Computer Assisted Methods in Engineering and Science, **21**: 5–14, 2014.

A semi-analytical method for identification of thin elastic plate parameters basing on LWM

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A new semi-analytical method, discussed in the presented paper, is composed of two stages. Stage A corresponds to the direct analysis, in which the Lamb Waves Measurements (LWM) technique enables obtaining an experimental set of points $\mathcal{D}(f^j,\ k^j)_{j=1}^J$, where f and k are frequency and wavenumber, respectively. After the preprocessing in the transform space an experimental approximate curve $\widetilde{k}_{\mathrm{exp}}(f\,|\,\mathcal{D})$ can be formulated. In Stage B the identification procedure is simulated as a sequence of direct analyses. The dimensionless Lamb Dispersion curves are computed by means of the dimensionless simulation curve $k_{\mathrm{sim}}(f\,|\,\mathbf{par})$, where the vector of plate parameters $\mathbf{par} = \{E, \nu, d, \rho\}$ is adopted, in which Young's modulus E, Poisson ratio ν, plate thickness d and density ρ are used. The main idea of the proposed approach is similar to that in the classical method of error minimization. In our paper we propose to apply the zero error value of relative criterion $Reky = 0$, cf. formula (15). The formula can be applied for the identification of a single plate parameter, assuming a fixed value of the other plate parameters. This approach was used in a case study, in which Stages A and B were analysed for an aluminum plate.

Keywords: Structure Health Monitoring, non-destructive method, Lamb waves, dispersion curve, modes of vibration, elastic isotropic and homogenous plate, identification of plate parameters.

1. Introduction

A new approach in engineering, called Structure Health Monitoring (SHM) has been under increasing development for some time now. SHM deals with structures and various processes, closely related to the life and maintenance of a variety of engineering structures. An important role for SHM is played by systems which on the base of monitoring or measurements can reflect the actual state (health) of structures. This permits the control the structure and warning against failures or dangerous events. Non-destructive methods of structure examination and 'on line' methods of information transmission are especially valuable for SHM, see [5, 7].

From among non-destructive methods, the application of ultrasonic waves is worth emphasizing for the evaluation of material properties and detection of various defects [12]. In what follows we are discussing a comparatively simple problem of the application of the Lamb Waves (LWs) propagation in thin, elastic and homogeneous plates, cf. [6, 13]. These waves are guided in the vibration plane, perpendicular to the plate midsurface and they are propagated along a comparatively long distance.

The LW equations make it possible to formulate the Dispersion Curve (DC) which can be written as a relation $k(f)$, where k is the wavenumber and f is the frequency of vibrations. Unfortunately, the implicit formulation of DCs as mentioned above is analytically impossible so only numerical methods can be applied. In the presented paper we are addressing the problem of identification of 'a priori' unknown plate properties, i.e. we are interested in the analysis of DCs written in a schematic form $k(f|\mathbf{par})$, where f – an independent variable, $\mathbf{par} = \{E, \nu, \rho, d\}$ – vector of given values of plate parameters: Young's modulus E, Poisson ratio ν, plate density and thickness ρ and d, respectively.

An approach commonly applied to the identification of vector **par** components is to find experimentally a $\mathrm{DC}_{\mathrm{exp}}$ and simulate numerically a corresponding $\mathrm{DC}_{\mathrm{sim}}$. Then a measure of distance $\|k_{\mathrm{sim}} - k_{\mathrm{exp}}\|$ is minimized applying different computer methods for variations of **par** components.

There is a variety of numerical methods supported on FEM, BEM and FDM. What seems especially numerically efficient are their modifications and combinations, see e.g. Local Interaction Simulation Approach/ Sharp Interface Model (LISA/SIM), Elastodynamic Finite Integration Technique (EFIT), cf. [2], Semi-Analytical FE (SEFE), cf. [10] and application of Spectral FEs [6]. Such approaches are rather "costly" since they need several iterations, related to a great number of operations.

