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Application of sensitivity analysis in microscale heat transfer

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In the paper, the thin metal film subjected to the ultrashort laser pulse has been analyzed. The heat conduction in the domain considered has been described by two-temperature model consisting of the system of two coupled parabolic equations determining the electron and lattice temperatures. The sensitivity analysis of electron and lattice temperatures with respect to the parameters appearing in mathematical description has been discussed. In particular, the changes of temperatures due to the changes of coupling factor G and the film thickness L have been estimated. At the stage of numerical computations in a case of basic as well as sensitivity problems solutions the explicit scheme of finite difference method has been used. In the final part of the paper the results of computations have been shown.

Keywords: microscale heat transfer, two-temperature model, sensitivity analysis, finite difference method.

1. Introduction

The differences between the macroscopic heat conduction equation that is based on the Fourier law and the models describing the microscale heat transfer occur because of the extremely short duration, extreme temperature gradients and very small geometrical dimensions of domain considered [11]. Microscale heat transfer can be described by many different models, in particular: the Cattaneo-Vernotte equation, the dual phase lag model, the two-temperature models and the Boltzmann equation. In this paper, the parabolic two-temperature model is presented. This model involves two energy equations determining the heat exchange in the electron gas and metal lattice. Sensitivity analysis of the problem discussed allows to estimate the influence of parameters perturbations on the course of thermal processes in the domain considered. In this article, the direct approach of sensitivity analysis [6] is applied (the sensitivity model results from the differentiation of energy equations and boundary-initial conditions with respect to parameter considered). In particular, the changes of electrons and phonons temperatures due to the coupling factor G and film thickness L perturbations have been estimated. In the final section the examples of computations are presented and the conclusions are formulated.

2. Formulation of the problem

A thin film of thickness L is considered. A surface $x = 0$ is irradiated by an ultrashort laser pulse (Fig. 1). The temporal and spatial evolution of the lattice and electrons temperatures in the irradiated metal is described by equations [1, 2]

$$C_e(T_e)\frac{\partial T_e(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[\lambda_e(T_e,T_l)\frac{\partial T_e(x,t)}{\partial x}\right] - G\left[T_e(x,t) - T_l(x,t)\right] + Q(x,t) \tag{1}$$

and

$$C_l \frac{\partial T_l(x,t)}{\partial t} = \lambda_l \frac{\partial^2 T_l(x,t)}{\partial x^2} + G\left[T_e(x,t) - T_l(x,t)\right], \tag{2}$$

where C_e (T_e), C_l are the volumetric specific heats of the electrons and lattice, respectively, $\lambda_e(T_e, T_l)$, λ_l are the thermal conductivities, G is the electron-phonon coupling factor related to the rate of the energy exchange between electrons and lattice. The internal heat source Q (x, t) connected with the laser action is given as [2]

$$Q(x,t) = \sqrt{\frac{\beta}{\pi}} \frac{1-R}{t_p \delta} I_0 \exp\left[-\frac{x}{\delta} - \beta \frac{(t-2\,t_p)^2}{t_p^2}\right], \tag{3}$$

where I_0 is the laser intensity, t_p is the characteristic time of laser pulse, δ is the optical penetration depth, R is the reflectivity of the irradiated surface and $\beta = 4\ln 2$.

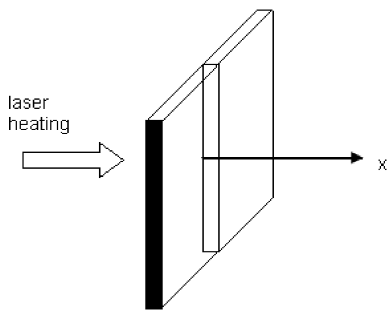

Fig. 1. Thin film.

