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A computational model for static and dynamic balancing of masses on rotating shafts

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Balancing is the process of improving the mass distribution of a body so that it rotates in its bearings without unbalanced centrifugal forces. It is thus critical to the performance of any high speed equipment. The problem is mathematically modeled and a genetic algorithm is presented for obtaining optimal solutions for balancing problems on rotating shafts. This is eventually converted into computer package titled BALANCER, developed using the VisualBASIC platform. Examples are presented to illustrate implementation of the methodology. The model was tested by using typical problems, correctly solved in the literature using conventional methods. The results of the three examples gave same match with those obtained from analytical approach. The accuracy of analysis using the model and the students' feedback suggest that integration of the software tool will be beneficial for improving students' performance in any dynamics course.

Keywords: modeling, balancing, rotating shaft, off-line, imbalance

Notations

m – Mass on the rotating shaft, kg
ω – Angular velocity of the rotating shaft, rad s^{-1}
r – Radius of the mass, mm
R – Resultant of centrifugal forces, N
θ – Angle of inclination of the mass to the horizontal, degree
$\sum H$ – Summation of all horizontal forces, N
$\sum V$ – Summation of all vertical forces, N
x – Distance from a reference plane, mm
M – Mass on the reference plane, kg

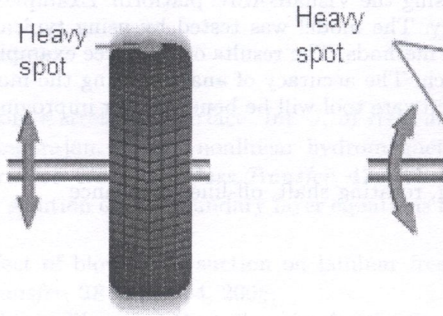
1. Introduction

Computer modeling of dynamic systems is a valuable tool for engineering analysis and design. It allows for active experimentation, design modification, and subsequent analysis without investment in raw materials and supplies. The "chalk and talk" style of teaching and instructing attempts to transmit knowledge from the teacher to a passive recipient. There is, however, a growing awareness among engineering educators that while this style of instruction is suitable for teaching engineering analysis, it has some limitations when it comes to nurturing creativity, synthesis and engineering design. Therefore a prudent combination of teaching by lectures and active learning techniques are perhaps the ideal way to enhance student comprehension and creativity [1]

Balancing is the process of improving the mass distribution of a body so that it rotates in its bearings without unbalanced centrifugal forces. It is thus critical to the performance of any high speed equipment [6]. Rotating machinery is commonly used in mechanical systems, such as machining tools, industrial turbo machinery, and aircraft gas turbine engines. Vibration caused by mass imbalance is a common problem in this machinery. Imbalance occurs if the principal axis of inertia of the shaft is not coincident with its geometric axis [7]. Higher speeds cause much greater centrifugal imbalance forces; and the trend of rotating equipment toward higher power clearly leads to higher operational speeds. For example, speeds as high as 30 000 rpm are not uncommon in today's high-speed machining applications; therefore, vibration control is essential in improving machining surface finish; achieving longer bearing, spindle and tool life resulting in the reduction of shutdowns [5].

There are two types of imbalance: static and dynamic. Static imbalance occurs when there is a heavy spot on the rotating mass; that is, radially away from the mass center: there is some point on a radius where the mass is greater than the others. As the mass rotates, there is a centrifugal force that points away from the center of the mass. If the mass has a heavy spot, then the sum of the forces does not equal zero, which causes the mass to roll unevenly and induces a vertical vibration [4].

In dynamic imbalance, the weight may be distributed evenly radially, but there is an imbalance in the width of the mass; so, when the sum of the moments about the center does not equal zero, it causes the mass to wobble as shown in Fig. 1b. Mechanics correct the imbalances by distributing lead weights on the tyre rim in such a way as to cancel the effects of the heavy spot(s). Unbalanced tyre does not wear evenly and need to be replaced sooner than a balanced tire.



(a) (b)

Fig. 1. Illustration of imbalance in tyres; (a) static (b) dynamic

2. Literature review

Imbalance is presented as a "heavy spot" on a rotating mass, where the mass center is apparently located away from the center of rotation. Balancing is thus, a procedure of removing the heavy spot, by drilling holes or adding weight. There are two general forms of balancing: "Static" and "Dynamic". Static balancing involves installing the component into a balancing machine and measuring the "heavy" point in relation to the center line while the part is rotating [3]. If the required balance correction is at a single axial point on the rotor the balance is said to be "Single-Plane". Single plane balancing is adequate for rotors which are short in length, such as pulleys and fans.

Dynamic or "Dual-Plane" balancing is required for components or assemblies of significant length. with some axial length can have two "heavy" points at opposing ends of the component, acting independently on the mass center line [8]. In order to balance the component, both planes must be corrected for center line error. Dynamic balancing is required for components such as shafts and multi-rotor assemblies.

The study is oriented towards the training and education aspects of balancing of masses on rotating shaft. In the teaching of balancing techniques to engineering students, analytical and graphical approaches are always employed. Meanwhile the two approaches are often cumbersome and prone to unnecessary errors.