A novelty of the presented paper lies in the formulation of the $\mathrm{DC}_{\mathrm{exp}}$ by the application of the method supported on the basis functions with a set of basis parameters. Then a dimensionless form of dispersion curves $\mathrm{DC}_{\mathrm{sim}}$ was adopted in order to formulate the zero value relative error *Rey* as a measure of distance between $\mathrm{DC}_{\mathrm{sim}}$ and $\mathrm{DC}_{\mathrm{exp}}$. The application of such a criterion enabled identification of the value of single plate parameters, fixing other parameters.

The presented approach was numerically proved on a case study in which the testing and computational processes are discussed.

2. LWS IN ELASTIC, HOMOGENEOUS AND ISOTROPIC THIN PLATES

2.1. Some basics on LWs

The 3D waves in elastic solids can be reduced to the analysis of 2D Lamb waves (LWs), guided in the Lamb plane, see Fig. 1. The plane of vibration (x_1, x_3) is perpendicular to the tested plate of the midsurface at (x_1, x_2) with dimensions $L \times d$ and plate thickness $d = 2h$.

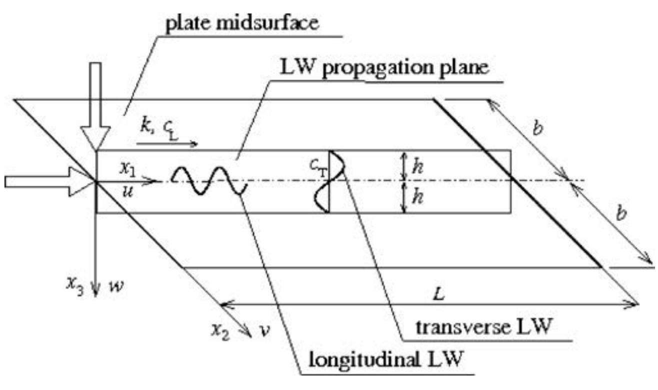


Fig. 1. Lamb wave propagation plane with longitudinal and transverse/shear LWs.

The following assumptions are adopted:

i) Plain strain at the LWs plane
i.e. the displacement v disappears and

$$v(\mathbf{x}) = 0 \quad \text{for all the points } \mathbf{x} \text{ of the LGW plane;} \tag{1}$$

ii) upper and lower plate stress free surfaces:

$$\sigma_{33} = \sigma_{31} = 0 \quad \text{at} \quad x_3 = \pm h. \tag{2}$$

There is the superposition of the longitudinal and transverse LWs. Applying the Helmholtz decomposition, the governing motion equations can be written in the following form:

$$\frac{\partial^2 \Phi}{\partial x_1^2} + \frac{\partial^2 \Phi}{\partial x_2^2} = \frac{1}{c_L^2}\frac{\partial^2 \Phi}{\partial t^2}, \tag{3$_1$}$$

$$\frac{\partial^2 \Psi}{\partial x_1^2} + \frac{\partial^2 \Psi}{\partial x_2^2} = \frac{1}{c_T^2}\frac{\partial^2 \Psi}{\partial t^2}, \tag{3$_2$}$$

where longitudinal and transverse velocities are written as:

$$c_L = \sqrt{\frac{E(1-\nu)}{2\rho(1+\nu)(1-2\nu)}} \qquad \text{for longitudinal modes,} \tag{4$_1$}$$

$$c_T = \sqrt{\frac{E}{2\rho(1+\nu)}} \qquad \text{for transverse modes.} \tag{4$_2$}$$

In formulas (4) the plate parameters E, ν, ρ and d are written.

