Computer Assisted Mechanics and Engineering Sciences, 14: 631–636, 2007.

The uncertain analysis of human pelvic bone

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(Received in the final form September 18, 2007)

Numerical modeling of the human pelvic bone is a complex process in which many important factors are taken into account. One of them concerns material properties. Numerical calculations require the characteristics of the material properties and the material parameters from the beginning. The material properties of the living body depend on age, health, gender, environment and many others factors. To determine correct material parameters, health details of a group of patients need to be taken into consideration. In this paper authors assumed interval values of the selected material parameters and proposed interval and fuzzy analysis of the pelvic bone.

Keywords: human pelvic bone, interval and fuzzy analysis, finite element method, material properties

1. Introduction

Bioengineering concerns many significant problems applied to the human body. The pelvic bone is one of the most important supporting elements in human pelvic joint but it is exposed to the injuries. Very often before and after surgical intervention the expertises about the stress, strain and displacement distributions in the pelvic bone are needed. For the safety of the patient there are only two possibilities available to derive mentioned values: model testing and numerical calculations. The numerical model should be prepared before numerical calculations [4–6]. Numerical calculations require the characteristics of the material properties and the material parameters from the beginning. Usually the literature is the source of the material parameters, but sometimes this data is not suitable for the implementation. This is a reason for the experimental investigations to identify these parameters [2, 3, 7]. It is well known that material properties of the living body depend on many factors: age, health, gender, environment and many others changing in time. As we are interested in results of analysis not only for a one patient but for a group of patients, we should assume an interval value of material parameters. In this paper the test of the interval and fuzzy analysis of the pelvic bone is presented. The interval and fuzzy analysis concerns material properties. The finite elements method is applied [1, 9, 10].

2. The interval and fuzzy mathematics basic definition

Essential assumptions and formulas concerning interval numbers and interval analysis can be found in [8]. According to the definition, an interval scalar consists of a single continuous domain in the domain of real numbers $\boldsymbol{R}$. The domain of interval scalars is denoted by $\boldsymbol{IR}$. The interval scalar is denoted by a boldface variable $\boldsymbol{x}$. A real closed interval scalar is defined as [8]

$$\boldsymbol{x} = \{x|(x \in \boldsymbol{R})(\underline{x} \leq x \leq \overline{x})\} \quad \text{or} \quad \boldsymbol{x} = [\underline{x}, \overline{x}] \tag{1}$$

where $\underline{x}$ and $\bar{x}$ are respectively the lower and upper bound of interval scalar. The set scalar is denoted by $\langle x \rangle$ and defined as

$$\langle x \rangle = \bigcup_i x_i \,. \tag{2}$$

The interval vector is denoted by $\{x\} \in \boldsymbol{IR}^n$. It describes the set of all vectors for which each vector component x_i belongs to its corresponding interval scalar $\boldsymbol{x}_i$,

$$\{\boldsymbol{x}\} = \{\{x\} | x_i \in \boldsymbol{x}_i\} \,. \tag{3}$$

The interval matrix $[\boldsymbol{X}] \in \boldsymbol{IR}^{n\times m}$ describes the set of all matrices for which each matrix component x_{ij} is contained within its corresponding interval scalar $\boldsymbol{x}_{ij}$:

$$[\boldsymbol{X}] = \{[X] | x_{ij} \in \boldsymbol{x}_{ij}\} \,. \tag{4}$$

The set matrix $\langle [X] \rangle$ describes the set of all possible matrices for which each matrix component x_{ij} is contained within its corresponding set scalar $\langle x_{ij} \rangle$,

$$\langle [X_{ij}] \rangle = \{[X_{ij}] | x_{ij} \in \langle x_{ij} \rangle\} \,. \tag{5}$$