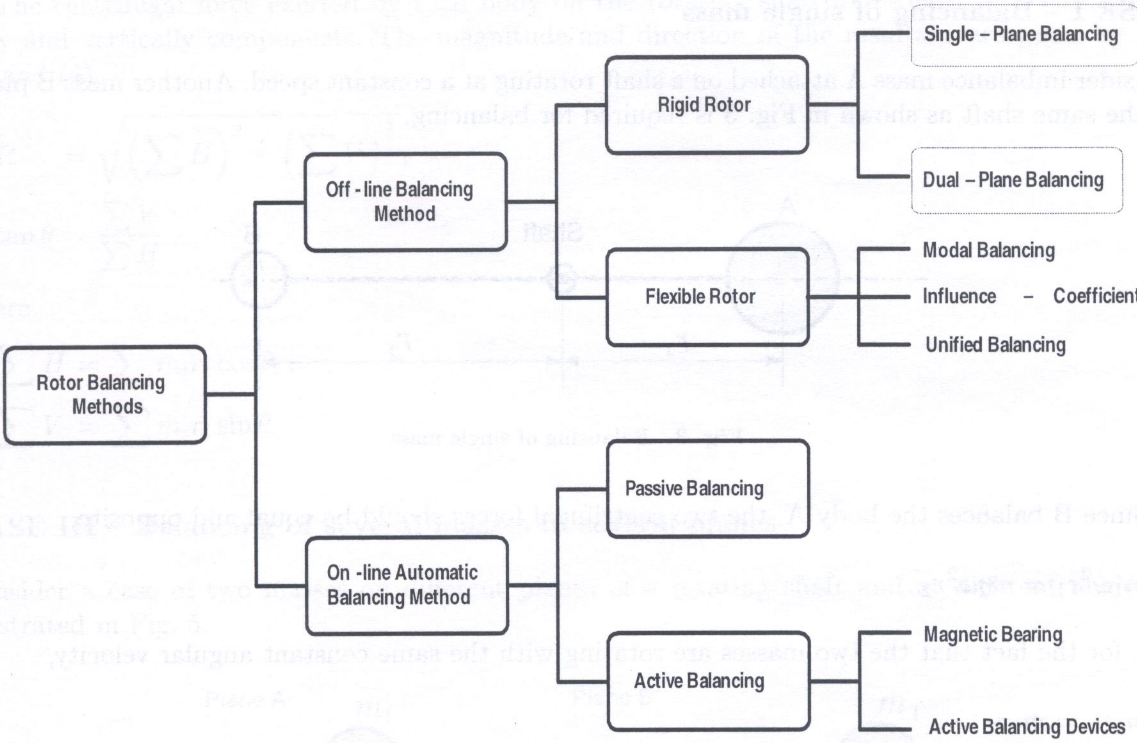


Fig. 2. Classification of balancing methods (Wowk [8])

A large body of literature is available on rotor or shaft balancing methods. A rough classification of the various balancing methods is shown in Fig. 2. The shaft balancing techniques can be classified as off-line balancing methods and real-time active balancing methods.

The real-time balancing methods can be classified into passive balancing and active balancing methods according to which kinds of balancing devices are used.

The off-line rigid rotor balancing method is very common in industrial applications. In this method, the rotor is modeled as rigid shaft without elastic deformation during operation. Theoretically, any imbalance distribution in a rigid rotor can be balanced in two different planes [8]. Methods for rigid rotors are easy to implement but can only be applied to low-speed rotors, where the rigid rotor assumption is valid. A simple rule of thumb is that rotors operating below 5000 rpm can be considered rigid rotors. It is well known that rigid rotor balancing methods cannot be applied to flexible rotor balancing. Therefore, researchers developed modal balancing, influence coefficient and unified balancing methods for flexible [2].

In this paper, both single plane and dual-plane balancing approaches were considered. The first is employed for the two cases of balancing single mass and several masses on a single plane while the later is employed for balancing several masses on several planes. Typical components that always require static or dynamic balancing include pulley assemblies, centrifugal rotors, compressor rotors, flywheels, impellers, fans, turbine rotors, precision shafts etc.

3. Theoretical background

When a mass is rotating at a certain distance from the centre of rotation, the disturbing effect due to its being out of balance is proportional to the mass and to its distance from the centre of rotation. The mass exerts some centrifugal force, whose effect is to bend the shaft, and to produce vibrations in it. In order to eliminate the centrifugal force, another mass is attached to the opposite side of the disturbing mass in the same plane of the shaft, at such a position that the center of gravity of the mass is now on the same line with that of the balance mass and center of the shaft.

CASE I – Balancing of single mass

Consider imbalance mass A attached on a shaft rotating at a constant speed. Another mass B placed on the same shaft as shown in Fig. 3 is required for balancing.

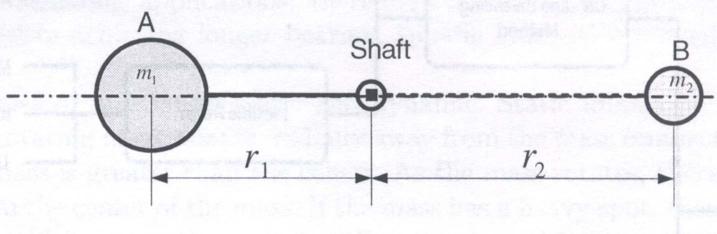


Fig. 3. Balancing of single mass

Since B balances the body A, the two centrifugal forces should be equal and opposite,

$$m_1\omega^2 r_1 = m_2\omega^2 r_2 \,. \tag{1}$$

But, for the fact that the two masses are rotating with the same constant angular velocity,

$$m_1 r_1 = m_2 r_2 \,. \tag{2}$$

CASE II – Balancing of several masses in a single plane

Consider any number of masses (say three) m_1, m_2 and m_3 attached to a shaft rotating in one plane. In order to balance these masses, another body mass m is attached to the same shaft as shown in Fig. 4.

