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# The granular computing in uncertain identification problems

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The paper is devoted to applications of evolutionary algorithms in identification of structures being under the uncertain conditions. Uncertainties can occur in boundary conditions, in material parameters or in geometrical parameters of structures and are modelled by three kinds of granularity: interval mathematics, fuzzy sets and theory of probability. In order to formulate the optimization problem for such a class of problems by means of evolutionary algorithms the chromosomes are considered as interval, fuzzy and random vectors whose genes are represented by: (i) interval numbers, (ii) fuzzy numbers and (iii) random variables, respectively. Description of evolutionary algorithms with granular representation of data is presented in this paper. Various concepts of evolutionary operator such as a crossover and a mutation and methods of selections are described. In order to evaluate the fitness functions the interval, fuzzy and stochastic finite element methods are applied. Several numerical tests and examples of identification of uncertain parameters are presented.

**Keywords:** evolutionary algorithms, granular computing, intervals, fuzzy sets, theory of probability, identification

## 1. INTRODUCTION

In the majority engineering cases it is not possible to determine exactly all parameters of the physical systems. It is necessary to introduce some uncertain parameters which describe the granular character of data. Representation of uncertain values may have different forms. It depends of the physical meaning of the considered problem and the assumed model of uncertainty. There are several formal frameworks in which information granules can be built [7] among them interval analysis [19], fuzzy sets [21] and random variables [3] can be considered.

The aim of an identification problem is to find some unknown parameters of a mechanical system having some measurements of physical quantities such as displacements or natural frequencies [4]. The identification problem is considered as the minimization of a functional which depends of an error between measured and computed physical quantities as e.g. displacements in the sensor points.

The evolutionary algorithms [1], as the global optimization technique for searching uncertain values, can be applied in finding the interval parameter [7], fuzzy models [8], fuzzy controllers [11], fuzzy rules [2], random parameters and others. In such algorithms, the chromosome consists of uncertain genes. Therefore, the evolutionary operators are modified for uncertain types of data.

This paper describes a new conception of application of the granular evolutionary algorithm in identification problems with uncertain parameters. The following systems are considered as the granular models (i) interval numbers, (ii) fuzzy numbers and (iii) random variables. The proposed granular evolutionary algorithm is examined for testing bench-mark, due to the optimal parameters of the algorithms (population size, probability of mutation and crossover) are found. Next, the algorithm is applied for identification problem in mechanical structures. The paper presents the application of the algorithm in finding the shape, material coefficients and boundary conditions of the mechanical structures.

## 2. GRANULAR EVOLUTIONARY ALGORITHM

The paper concerns the granular evolutionary algorithm with granular operators and granular representation of the data. The chromosomes contain granular genes. Each gene decides about the heredity of one or a few characteristics. The individuals can be modified by means of the granular operators. The evolutionary operators generate new chromosomes. The next step is the operator of the selection. It creates a new generation, which contains better chromosomes. All steps are repeated until the stop condition is fulfilled (Fig. 1).

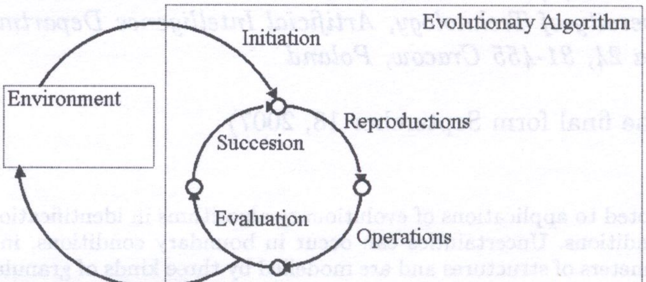

**Fig. 1.** The flow chart of the granular evolutionary algorithm

In the granular evolutionary algorithm an individual expresses a granular solution. In each generation the granular evolutionary algorithm contains a population of solutions. Each solution is evaluated, and as the result a granular value of the fitness function is obtained.

### 2.1. The granular representation of chromosomes

### *2.1.1. The interval chromosome*

In most cases the evolutionary algorithm has the genes as the real values. The granular algorithm works on the granular data, so the gene should be modified to granular data. In the paper the following cases are considered: (i) interval genes, (ii) fuzzy genes and (iii) random genes.

In the interval case the gen $[x] = [\underline{x}, \bar{x}]$ is described by the central value $cv([x]) = (\underline{x} + \bar{x})/2$ and the radius $r([x]) = (\underline{x} - \bar{x})/2$.

