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Identification of thermal properties of solids by means of solving the inverse problem using evolutionary algorithms and optimal dynamic filtration

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The aim of the paper is to combine the evolutionary algorithms method, optimal dynamic filtration method and measurement data for the simultaneous identification of the thermal properties or their temperature characteristics of anisotropic solids. The idea of the proposed method depends on measuring the time-dependent temperature distribution at selected points of the sample and identification of the thermal parameters (heat conductivity and specific heat) by solving a transient inverse heat conduction problem. In the paper the discrete mathematical model has been formulated basing on the control volume method. The inverse problem was solved by using a hybrid method. Information about measurement data which are necessary to solve the inverse heat conduction problem was obtained by solving the direct heat conduction problem. The chosen results of analysis have been presented.

Keywords: control volume method, discrete mathematical model, parameter inverse problem, evolutionary algorithms method, optimal dynamic filtration method, thermal properties of anisotropic materials

1. Introduction

The knowledge of the exact values of the thermal properties of materials including thermal conductivity k and specific heat c and their temperature characteristics is a very important factor taking into account:

- the usage of materials of the required thermal properties,
- the production of new materials with the required properties,
- mathematical modelling of a wide range of thermal processes.

Therefore, we can observe the development and implementation of new, more accurate methods (in particular transient methods) for the determination of the thermal properties of materials [3, 4, 6, 7, 9]. Those methods require more sophisticated equipment and a mathematical background, but enable us to determine simultaneously more than one thermal parameter. The idea of the presented inverse approach for the identification of the thermal parameters is to measure the time-dependent temperature distribution at selected points in the sample and to estimate the thermal properties by solving the inverse heat conduction problem. The inverse heat conduction problem is generally solved in two stages.

In the first stage, basing on a suitably formulated mathematical model, the direct heat conduction problem is solved. Auxiliary measurements and their characteristics (for example the temperature field) are also determined. These quantities will be used in the second stage of solving the algorithm.

In the second stage, making use of measurement data and previously determined quantities, the inverse problem is solved and the final quantities are determined.

Generally the final effect of solving the inverse heat conduction parameter problem are thermo-physical and technological parameters. Those parameters include generally: heat flux, heat transfer coefficient or ambient temperature, geometrical parameters such as dimensions and/or the configuration of the device under investigation and the thermo-physical characteristics such as the thermal properties or their temperature characteristics of the investigated material. The specific feature of the considered problem is that the objective function does not depend on the estimated variables in the explicit way.

In order to find the relationship between the identified variables and changes of the objective function we must solve the direct transient heat conduction boundary problem. In this paper to solve the direct boundary problem, the mathematical model has been formulated basing on the control volume method. The inverse problem was formulated as an optimisation problem and solved by using the hybrid method. Thus by combining evolutionary algorithms method, the optimal dynamic filtration method and measurement data the effective tool was obtained for the simultaneous identification of the thermal properties of materials and their temperature characteristics.

Information about input data required to solve the inverse heat conduction problem was obtained by solving the direct heat conduction problem.

2. FORMULATION OF THE MATHEMATICAL MODEL AND SOLUTION OF THE DIRECT HEAT CONDUCTION PROBLEM

In the paper two identical samples located on the both sites of the electric heater were considered. Hence the problem becomes symmetrical and we can analyse only one of the samples (Figs. 1 and 2).