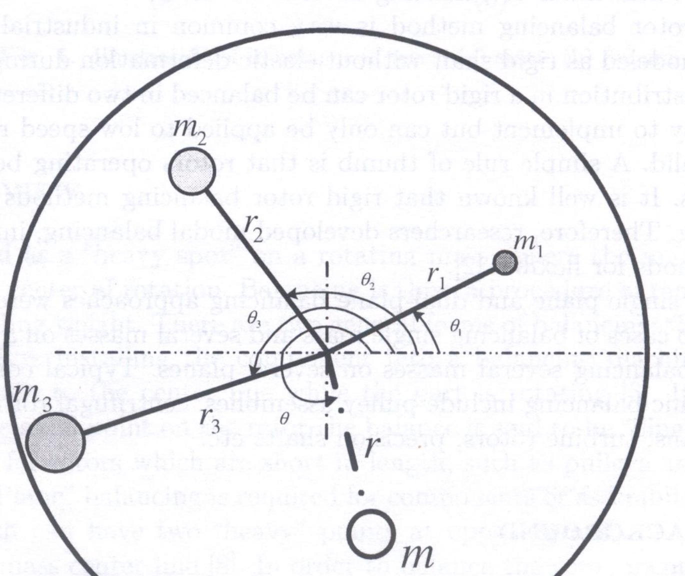


Fig. 4. Three masses of single plane

The centrifugal force exerted by each body on the rotating shaft is resolved into the horizontally and vertically components. The magnitude and direction of the resultant are given by Eqs. (3) and (4),

$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2}\,, \tag{3}$$

$$\tan\theta = \frac{\sum V}{\sum H}\,, \tag{4}$$

where

$$\sum H = \sum m_i r_i \cos\theta_i\,, \tag{5}$$

$$\sum V = \sum m_i r_i \sin\theta_i\,. \tag{6}$$

CASE III – Balancing of several masses in several planes

Consider a case of two masses on different planes of a rotating shaft and at different angles, as illustrated in Fig. 5.

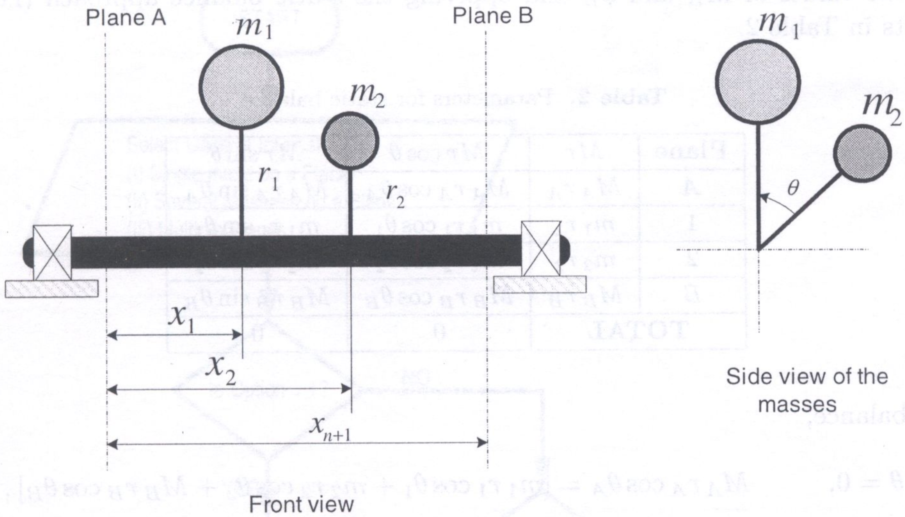


Fig. 5. Two masses on separate planes

The unbalance mass couples, about a chosen reference plane, are set against the correcting balancing mass couple, that is

$$mrx = \sum m_i r_i x_i\,. \tag{7}$$

This is usually analyzed by highlighting the known and unknown parameters in tabular form as stated in Table 1.

With the addition of the balancing mass, the system is set to equilibrium. This implies both static and dynamic balance for the system.

$$\sum mrx\cos\theta = 0, \qquad M_B r_B x_B \cos\theta_B = [m_1 r_1 x_1 \cos\theta_1 + m_2 r_2 x_2 \cos\theta_2]\,, \tag{8}$$

and

$$\sum mrx\sin\theta = 0, \qquad M_B r_B x_B \sin\theta_B = [m_1 r_1 x_1 \sin\theta_1 + m_2 r_2 x_2 \sin\theta_2]\,, \tag{9}$$

Table 1. Parameters of the system of two masses in different planes

Plane	Mass (m)	Radius (r)	Distance From Plane A (x)	mrx	Angle θ	$mrx\cos\theta$	$mrx\sin\theta$
A	M_A	r_A	0	0	θ_A	0	0
1	m_1	r_1	x_1	$m_1r_1x_1$	θ_1	$m_1r_1x_1\cos\theta_1$	$m_1r_1x_1\sin\theta_1$
2	m_2	r_2	x_2	$m_2r_2x_2$	θ_2	$m_2r_2x_2\cos\theta_2$	$m_2r_2x_2\sin\theta_2$
B	M_B	r_B	x_B	$M_Br_Bx_B$	θ_B	$M_Br_Bx_B\cos\theta_B$	$M_Br_Bx_B\sin\theta_B$
TOTAL						0	0

The resultant mrx for B,

$$M_B\,r_B\,x_B = \sqrt{(M_B\,r_B\,x_B\cos\theta_B)^2 + (M_B\,r_B\,x_B\sin\theta_B)^2} \tag{10}$$

and

$$\phi_B = \left[\left(\tan^{-1}\left(\frac{M_B\,r_B\,x_B\sin\theta_B}{M_B\,r_B\,x_B\cos\theta_B}\right)\right)\right]. \tag{11}$$

Substituting the values of M_B and ϕ_B and applying the static balance approach (i.e. mr values) give the results in Table 2.