Taking into account the short period of laser heating, heat losses from the front and the back surfaces of the thin film can be neglected [2], this means

$$q_e(0,t) = q_e(L,t) = q_l(0,t) = q_l(L,t) = 0, \tag{4}$$

where q_e (x, t), q_l (x, t) are the heat fluxes for electron and lattice systems, respectively. The initial conditions are in the form

$$t = 0\colon T_e(x,0) = T_l(x,0) = T_p. \tag{5}$$

To define the thermal conductivity λ_e and heat capacity C_e of electrons the following formulas are widely used [2, 7, 9, 10]

$$\lambda_e(T_e, T_l) = \lambda_0 \frac{T_e}{T_l} \tag{6}$$

and

$$C_e(T_e) = \gamma T_e, \tag{7}$$

where λ_0, γ are the material constants and C_l, λ_l (see, for example, Eq. (2)) are constant values.

It should be pointed out that the simple form of dependences (6), (7) and constant values of C_l and λ_l are only suitable for low laser intensity [7].

3. Sensitivity analysis – direct approach

To estimate the changes of electrons and phonons temperatures due to the parameter considered (e.g., coupling factor G, layer thickness L) the direct approach of sensitivity method is applied [4, 6]. This approach is connected with the differentiation of governing equations with respect to the parameter considered.

So, the differentiation of Eqs. (1) and (2) with respect to the coupling factor G gives

$$\begin{aligned}\frac{\mathrm{d}\,C_e(T_e)}{\mathrm{d}\,T_e}\frac{\partial\,T_e}{\partial\,G}\frac{\partial\,T_e}{\partial\,t} + C_e(T_e)\frac{\partial}{\partial\,t}\left(\frac{\partial\,T_e}{\partial\,G}\right) \\ = \frac{\partial}{\partial\,x}\left[\left(\frac{\partial\,\lambda_e(T_e,\,T_l)}{\partial\,T_e}\frac{\partial\,T_e}{\partial\,G} + \frac{\partial\,\lambda_e(T_e,\,T_l)}{\partial\,T_l}\frac{\partial\,T_l}{\partial\,G}\right)\frac{\partial\,T_e}{\partial\,x} + \lambda_e(T_e,\,T_l)\frac{\partial}{\partial\,x}\left(\frac{\partial\,T_e}{\partial\,G}\right)\right] \\ -(T_e - T_l) - G\left(\frac{\partial\,T_e}{\partial\,G} - \frac{\partial\,T_l}{\partial\,G}\right)\end{aligned} \tag{8}$$

and

$$C_l\frac{\partial}{\partial\,t}\left(\frac{\partial\,T_l}{\partial\,G}\right) = \lambda_l\frac{\partial^2}{\partial\,x^2}\left(\frac{\partial\,T_l}{\partial\,G}\right) + (T_e - T_l) + G\left(\frac{\partial\,T_e}{\partial\,G} - \frac{\partial\,T_l}{\partial\,G}\right). \tag{9}$$

Introducing

$$U_e = \frac{\partial\,T_e}{\partial\,G}, \qquad U_l = \frac{\partial\,T_l}{\partial\,G} \tag{10}$$

into Eqs. (8) and (9) one can obtain

$$\begin{aligned}\frac{\mathrm{d}\,C_e(T_e)}{\mathrm{d}\,T_e}U_e\frac{\partial\,T_e}{\partial\,t} + C_e(T_e)\frac{\partial\,U_e}{\partial\,t} \\ = \frac{\partial}{\partial\,x}\left[\left(\frac{\partial\,\lambda_e(T_e,\,T_l)}{\partial\,T_e}U_e + \frac{\partial\,\lambda_e(T_e,\,T_l)}{\partial\,T_l}U_l\right)\frac{\partial\,T_e}{\partial\,x} + \lambda_e(T_e,\,T_l)\frac{\partial\,U_e}{\partial\,x}\right] \\ -(T_e - T_l) - G\,(U_e - U_l)\end{aligned} \tag{11}$$

and

$$C_l\frac{\partial\,U_l}{\partial\,t} = \lambda_l\frac{\partial^2 U_l}{\partial\,x^2} + (T_e - T_l) + G\,(U_e - U_l)\,. \tag{12}$$

