sum_{\substack{i=1 \\ \underline{v}_{ji} \prec 0}}^{I} \underline{v}_{ji}\overline{Z}_i + \underline{\theta}_j, \tag{9}$$

$$\overline{P}_j = \sum_{\substack{i=1 \\ \overline{v}_{ji} \geq 0}}^{I} \overline{v}_{ji}\overline{Z}_i + \sum_{\substack{i=1 \\ \overline{v}_{ji} \prec 0}}^{I} \overline{v}_{ji}\underline{Z}_i + \overline{\theta}_j. \tag{10}$$

Here, $\left[\underline{\theta}_j, \overline{\theta}_j\right]$ are the bias weights of the hidden layer. Then the output of the hidden unit can be evaluated as

$$\widetilde{U}_j = \left[f\left(\underline{P}_j\right), f\left(\overline{P}_j\right)\right] = \left[\underline{U}_j, \overline{U}_j\right],$$

where f is the unipolar activation function defined by $f(net) = 1/\left[1 + \exp(-\gamma\, net)\right]$.

Similarly the total input from hidden to the output unit is calculated as

$$\widetilde{Y}_m = \left[\underline{Y}_m, \overline{Y}_m\right] = \left[\underline{w}_{mj}, \overline{w}_{mj}\right] \cdot \left[\underline{U}_j, \overline{U}_j\right] + \left[\underline{\theta}_m, \overline{\theta}_m\right],$$

$$\underline{Y}_m = \sum_{\substack{m=1 \\ \underline{w}_{mj} \geq 0}}^{M} \underline{w}_{mj}\underline{U}_j + \sum_{\substack{m=1 \\ \underline{w}_{mj} \prec 0}}^{M} \underline{w}_{mj}\overline{U}_j + \underline{\theta}_m, \tag{11}$$

$$\overline{Y}_m = \sum_{\substack{m=1 \\ \overline{w}_{mj} \geq 0}}^{M} \overline{w}_{mj}\overline{U}_i + \sum_{\substack{m=1 \\ \overline{w}_{mj} \prec 0}}^{M} \overline{w}_{mj}\underline{U}_j + \overline{\theta}_m, \tag{12}$$

where $\left[\underline{\theta}_m, \overline{\theta}_m\right]$ are the bias weights of the output layer. Finally, the response of the net is given as

$$\widetilde{O}_m = \left[f\left(\underline{Y}_m\right), f\left(\overline{Y}_m\right)\right] = \left[\underline{O}_m, \overline{O}_m\right].$$

The error value is computed as

$$\widetilde{E} = \frac{1}{2}\left[\left(\underline{d}_m - \underline{O}_m\right)^2 + \left(\overline{d}_m - \overline{O}_m\right)^2\right], \quad m = 1, 2, ..., M \tag{13}$$

for the present neural network as shown in Fig. 2. From the cost function (13), a learning rule can be derived for the interval weight $(\widetilde{v}_{ji})$ between the hidden and the input layer. The interval weights are updated as

$$\begin{gathered} \widetilde{v}_{ji}^{(New)} = \left[\underline{v}_{ji}^{(New)}, \overline{v}_{ji}^{(New)}\right] = \left[\underline{v}_{ji}^{(old)}, \overline{v}_{ji}^{(old)}\right] + \left[\Delta\underline{v}_{ji}, \Delta\overline{v}_{ji}\right], \\ j = 1, 2, \ldots, J \qquad \text{and} \qquad i = 1, 2, \ldots, I, \end{gathered} \tag{14}$$

where change in weights are calculated as

$$\begin{gathered} \Delta\widetilde{v}_{ji} = \left[\Delta\underline{v}_{ji}, \Delta\overline{v}_{ji}\right] = \left[-\eta\frac{\partial\widetilde{E}}{\partial\underline{v}_{ji}}, -\eta\frac{\partial\widetilde{E}}{\partial\overline{v}_{ji}}\right], \\ j = 1, 2, ..., J \qquad \text{and} \qquad i = 1, 2, ..., I. \end{gathered} \tag{15}$$

Consequently, output layer weights $(\widetilde{w}_{mj})$ between the output layer and the hidden layer are adjusted as

$$\begin{gathered} \widetilde{w}_{ji}^{(New)} = \left[\underline{w}_{ji}^{(New)}, \overline{w}_{ji}^{(New)}\right] = \left[\underline{w}_{ji}^{(old)}, \overline{w}_{ji}^{(old)}\right] + \left[\Delta\underline{w}_{ji}, \Delta\overline{w}_{ji}\right], \\ m = 1, 2, ..., M, \qquad \text{and} \qquad j = 1, 2, ..., J, \end{gathered} \tag{16}$$

where change in weights are now calculated as

$$\begin{gathered} \Delta\widetilde{w}_{mj} = \left[\Delta\underline{w}_{mj}, \Delta\overline{w}_{mj}\right] = \left[-\eta\frac{\partial\widetilde{E}}{\partial\underline{w}_{mj}}, -\eta\frac{\partial\widetilde{E}}{\partial\overline{w}_{mj}}\right], \\ m = 1, 2, ..., M, \qquad \text{and} \qquad j = 1, 2, ..., J \end{gathered} \tag{17}$$

and η is the learning constant. While modifying $\underline{v}_{ji}$, $\overline{v}_{ji}$ and $\underline{w}_{mj}$, $\overline{w}_{mj}$ by Eqs. (14)–(17), it is undesirable but possible sometimes that $\underline{v}_{ji} > \overline{v}_{ji}$ and $\underline{w}_{mj} > \overline{w}_{mj}$. In order to cope with this situation, the interval weights from input to hidden layer and from hidden to output layer are determined as

$$v_{ji}^{(New)} = \left[\min\left\{\underline{v}_{ji}^{(New)}, \overline{v}_{ji}^{(New)}\right\}, \max\left\{\underline{v}_{ji}^{(New)}, \overline{v}_{ji}^{(New)}\right\}\right], \tag{18}$$

$$w_{mj}^{(New)} = \left[\min\left\{\underline{w}_{mj}^{(New)}, \overline{w}_{mj}^{(New)}\right\}, \max\left\{\underline{w}_{mj}^{(New)}, \overline{w}_{mj}^{(New)}\right\}\right]. \tag{19}$$

In the similar fashion the interval biases $\widetilde{\theta}_j$ and $\widetilde{\theta}_m$ are also updated.