Table 2. Parameters for static balance

Plane	Mr	$Mr\cos\theta$	$Mr\sin\theta$
A	$M_A\,r_A$	$M_A\,r_A\cos\theta_A$	$M_A\,r_A\sin\theta_A$
1	$m_1\,r_1$	$m_1\,r_1\cos\theta_1$	$m_1\,r_1\sin\theta_1$
2	$m_2\,r_2$	$m_2\,r_2\cos\theta_2$	$m_2\,r_2\sin\theta_2$
B	$M_B\,r_B$	$M_B\,r_B\cos\theta_B$	$M_B\,r_B\sin\theta_B$
TOTAL		0	0

For static balance,

$$\sum mr\cos\theta = 0, \qquad M_A\,r_A\cos\theta_A = [m_1\,r_1\cos\theta_1 + m_2\,r_2\cos\theta_2 + M_B\,r_B\cos\theta_B], \tag{12}$$

$$\sum mr\sin\theta = 0, \qquad M_A\,r_A\sin\theta_A = [m_1\,r_1\sin\theta_1 + m_2\,r_2\sin\theta_2 + M_B\,r_B\sin\theta_B]. \tag{13}$$

The resultant mr for A

$$M_A\,r_A = \sqrt{(M_A\,r_A\cos\theta_A)^2 + (M_A\,r_A\sin\theta_A)^2} \tag{14}$$

and

$$\phi_A = \left[\left(\tan^{-1}\left(\frac{M_A\,r_A\sin\theta_A}{M_A\,r_A\cos\theta_A}\right)\right)\right]. \tag{15}$$

4. Methodology

The main approach of any computer program effort is to translate the problem solving algorithm into instructions that the computer can handle. Before embarking on any translation, the problem on hand has to be studied and properly identified.

4.1. Algorithm and flowcharting

From the theory, the balancing mass in a single plane can be handled by static balancing approach. Equations (3) and (4) provide the magnitude and angular position of the balancing mass respectively. The case of several masses on several planes can be handled by dynamic balancing approach where Eqs. (10), (11), (14) and (15) are used in evaluating the magnitude and angular position of the balanced masses on the specified planes A and B. The theory assisted in the development of an algorithm which was eventually used to design a flowchart for the computer model. The flowchart is illustrated in Fig. 6. This consists of the main module which is made up of the three subroutines. Each subroutine handles each of the three cases highlighted in Section 3. The detailed flowcharts of the three subroutines are illustrated in Figs. 7–9.

4.2. Program coding

The flowchart was eventually developed into a computer model, named "Balancer 1.0". The coding of the model was made on the platform of VisualBASIC 6.0. An executable version was created for proper operation on any system with or without availability of VisualBASIC compiler.