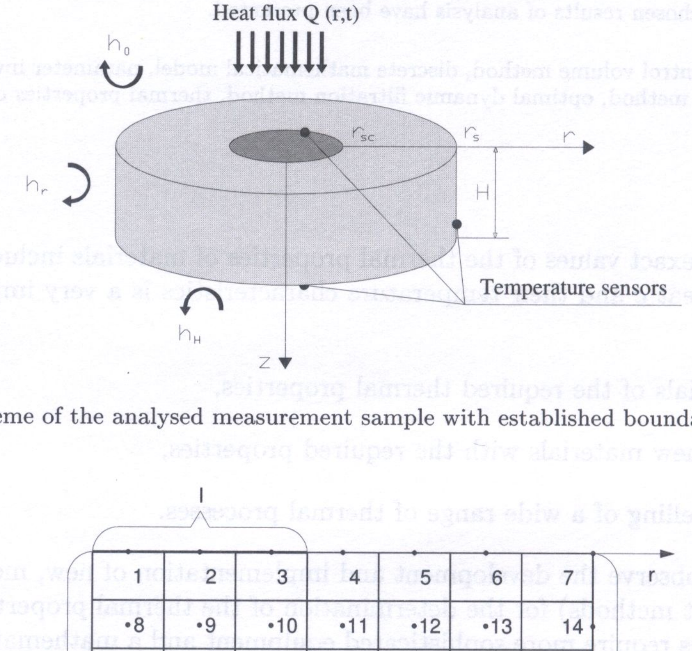

Fig. 1. Scheme of the analysed measurement sample with established boundary conditions

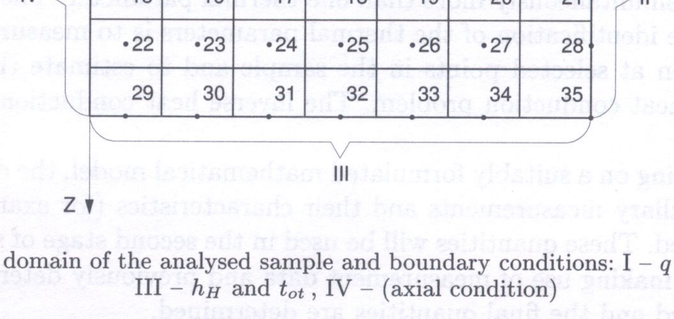

Fig. 2. The discrete domain of the analysed sample and boundary conditions: I – q W/m^2, II – h_r and t_{ot}, III – h_H and t_{ot}, IV – (axial condition)

Two-dimensional heat conduction in the analysed process can be described by the well-known differential equation of the form

$$c\rho\frac{\partial t}{\partial \tau}=\frac{\partial}{\partial z}\left(k_z\frac{\partial t}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(k_r\frac{\partial t}{\partial r}\right),\qquad (r,z,\tau)\in(0,r_s)\times(0,H)\times(0,\tau_k). \tag{1a}$$

The boundary conditions for that case can be written in the form

$$k_z\left.\frac{\partial t}{\partial z}\right|_{z=0}=\dot{q},\qquad 0\le r\le r_{sc}, \tag{1b}$$

$$k_z\left.\frac{\partial t}{\partial z}\right|_{z=H}=h_H(t-t_{ot}),\qquad 0\le r\le r_s, \tag{1c}$$

$$k_r\left.\frac{\partial t}{\partial r}\right|_{r=\pm r_s}=h_r(t-t_{ot}),\qquad 0\le z\le H, \tag{1d}$$

$$\lim_{r\to 0}k_r\frac{\partial t}{\partial r}=0,\qquad 0\le z\le H. \tag{1e}$$

The initial temperature in the sample is known and kept uniform, so the initial condition concerning the boundary heat conduction problem is taken as

$$t(r,z,\tau{=}0)=t_{ot},\qquad (r,z)\in(0,r_s)\times(0,H). \tag{1f}$$

Basing on the control volume method, the discrete mathematical model of the transient temperature field within the sample (Eqs. (1)), for the necessity of the dynamic filtration method is convenient to be expressed in the following matrix equation,

$$\mathbf{y}_{k+1}=F(\mathbf{y}_k\,,\tau_k), \tag{2}$$

where $\mathbf{y}_k$, $\mathbf{y}_{k+1}$ are the augmented vectors of state at the time step k and $k+1$, that includes the temperature field and unknown thermal parameters, i.e. N nodal temperatures written as the vector $\mathbf{t}$ and M determined parameters written as the vector $\mathbf{p}$, F is a function of the state which defines the relation between the vectors of state in two adjacent time steps.

So the augmented vector of state $\mathbf{y}$ can be written as follows,

$$\mathbf{y}=\left[\mathbf{t}^T\ \mathbf{p}^T\right]^T, \tag{3}$$

where N nodal temperatures vector $\mathbf{t}$ and M determined parameters vector $\mathbf{p}$ can be written as

$$\mathbf{t}=[t_1\,,\ldots,t_N]^T\qquad\text{and}\qquad \mathbf{p}=[k_r\,,k_z\,,c]^T. \tag{4}$$

Generally Eq. (2) can be non-linear and the value of M depends on the functional form of heat conductivity k or specific heat c.