4. Results and discussion

Although the developed method has been used for different storey shear structure but here only two-storey shear structure has been reported to understand the methodology. To investigate the present method numerical experiment has been shown for two-storey lumped mass structure to identify interval stiffness parameters. So, we consider the floor masses for two-storey shear structure in interval form as $[\underline{m}_1, \overline{m}_1]$ and $[\underline{m}_2, \overline{m}_2]$. Similarly the stiffness parameter may also be written in interval form as $\left[\underline{k}_1, \overline{k}_1\right]$ and $\left[\underline{k}_2, \overline{k}_2\right]$. For the present investigation, the masses are assumed to be constant i.e., $\underline{m}_1 = \overline{m}_1$, $\underline{m}_2 = \overline{m}_2$. One may note for identifying the interval stiffness parameters, we need to have interval responses in the input nodes. In practical application due to error in measurements, we may have the response data in interval form. It is worth mentioning that the response may actually be obtained from some experiments. But here the analyses have been shown by numerical simulations only. In this respect one may also see that the procedure has been discussed with constant masses but with interval stiffness parameters. In order to get the set of data of interval responses and interval stiffness parameters, the problem has to be solved first as forward vibration problem. For this the initial design (structural) parameters in interval form are randomized [26] and training sets of initial interval stiffness parameters are generated. For the above sets of initial interval stiffness parameters, the set of corresponding responses in interval form are generated from Eq. (6) for ambient vibration and from Eq. (5) for the other case (after solving Eq. (8) for $\underline{y}$ or $\overline{y}$). Now, the mentioned neural net is trained with the interval responses that are generated from the structural parameters. When the neural net is converged (or trained) the converged neural weight matrices $\widetilde{v}_{ji}$ and $\widetilde{w}_{mj}$ for hidden and output layer are stored. In order to get the interval responses for ambient vibration problem, Eq. (6) is used and for forced vibration problem Eqs. (5) and (8) are used. The neural network training is done till a desired accuracy is achieved. Now, we will identify the stiffness parameters in interval form using the interval form of the maximum absolute response. The methodology has been discussed by giving the results for following five cases.

Case (i): ambient vibration: interval response with crisp initial condition.

Case (ii): ambient vibration: interval response with initial condition in interval form.

Case (iii): forced vibration: interval response with the forcing function in crisp form.

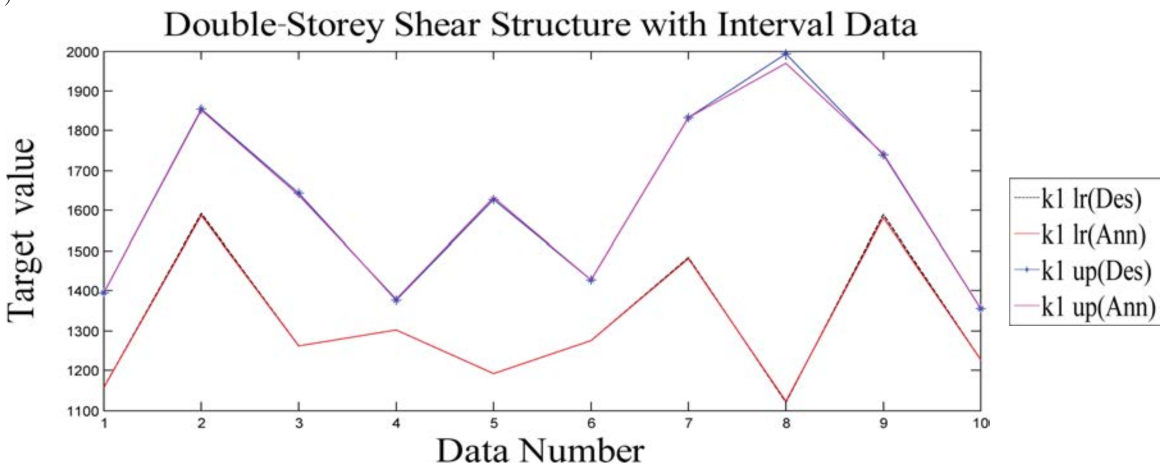
Case (iv): forced vibration: interval response with the forcing function in interval form.

Case (v): ambient and forced vibration: for testing of the method with the data which are not used (seen) in the training.

A set of computer programs have been written and tested for variety of experiments for different cases and it is a gigantic task to incorporate all the results. But few of them are reported to understand the methodology. All the parameters are taken in consistent units and the data for the initial interval stiffness parameters are considered for the academic illustrations. The input layer will have the maximum absolute interval responses for ambient as well as for forced vibration and output layer contains the corresponding interval stiffness parameters of the system. As such, the input layer will have the nodes as $\left\{\widetilde{X}_1 = [\underline{X}_1, \overline{X}_1] \quad \text{and} \quad \widetilde{X}_2 = [\underline{X}_2, \overline{X}_2]\right\}$ output layer will have the nodes as $\left\{\widetilde{k}_1 = [\underline{k}_1, \overline{k}_1] \quad \text{and} \quad \widetilde{k}_2 = [\underline{k}_2\,, \overline{k}_2]\right\}$ for two-storey shear structure. This neural network architecture has been maintained for all the cases.