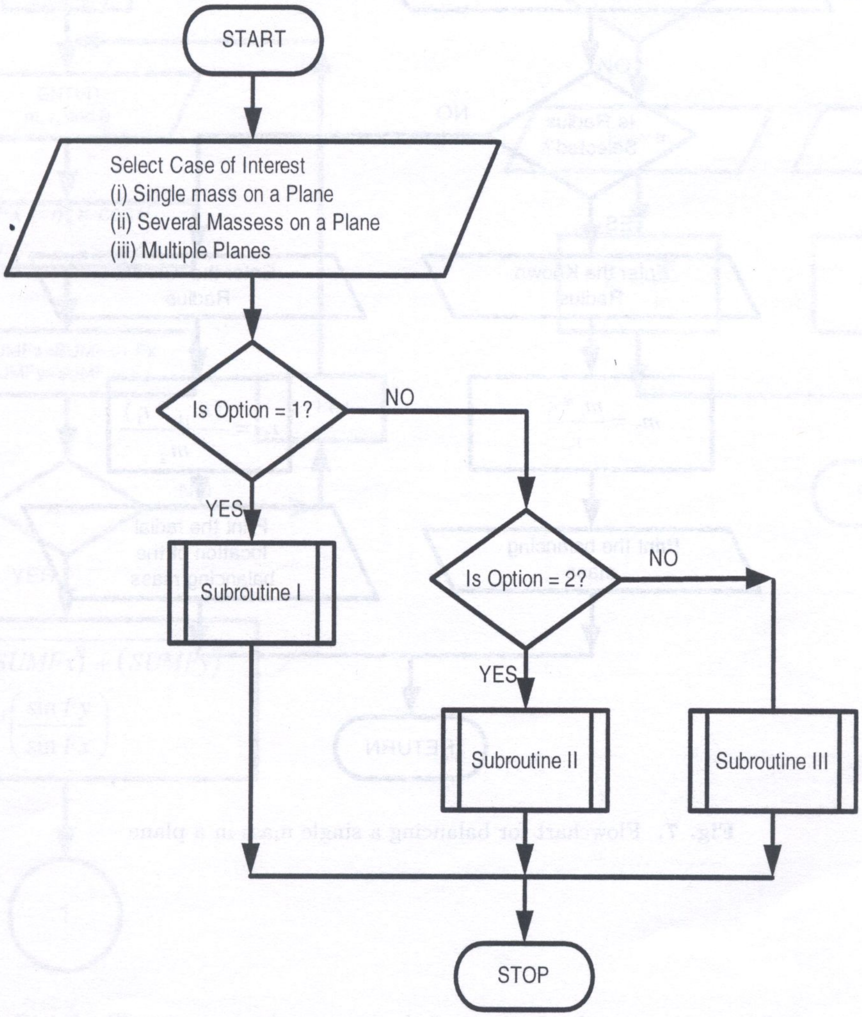


Fig. 6. Flowchart of the main module

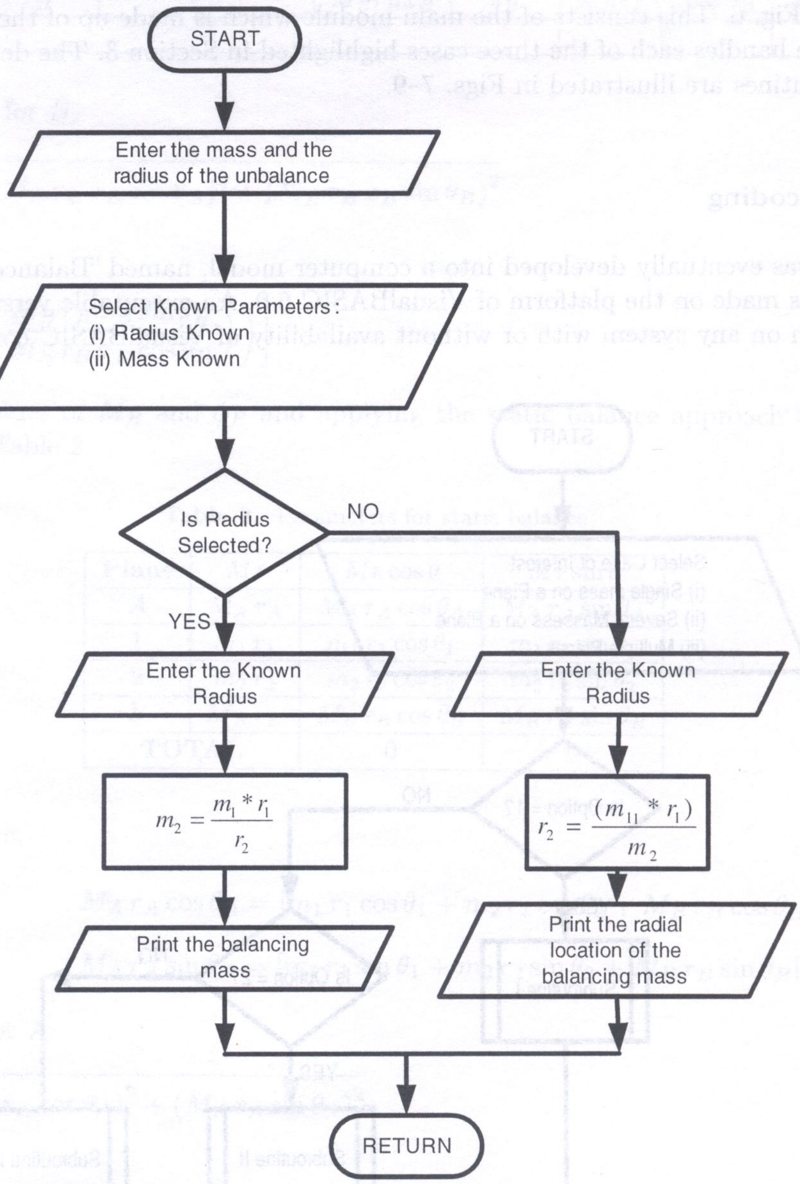


Fig. 7. Flowchart for balancing a single mass in a plane

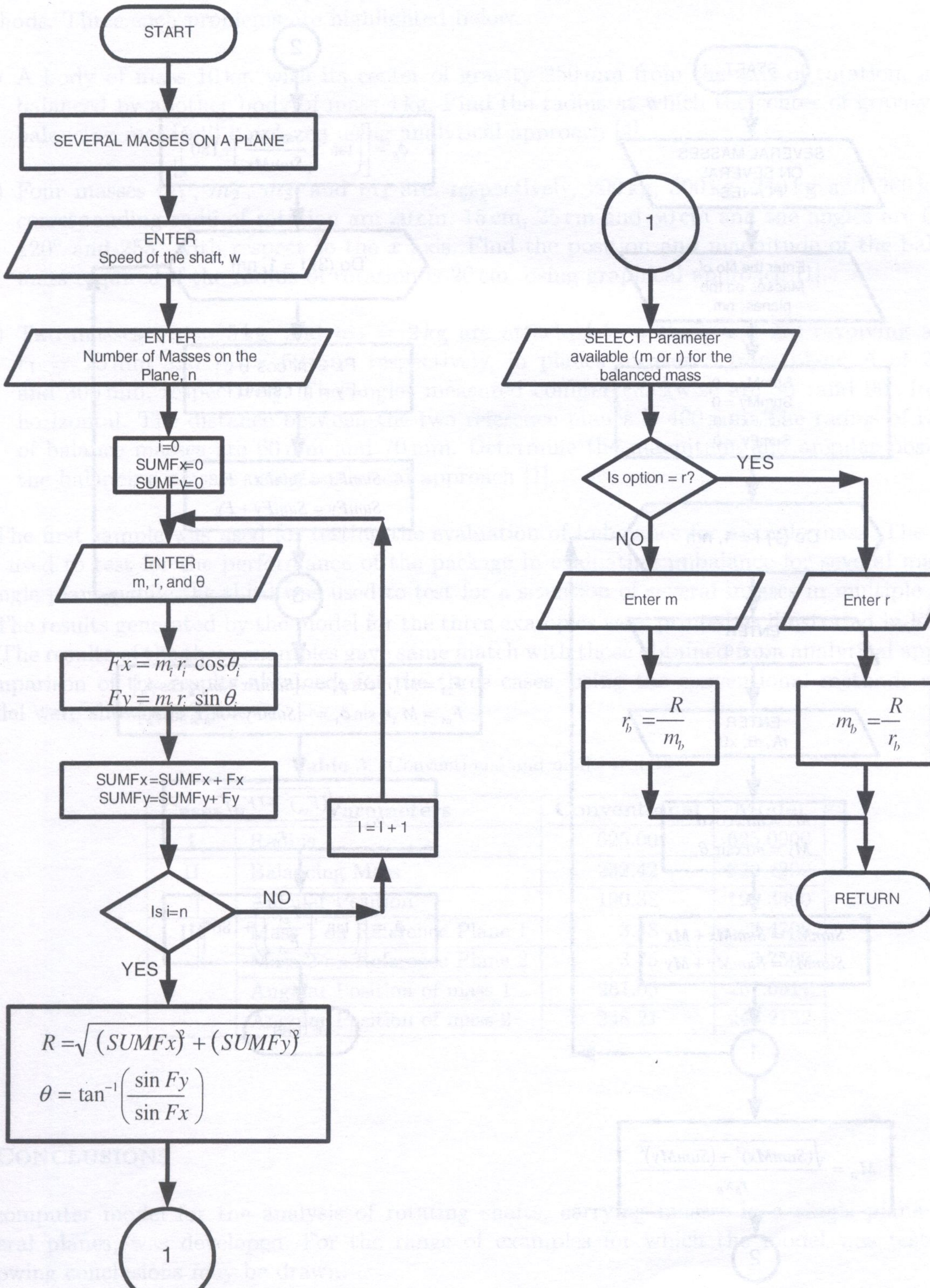


Fig. 8. Flowchart of subroutine for balancing several masses in several planes