Therefore the interval chromosome can be expressed by

$$[[x_1], [x_2], \ldots, [x_i], \ldots, [x_n]]. \tag{1}$$

The interval chromosome can be replaced by the real-coded chromosome

$$[(cv_1 \, , r_1) \, , (cv_2 \, , r_2) \, , \ldots \, , (cv_i \, , r_i) \, , \ldots \, , (cv_n \, , r_n)] \tag{2}$$

where $[x_i] = (cv_i \, , r_i)$.

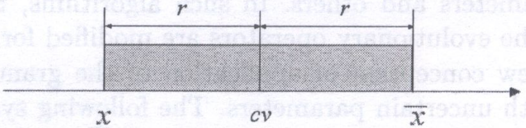

**Fig. 2.** The interval representation of the gene

### *2.1.2. The fuzzy chromosome*

In the fuzzy case the gene **x** can be considered as a fuzzy set (Fig. 3a). The fuzzy set is considered as a set of pairs of the $x$and the density function $\mu(x)$. When the fuzzy set is convex and normal and the density function is continuous, the fuzzy set is the fuzzy number (Fig. 3b). The concept of $\alpha$-cuts plays the important role in the theory of fuzzy sets. An $\alpha$-cut of a fuzzy number $A$ is a interval that contains all the numbers of $A$ that have the membership value of $A$ greater than or equal to $\alpha$. In this case the fuzzy number can be replaced by a set of the interval values, which are stretched on the adequate levels ($\alpha$-cuts) of the fuzzy value. The number of the $\alpha$-cuts can be arbitrary. Figure 3c shows an example of the replacement of the fuzzy number by 5 interval numbers.