After some mathematical operations one obtains

$$\begin{aligned}C_e(T_e)\frac{\partial\,U_e}{\partial\,t} &= \frac{\partial}{\partial\,x}\left(\lambda_e\frac{\partial\,U_e}{\partial\,x}\right) - G\,(U_e - U_l) \\ &\quad + \frac{\partial}{\partial\,x}\left[(\lambda_{e,e}U_e + \lambda_{e,l}U_l)\frac{\partial\,T_e}{\partial\,x}\right] - (T_e - T_l) - C_{e,e}U_e\frac{\partial\,T_e}{\partial\,t}, \\ C_l\frac{\partial\,U_l}{\partial\,t} &= \lambda_l\frac{\partial^2 U_l}{\partial\,x^2} + (T_e - T_l) + G\,(U_e - U_l)\,,\end{aligned} \tag{13}$$

where

$$C_{e,e} = \frac{\mathrm{d}\,C_e(T_e)}{\mathrm{d}\,T_e}, \qquad \lambda_{e,e} = \frac{\partial\,\lambda_e(T_e,\,T_l)}{\partial\,T_e}, \qquad \lambda_{e,l} = \frac{\partial\,\lambda_e(T_e,\,T_l)}{\partial\,T_l}. \tag{14}$$

Additionally, the differentiation of boundary and initial conditions with respect to the coupling factor G gives

$$\frac{\partial\, q_e(0,t)}{\partial\, G} = \frac{\partial\, q_e(L,\ t)}{\partial\, G} = \frac{\partial\, q_l(0,\ t)}{\partial\, G} = \frac{\partial\, q_l(L,\ t)}{\partial\, G} = 0, \tag{15}$$

$$t = 0\colon U_e(x,0) = U_l(x,0) = 0. \tag{16}$$

Summing up, the equations (13) supplemented by boundary and initial conditions (15), (16) create the additional problem connected with the sensitivity analysis of temperature fields T_e and T_l with respect to the coupling factor G.

In the case of shape parameter L (film thickness) the concept of material derivative [4, 6] is used

$$\frac{\mathrm{D}T}{\mathrm{D}L} = \frac{\partial T}{\partial L} + \frac{\partial T}{\partial x}v, \tag{17}$$

where $v = v(x, L)$ is the velocity associated with design parameter L.

From Eq. (17) it follows that

$$\frac{\mathrm{D}}{\mathrm{D}L}\left(\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial L}\left(\frac{\partial T}{\partial x}\right) + \frac{\partial^2 T}{\partial x^2}v = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial L}\right) + \frac{\partial^2 T}{\partial x^2}v \tag{18}$$

and

$$\frac{\partial}{\partial x}\left(\frac{\mathrm{D}T}{\mathrm{D}L}\right) = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial L} + \frac{\partial T}{\partial x}v\right) = \frac{\partial}{\partial x}\left(\frac{\mathrm{D}T}{\mathrm{D}L}\right) + \frac{\partial^2 T}{\partial x^2}v + \frac{\partial T}{\partial x}\frac{\partial v}{\partial x} \tag{19}$$

therefore

$$\frac{\mathrm{D}}{\mathrm{D}L}\left(\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\mathrm{D}T}{\mathrm{D}L}\right) - \frac{\partial T}{\partial x}\frac{\partial v}{\partial x}. \tag{20}$$

In similar way the following can be achieved:

$$\frac{\mathrm{D}}{\mathrm{D}L}\left(\frac{\partial T}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\mathrm{D}T}{\mathrm{D}L}\right). \tag{21}$$

Using formula (20) one can obtain

$$\frac{\mathrm{D}}{\mathrm{D}L}\left(\frac{\partial^2 T}{\partial x^2}\right) = \frac{\mathrm{D}}{\mathrm{D}L}\left[\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)\right] = \frac{\partial}{\partial x}\left[\frac{\mathrm{D}}{\mathrm{D}L}\left(\frac{\partial T}{\partial x}\right)\right] - \frac{\partial^2 T}{\partial x^2}\frac{\partial v}{\partial x} \tag{22}$$

and next

$$\begin{aligned}\frac{\mathrm{D}}{\mathrm{D}L}\left(\frac{\partial^2 T}{\partial x^2}\right) &= \frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left(\frac{DT}{DL}\right) - \frac{\partial T}{\partial x}\frac{\partial v}{\partial x}\right] - \frac{\partial^2 T}{\partial x^2}\frac{\partial v}{\partial x} \\ &= \frac{\partial^2}{\partial x^2}\left(\frac{\mathrm{D}T}{\mathrm{D}L}\right) - 2\frac{\partial^2 T}{\partial x^2}\frac{\partial v}{\partial x} - \frac{\partial T}{\partial x}\frac{\partial^2 v}{\partial x^2}.\end{aligned} \tag{23}$$