The potentials correspond to the following functions:

$$\Phi = (A_1 \sin px_3 + A_2 \cos px_3)\exp[i(kx_1 - \omega t)], \tag{5$_1$}$$

$$\Psi = (B_1 \sin qx_3 + B_2 \cos qx_3)\exp[i(kx_1 - \omega t)], \tag{5$_2$}$$

where besides the parameters p and q:

$$p^2 = \frac{\omega^2}{c_L^2} - k^2, \qquad q^2 = \frac{\omega^2}{c_T^2} - k^2, \tag{6}$$

the factors A_1, A_2, B_1, B_2 are related to boundary conditions (2). The other parameters are: k – wavenumber, $\omega = 2\pi f$ – angular velocity ω related to frequency f.

From the condition of nonzero values of the factors A_1, A_2, B_1, B_2 the following, two well known equations of LWs can be derived. They correspond to symmetric and anti-symmetric modes S and A, see Fig. 2.

$$\text{S:} \quad \frac{\operatorname{tg} q\,h}{tg\,p\,h} = -\frac{4k^2qp}{(k^2-q^2)^2}, \tag{7S}$$

$$\text{A:} \quad \frac{\operatorname{tg} q\,h}{tg\,p\,h} = -\frac{(k^2-q^2)^2}{4k^2qp}. \tag{7A}$$

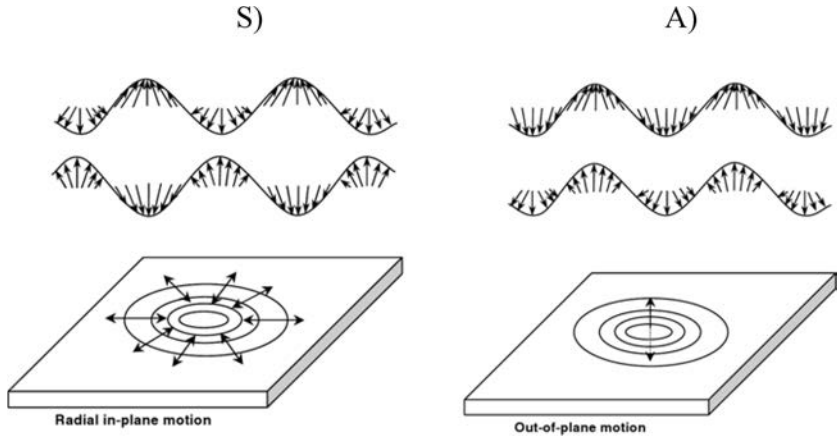


Fig. 2. LW modes a) Symmetric (radial in-plane motion), b) Anti-symmetric mode (out of-plane motion).

3. Dimensionless Lamb equations

3.1. Derived form of dimensionless dispersion equations

Instead of Eqs. (7) commonly applied, Armikulova in her M.Sc. Thesis [4] formulated the dimensionless Lamb dispersion equations in the implicit form:

$$\text{(S):}\ (\xi^2 - y^2)^2 \sin x \cos y + 4xy\xi^2 \cos x \sin y = 0, \tag{8S}$$

$$\text{(A):}\ (\xi^2 - y^2)^2 \sin y \cos x + 4xy\xi^2 \cos y \sin x = 0, \tag{8A}$$

where the following dimensionless variables are used:

$$x = (\Omega^2 - \xi^2)^{1/2}, \qquad y = (\Omega^2 \kappa^{-2} - \xi^2)^{1/2}, \tag{9}$$

using the dimensionless frequency Ω, wavenumber ξ and ratio of velocities κ:

$$\Omega = \omega h / c_T, \qquad \xi = kh, \qquad \kappa = c_L / c_T. \tag{10}$$

In [4], the power series and Poisson ratio ν as a small parameter were applied for the analysis of the dimensionless DCs. These curves can be simulated numerically. The plots of dimensionless DCs for the S_0 and A_0 modes are shown in Fig. 3 for the fixed value $\nu = 0.25$.