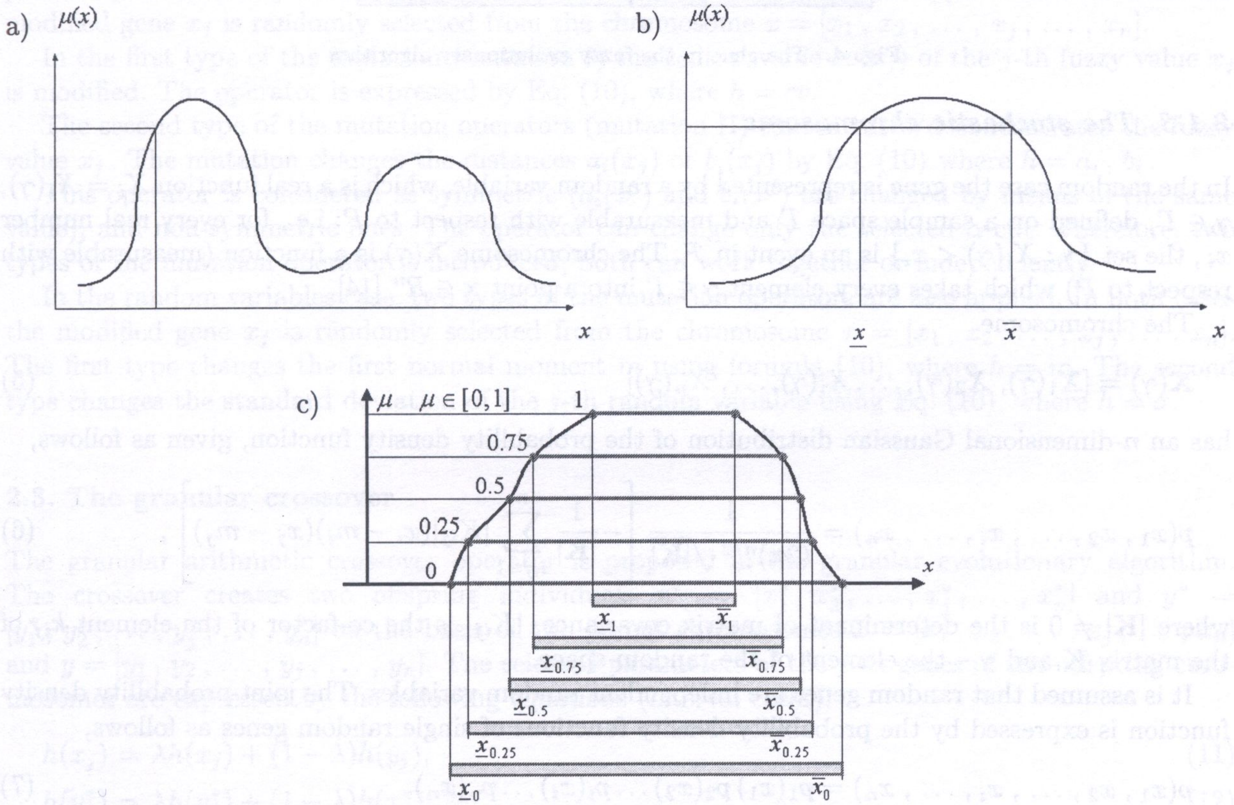

**Fig. 3.** The fuzzy a) set, b) number, c) corresponded intervals

This approach has some advantages. For each $\alpha$-cut the very good known interval arithmetic operators are used. It is possible to obtain different forms of the fuzzy values due to the generation of a few $\alpha$-cuts and corresponding them interval values $[\underline{x};\overline{x}]$. The forms can be symmetric or not symmetric. They describe some characteristic forms of the fuzzy values, and permit to build a new form of the fuzzy value too. Finally, each gene $x$ is expressed as the real value: the *central value* $cv(x)$ (Fig. 4) and a set of parameters $a_i(x)$ and $b_i(x)$ which define distances between $cv(x)$ and the boundaries of the intervals (Fig. 4). It is possible to introduce the constraints on the $cv(x)$ and non-symmetric constraints on the widths of the intervals using the central value $cv(x)$.

Therefore, the fuzzy chromosome can be expressed by

$$[\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_i, \ldots, \mathbf{x}_n]. \tag{3}$$

The fuzzy chromosome can be replaced by the real-coded chromosome (for 2 $\alpha$-cuts)

$$[(a_1^1, a_1^2, cv_1, b_1^2, b_1^1), (a_2^1, a_2^2, cv_2, b_2^2, b_2^1), \ldots, (a_i^1, a_i^2, cv_i, b_i^2, b_i^1), \ldots, (a_n^1, a_n^2, cv_n, b_n^2, b_n^1)] \tag{4}$$

where $\mathbf{x}_i = (a_i^1, a_i^2, cv_i, b_i^2, b_i^1)$.

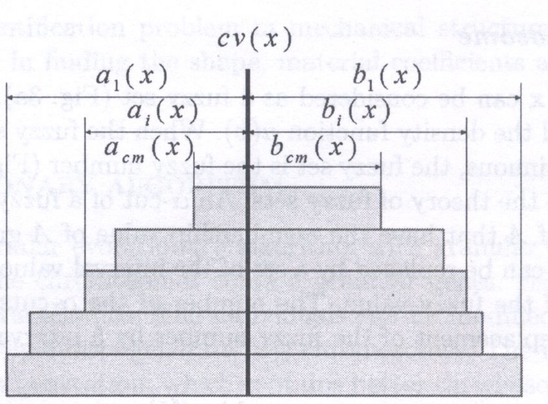

**Fig. 4.** The gene in the fuzzy evolutionary algorithm

### *2.1.3. The stochastic chromosome*

In the random case the gene is represented by a random variable, which is a real function $X_i = X_i(\gamma)$, $\gamma \in \Gamma$, defined on a sample space $\Gamma$ and measurable with respect to $P$: i.e., for every real number $x_i$, the set $\{\gamma : X_i(\gamma) < x_i\}$ is an event in $\mathcal{F}$. The chromosome $\mathbf{X}(\gamma)$ is a function (measurable with respect to $P$) which takes every element $\gamma \in \Gamma$ into a point $\mathbf{x} \in R^n$ [14].

The chromosome

$$\mathbf{X}(\gamma) = [X_1(\gamma), X_2(\gamma), \ldots, X_i(\gamma), \ldots, X_n(\gamma)] \tag{5}$$

has an $n$-dimensional Gaussian distribution of the probability density function, given as follows,

$$p(x_1, x_2, \ldots, x_i, \ldots, x_n) = \frac{1}{(2\pi)^{n/2}\sqrt{|\mathbf{K}|}} \left[ -\frac{1}{|\mathbf{K}|} \sum_{i,j=1}^{n} |K_{ij}|(x_i - m_i)(x_j - m_j) \right], \tag{6}$$

where $|\mathbf{K}| \neq 0$ is the determinant of matrix covariance, $|K_{ij}|$ is the co-factor of the element $k_{ij}$ of the matrix $\mathbf{K}$ and $\gamma$ – the element of the random space.

It is assumed that random genes are independent random variables. The joint probability density function is expressed by the probability density functions of single random genes as follows,

$$p(x_1, x_2, \ldots, x_i, \ldots, x_n) = p_1(x_1)\, p_2(x_2) \ldots p_i(x_i) \ldots p_n(x_n), \tag{7}$$

where

$$p_i(x_i) = N(m_i, \sigma_i) = \frac{1}{\sigma_i\sqrt{2\pi}} \exp\left[ -\frac{(x_i - m_i)^2}{2\sigma_i^2} \right] \tag{8}$$

is the probability density function of the random gene $X_i(\gamma)$.