As mentioned earlier for case (i), the system is subjected first to crisp initial condition expressed by the vector (with zero displacement) as $\{\dot{x}(0)\} = \{10 \quad -10\}^T$. Two examples in case (i) have been solved. For the first example, a double-storey shear structure is taken where the masses are $\underline{m}_1 = \overline{m}_1 = 1$ and $\underline{m}_2 = \overline{m}_2 = 1$ and the initial stiffness parameters are within the range $\widetilde{k}_1 = [1000,\ 2000]$ and $\widetilde{k}_2 = [1000,\ 2000]$. In the second problem the masses are taken to be the same as that of the first one and the stiffness parameters vary within the range $\widetilde{k}_1 = [2200,\ 3200]$ and $\widetilde{k}_2 = [1100,\ 2100]$. From these initial interval stiffness parameters we have generated 40 data for both stiffness and responses in interval form. These 40 numbers of data are used as training patterns. Here the input layer contains two (interval) input neurons and output layer contains two (interval) output neurons. Various numbers of hidden nodes were considered and the program was executed. After few runs it can be seen that six hidden nodes are sufficient to get the desirable result. As such for the first problem, with accuracy of 0.001, the desired and ANN results for 10 numbers of data chosen from 40 data have been plotted in Figs. 3a and 3b. For the second problem, again 10 data are summarized in Table 1.

a)



b)

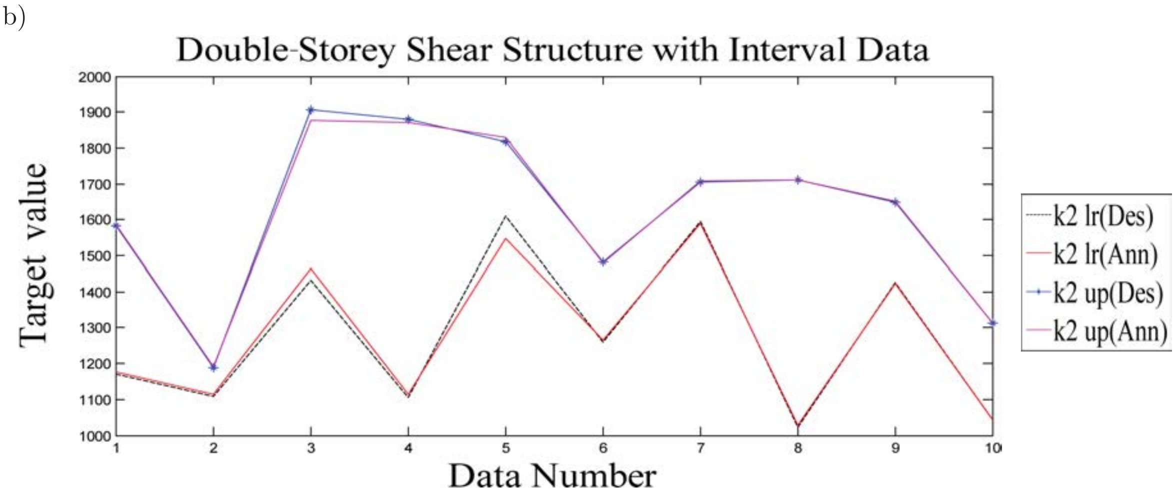


Fig. 3. Comparison of Desired and ANN value for ambient vibration with crisp initial condition for: a) $\underline{k}_1$, $\overline{k}_1$, b) $\underline{k}_2$, $\overline{k}_2$, for case (i), eg. (1).

Table 1. Comparison of Desired and ANN value for ambient vibration with crisp initial condition for $\underline{k}_1$, $\overline{k}_1$, $\underline{k}_2$, $\overline{k}_2$ for case (i), eg. (2).

	Data no									
	1	2	3	4	5	6	7	8	9	10
$\underline{k}_1$ (Ann)	2415	2300	2925	2290	2567	2575	2427	2803	2693	2361
$\underline{k}_1$ (Des)	2416	2297	2982	2286	2566	2569	2439	2798	2694	2362
Deviation [%]	0.04	−0.13	1.91	−0.17	−0.04	−0.23	0.49	−0.18	0.04	0.04
$\overline{k}_1$ (Ann)	2936	3125	3018	2398	2625	2784	2844	2848	3036	2564
$\overline{k}_1$ (Des)	2935	3171	3067	2416	2598	2762	2885	2852	2989	2568
Deviation [%]	−0.03	1.45	1.6	0.75	−1.04	−0.8	1.42	0.14	−1.57	0
$\underline{k}_2$ (Ann)	1401	1202	1393	1340	1637	1193	1495	1210	1215	1874
$\underline{k}_2$ (Des)	1402	1201	1394	1337	1631	1191	1505	1205	1212	1884
Deviation [%]	0.07	−0.08	0.07	−0.22	−0.37	−0.17	0.66	−0.41	−0.25	0.53
$\overline{k}_2$ (Ann)	1890	1710	2013	1517	1845	1878	1555	1758	1227	2004
$\overline{k}_2$ (Des)	1882	1714	2074	1542	1805	1868	1543	1765	1220	2044
Deviation [%]	−0.43	0.23	2.94	1.62	−2.22	−0.54	−0.78	0.4	−0.57	1.96

In case (ii), two problems have been solved for two-storey shear structure. Here, the system is subjected to initial condition expressed by the vector (with zero displacement) in interval form as $\left\{\underline{\dot{x}(0)}\ \ \overline{\dot{x}(0)}\right\} = \{(8,\ 10)\quad(-10,\ -8)\}^T$. The masses are kept constant for both the problems and are taken as $\underline{m}_1 = \overline{m}_1 = 1$ and $\underline{m}_2 = \overline{m}_2 = 1$. The initial interval stiffness parameter for the first problem is considered as $\widetilde{k}_1 = [1000,\ 2000]$ and $\widetilde{k}_2 = [1000,\ 2000]$ and for the second example the initial interval stiffness parameters are taken as $\widetilde{k}_1 = [2200,\ 3200]$ and $\widetilde{k}_2 = [1100,\ 2100]$. In this case, 50 numbers of data for both responses and structural parameters are generated from these initial interval stiffness parameters. The neural network architecture is similar to case (i). Again various numbers of hidden nodes are taken as per the desired accuracy and finally eight hidden nodes are found to be sufficient to get an accuracy of 0.001. After training with 50 numbers of data, we incorporate 10 numbers of data for comparison of the desired and ANN values for the first problem in Table 2. For the second problem, comparison between the desired and ANN values for 10 data chosen from 50 numbers of data are plotted in Figs. 4a and 4b.