Additionally the special arithmetic for interval numbers is applied,

$$[\underline{a}, \bar{a}] + [\underline{b}, \bar{b}] = [\underline{a} + \underline{b}, \bar{a} + \bar{b}], \tag{6}$$

$$[\underline{a}, \bar{a}] - [\underline{b}, \bar{b}] = [\underline{a} - \bar{b}, \bar{a} - \underline{b}], \tag{7}$$

$$[\underline{a}, \bar{a}] \cdot [\underline{b}, \bar{b}] = [\min(\mathbf{M}), \max(\mathbf{M})] \quad \text{where} \quad \mathbf{M} = \{\underline{a} \cdot \underline{b};\ \underline{a} \cdot \bar{b};\ \bar{a} \cdot \underline{b};\ \bar{a} \cdot \bar{b}\}, \tag{8}$$

$$[\underline{a}, \bar{a}] / [\underline{b}, \bar{b}] = [\min(\mathbf{D}), \max(\mathbf{D})] \quad \text{where} \quad \mathbf{D} = \{\underline{a}/\underline{b};\ \underline{a}/\bar{b};\ \bar{a}/\underline{b};\ \bar{a}/\bar{b}\}. \tag{9}$$

Let us consider the FE analysis as black-box function $f(\{x\})$ of non-deterministic model properties assembled in a parameter vector $\{x\}$ and resulting in output vector $\{y\}$. The input parameter vector is contained within an interval vector $\{\boldsymbol{x}\}$. The interval FE procedure is numerically equivalent to searching for a solution of the following form,

$$\langle \{y\} \rangle = \{\{y\} | (\{x\} \in \{\boldsymbol{x}\})(\{y\} = f(\{x\}))\} \,. \tag{10}$$

The discussions are focused on calculating an interval vector that approximates the exact solution set. This hypercube approximation describes a range for each vector component. A few different numerical solution strategies to calculate a hypercube approximation of the exact solution set can be distinguished. In the presented work the vertex analysis is applied.

The fuzzy sets are considered as a set of pairs x and $\mu(x)$. The type of the fuzzy sets is the fuzzy number. The fuzzy number can be represented by the sets of alpha-cuts (Fig. 1). The fuzzy operators can be determined, due to the applied interval arithmetic for each alpha-cut.

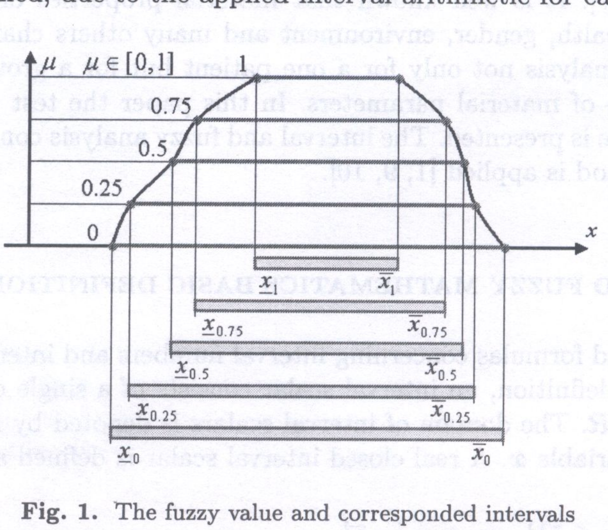

Fig. 1. The fuzzy value and corresponded intervals

3. The interval finite element method

The interval finite element method (IFEM) is a relatively new computational tool for analysing physical problems featuring uncertain system parameters. The governing algebraic equations after spatial discretization are similar in the form to those obtained from the deterministic finite element method (FEM) and give n-dimensional system of linear equations