It is also easy to check that the calculations of material derivative of the product or quotient of two functions are the same as in the case of ordinary derivatives.

The formulas presented above are necessary to accomplish the shape sensitivity analysis. So, differentiation of equations (1) and (2) with respect to the shape parameter L gives [8]

$$\frac{\mathrm{D}C_e(T_e)}{\mathrm{D}L}\frac{\partial T_e}{\partial t} + C_e(T_e)\frac{\mathrm{D}}{\mathrm{D}L}\left(\frac{\partial T_e}{\partial t}\right) = \frac{\mathrm{D}}{\mathrm{D}L}\left[\frac{\partial}{\partial x}\left[\lambda_e(T_e, T_l)\frac{\partial T_e}{\partial x}\right]\right] - G\left(\frac{\mathrm{D}T_e}{\mathrm{D}L} - \frac{\mathrm{D}T_l}{\mathrm{D}L}\right) + \frac{\mathrm{D}Q}{\mathrm{D}L} \tag{24}$$

and

$$C_l \frac{\mathrm{D}}{\mathrm{D}L}\left(\frac{\partial T_l}{\partial t}\right) = \lambda_0 \frac{\mathrm{D}}{\mathrm{D}L}\left(\frac{\partial^2 T_l}{\partial x^2}\right) + G\left(\frac{\mathrm{D}T_e}{\mathrm{D}L} - \frac{\mathrm{D}T_l}{\mathrm{D}L}\right). \tag{25}$$

Using the rules of material derivative calculations, the first term on the right-hand side of Eq. (24) takes the form

$$\begin{aligned}\frac{\mathrm{D}}{\mathrm{D}L}\left[\frac{\partial}{\partial x}\left[\lambda_e(T_e,T_l)\frac{\partial T_e}{\partial x}\right]\right] &= \frac{\partial}{\partial x}\left[\frac{\mathrm{D}\lambda_e(T_e,T_l)}{\mathrm{D}L}\frac{\partial T_e}{\partial x}\right] \\ &+ \frac{\partial}{\partial x}\left[\lambda_e(T_e,T_l)\frac{\partial}{\partial x}\left(\frac{\mathrm{D}T_e}{\mathrm{D}L}\right)\right] - 2\frac{\partial}{\partial x}\left[\lambda_e(T_e,T_l)\frac{\partial T_e}{\partial x}\right]\frac{\partial v}{\partial x} - \lambda_e(T_e,T_l)\frac{\partial T_e}{\partial x}\frac{\partial^2 v}{\partial x^2}.\end{aligned} \tag{26}$$

From Eq. (1) one obtains

$$\frac{\partial}{\partial x}\left[\lambda_e(T_e,T_l)\frac{\partial T_e}{\partial x}\right] = C_e(T_e)\frac{\partial T_e}{\partial t} + G\,(T_e - T_l) - Q. \tag{27}$$

When we introduce Eq. (27) into (26), Eq. (24) takes the following form:

$$\begin{aligned}C_e(T_e)\frac{\partial V_e}{\partial t} &= \frac{\partial}{\partial x}\left[\lambda_e(T_e,T_l)\frac{\partial V_e}{\partial x}\right] - G\,(V_e - V_l) \\ &+ \frac{\partial}{\partial x}\left[\frac{\mathrm{D}\lambda_e(T_e,T_l)}{\mathrm{D}L}\frac{\partial T_e}{\partial x}\right] - 2\left[C_e(T_e)\frac{\partial T_e}{\partial t} + G\,(T_e - T_l) - Q\right]\frac{\partial v}{\partial x} \\ &- \lambda_e(T_e,T_l)\frac{\partial T_e}{\partial x}\frac{\partial^2 v}{\partial x^2} + \frac{\mathrm{D}Q}{\mathrm{D}L} - \frac{\mathrm{D}C_e(T_e)}{\mathrm{D}L}\frac{\partial T_e}{\partial t},\end{aligned} \tag{28}$$

where $V_e = \mathrm{D}T_e/\mathrm{D}L$, $V_l = \mathrm{D}T_l/\mathrm{D}L$ are the sensitivity functions.