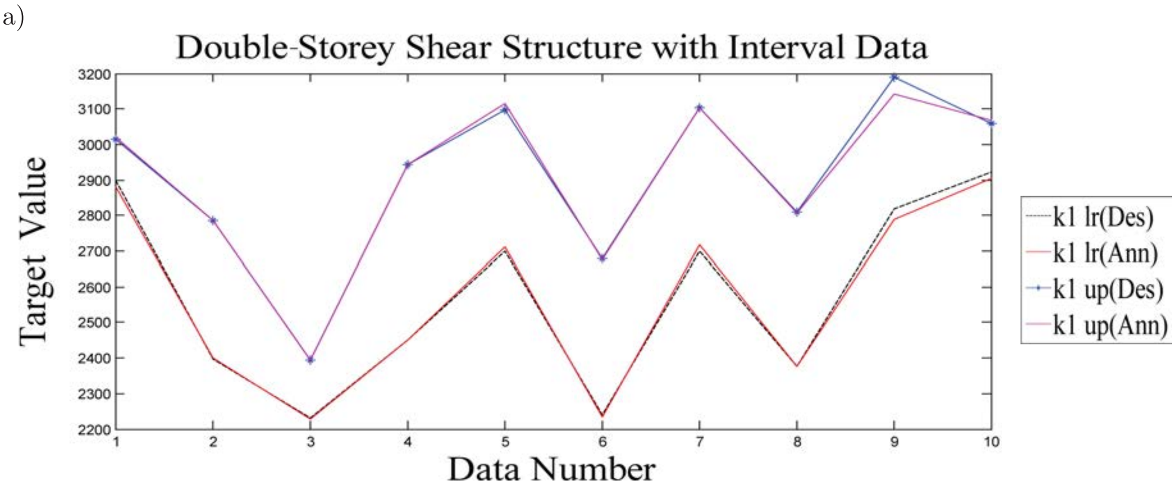
Similarly for the problem with the considered horizontal displacement function, the identification of interval stiffness from interval responses with zero initial condition and the forcing function in crisp form are considered in case (iii). The forcing function vector in crisp form is defined as $F(t) = \{100\sin(1.6\pi t + \pi)100\sin(1.6\pi t)\}^T$. Again, two problems have been considered for this case. The initial interval stiffness parameter used to train the first problem have values as $\widetilde{k}_1 = [2000,\ 3000]$ and $\widetilde{k}_2 = [1000,\ 2000]$ and for the second problem, the initial interval stiffness parameters are considered as $\widetilde{k}_1 = [2200,\ 3200]$ and $\widetilde{k}_2 = [1100,\ 2100]$. The masses are kept constant as that of the above cases. Here, 60 data for both responses and stiffness parameters have been generated using these initial interval structural parameters. These 60 data are used for training with 10 hidden nodes so as to get an accuracy of 0.001. After training, 10 data chosen from 60 data are again plotted in Figs. 5a and 5b in order to compare the desired and ANN values for the first problem. Similarly the results for second example are included in Table 3.

Next, in case (iv) the forcing function vector in interval form with zero initial condition is defined as $\widetilde{F}(t) = \{(80\sin(1.6\pi t + \pi)\ ,\ 100\sin(3.2\pi t + \pi))\quad(80\sin(1.6\pi t)\ ,\ 100\sin(3.2\pi t))\}^T$. Again two problems have been solved considering the masses and stiffnesses as the previous cases. Here, 80 data

Table 2. Comparison of Desired and ANN value for ambient vibration with interval initial condition for $\underline{k}_1$, $\overline{k}_1$, $\underline{k}_2$, $\overline{k}_2$ for case (ii), eg. (1).

	Data no									
	1	2	3	4	5	6	7	8	9	10
$\underline{k}_1$ (Ann)	1144	1451	1063	1349	1513	1420	1075	1223	1129	1184
$\underline{k}_1$ (Des)	1145	1427	1060	1351	1513	1402	1076	1240	1123	1184
Deviation [%]	0.09	−1.68	−0.28	0.15	0	−1.28	0.09	1.37	−0.53	0
$\overline{k}_1$ (Ann)	1251	1836	1617	1953	1930	1493	1496	1360	1850	1378
$\overline{k}_1$ (Des)	1250	1853	1622	1913	1955	1501	1499	1348	1910	1379
Deviation [%]	−0.08	0.92	0.31	−2.09	1.28	0.53	0.2	−0.89	3.14	0.07
$\underline{k}_2$ (Ann)	1401	1202	1393	1340	1637	1193	1495	1210	1215	1874
$\underline{k}_2$ (Des)	1402	1201	1394	1337	1631	1191	1505	1205	1212	1884
Deviation [%]	0.07	−0.08	0.07	−0.22	−0.37	−0.17	0.66	−0.41	−0.25	0.53
$\overline{k}_2$ (Ann)	1890	1710	2013	1517	1845	1878	1555	1758	1227	2004
$\overline{k}_2$ (Des)	1882	1714	2074	1542	1805	1868	1543	1765	1220	2044
Deviation [%]	−0.43	0.23	2.94	1.62	−2.22	−0.54	−0.78	0.4	−0.57	1.96

a)



b)