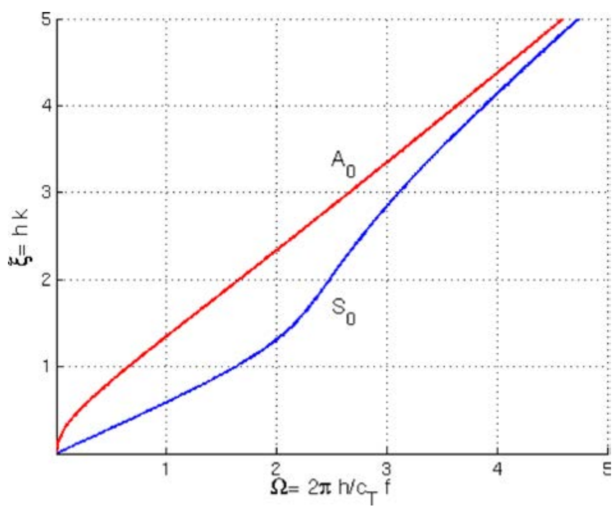
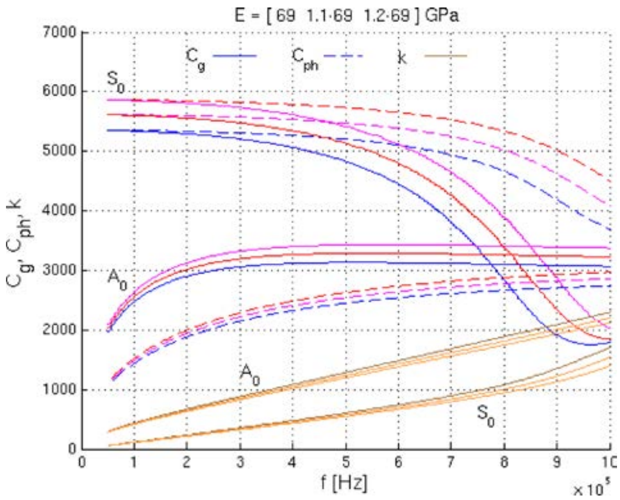


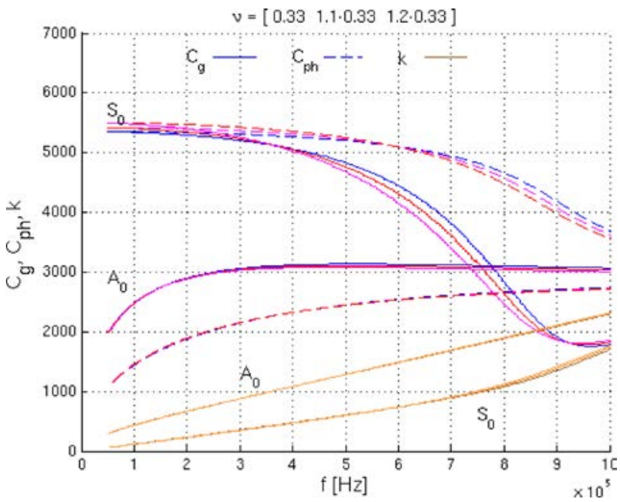
Fig. 3. Dimensionless dispersion curves for Lamb modes A_0 and B_0.

The function $\Omega(\xi)$ was derived in an analytical form, but in further computations its inverse form $\xi(\Omega)$ is needed, see Fig. 4. The corresponding curve was formulated numerically for discrete points, due to one-to-one correspondence of coordinates $(\xi,\, \Omega)$.

a)



b)



c)