It can be seen that if the random genes $X_i(\gamma)$, $i = 1, 2, \ldots, n$, are random independent Gaussian variables, two moments describe the probability density function of the random variable $X_i(\gamma)$.

Therefore the stochastic chromosome (5) can be replaced by

$$[(m_1, \sigma_1), (m_2, \sigma_2), \ldots, (m_i, \sigma_i), \ldots, (m_n, \sigma_n)] \tag{9}$$

where $m_i$ – the mean value and $\sigma_i$ – standard deviation.

## 2.2. The granular mutation

In the interval case two types of the mutation operators are applied. In both cases the modified gene $x_j$ is randomly selected from the chromosome $x = [x_1, x_2, \ldots, x_j, \ldots, x_n]$.

In the first type of the mutation (mutation I) the central value $cv(x_j)$ of the $j$-th interval value $x_j$ is modified. The operator is expressed by the following equation,

$$h(x_j^*) = h(x_j) + G_h\,, \tag{10}$$

where $h(y) = cv(y)$ for each gene $y$, $G_h$ – random value (with Gaussian distribution), $j = 1 \ldots n$ is the number of the gene.

The second type of the mutation operators (mutation II) concentrates/deconcentrates the interval value. The mutation changes the radius $r(x_j)$ according to Eq. (10), where $h(y) = r(y)$ for each gene $y$.

Therefore, two types of the mutation operator are introduced, both can work together or independently. In the fuzzy case two types of the mutation operators are also applied. In both cases the modified gene $x_j$ is randomly selected from the chromosome $x = [x_1\,, x_2\,, \ldots\,, x_j\,, \ldots\,, x_n]$.

In the first type of the mutation (mutation I) the central value $cv(x_j)$ of the $j$-th fuzzy value $x_j$ is modified. The operator is expressed by Eq. (10), where $h = cv$.

The second type of the mutation operators (mutation II) concentrates/deconcentrates the fuzzy value $x_j$. The mutation changes the distances $a_i(x_j)$ or $b_i(x_j)$ by Eq. (10) where $h = a_i\,, b_i$.

This operator is considered as symmetric ($a_i(x^j)$ and $b_i(x^j)$ are changed by means of the same value), and non-symmetric ones. The operator can change only the selected $\alpha$-cut. Therefore, two types of the mutation operator is introduced, both can work together or independently.

In the random variables case, two types of the mutation operators are also applied. In both cases the modified gene $x_j$ is randomly selected from the chromosome $x = [x_1\,, x_2\,, \ldots\,, x_j\,, \ldots\,, x_n]$. The first type changes the first normal moment $m$ using formula (10), where $h = m$. The second type changes the standard deviation of the $j$-th random variable using Eq. (10), where $h = \sigma$.

### 2.3. The granular crossover

The granular arithmetic crossover operator is proposed in the granular evolutionary algorithm. The crossover creates two offspring individuals $x^* = [x_1^*\,, x_2^*\,, \ldots\,, x_j^*\,, \ldots\,, x_n^*]$ and $y^* = [y_1^*\,, y_2^*\,, \ldots\,, y_j^*\,, \ldots\,, y_n^*]$ on the basis of two parent chromosomes $x = [x_1\,, x_2\,, \ldots\,, x_j\,, \ldots\,, x_n]$ and $y = [y_1\,, y_2\,, \ldots\,, y_j\,, \ldots\,, y_n]$. The selected parameters of the $j$-th genes of the offspring chromosomes are expressed by the following equations (interval cases),

$$h(x_j) = \lambda h(x_j) + (1 - \lambda) h(y_j), \tag{11}$$

$$h(y_j^*) = \lambda h(y_j^*) + (1 - \lambda) h(x_j^*), \tag{12}$$

where $h = cv, r$, and $\lambda \in [0, 1]$ is a random value with the uniform distribution.

In the fuzzy case the selected parameters of the $j$-th genes of the offspring chromosomes are expressed by Eqs. (11) and (12), where $h = cv, a_i\,, b_i$.

In the random variables case the offspring chromosomes are expressed by Eq. (11) and (12), where $h = m, \sigma$.

### 2.4. The granular selection

The last modified operator for the interval, fuzzy values and random variables is the selection operator. This operator is constructed on the basis of a well known tournament selection. In this selection the fitness function values $f$ are compared, and the better chromosome wins more often. Therefore the special strategy of comparison of two granular values is proposed.

In interval and fuzzy cases the special conditions are constructed,

$$h(f_1) < h(f_2). \tag{13}$$

where $h = cv, r$.

In the fuzzy case the condition (13) is checked, where $h = a_1, b_1, r$. In the stochastic case the condition (13) is checked, where $h = m, \sigma$.