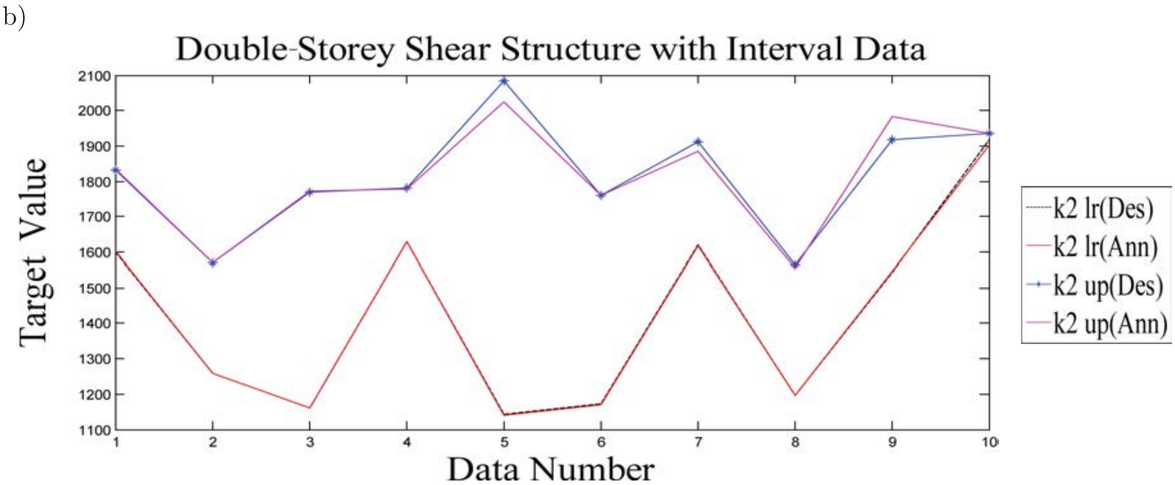
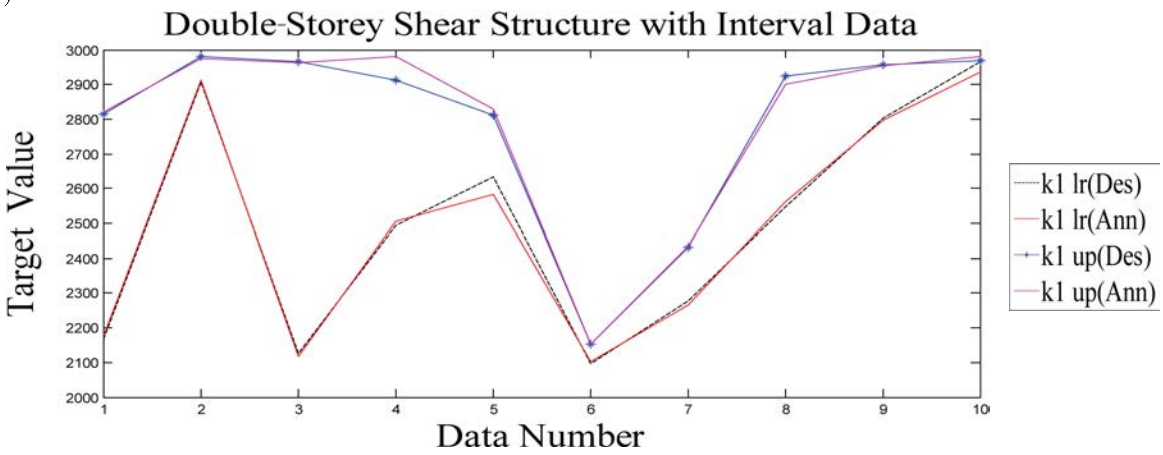


Fig. 4. Comparison of Desired and ANN value for ambient vibration with interval initial condition for: a) $\underline{k}_1$, $\overline{k}_1$, b) $\underline{k}_2$, $\overline{k}_2$, for case (ii), eg. (2).

a)



b)

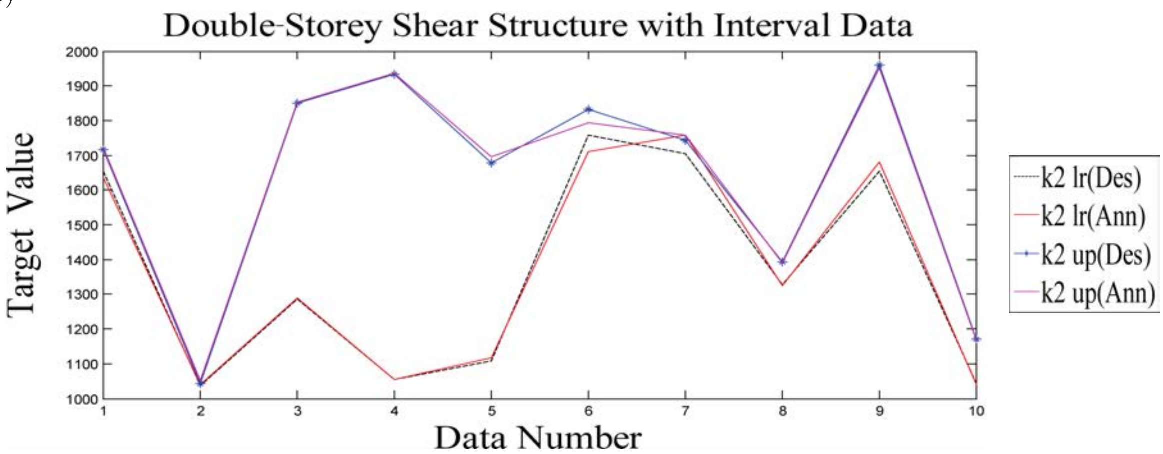


Fig. 5. Comparison of Desired and ANN value for forced vibration with crisp forcing function for: a) $\underline{k}_1$, $\overline{k}_1$, b) $\underline{k}_2$, $\overline{k}_2$, for case (iii), eg. (1).