3. Formulation of the inverse hybrid problems

As said before the identification of the thermal properties of solids and their temperature characteristics is based on the solution of the inverse heat conduction coefficient problem and on searching for the value of one or more coefficients that describe the thermal properties of the solid material. The inverse (identification) problem was formulated as an optimization (minimization) problem of properly formulated objective functions. In this paper the modified efficient parallel computational scheme for system parameters identification was applied. The modification is achieved by the joint evolutionary algorithms method with the optimal dynamic filtration method. In the order each time interval between time steps k and $k+1$ was divided into some shorter time subintervals and the temperature was measured in these time sub-steps. This observation was used to solve the inverse heat conduction problem by means of the evolutionary algorithms method.

Next the results obtained by solving the evolutionary algorithms method and measuring results in time step $k+1$ in the optimal dynamic filtration method were applied to determine of the augmented vector of state $\hat{\mathbf{y}}_{k+1}$ in the time step $k+1$. In the evolutionary algorithms method the objective function (evaluative function) was assumed in the following form,

$$f(\mathbf{p}) = \sum_{k=1}^{kk} \sum_{p=1}^{MP} (t_{k,p}^{cal} - t_{k,p}^{meas})^2 \Rightarrow \min\,, \tag{5}$$

where MP – total number of sensors within the sample, kk – number of time sub-steps during the measurements of the transient temperature, $t_{k,p}^{cal}$, $t_{k,p}^{meas}$ – calculated and measured temperature at the kth time sub-step and pth point of the sample,

For the linear temperature dependence of k and c

$$k_r = k_{r0} + k_{r1}t\,, \qquad k_z = k_{z0} + k_{z1}t\,, \qquad c = c_0 + c_1 t\,. \tag{6}$$

The vector $\mathbf{p}$ can be expressed in the following form,

$$\mathbf{p} = [k_{r0}\,, k_{r1}\,, k_{z0}\,, k_{z1}\,, c_0\,, c_1]^T\,. \tag{7}$$

In the optimal dynamic filtration method the objective function (risk function) can be written in the following form,

$$\begin{aligned} R(\mathbf{y}, \tilde{\mathbf{Y}}_{k+1}) = {} & \left(F_{k+1/k}(\hat{\mathbf{y}}_k) - \tilde{\mathbf{Y}}_{k+1}\right)^T \mathbf{V}_{k+1}^{-1} \left(F_{k+1/k}(\hat{\mathbf{y}}_k) - \tilde{\mathbf{Y}}_{k+1}\right) \\ & + \left((\mathbf{y} - \hat{\mathbf{y}}_k)^T \left[\mathbf{W}_{k/k}\right]^{-1} (\mathbf{y} - \hat{\mathbf{y}}_k)\right) \end{aligned} \tag{8}$$

where $\tilde{\mathbf{Y}}$ – vector of measured temperatures, $F(\hat{\mathbf{y}}_k) = \hat{\mathbf{y}}_{k+1/k}$ – prediction vector of measured temperatures, $\mathbf{W}$ – covariance matrix of estimated errors, $\mathbf{V}$ – covariance matrix of measurement errors.

4. Hybrid solution of the identification problem

This problem can be solved by means of the traditional optimization method, based on sensitivity information. But such approaches have many faults, for example: they seek the optima in the neighbourhood of the starting point and if the sensor position within the sample is not optimal there is a large probability of convergence to a local optimum etc. Therefore, in this paper the optimization problem was solved by applying the hybrid method. At first applying the evolutionary algorithms method to solve the inverse problem within some time subintervals between the time steps k and $k+1$ and the vector $\mathbf{p}$ was determined. Next this vector was used as entire data for the optimal dynamic filtration method and the augmented vector was determined in time step $k+1$. The evolutionary algorithms method enables us among others:

- assuming to solve inverse problem a wide entire range of research parameters (it is very important in the case of unknown thermal parameters of the material),
- find always the global extremum and in this way to come closer to model value of identified parameters .

The second point is very important in the case of determining the thermal properties of solid materials, because the objective function (5) has a specific shape.

Thus using in this case only the standard optimisation method we can often obtain uncorrected final results.