## 2.5. The granular fitness function

One of the most important steps of the evolutionary algorithm is the evaluation of the fitness function. If the design variables are deterministic, the fitness function result is also deterministic. In the case of solving the granular optimization problems, the problem of evaluating the fitness function is much more complicated. A few ways to estimate the results are possible in this case. In the case of simple mathematical functions the basic arithmetic operators $\{+; -; *; /\}$ for granular representation are used.

Unfortunately, in many cases the fitness function can be examined after solving the interval/fuzzy/stochastic boundary-value problem. The boundary-value problems can be solved by means of the interval/fuzzy/stochastic boundary element method or the interval/fuzzy/stochastic finite element method.

## 3. TESTING THE GRANULAR EVOLUTIONARY ALGORITHM

The aim of the test is to find the vector $\mathbf{x}$ which minimizes the granular function

$$f = f(\mathbf{x}) = \sum_{i=1}^{n} \left[ \frac{1}{\pi} |x_i - 0.7|^{\frac{\pi}{3}} \left( \frac{\pi}{2} - \cos\left( 2\pi p |x_i - 0.7|^{\frac{\pi}{3}} \right) \right) \right] \tag{14}$$

where $n$ – the number of granular design decision variables $x_i$, $p$ – the number of the optimum.

In the first step of examination, the best (optimal) probabilities of mutation ($pm$) and crossover ($pc$) operators were searched. In the second stage the best population size ($ps$) was searched. For each combination ($n = 1 \ldots 5, p = 1 \ldots 5$) the 10000 independent experiments were run. The optimal

**Table 1.** The optimal probabilities *pm*, *pc* and population size *ps* of the granular evolutionary algorithm (interval case)

| $n$ | $p$ | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | | | 2 | | | 3 | | | 4 | | | 5 | | |
| | $pm$ | $pc$ | $ps$ | $pm$ | $pc$ | $ps$ | $pm$ | $pc$ | $ps$ | $pm$ | $pc$ | $ps$ | $pm$ | $pc$ | $ps$ |
| 1 | 0.4 | 0.2 | 4 | 0.4 | 0.2 | 3 | 0.3 | 0.1 | 3 | 0.3 | 0.1 | 4 | 0.3 | 0.1 | 3 |
| 2 | 0.4 | 0.1 | 4 | 0.4 | 0.1 | 4 | 0.4 | 0.1 | 4 | 0.3 | 0.1 | 4 | 0.3 | 0.1 | 4 |
| 3 | 0.3 | 0.1 | 4 | 0.3 | 0.1 | 5 | 0.3 | 0.1 | 4 | 0.2 | 0.1 | 4 | 0.2 | 0.1 | 5 |
| 4 | 0.3 | 0.1 | 5 | 0.2 | 0.1 | 4 | 0.2 | 0.1 | 4 | 0.2 | 0.1 | 4 | 0.2 | 0.1 | 4 |
| 5 | 0.3 | 0.1 | 5 | 0.2 | 0.1 | 5 | 0.2 | 0.1 | 4 | 0.3 | 0.1 | 4 | 0.2 | 0.1 | 5 |

**Table 2.** The optimal probabilities *pm*, *pc* and population size *ps* of the granular evolutionary algorithm (fuzzy case, 2 $\alpha$-cuts)

| $n$ | $p$ | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | | | 2 | | | 3 | | | 4 | | | 5 | | |
| | $pm$ | $pc$ | $ps$ | $pm$ | $pc$ | $ps$ | $pm$ | $pc$ | $ps$ | $pm$ | $pc$ | $ps$ | $pm$ | $pc$ | $ps$ |
| 1 | 0.4 | 0.1 | 5 | 0.4 | 0.1 | 4 | 0.3 | 0.1 | 5 | 0.3 | 0.1 | 5 | 0.3 | 0.1 | 6 |
| 2 | 0.4 | 0.1 | 4 | 0.4 | 0.1 | 5 | 0.4 | 0.1 | 5 | 0.4 | 0.1 | 7 | 0.4 | 0.1 | 8 |
| 3 | 0.5 | 0.2 | 11 | 0.5 | 0.2 | 15 | 0.5 | 0.2 | 15 | 0.5 | 0.2 | 18 | 0.5 | 0.2 | 19 |
| 4 | 0.5 | 0.2 | 10 | 0.5 | 0.2 | 15 | 0.5 | 0.2 | 18 | 0.5 | 0.2 | 18 | 0.5 | 0.2 | 20 |
| 5 | 0.5 | 0.2 | 11 | 0.5 | 0.2 | 16 | 0.5 | 0.2 | 19 | 0.5 | 0.2 | 21 | 0.5 | 0.2 | 23 |