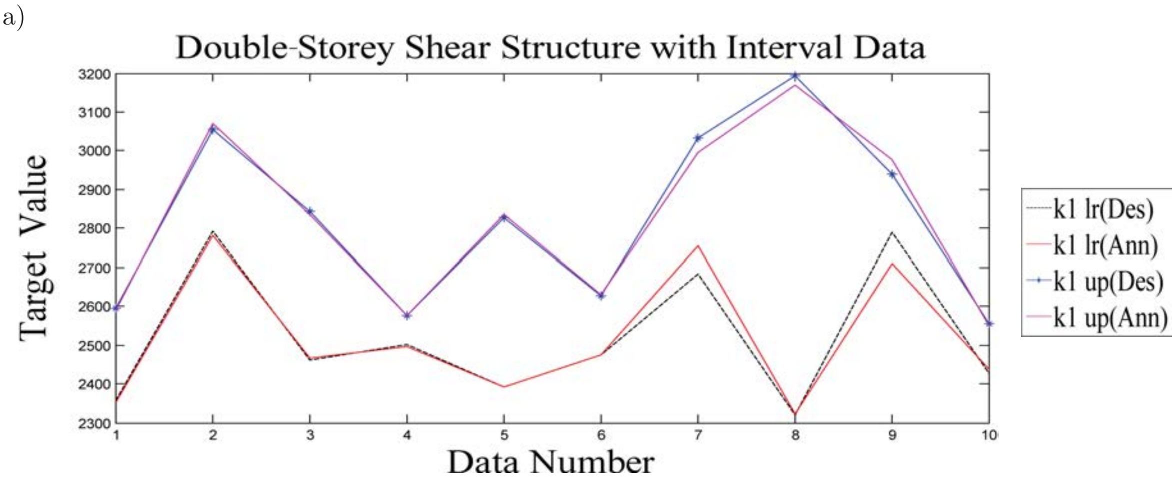
Table 3. Comparison of Desired and ANN value for forced vibration with crisp forcing function for $\underline{k}_1$, $\overline{k}_1$, $\underline{k}_2$, $\overline{k}_2$ for case (iii), eg. (2).

	Data no									
	1	2	3	4	5	6	7	8	9	10
$\underline{k_1}$ (Ann)	1070	1242	1357	1246	1022	1066	1121	1693	1729	1574
$\underline{k_1}$ (Des)	1070	1245	1363	1242	1025	1053	1132	1659	1742	1575
Deviation [%]	0	0.24	0.44	−0.32	0.29	−1.23	0.97	−2.05	0.75	0.06
$\overline{k_1}$ (Ann)	1111	1798	1401	1812	1407	1085	1183	1898	1939	1657
$\overline{k_1}$ (Des)	1111	1780	1390	1831	1404	1096	1179	1942	1956	1658
Deviation [%]	0	−1.01	−0.79	1.04	−0.21	1	−0.34	2.27	0.87	0.06
$\underline{k_2}$ (Ann)	1161	1330	1455	1337	1123	1145	1220	1821	1772	1684
$\underline{k_2}$ (Des)	1170	1345	1463	1342	1125	1153	1232	1759	1842	1675
Deviation [%]	0.77	1.12	0.55	0.37	0.18	0.69	0.97	−3.52	3.8	−0.54
$\overline{k_2}$ (Ann)	1211	1888	1494	1944	1514	1195	1282	2043	2018	1767
$\overline{k_2}$ (Des)	1211	1880	1490	1931	1504	1196	1279	2042	2056	1758
Deviation [%]	0	−0.43	−0.27	−0.67	−0.66	0.08	−0.23	−0.05	1.85	−0.51

Table 4. Comparison of Desired and ANN value for forced vibration with crisp forcing function for $\underline{k}_1$, $\overline{k}_1$, $\underline{k}_2$, $\overline{k}_2$ for case (iii), eg. (2).

	Data no									
	1	2	3	4	5	6	7	8	9	10
$\underline{k}_1$ (Ann)	2155	2416	2118	2338	2505	2357	2076	2198	2157	2155
$\underline{k}_1$ (Des)	2145	2427	2060	2351	2513	2402	2076	2240	2123	2184
Deviation [%]	−0.47	0.45	−2.82	0.55	0.32	1.87	0	1.88	−1.6	1.33
$\overline{k}_1$ (Ann)	2251	2879	2625	2955	2954	2499	2497	2362	2863	2370
$\overline{k}_1$ (Des)	2250	2853	2622	2913	2955	2501	2499	2348	2910	2379
Deviation [%]	−0.04	−0.91	−0.11	−1.44	0.03	0.08	0.08	−0.6	1.62	0.38
$\underline{k}_2$ (Ann)	1070	1240	1370	1244	1029	1051	1132	1689	1714	1571
$\underline{k}_2$ (Des)	1070	1245	1363	1242	1025	1053	1132	1659	1742	1575
Deviation [%]	0	0.4	−0.51	−0.16	−0.39	0.19	0	−1.81	1.61	0.25
$\overline{k}_2$ (Ann)	1105	1757	1370	1835	1397	1101	1180	1913	1931	1692
$\overline{k}_2$ (Des)	1111	1780	1390	1831	1404	1096	1179	1942	1956	1658
Deviation [%]	0.54	1.29	1.44	−0.22	0.5	−0.46	−0.08	1.49	1.28	−2.05

a)



b)