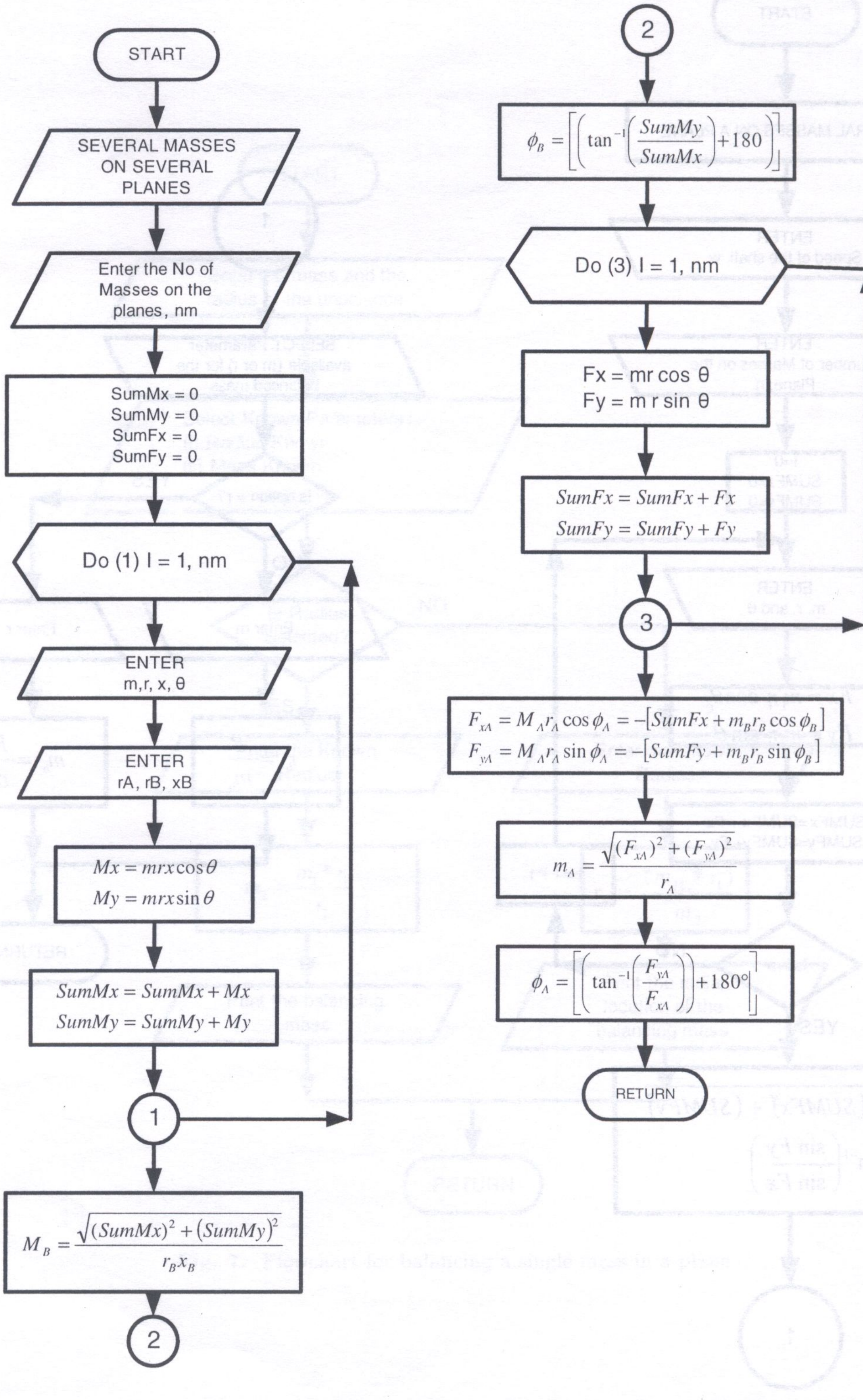


Fig. 9. Flowchart of subroutine for balancing several masses in several planes

5. Verification of the Model

The model was tested by using typical problems, correctly solved in the literature using conventional methods. Three such problems are highlighted below.