4.1. Applying the evolutionary algorithms method to solution of the inverse problem

Evolutionary algorithms, as well genetic algorithms are stochastic methods, basing on Darwin's theory, i.e. the natural selection and improvement of species which takes place in the organic world [1]. Classical genetic algorithms are based on coded binary chromosomes and use binary operators of selection, crossover and mutation. Every chromosome is estimated using the fitness function and a part of them is taken into the next generation. Classical genetic algorithms cannot be directly applied in real often very complex problems because we have to modify the problem and modify the algorithms.

Evolutionary algorithms are modified genetic algorithms, where the population of chromosomes is not coded binary but a floating point representation [5, 8]. They use the following operators: *crossovers* (simple, arithmetical or heuristic) and *mutation* (uniform, non-uniform or boundary). But to select a new population, the following probability of operators can be used: proportional selection, ranking selection or tournament selection.

In order to improve the accuracy of the evolutionary algorithm method in this paper, different genetic algorithm operators (*selection, crossover and mutation*) have been used.

As mentioned above, the identification problem was formulated as a minimization of the sum of differences between the calculated transient temperature obtained in the mathematical model (for the assumed thermal parameters of the material) and the measured temperature obtained by a numerical experiment at the test stand, Eq. (5).

In the present paper it has been assumed that the chromosome represents the vector which contains the genes x_i. These genes represent the thermal properties (thermal conductivity and specific heat or their temperature characteristics) of the researched material. These values of the genes x_i belong to the real space and each of them has the following restriction,

$$x_{ip} \leq x_i \leq x_{ik} \,. \tag{9}$$

In order to create the offspring, a crossover operation of the selected parents was applied: at first *heuristic crossover* and next *arithmetical crossover.*

During the *heuristic crossover* the operator produces an offspring, taking a random number ($\xi \in [0; 1]$) and two parents according to the formula

$$x_3 = x_2 + \xi(x_2 - x_1) \qquad \text{if} \quad f(x_2) \leq f(x_1), \tag{10a}$$

or

$$x_3 = x_2 - \xi(x_2 - x_1) \qquad \text{if} \quad f(x_2) \geq f(x_1). \tag{10b}$$

During the *arithmetical crossover* the operator produces two offspring from a linear combination of two parents as follows,

$$x_3^1 = \alpha x_1 + (1 - \alpha)x_2 \qquad \text{and} \qquad x_3^2 = \alpha x_2 + (1 - \alpha)x_1 \,, \tag{11}$$

where coefficient $\alpha \in [0, 1]$.

In the presented paper the *uniform mutation* and *Gauss mutation* were applied.

In the uniform mutation the offspring is allowed to move freely within a feasible domain and a new gene x_i^{new} takes its value within the range $[x_{il}\,, x_{ir}]$.

In the Gauss mutation a random number is described by means of the following equation,

$$Q = \mu \left(\sum_{i=1}^{12} (\xi_i - \sigma) \right), \tag{12}$$

where μ – expected value, σ – standard deviation.

To create a new generation, a *mixed selection* (soft and hard) was applied. At first a (soft) proportional selection and next a (hard) elitist model was applied.

4.2. Solution of the inverse heat conduction problem by means of the Kalman filter method

The dynamic filtration method requires a relationship between the results of temperature measurements $\mathbf{z}$ (the so-called vector of observations) in selected MP sensors located within the sample and the vector of state $\mathbf{y}$ at a given time step $k+1$, that can be written as

$$\mathbf{z}_{k+1} = \mathbf{H}\mathbf{y}_{k+1} + \mathbf{v}_{k+1} \tag{13}$$

where the matrix $\mathbf{H}$ is called the matrix of measurements, that is consist of elements equal to unity and zero because the augmented state vector always includes the temperatures from the measurement vector. The vector $\mathbf{v}_{k+1}$ represents measurement errors of the covariance matrix $\mathbf{V}_{k+1}$.

In the analysed case the function of the state $F(\mathbf{y})$ is non-linear with respect to the state variables which disables us to use the linear version of the discrete Kalman filter, so the function $F(\mathbf{y})$ must be first linearized. As the result of linearization it is not necessary to use an iterative procedure within a unique time step (but there is always the danger that the calculation process may be divergent, especially when the initial guess of the state vector is far from the real one).