In similar way the differentiation of Eq. (25) has been done

$$C_l\frac{\partial V_l}{\partial t} = \lambda_0\frac{\partial^2 V_l}{\partial x^2} + G\,(V_e - V_l) - 2\left(C_l\frac{\partial T_l(x,t)}{\partial t} - G\,(T_e - T_l)\right)\frac{\partial v}{\partial x} - \lambda_0\frac{\partial T_l}{\partial x}\frac{\partial^2 v}{\partial x^2}. \tag{29}$$

Differentiation of equations (4) and (5) leads to the following conditions:

$$\begin{cases} x = 0: & \lambda_e(T_e,T_l)\dfrac{\partial V_e}{\partial x} + \lambda_0\dfrac{\mathrm{D}\lambda_e(T_e,T_l)}{\mathrm{D}L}\dfrac{\partial T_e}{\partial x} = 0 \\ x = L: & -\lambda_e(T_e,T_l)\dfrac{\partial V_e}{\partial x} - \lambda_0\dfrac{\mathrm{D}\lambda_e(T_e,T_l)}{\mathrm{D}L}\dfrac{\partial T_e}{\partial x} = 0 \end{cases} \tag{30}$$

and

$$\begin{cases} x = 0: & \lambda_0\dfrac{\partial V_l}{\partial x} = 0 \\ x = L: & -\lambda_0\dfrac{\partial V_l}{\partial x} = 0 \end{cases} \tag{31}$$

while

$$t = 0: \quad V_e\,(x,0) = V_l\,(x,0) = \frac{\mathrm{D}T_p}{\mathrm{D}L} = 0\ . \tag{32}$$

To complete the shape sensitivity analysis of electron and lattice temperatures with respect to the thickness L, the following definition of velocity associated with design parameter L can be accepted:

$$v = \frac{x}{L}, \qquad 0 \le x \le L. \tag{33}$$

4. Method of solution

The basic problem described in Sec. 2 and the additional problems connected with the sensitivity analysis presented in Sec. 3 have been solved using the explicit scheme of finite difference method [3]. It should be pointed out that the structures of basic equations (1), (2) and additional equations (13), (28), (29) connected with the sensitivity analysis are similar. So, these equations for $k = 1, 2, 3$ can be written in the form

$$C_e(T_e)\frac{\partial W_{ek}}{\partial t} = \frac{\partial}{\partial x}\left[\lambda_e(T_e, T_l)\frac{\partial W_{ek}}{\partial x}\right] + Z_{ek} \tag{34}$$

and

$$C_l\frac{\partial W_{lk}}{\partial t} = \lambda_l\frac{\partial^2 W_{lk}}{\partial x^2} + Z_{lk}, \tag{35}$$

where $W_{e1} = T_e$, $W_{l1} = T_l$, $W_{e2} = U_e$, $W_{l2} = U_l$, $W_{e3} = V_e$, $W_{l3} = V_l$, while

$$Z_{e1} = -G\,(T_e - T_l) + Q, \qquad Z_{l1} = G\,(T_e - T_l) \tag{36}$$

and

$$\begin{aligned} Z_{e2} &= -G\,(U_e - U_l) + \frac{\partial}{\partial\, x}\left[(\lambda_{e,e}U_e + \lambda_{e,l}U_l)\,\frac{\partial\, T_e}{\partial\, x}\right] - (T_e - T_l) - C_{e,e}\,U_e\frac{\partial\, T_e}{\partial\, t}, \\ Z_{l2} &= (T_e - T_l) + G\,(U_e - U_l)\,, \end{aligned} \tag{37}$$