**Table 3.** The optimal probabilities *pm*, *pc* and population size *ps* of the granular evolutionary algorithm (fuzzy case, 3 $\alpha$-cuts)

| $n$ | $p$ | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | | | 2 | | | 3 | | | 4 | | | 5 | | |
| | *pm* | *pc* | *ps* | *pm* | *pc* | *ps* | *pm* | *pc* | *ps* | *pm* | *pc* | *ps* | *pm* | *pc* | *ps* |
| 1 | 0.4 | 0.1 | 4 | 0.3 | 0.1 | 6 | 0.3 | 0.1 | 8 | 0.3 | 0.1 | 9 | 0.3 | 0.1 | 11 |
| 2 | 0.4 | 0.1 | 4 | 0.4 | 0.1 | 6 | 0.4 | 0.1 | 6 | 0.4 | 0.1 | 9 | 0.4 | 0.1 | 11 |
| 3 | 0.4 | 0.2 | 8 | 0.4 | 0.2 | 14 | 0.4 | 0.2 | 13 | 0.4 | 0.2 | 13 | 0.4 | 0.2 | 15 |
| 4 | 0.5 | 0.2 | 10 | 0.5 | 0.2 | 19 | 0.4 | 0.2 | 17 | 0.4 | 0.2 | 21 | 0.5 | 0.2 | 22 |
| 5 | 0.5 | 0.2 | 11 | 0.5 | 0.2 | 16 | 0.5 | 0.2 | 19 | 0.5 | 0.2 | 24 | 0.5 | 0.2 | 27 |

**Table 4.** The optimal probabilities *pm*, *pc* and population size *ps* of the granular evolutionary algorithm (stochastic case)

| $n$ | $p$ | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | | | 2 | | | 3 | | | 4 | | | 5 | | |
| | *pm* | *pc* | *ps* | *pm* | *pc* | *ps* | *pm* | *pc* | *ps* | *pm* | *pc* | *ps* | *pm* | *pc* | *ps* |
| 1 | 0.4 | 0.1 | 4 | 0.4 | 0.1 | 5 | 0.3 | 0.1 | 4 | 0.3 | 0.1 | 9 | 0.3 | 0.1 | 11 |
| 2 | 0.3 | 0.1 | 4 | 0.3 | 0.1 | 6 | 0.3 | 0.1 | 9 | 0.4 | 0.1 | 9 | 0.4 | 0.1 | 11 |
| 3 | 0.2 | 0.1 | 4 | 0.2 | 0.1 | 5 | 0.2 | 0.1 | 5 | 0.4 | 0.2 | 13 | 0.4 | 0.2 | 15 |
| 4 | 0.2 | 0.1 | 4 | 0.2 | 0.1 | 5 | 0.2 | 0.1 | 5 | 0.4 | 0.2 | 21 | 0.5 | 0.2 | 22 |
| 5 | 0.2 | 0.1 | 4 | 0.2 | 0.1 | 5 | 0.2 | 0.1 | 4 | 0.5 | 0.2 | 24 | 0.5 | 0.2 | 27 |

probabilities and population size of granular evolutionary algorithms (interval, fuzzy and stochastic) are included in Tables 1–4.

## 4. THE IDENTIFICATION OF THE INTERVAL DEFECTS PARAMETERS

The aim of the identification problem is to find the parameters which define the circular defect: $x$, $y$ and $r$ (Fig. 5). The plate was restrained and loaded by the interval continuous traction $\mathbf{q}$. The actual interval parameters of the defect are included in Table 5.

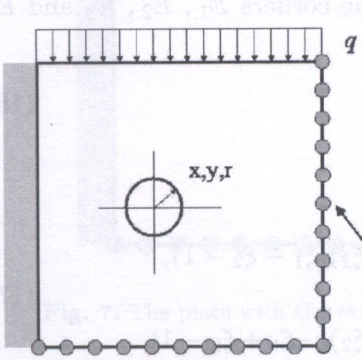

**Fig. 5.** The plate with the sensor points

**Table 5.** The results (interval case)

| | actual values | | found values | |
|---|---|---|---|---|
| | lower value | upper value | lower value | upper value |
| $x$ | 2.90 | 3.10 | 2.92 | 3.11 |
| $y$ | 2.90 | 3.10 | 2.97 | 3.07 |
| $r$ | 1.90 | 2.10 | 1.87 | 2.10 |
| $f$ | 0.00 | 0.00 | 0.21 | 0.87 |

Chromosome takes the form: $ch = [x_1, x_2, x_3] = [x, y, r]$, where $x$ and $y$ (the coordinates of the centre) and $r$ (the radius) are interval values. The material parameter $E$ and loading $\mathbf{q}$ were also assumed as the interval values. 21'points on the boundary were selected as the sensor points (Fig. 6).