$$[A][x] = [B] \tag{11}$$

what can be expressed more detailed as

$$\begin{bmatrix} [a_{11}\,,\,\bar{a}_{11}] & [a_{12}\,,\,\bar{a}_{12}] & [a_{13}\,,\,\bar{a}_{13}] & \cdots & [a_{1n}\,,\,\bar{a}_{1n}] \\ [a_{21}\,,\,\bar{a}_{21}] & [a_{22}\,,\,\bar{a}_{22}] & [a_{23}\,,\,\bar{a}_{23}] & \cdots & [a_{2n}\,,\,\bar{a}_{2n}] \\ [a_{31}\,,\,\bar{a}_{31}] & [a_{32}\,,\,\bar{a}_{32}] & [a_{33}\,,\,\bar{a}_{33}] & \cdots & [a_{3n}\,,\,\bar{a}_{3n}] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ [a_{n1}\,,\,\bar{a}_{n1}] & [a_{n2}\,,\,\bar{a}_{n2}] & [a_{n3}\,,\,\bar{a}_{n3}] & \cdots & [a_{nn}\,,\,\bar{a}_{nn}] \end{bmatrix} \begin{bmatrix} [x_1\,,\,\bar{x}_1] \\ [x_2\,,\,\bar{x}_2] \\ [x_3\,,\,\bar{x}_3] \\ \vdots \\ [x_n\,,\,\bar{x}_n] \end{bmatrix} = \begin{bmatrix} [b_1\,,\,\bar{b}_1] \\ [b_2\,,\,\bar{b}_2] \\ [b_3\,,\,\bar{b}_3] \\ \vdots \\ [b_n\,,\,\bar{b}_n] \end{bmatrix} \tag{12}$$

In the interval case, the elements of matrix $[A]$ and vector $[B]$ are interval. Therefore the vector $[x]$ is interval as well.

The result of the system of linear equations is the *hull* (Fig. 2). For example:

$$\begin{bmatrix} [1,2] & [2,4] \\ [2,4] & [1,2] \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} [-1,1] \\ [1,2] \end{bmatrix}. \tag{13}$$

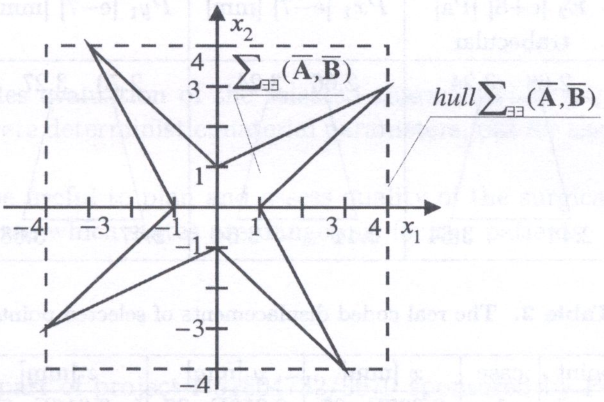

Fig. 2. Geometric interpretation of the interval system of equations solution

4. The interval and fuzzy analysis of human pelvic bone

The human pelvic bone is restrained in pubic symphysis and on contact surface with sacral bone. It is loaded with force F acting in artificial acetabulum. Two cases of the linear elastic analysis were carried out. In the first case the material parameters are not position-depended. In the second case the selected material parameters depend on the position in the bone.

For both cases the interval and fuzzy (two alpha-cuts-trapezoid) approaches are applied.

It was assumed that for the interval analysis, the Young moduli of trabecular bone (in both cases) was constant and equal [1.8E8;2.2E8]. The Young moduli of the cortical bone (in the first case) was modelled as the interval [1.8E10;2.2E10]. In the second case the Young moduli of the cortical bone was equal to the interval [1.8E10;2.2E10] in zone A (Fig. 3) and was equal to [0.9E10;1.1E10] in the bound B (between P_1 and P_3). In space between zone A and the bound B, the Young moduli was generated with the linear weight function.