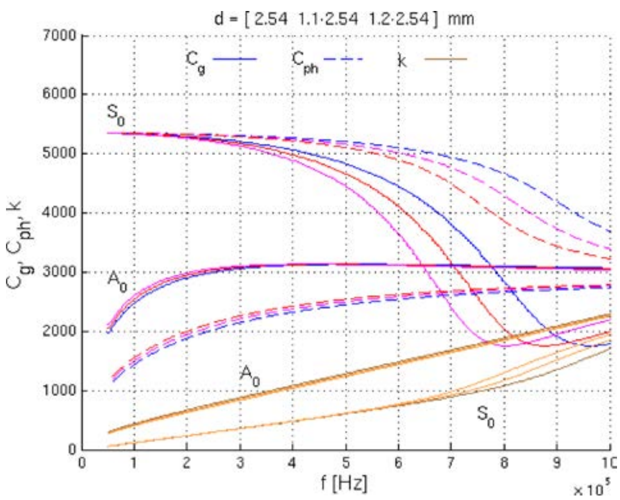


Fig. 4. Families of curves for 10% and 20% changes of LDC of $E_{\text{ref}} = 69.0$ GPa, $\nu_{\text{ref}} = 0.33$, $d = 2.54$ mm, $\rho_{\text{ref}} = 2700$ kg/m and for $f \in (0, 1.0]$ MHz.

3.2. Application of dimensionless Lamb Dispersion curves for drawing LDCs for changes of material parameters

Besides the wavenumber k, also two additional forms of wave physical velocities, are used:

1) k [m^{-1}] – wavenumber, $(11)_1$

2) c_{ph} [m/s] $= \omega/k = f/(2\pi k)$ – phase velocity, $(11)_2$

3) c_g [m/s] $= \partial\omega/\partial k$ – group velocity. $(11)_3$

Dimensionless Lamb Dispersion curves can be easily applied for drawing the LDCs for different physical velocities. Adopting a reference set of plate parameters $\{E,\ \nu,\ d,\ \rho\}_{\mathrm{ref}}$, the sensitivity of Lamb equations to changes of selected parameters can be evaluated. Figures 4a, 4b, 4c, corresponding to the families of dispersion curves DC(E), DC(ν), DC(d), were computed for 10% and 20% changes of E_{ref}, ν_{ref}, d_{ref}. The values of a reference aluminum plate parameter were taken from [10]. Following the paper mentioned above only A_0 and B_0 modes were selected for the frequencies range $f \in (0, 1.0]$ MHz.

4. Identification of plate parameters

The total analysis can be divided into two general stages: **Stage A** (Direct analysis) and **Stage B** (Inverse analysis).

4.1. Stage A

Stage A (Direct analysis) corresponds to three essential steps discussed in [11]. These steps are related to carrying out a laboratory test with excitation and propagation of the LWs (Essential Step I). The signals are transmitted from sensors and preprocessed in the Essential Step II. Then, in Step III the time signals are transformed into 2-B scans. Applying 2D-FFT and searching local maxima in the transform space the points corresponding to different vibration modes points at dispersion curves can be found [1]. Thus, the experimental set of points can be formulated:

$$\mathcal{D} = (f^j, k^j|m), \tag{11}$$

where $j = 1, 2, \ldots, J$ – numbers of dispersion points, m – number of vibration mode.

Having data set (11), an approximate experimental curve can be formulated and parameters of these curves are computed by means of the Least Square Method (LSM):

$$\widetilde{k}_{\mathrm{exp}}(f\,|\mathbf{BF}_{\mathrm{ref}}(f)\ ,\ \alpha,\ \mathcal{D}_{\mathrm{exp}}) \xrightarrow{\text{LMS}} \widetilde{\alpha}_{\mathrm{exp}}, \tag{12}$$

where $\widetilde{\alpha}_{\mathrm{exp}}$ – vector of the approximate, experimental curve parameters. The vector of reference basis functions $\mathbf{BF}_{\mathrm{ref}}(f)$ is found by means of numerical analysis. In the identification analysis only selected modes of vibrations are used, e.g. the basic modes A_0 and S_0, see [3], or even a single mode A_0 is recommended in [12].