The plate was loaded by $\mathbf{q} = [99.8; 100.2]$ kN. The Young module is contained in the interval $[2\text{e}+11 - 2\%, 2\text{e}+11 + 2\%]$.

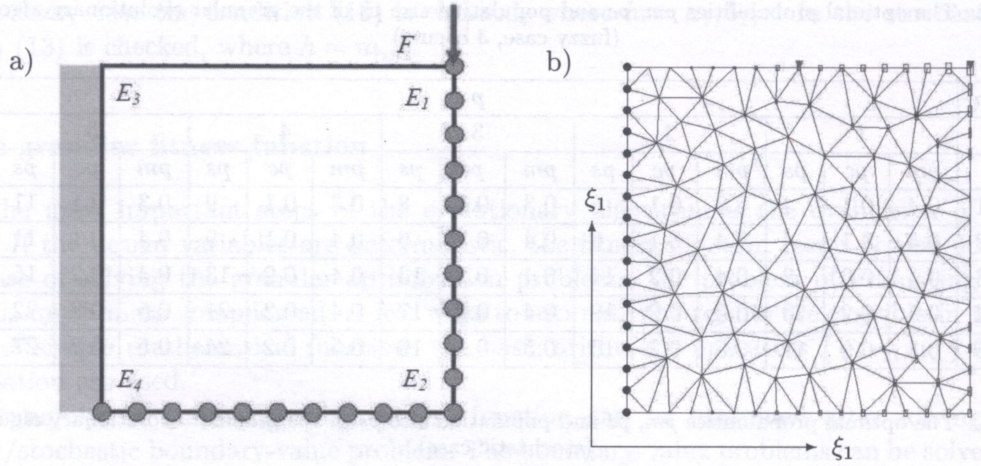

**Fig. 6.** The plate with the fuzzy material distribution a) sensor points, b) FEM discretization

The identification problem is expressed by the minimization problem of the displacement function

$$f = \sum_k (u_k - \hat{u}_k)^2 \tag{15}$$

where $\hat{u}_k$ – the measured displacement, $u_k$ – the displacement computed for the structure with defects generated by the evolutionary algorithm, $k$ – the number of the sensor points.

The following values of the granular evolutionary algorithm were received: the population size 7, the probability of mutation 0.3, the probability of crossover 0.1, the number of generations 1000. The actual and found parameters are included in Table 5.

## 5. THE IDENTIFICATION OF THE FUZZY MATERIAL DISTRIBUTION

Consider a 2-D elastic body in the plane stress loaded by force $F$. The material of the body is isotropic but homogeneous. After the discretization by means of the finite element method each finite element of the body can have different material properties. In each element the Young moduli is equal to the mean values of Young moduli in the nodes of the elements. The Young moduli $E_l$ of the $l$-th nodes are computed on the basis of the Young moduli in the corners $E_1$, $E_2$, $E_3$ and $E_4$ (Fig. 6a) by using linear approximation

$$E_l = \sum_r E_r M_r \tag{16}$$

where $M_l = 1, 2, 3, \ldots, 4$ – play the role of interpolation functions

$$\begin{aligned} M_1 &= \frac{1}{4}(1-\xi_1)(1-\xi_2)(-\xi_1-\xi_2-1), & M_2 &= \frac{1}{4}(1+\xi_1)(1-\xi_2)(\xi_1-\xi_2-1), \\ M_3 &= \frac{1}{4}(1+\xi_1)(1+\xi_2)(\xi_1+\xi_2-1), & M_4 &= \frac{1}{4}(1-\xi_1)(1+\xi_2)(-\xi_1+\xi_2-1), \end{aligned} \tag{17}$$

where $\xi_1$ and $\xi_2$ are coordinates.

The aim of the identification problem is to find the Young moduli in the corners. The actual fuzzy values of Young's moduli parameters of the corners are equal to $E_1$, $E_2$, $E_3$ and $E_4$ (Table 6). The force is assumed to be a fuzzy one: $F = [1.00; 0.50; 10.00; 0.50; 1.00]$[kN].

21 boundary nodes were selected as the sensor points (Fig. 6a). The following values of the granular evolutionary algorithm parameters were assumed: the population size 15, the probability of mutation 0.4, the probability of crossover 0.1, the number of generations 400. The results are presented in the Table 6.