$$\begin{aligned} Z_{e3} = -G\,(V_e - V_l) &+ \frac{\partial}{\partial\, x}\left[\frac{\mathrm{D}\lambda_e(T_e, T_l)}{\mathrm{D}L}\frac{\partial\, T_e}{\partial\, x}\right] \\ &- \frac{2}{L}\left[C_e(T_e)\frac{\partial\, T_e}{\partial\, t} + G\,(T_e - T_l) - Q\right] - \frac{\mathrm{D}Q}{\mathrm{D}L} - \frac{\mathrm{D}C_e(T_e)}{\mathrm{D}L}\frac{\partial\, T_e}{\partial\, t}, \end{aligned} \tag{38}$$

$$Z_{l3} = G\,(T_e - T_l)\,.$$

A time grid

$$t^0 < t^1 < ... < t^{f-2} < t^{f-1} < t^f < ... < t^F < \infty \tag{39}$$

with constant time step Δt is introduced and a mesh with constant step h is used.

For transition $t^{f-1} \to t^f$ the following approximation of Eqs. (34) and (35) is proposed:

$$C_{ei}^{f-1}\frac{W_{ek,i}^f - W_{ek,i}^{f-1}}{\Delta t} = \frac{1}{h}\left[\lambda_{ek,i+0.5}^{f-1}\frac{W_{ek,i+1}^{f-1} - W_{ek,i}^{f-1}}{h} - \lambda_{ek,i-0.5}^{f-1}\frac{W_{ek,i}^{f-1} - W_{ek,i-1}^{f-1}}{h}\right] + Z_{ek,i}^{f-1} \tag{40}$$

and

$$C_l\frac{W_{lk,i}^f - W_{lk,i}^{f-1}}{\Delta t} = \lambda_l\frac{W_{lk,i-1}^{f-1} - 2W_{lk,i}^{f-1} + W_{lk,i+1}^{f-1}}{h^2} + Z_{lk,i}^{f-1}. \tag{41}$$

From Eqs. (40) and (41) results that

$$\begin{aligned} W_{ek,i}^f = W_{ek,i}^{f-1}&\left(1 - \frac{\left(\lambda_{ek,i}^{f-1} + \lambda_{ek,i+1}^{f-1}\right)\Delta t}{2h^2C_{ei}^{f-1}} - \frac{\left(\lambda_{ek,i}^{f-1} + \lambda_{ek,i-1}^{f-1}\right)\Delta t}{2h^2C_{ei}^{f-1}}\right) \\ &+ W_{ek,i+1}^{f-1}\left(\frac{\left(\lambda_{ek,i}^{f-1} + \lambda_{ek,i+1}^{f-1}\right)\Delta t}{2h^2C_{ei}^{f-1}}\right) + W_{ek,i-1}^{f-1}\left(\frac{\left(\lambda_{ek,i}^{f-1} + \lambda_{ek,i-1}^{f-1}\right)\Delta t}{2h^2C_{ei}^{f-1}}\right) + Z_{ek,i}^{f-1}\left(\frac{\Delta t}{C_{ei}^{f-1}}\right) \end{aligned} \tag{42}$$

and

$$W_{lk,i}^{f} = W_{lk,i}^{f-1}\left(1 - \frac{2\lambda_l \Delta t}{h^2 C_l}\right) + W_{lk,i+1}^{f-1}\left(\frac{\lambda_l\,\Delta t}{h^2 C_l}\right) + W_{lk,i-1}^{f-1}\left(\frac{\lambda_l \Delta t}{h^2 C_l}\right) + Z_{lk,i}^{f-1}\left(\frac{\Delta t}{C_l}\right). \tag{43}$$

It should be pointed out that the stability criteria should be fulfilled, this means that

$$\begin{aligned} &1 - \frac{\left(\lambda_{ek,i}^{f-1} + \lambda_{ek,i+1}^{f-1}\right)\Delta t}{2h^2 C_{ei}^{f-1}} - \frac{\left(\lambda_{ek,i}^{f-1} + \lambda_{ek,i-1}^{f-1}\right)\Delta t}{2h^2 C_{ei}^{f-1}} \geq 0, \\ &1 - \frac{2\lambda_l \Delta t}{h^2 C_l} \geq 0. \end{aligned} \tag{44}$$