The main two steps of the dynamic filtration procedure are prediction and correction [3, 6]:

- **prediction**: The prediction vector of state $\hat{\mathbf{y}}_{k+1/k}$ is given by the relationship

$$\hat{\mathbf{y}}_{k+1/k} = F_{k+1,k}(\hat{\mathbf{y}}_k), \tag{14}$$

- **correction**: the correction vector of state $\tilde{\mathbf{y}}_{k+1}$ has the form

$$\hat{\mathbf{y}}_{k+1} = \hat{\mathbf{y}}_{k+1/k} + \mathbf{K}_{k+1}[\mathbf{z}_{k+1} - \mathbf{H}\hat{\mathbf{y}}_{k+1/k}] \tag{15}$$

and the covariance matrix of the prediction of assessed errors can be written as

$$\mathbf{W}_{k+1/k} = \mathbf{G}_{k+1/k}\mathbf{W}_{k,k}\mathbf{G}^T_{k+1/k}\,. \tag{16}$$

The elements of the matrix $\mathbf{G}$ represent derivatives of the state function F_i with respect to the elements of the state vector $\hat{\mathbf{y}}_k$, where $\mathbf{K}_{k+1}$ is the so-called Kalman gain matrix and has the form

$$\mathbf{K}_{k+1} = \mathbf{W}_{k+1/k}\mathbf{H}^T\left[\mathbf{H}\mathbf{W}_{k+1/k}\mathbf{H}^T + \mathbf{V}_{k+1}\right]^{-1} \tag{17}$$

but the covariance matrix of estimate errors has the form

$$\mathbf{W}_{k+1} = \mathbf{W}_{k+1/k} - \mathbf{W}_{k+1/k}\mathbf{H}^T\left[\mathbf{H}\mathbf{W}_{k+1/k}\mathbf{H}^T + \mathbf{V}_{k+1}\right]^{-1}\mathbf{H}\mathbf{W}_{k+1/k}\,. \tag{18}$$

5. Selected results of the researches

The accuracy of the proposed method was verified by a numerical experiment. In order to solve the inverse problem for numerical experiment, the (disturbed data) simulated "measured" temperature t_i^{meas} was obtained by adding a noise term $(\omega \cdot \nu)$ to the results of solution of the direct boundary heat conduction problem (for given values of k and c) t_i according to the relationship

$$t_i^{meas} = t_i + \omega \cdot \nu \tag{19}$$

where ν is the standard deviation of measurement errors. For normally distributed errors (Gaussian distributed noise) with 99% confidence for the measured data, ω lies in the range $-2.576 \leq \omega \leq 2.576$, and the value of ω is calculated by a random generator [2].

The analysed material was organic glass (density $\rho = 1180\,\mathrm{kg/m^3}$). The samples were made in the shape of a cylinder with the diameter $d = 71.9\,\mathrm{mm}$, the thickness $H = 12.45\,\mathrm{mm}$. The power of the heater varied during the each experiment. The sample value of power P is 4.8 W, with a heat flux density of $q = 591.4\,\mathrm{W/m^2}$. Measurement sensors were located on the internal ($z = 0$), external ($z = H$) and cylindrical ($r = r_s$) surfaces of the sample. The simulated measurement data were obtained by disturbances by the different error ν (Eq. 19) of the results of the solution the direct heat conduction problem.

The temperature within the sample was measured at $\Delta\tau = 1\,\mathrm{s}$ and this time step was used for the method of evolutionary algorithms, but in the case of the optimal filtering method $\Delta\tau = 6\,\mathrm{s}$ was used.

As the first case the constant, i.e. temperature independent values of conductivity k and specific heat c are studied. The thermal properties assumed for the numerical experiment are: conductivity $k_r = 0.182\,\mathrm{W/mK}$, $k_z = 0.162\,\mathrm{W/mK}$ and specific heat $c = 1500\,\mathrm{J/kgK}$. The following entire ranges of research parameters were assumed as $k_r \in [0;1]$, $k_z \in [0;1]$ and $c = [500;3000]$. After calculations by means of the evolutionary algorithms method the following results were obtained: $k_r = 0.177\,\mathrm{W/mK}$, $k_z = 0.157\,\mathrm{W/mK}$ and $c = 1462\,\mathrm{J/kgK}$. Next using this data the dynamic filtration method was used and the following value were obtained $k_r = 0.1818\,\mathrm{W/mK}$, $k_z = 0.1619\,\mathrm{W/mK}$ and $c = 1498\,\mathrm{J/kgK}$. So after two time steps correct results were obtained.