(i) A body of mass 10 kg, with its center of gravity 250 mm from the axis of rotation, is to be balanced by another body of mass 4 kg. Find the radius at which the center of gravity of the balancing mass will be placed using analytical approach [4].

(ii) Four masses m_1, m_2, m_3, and m_4 are, respectively, 200 kg, 300 kg, 240 kg and 260 kg. The corresponding radii of rotation are 20 cm, 15 cm, 25 cm and 30 cm and the angles are 0°, 45°, 120° and 255° with respect to the x axis. Find the position and magnitude of the balancing mass required if the radius of rotation is 20 cm, using graphical approach [1].

(iii) Two masses $m_1 = 5$ kg, and $m_2 = 2$ kg are attached to a shaft and are revolving at radii $r_1 = 75$ mm and $r_2 = 50$ mm, respectively, in planes measured from plane A of 200 mm and 300 mm, respectively. The angles measured counter clockwise are 30° and 90° from the horizontal. The distance between the two reference planes is 400 mm. The radius of rotation of balance masses are 60 mm and 70 mm. Determine the magnitude and angular position of the balancing masses using analytical approach [1].

The first sample was used for testing the evaluation of imbalance for a single mass. The second was used to test for the performance of the package in evaluating imbalance for several masses in a single plane, while the third was used to test for a situation of several masses in multiple planes.

The results generated by the model for the three examples were printed as illustrated in Figs. 10–12. The results of the three examples gave same match with those obtained from analytical approach. Comparison of the results obtained, for the three cases, using the conventional methods and the model were shown in Table 3.

Table 3. Conventional and model results

Cases	Parameters	Conventional	Model
I	Radius	625.00	625.0000
II	Balancing Mass	232.42	232.4200
	Angular Position	190.38	190.3800
III	Mass 1 on Reference Plane 1	3.48	3.4799
	Mass 2 on Reference Plane 2	3.75	3.7500
	Angular Position of mass 1	261.05	261.0517
	Angular Position of mass 2	248.21	248.2132

6. Conclusions

A computer model for the analysis of rotating shafts, carrying masses in a single plane and in several planes, was developed. For the range of examples for which the model was tested, the following conclusions may be drawn:

It generates the results that are in perfect agreement with the conventional approaches.

The analysis period is very much shorter compared to the time always consumed by the analytical and graphical approaches.

The ease of computer usage will reduce frustration for students and provide more time to think about the principles of the problems.