5. Results of computations

At first, the gold film of thickness $L = 100$ nm (1 nm $= 10^{-9}$ m) is considered. The layer is subjected to a short-pulse laser irradiation ($R = 0.93$, $I_0 = 13.4$ J/m^2, $t_p = 0.1$ ps, $\delta = 15.3$ nm). Thermophysical parameters are as follows: $\lambda_l = \lambda_0$, $\lambda_e = \lambda_0 T_e/T_l$, where $\lambda_0 = 315$ W/(mK), $C_l = 2.5$ MJ/(m^{3}K), where $\gamma = 62.9$ J/(m^{3}K^2), $G = 2.6 \cdot 10^{16}$ W/(m^{3}K) [5]. Initial temperature equals to $T_p = 300$ K.

The problem is solved using finite difference method under the assumption that $\Delta t = 0.00001$ ps and $h = 1$ nm. Figure 2 shows the comparison of numerical results for thin gold film ($x = 0$) with experimental data presented in [2]. The line and the symbols represent calculated temperature of electrons and experimental data, respectively. One can see that the obtained results and measured temperatures are in good agreement.

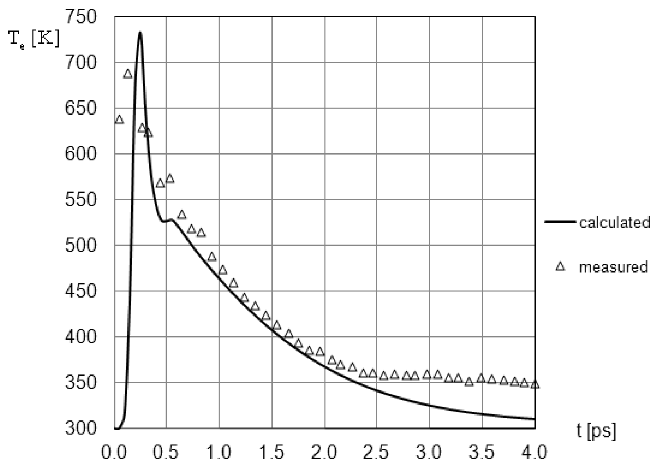

Fig. 2. Comparison of calculated electron temperature with experimental data for 100 nm gold film.

Knowledge of sensitivity functions U_e, U_l, V_e, V_l allows, among others, to estimate the temperature changes due to the parameters G and L perturbations, this means

$$\Delta T_e\,(x,t,2\Delta G) = 2U_e\,(x,t,G)\,\Delta G, \qquad \Delta T_l\,(x,t,2\Delta G) = 2U_l\,(x,t,G)\,\Delta G, \tag{45}$$

$$\Delta T_e\,(x,t,2\Delta L) = 2V_e\,(x,t,L)\,\Delta L, \qquad \Delta T_l\,(x,t,2\Delta L) = 2V_l\,(x,t,L)\,\Delta L. \tag{46}$$

In Figs. 3 and 4 the changes of temperatures T_e, T_l due to the changes of parameters $G(\Delta G = 0.1G)$ and L ($\Delta L = 0.1L$) are presented.