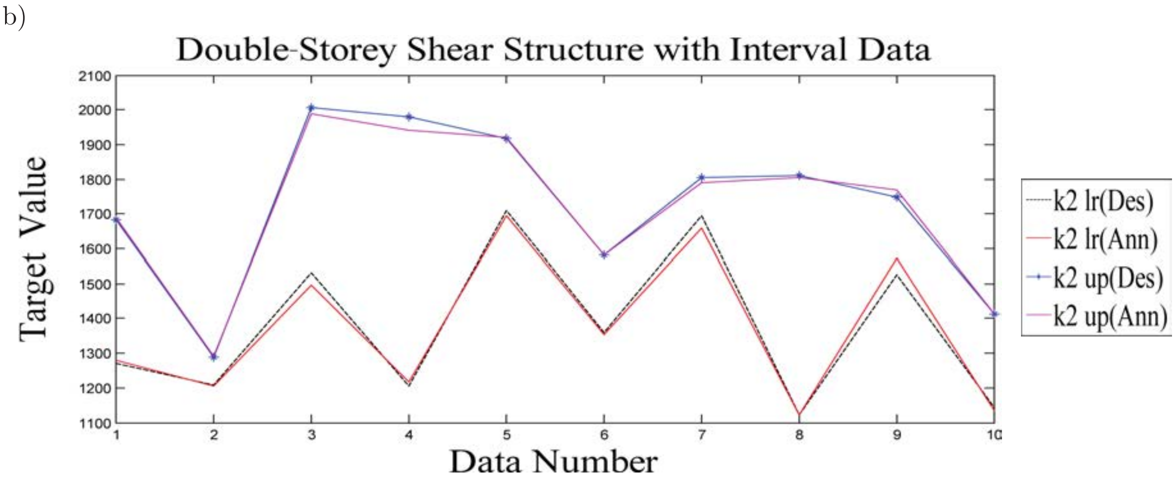


Fig. 6. Comparison of Desired and ANN value for forced vibration with interval forcing function for: a) $\underline{k}_1$, $\overline{k}_1$, b) $\underline{k}_2$, $\overline{k}_2$, for case (iv), eg. (2).

are used to train with 15 hidden nodes so as to get an accuracy of 0.001. Comparison between the desired and ANN values for 10 data chosen from 80 numbers of data have been incorporated in Table 4 for first problem. Similarly the results for second problem have been plotted in Figs. 6a and 6b.

Finally in case (v), two examples for testing the data which are not used (seen) during the training are considered for both ambient and forced vibration. These test data are fed into the neural network along with the stored (converged) weights to generate corresponding stiffness parameters. For the first problem, interval response with initial condition in interval form and for second problem, the interval responses with the forcing function in interval form are considered for testing. Here 10 numbers of data are taken for testing using the stored converged weights of training. Comparison between the test values of desired and ANN for ambient vibration with the initial condition in interval form for 10 numbers of data are plotted in Figs. 7a and 7b. Again comparison between the test values of desired and ANN for forced vibration with the forcing function for 10 data in interval form have been plotted in Figs. 8a and 8b.

It may be seen that the neural results are comparable with the desired and the deviations in percentage between them have also been shown in all the tables.

a)

b)

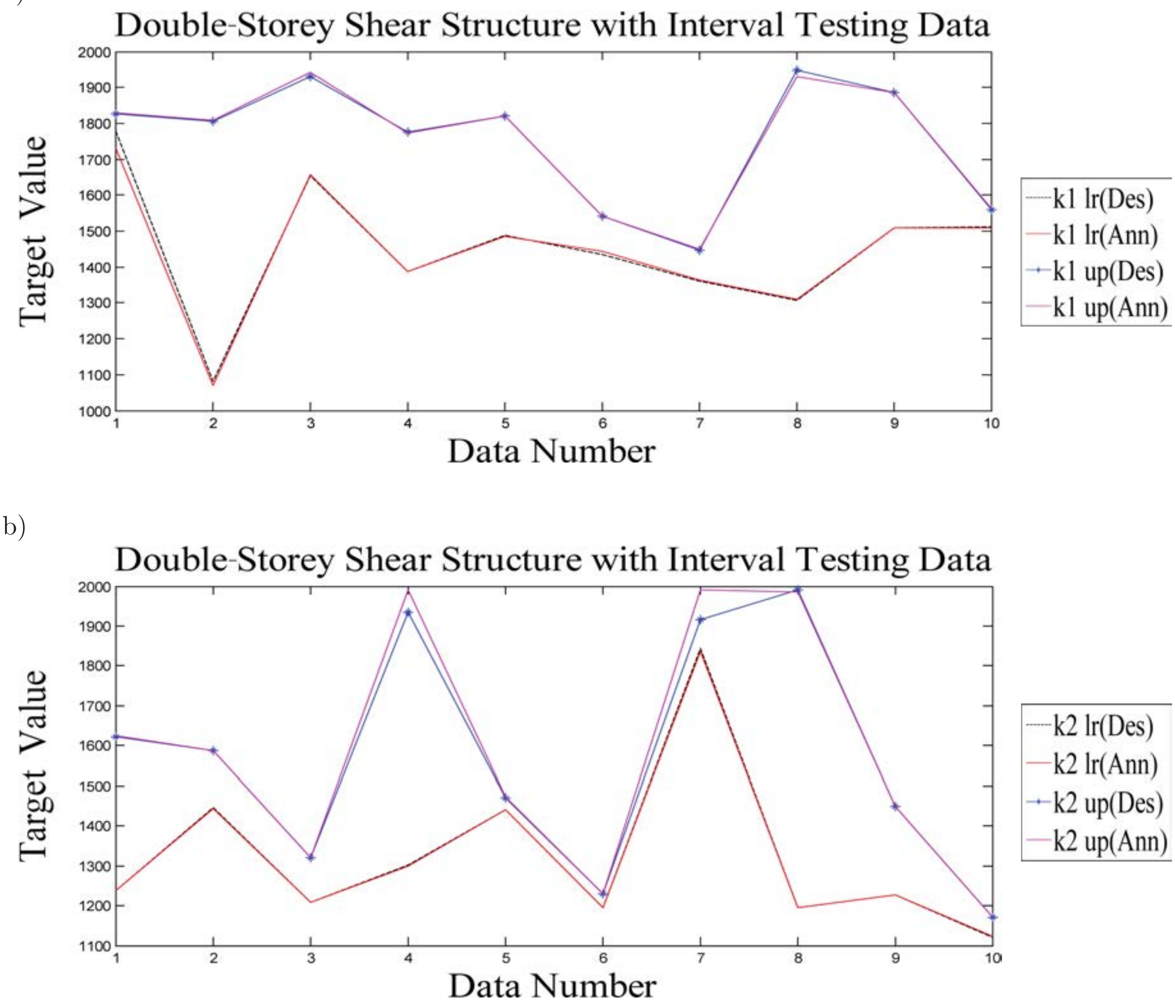


Fig. 7. Comparison of Desired and ANN value of testing data for ambient vibration with interval initial condition for: a) $\underline{k}_1$, $\overline{k}_1$, b) $\underline{k}_2$, $\overline{k}_2$, for case (v), eg. (1).