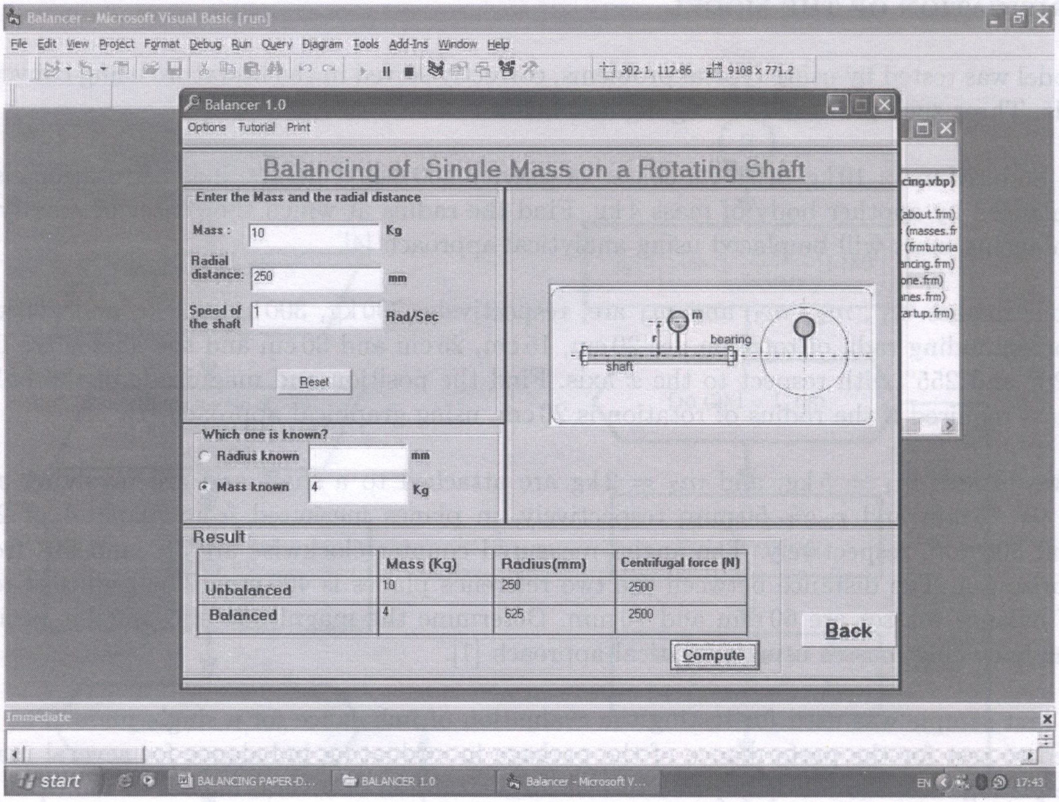


Fig. 10. Balancing of a single mass on a rotating shaft

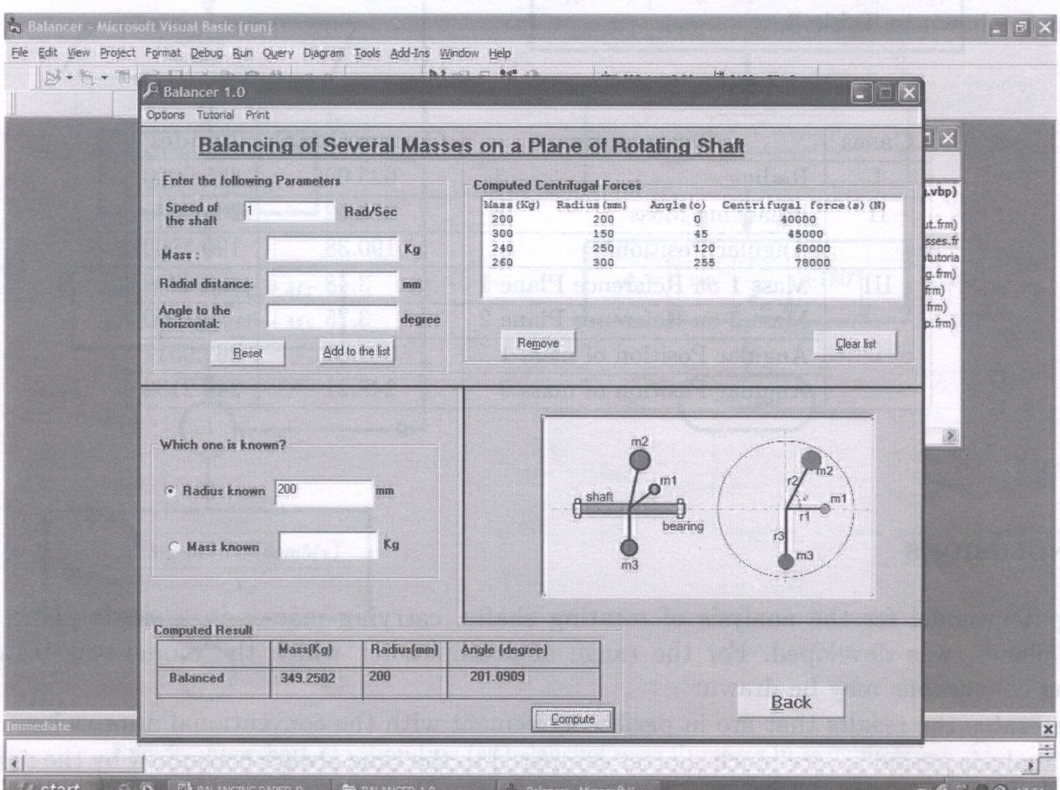


Fig. 11. Balancing of several masses in a plane of rotating shaft

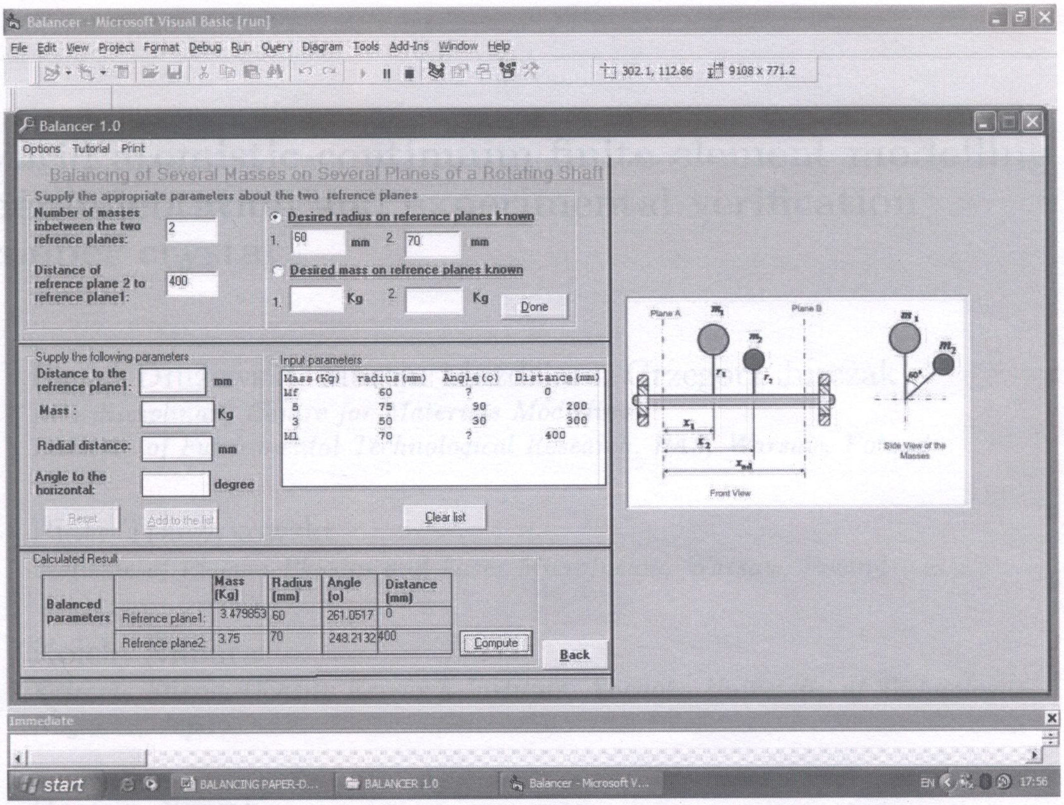


Fig. 12. Balancing of several masses in several planes

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