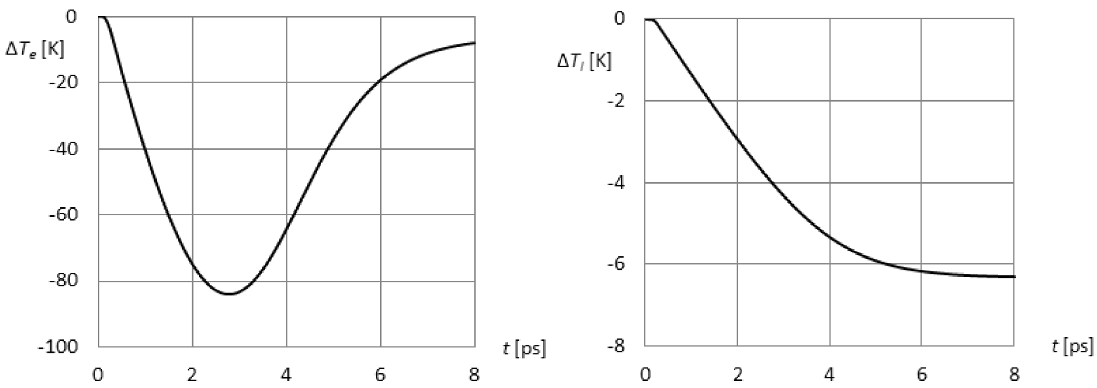

Fig. 3. History of functions ΔT_e $(0,\, t,\, 2\Delta G)$ and ΔT_l $(0,\, t,\, 2\Delta G)$.

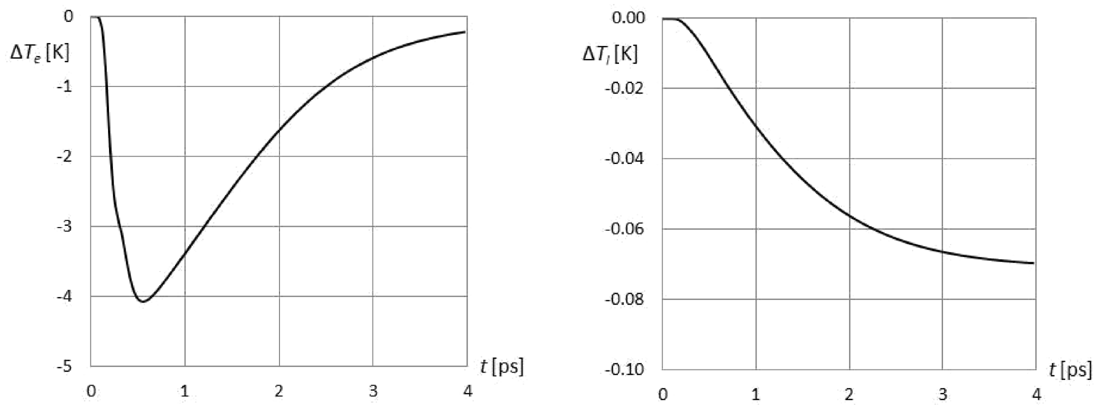

Fig. 4. History of functions ΔT_e $(0,\, t,\, 2\Delta L)$ and ΔT_l $(0,\, t,\, 2\Delta L)$.

6. Conclusions

Thin metal film subjected to the laser pulse has been considered. The process analyzed was described by the two-temperature parabolic model created by the system of two coupled Fourier equations supplemented by the boundary and initial conditions. The changes of electron and lattice temperatures due to the perturbations of film thickness L and coupling factor G have been discussed. For this purpose, the direct approach of sensitivity analysis has been used. As it can be seen in Fig. 3, the change of coupling factor G in the range from $G - \Delta G$ to $G + \Delta G$, where ΔG corresponds to 10% of the basic value of the parameter G, resulted in a maximum change of electron temperature equal to 82 K and the maximum change of lattice temperature equal to 6.2 K. The variation of the film thickness L from $L - \Delta L$ to $L + \Delta L$ caused the maximum change of electron temperature equal to 4.1 K and maximum change of lattice temperature equal to 0.068 K (Fig. 4). Thus, the perturbation of coupling factor G causes essentially greater temperature changes than the perturbation of film thickness L. In addition, the changes of these parameters affected mainly on electrons temperature, especially at the stage of initial steps of heating process.

It should be pointed out that the determination of temperatures and sensitivity functions allows one, among others, to identify the film thickness L and the parameter G using the gradient methods under the assumption that the temperature history on the irradiated surface is known. The other application of sensitivity functions is connected with the transformation of basic solution on the solution corresponding to other parameters of the process considered (the Taylor formula should be applied). Sensitivity analysis methods presented in this article may be used to estimate the changes in temperature caused by the changes in other parameters, for example, the thermal conductivity or the volumetric specific heat of lattice.

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