In the numerical experiment for the different linear temperature dependences of k_r, k_z and c, the direct heat conduction problem was solved for a sample with the same geometrical parameters and power of the heater as in the case of the analysed constant properties, but for optimal filtering method $\Delta\tau = 20\,\mathrm{s}$ was used. The influence of the measurement errors $\upsilon = [0.01 \div 0.1]$ (for the given value of the time step) on the results of identification was examined. In the paper the simulated measurement temperatures of heated and other surfaces of the sample and disturbances by an error $\nu = 0.05\,\mathrm{K}$ are shown in Fig. 3.

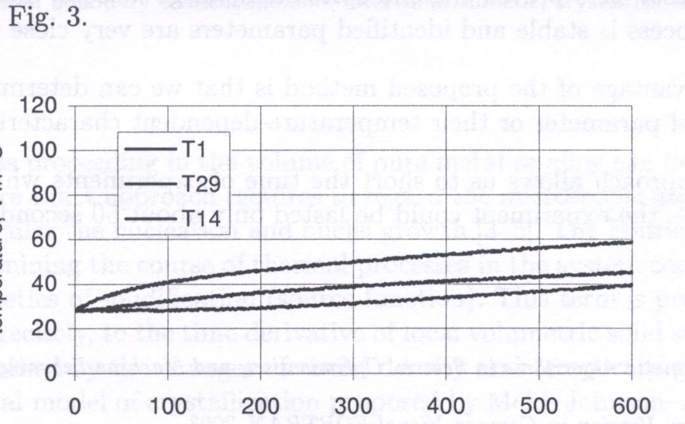

Fig. 3. Temperature distribution on the surfaces of the sample as a function of time

The direct heat conduction problem presented in Fig. 3 was solved for the following temperature dependence of k_r, k_z and c,

$$\begin{aligned} k_r(t) &= k_{r0} + k_{r1}t = 0,2 + 0,05t, \\ k_z(t) &= k_{z0} + k_{z1}t = 0,16 + 0,02t, \\ c &= c_0 + c_1 t = 1500 + 5t. \end{aligned} \tag{20}$$

The results obtained in the sequence of time steps during the calculation by means of the proposed algorithm have been presented in Table 1.

So, in case of the linear temperature dependences of parameters after four time steps correct results were obtained.

Table 1. Results of the parameter identification

Evolutionary algorithms method												
number of time substeps	Entire value of parameters						Results of estimation					
	k_{r0}	k_{r1}	k_{z0}	k_{z1}	c_0	c_1	k_{r0}	k_{r1}	k_{z0}	k_{z1}	c_0	c_1
20	[0;1]	[0;0.1]	[0;1]	[0;0.1]	[500;3000]	[0;8]	0.175	0.07	0.14	0.025	1450	8.000
20	[0;0.24]	[0;0.1]	[0;0.2]	[0;0.08]	[1200;1600]	[4;8]	0.185	0.06	0.17	0.019	1509	7.500

Optimal dynamic filtration method												
time step number	Initial data of parameters						Results of estimation					
	k_{r0}	k_{r1}	k_{z0}	k_{z1}	c_0	c_1	k_{r0}	k_{r1}	k_{z0}	k_{z1}	c_0	c_1
1	0.175	0.07	0.14	0.025	1450	8.000	0.187	0.061	0.15	0.018	1494	6.030
2	0.185	0.06	0.17	0.019	1509	7.000	0.2003	0.0498	0.1599	0.0199	1501	5.010

6. Final conclusions

The final conclusions may be listed as follows:

- The inverse heat conduction solution problem based on the connection control volume method, measurement data and the hybrid method constitutes a very effective tool for identification of the thermo-physical parameters or their temperature dependent characteristics of various solid materials.
- The numerical tests have proved that, in spite of the large noise of measured temperatures the identification process is stable and identified parameters are very close to real values.
- The essential advantage of the proposed method is that we can determine simultaneously more than one physical parameter or their temperature-dependent characteristics (i.e. k, c),
- The proposed approach allows us to short the time of experiments which is another advantage of this method — the experiment could be lasted only about 50 seconds.

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