4.2. Stage B

In the presented paper, **Stage B** of computer simulations is related to the Essential Step IV in [11]. In this step the inverse analysis is carried out in a classical way. This means that a sequence of direct analyses is iteratively made. In order to do it the Lamb $\mathrm{DC}_{\mathrm{sim}}$ are computed using the dimensionless equations and procedures taken from [4]. The $\mathrm{LDC_{sim}}$ for a selected plate parameter can be written in the shortened form:

$$k^J(f^J|y^r,\mathbf{spar}),\quad \text{for} \quad j=1,2,\ldots,J \quad \text{and} \quad r=1,2,\ldots,R, \tag{13}$$

where y^r – selected plate parameter at the r-th iteration step, **spar** – vector of plate stored (fixed) parameters, others than y^r. For example if $y^r = E^r$ than $\mathbf{spar} = \{\nu_{\text{fix}},\ \rho_{\text{fix}},\ d_{\text{fix}}\}$.

Then a range of searched parameters is adopted, i.e. $y^r \in (y^r_{\min},\ y^r_{\max})$, where y^r is the identified plate parameter. The range is uniformly divided into S subranges to have S points for estimating the 0-value relative error Rky for $y^r_{\min} \le y^r_{\text{ident}} \le y^r_{\max}$. The adopted relative error measure for the wavenumber Rky is:

$$Rek(y^r) = \frac{1}{J}\sum_{j=1}^{J}(k^j(y^r)/\widetilde{k}^j_{\exp} - 1) \times 100\%. \tag{14}$$

Then we explore the zero value criterion $Reky = 0$ for estimating an identified value of the plate parameter y_{ident}:

$$Rek(y^r) = 0 \overset{\text{reg falsi}}{\longrightarrow} y^r_{\text{ident}}. \tag{15}$$

The algorithm sketched above needs an evaluation of the selected parameter range. This can be easily done by looking at the changes of the error (14) signs. Then, we can adopt a corresponding range and end the iteration by finding the zero value of the function $Rek(y^r) = 0$ by the simple "regula falsi" algorithm.

5. Case study

5.1. A selected case study

A number of test studies was carried out by Ł. Ambrozinski in years 2011–2013. From among them we adopted the data discussed in [3, 8]. In Fig. 5a, 5b we show plots corresponding to the Essential Step III, described in [11].

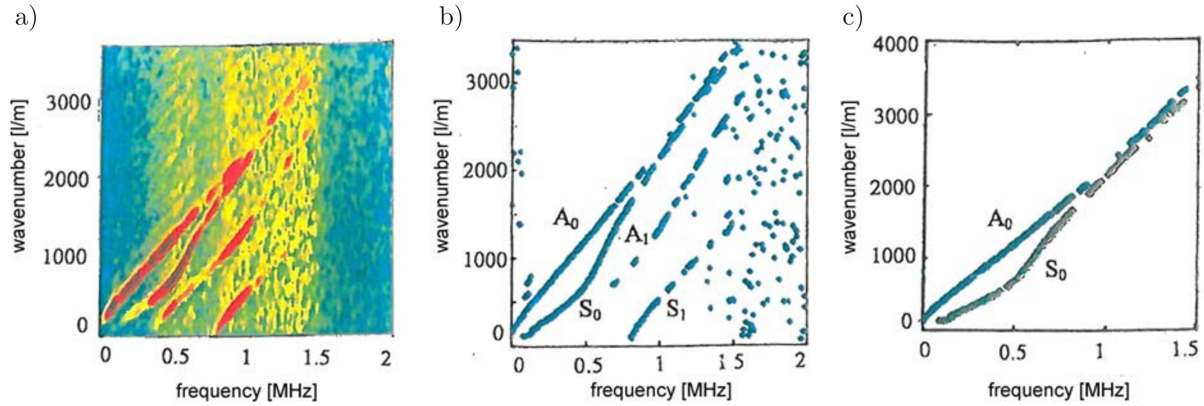


Fig. 5. Experimental $\mathrm{DC_{exp}}$ for the aluminum plate of thickness $d = 4.0$ mm: a) experimental points, b) points found for local maxima at the bit plane, c) selection of vibration modes for the numerical analysis.