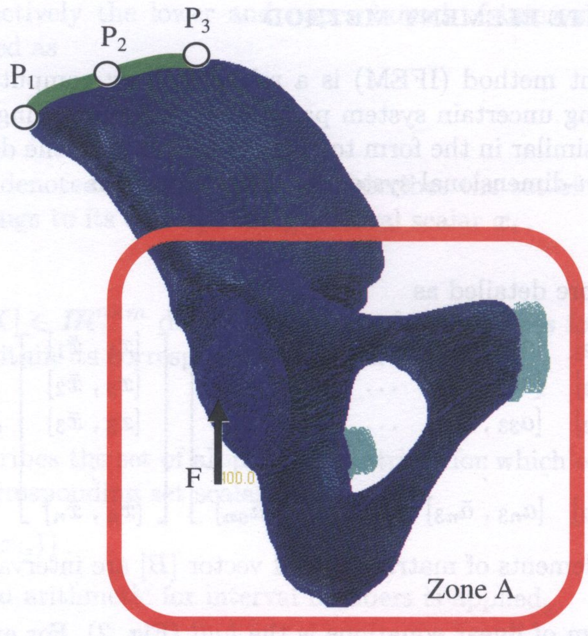

Fig. 3. The model of the human pelvic bone

Table 1. The fuzzy material parameters and displacements of point P_1

E_1 [e+10] [Pa] cortical	E_2 [e+8] [Pa] trabecular	Px_1 [e−7] [mm]	Py_1 [e−7] [mm]	Pz_1 [e−7] [mm]
1.8 2.2	2.66 3.24	2.66 3.24	2.70 3.27	−3.34 −2.74
1.6 2.4	2.44 3.64	2.44 3.64	2.47 3.68	−3.76 −2.51

Table 2. The real coded displacements of selected points

point	case	x [mm]	y [mm]	z [mm]
P_1	1	2.927E−07	2.953E−07	−3.016E−07
	2	2.927E−07	2.953E−07	−3.015E−07
P_2	1	3.932E−07	1.337E−07	−2.920E−07
	2	3.932E−07	1.337E−07	−2.929E−07
P_3	1	4.194E−07	3.731E−08	−2.663E−07
	2	4.194E−07	3.733E−08	−2.662E−07

Table 3. The interval displacements of selected points

point	case	x [mm]		y [mm]		z [mm]	
		$\underline{x}$	$\overline{x}$	$\underline{y}$	$\overline{y}$	$\underline{z}$	$\overline{z}$
P_1	1	2.666E−07	3.247E−07	2.690E−07	3.277E−07	−3.347E−07	−2.744E−07
	2	2.665E−07	3.247E−07	2.690E−07	3.276E−07	−3.348E−07	−2.745E−07
P_2	1	3.580E−07	4.363E−07	1.217E−07	1.484E−07	−3.240E−07	−2.658E−07
	2	3.580E−07	4.362E−07	1.217E−07	1.484E−07	−3.241E−07	−2.658E−07
P_3	1	3.819E−07	4.654E−07	3.390E−08	4.151E−08	−2.955E−07	−2.424E−07
	2	3.819E−07	4.654E−07	3.388E−08	4.150E−08	−2.955E−07	−2.424E−07

In the fuzzy analysis case, the Young moduli of the trabecular bone (in both cases) was constant and equal to the fuzzy number (see Table 1). First the Young moduli of the cortical bone was modelled as the fuzzy number (see Table 1). The values of the cortical bone in zone A and the bound B (Fig. 3) in the second case are shown in Table 1. The space between zone A and the bound B was determined with linear function.

The rest of parameter were assumed as the determine numbers.

The results: real-coded, interval and fuzzy displacements in selected points are shown in Tables 2 and 3 and in Fig. 4.

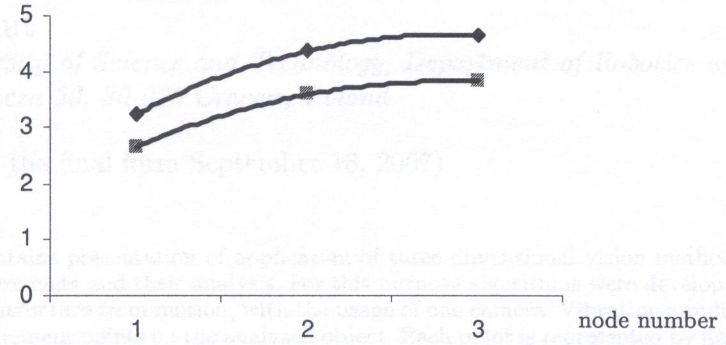

Fig. 4. The interval (lower and upper) x-displacements of front $P_1 - P_3$

5. Conclusions

Arithmetic analysis enables evaluation of the selected characteristics (strain, stress and displacements) not only for a discrete deterministic material parameters, but for assumed interval. It satisfies reality more precisely.

Obtained results can be useful to plan and assess quality of the surgical intervention.

The surgeons can observe which states are dangerous for the patients.

Acknowledgement

The work was done as a part of project N51804732/3670 sponsored by Polish Ministry of Science and Higher Education.

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