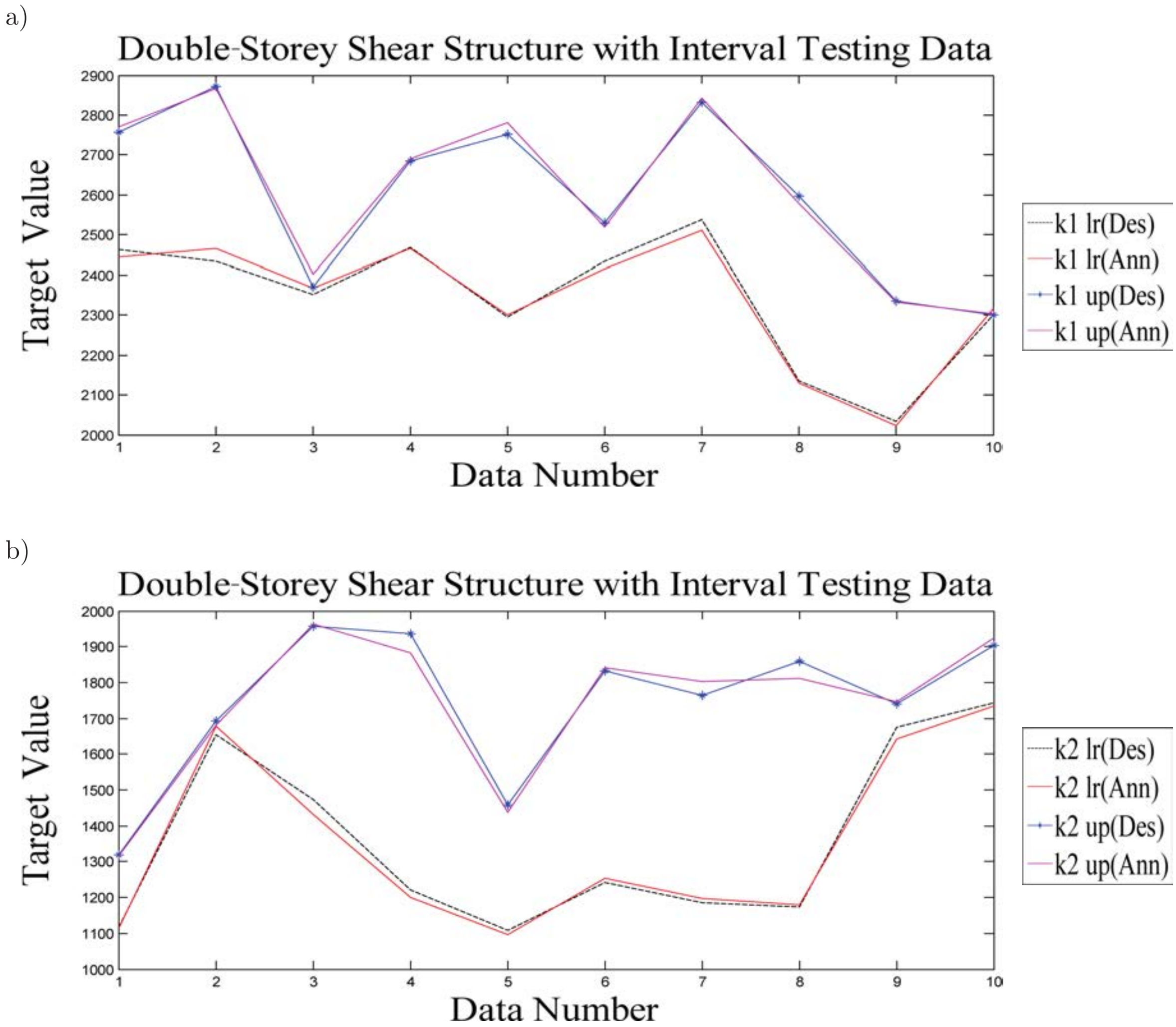


Fig. 8. Comparison of Desired and ANN value of testing data for forced vibration with interval forcing function for: a) $\underline{k}_1$, $\overline{k}_1$, b) $\underline{k}_2$, $\overline{k}_2$, for case (v), eg. (2).

5. Conclusion

Protection of various structures against the effect of earthquake is an interdisciplinary research where the knowledge, skills and experience of earthquake along with structural engineers assisted by architects, art historians, material scientists and applied mathematicians are required. Health monitoring, system identification, theoretical and experimental assessment of structural performance, design, testing and implementation of retrofit are some of the main steps of any modern earthquake protection methodology for conservation of structures. As such after a long span of time, the structures deteriorate due to application of various manmade and natural hazards. So, it is a challenging task to know the present health of the above structures to avoid failure. Hence, the present study demonstrates application of IANN with solution of forward vibration problem for the identification of structural parameters of multi-storey shear buildings utilizing only design parameters of the system by a proposed IANN methodology. The present study considers example problems of two-storey shear structure with the use of interval artificial neural network (IANN) models which can handle interval data. It is assumed that only the stiffness is changed and the mass remains the same. The values of the responses in interval form may be obtained by available experiments and using these, one may get the parameter values by IANN. Although to train the new ANN model, set of data are generated numerically beforehand. As such converged IANN model gives the present stiffness parameter values in interval form for each floor. Thus one may predict the health of the structure in interval form from the knowledge of the identified stiffness parameters in interval form. Corresponding example problems have been solved and related results are reported to show the reliability and powerfulness of the model.

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