According to suggestions from the book [12] we focus on a single vibration mode A_0. Then following the carried out tests and recommendations in [10] our analysis was restricted to the lower range of frequencies $f \in [0.05,\ 1.0]$ MHz, see Fig. 6. It was shown that for higher values of $f > 0.8$ MHz a number of noisy points appears. Altogether $J = 1700$ points were selected taking into account also noisy patterns. For the cloud of these points the Least Square Method was applied, cf. relations (12). On the base of extensive numerical analysis the following five basis functions were found:

$$\widetilde{k}_{\text{exp}} = \boldsymbol{\alpha}^T_{(1\times5)} \cdot \mathbf{BF}_{(5\times1)} = \alpha_1 + \alpha_2 f + \alpha_3 f^2 + \alpha_4(1/f) + \alpha_5 \text{th}(0.65f), \tag{16}$$

where the computed values of $\widetilde{\alpha}_{i_{\text{exp}}}$ equal:

$$\widetilde{\alpha}_{1\,\text{exp}} = 183.5, \quad \widetilde{\alpha}_{2\,\text{exp}} = 194.4, \quad \widetilde{\alpha}_{3\,\text{exp}} = 1.00, \quad \widetilde{\alpha}_{4\,\text{exp}} = -22.0, \quad \widetilde{\alpha}_{5\,\text{exp}} = 24.9. \tag{17}$$

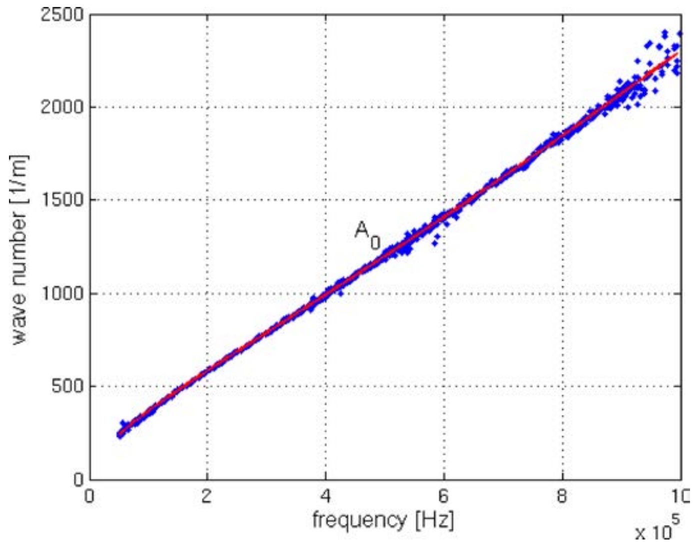


Fig. 6. A part of dispersion curve, mode A_0 taken from [3].

5.2. Discussion of numerical results

The formulated experimental curve (16) is a base for computing the relative error (14). At the beginning, the evaluation of elastic modulus is made, adopting the nominal values of aluminum plate parameters from [3]. The corresponding values are: $E = 68.0$ GPa, $\nu = 0.3$, $d = 4.0$ mm and $\rho = 2700$ kg/m^3. Assuming the range $E \in (64.0,\ 70.0)$ MHz and adopting the values of other plate parameters as equal to nominal values, the 'regula falsi' method was applied. After three iterations the identification by *Reky* criterion gave:

$$E^{\text{Rek}E}_{\text{ident}} = 65.915 \text{ GPa}. \tag{18}$$

The approach discussed above can be applied to the identification of other plate parameters. For instance, if we want to identify the Poisson ratio $\nu \in (0.26,\ 0.32)$ or $d \in (3.5,\ 4.5)$ mm, then the computed values of these parameters ν equal:

$$\begin{aligned} &\nu^{\text{Rek}\nu}_{\text{ident}} = 0.29991 \quad \text{for} \quad E_{\text{fix}} = 65.915 \text{ GPa}, \quad d_{\text{fix}} = 4.0 \text{ mm}, \quad \rho_{\text{fix}} = 2700 \text{ kg/m}^3, \\ &\nu_{\text{fix}} = 0.29991 \quad \text{for} \quad E_{\text{fix}} = 65.915 \text{ GPa}, \quad d^{\text{Rek}d}_{\text{ident}} = 4.00052 \text{ mm}, \quad \rho_{\text{fix}} = 2700 \text{ kg/m}^3. \end{aligned} \tag{19}$$

The nominal values of plate parameters mentioned above were used for the identification of plate parameters applying the LINSA method, cf. [3]. After 5–7 iterations it was found that:

$$E^{\text{LINSA}}_{\text{ident}} = 65.45 \text{ GPa} \quad \text{for nominal values of other parameters.} \tag{20}$$

At the end of this discussion let us assume that the fixed value of elasticity modulus equals (20). Then, applying the *Reky* criterion we can identify ν parameter:

$$E_{\text{fix}} = E_{\text{ident}}^{\text{LINSA}} = 65.45 \text{ GPa}, \quad \nu_{\text{ident}}^{\text{Rek}\nu} = 0.2908, \quad d_{\text{fix}} = 4.0 \text{ mm}, \quad \rho_{\text{fix}} = 2700 \text{ kg/m}^3. \tag{21}$$

The application of the SAFE method in [10] needed 13–17 iterations for the identification of E and ν.

A conclusion from the obtained results (21) is that the decrease of the modulus E of about 0.7% influences the decrease of the parameter ν of about 3.0% if the *Reky* criterion method is applied. Such a relation can be caused by non linear relations between E and ν.

It is worth emphasizing that if the Finite Element Method was applied for the elastic modulus E and ratio ν, the identification process (other fixed plate parameters are written in Fig. 4) needed 13–17 iterations. From the point of view of the iteration numbers to fulfil the Lamb equations, the LINSA method seems to be better than the application of SAFE method code for the identification of E^{SAFE}.

6. Some general remarks and final conclusions

1. A semi-analytical method, applied in the presented paper, is supported on three basic algorithms: i) Construction of an experimental approximate curve $\widetilde{k}_{\text{exp}}(f|\mathcal{D}_{exp})$ on the base of the data set $\mathcal{D}_{exp}$, taken from the LW measurement results; ii) Semi-analytical simulation of the Lamb dispersion curve $k(f|\mathcal{D}_{exp})$, computed by means of the procedure formulated for the dimensionless DC and applying procedure taken from [4]; iii) The identification of a selected plate parameter y^r is made by fixing values of other plate parameters in the *Reky* criterion method. This method corresponds to zero value of the error measure function $Rey(y^r) = 0$, in which the identified value of parameter y^r_{ident} is computed by the 'regula falsi' method.

2. The proposed approach is in fact supported on the analysis of a sequence of direct problems, but the corresponding identification criterion *Reky* seems to be very simple and does not need complex algorithms.

3. The problem of sensitivity of plate parameters on perturbation of other parameters has been only marked in Point 5.2 of the presented paper. This problem seems to be very difficult for the analysis since the corresponding optimizing process should be carried out in 4D space with many minima.

4. It follows from the experience of the first two authors of this paper that the application of the Artificial Neural Networks could be very promising [14]. That is why a corresponding paper with the application of ANNs to the plate parameters identification has been submitted for publication in a scientific journal [15].

Acknowledgements

The financial support of the Polish National Science Center, Grant No. UMO-2011/01/B/ ST8/07210, AGH No.18.18.130.384, is gratefully acknowledged.

The authors of the present paper are very grateful to Prof. T. Uhl, supervisor of the grant, for his constructive comments.

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