**Table 6.** The results (fuzzy case)

| | actual values | | | | | found values | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $cv$ | $b_2$ | $b_1$ | $a_1$ | $a_2$ | $cv$ | $b_2$ | $b_1$ |
| $E_1$ | 2.00e7 | 1.00e7 | 2.00e11 | 1.00e7 | 2.00e7 | 2.00e7 | 1.06e7 | 1.98e11 | 1.12e7 | 2.12e7 |
| $E_2$ | 4.00e7 | 2.00e7 | 1.95e11 | 1.00e7 | 1.00e7 | 3.86e7 | 2.06e7 | 1.92e11 | 1.10e7 | 1.08e7 |
| $E_3$ | 3.00e7 | 0.50e7 | 2.05e11 | 2.00e7 | 3.00e7 | 3.12e7 | 0.50e7 | 1.99e11 | 2.09e7 | 3.22e7 |
| $E_4$ | 2.00e7 | 1.50e7 | 2.00e11 | 0.50e7 | 4.00e7 | 1.80e7 | 1.48e7 | 2.02e11 | 0.53e7 | 4.08e7 |
| $f$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.04 | 0.21 | 0.02 | 0.12 |

## 6. THE IDENTIFICATION OF STOCHASTIC BOUNDARY CONDITIONS

Consider a two-dimensional elastic structure (plane stress) with prescribed boundary conditions (Fig. 8). The following parameters of the structure: (i) geometry, (ii) material properties and (iii) boundary conditions, can be modelled by using the stochastic approach. In this example loads $X_i(\gamma) i = 1, 2, \ldots, n$, are random variables. The rest of the parameters are deterministic.

The aim of this test is to find $n=2$ random loads: $F_1(\gamma)$ and $F_2(\gamma)$ (Fig. 8). The actual stochastic parameters of the load $F_1(\gamma)$ is described by: $(m_1, \sigma_1)$, where $m_1 = 10.0$ [kN], and $\sigma_1 = 0.167$ [kN]. The actual stochastic parameters of the load $F_2(\gamma)$ is described by: $(m_2, \sigma_2)$, where $m_2 = 15.0$ [kN], and $\sigma_2 = 0.167$ [kN]. The loads are independent random variables.

The stochastic chromosome $\mathbf{X}(\gamma) = [F_1(\gamma), F_2(\gamma)]$ is replaced by deterministic one containing moments of $F_i(\gamma)$, $ch = [(m_1, \sigma_1)(m_2, \sigma_2)]$.

21 boundary nodes as the sensor points were selected (Fig. 7).

The following parameter values of the stochastic evolutionary algorithm were assumed: the population size ($ps$) 5, the mutation probability ($pm$) 0.4, the crossover probability ($pc$) 0.1, the number of generations was equal to 500.

The actual and found parameters are included in Table 7.

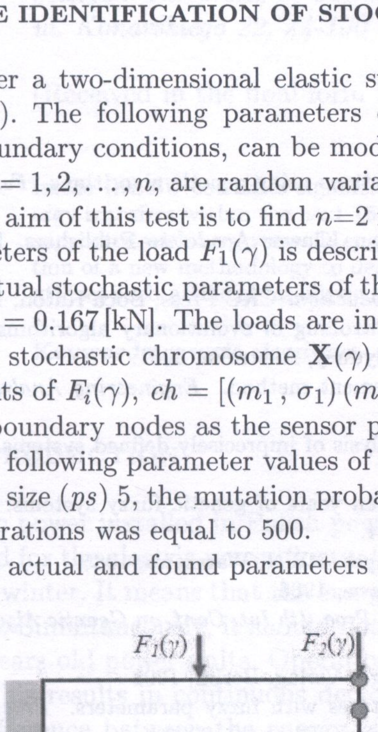

**Fig. 7.** The plate with the random loads

**Table 7.** The results (stochastic case)

| | actual values | | found values | |
|---|---|---|---|---|
| | $m$ | $\sigma$ | $m$ | $\sigma$ |
| $F_1(\gamma)$ | 10.000 | 0.167 | 9.982 | 0.177 |
| $F_2(\gamma)$ | 15.000 | 0.167 | 14.902 | 0.159 |
| $f$ | 0.000 | 0.000 | 0.202 | 0.023 |

## 7. CONCLUSIONS

An effective intelligent technique based on the granular evolutionary algorithm has been presented. This approach can be applied in the optimization and the identification of systems that are in the uncertain conditions. This approach is very promising for reliability optimization in which the safety of a system is estimated and represented by the probability of its failure, i.e. the occurrence of an ultimate limit state manifesting itself.

The more general approaches will be developed in future. The other types of probability density function (PDF) with a greater number of moments will be examined. The dependences between random numbers will be also examined.

In the general case uncertain conditions have the granular form [3]. The models based on the interval and fuzzy numbers were used instead of stochastic approach presented in this paper. The models based on the perturbation numbers will be presented in the future.

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