Computer Assisted Mechanics and Engineering Sciences, 14: 745–752, 2007.


# Ant colony optimization in project management

Bożena Skołud, Bożena Marcińczyk
*Silesian University of Technology,*
*Institute of Engineering Processes Automation and Integrated Manufacturing Systems,*
*ul. Konarskiego 18A, 44-100 Gliwice, Poland*

(Received in the final form September 18, 2007)

This paper presents an Ant Colony Optimization (ACO) approach to the resource-constrained project scheduling problem (RCPSP). RCPSP as a generalization of the classical job shop scheduling problem belongs to the class of NP-hard optimization problems. Therefore, the use of heuristic solution procedures when solving large problem is well-founded. Most of the heuristic methods used for solving resource-constrained project scheduling problems either belong to the class of priority rule based methods or to the class of metaheuristic based approaches. ACO is a metaheuristic method in which artificial ants build solutions by probabilistic selecting from problem-specific solutions components influenced by a parametrized model of solution, called pheromone model. In ACO several generations of artificial ants search for good solution. Every ant builds a solution step by step going through several probabilistic decisions. If ant find a good solution mark their paths by putting some amount of pheromone (which is guided by some problem specific heuristic) on the edges of the path.

**Keywords:** resources constrained scheduling problem, project scheduling, multi-project scheduling, ant colony optimization, swarm intelligence

## 1. BACKGROUND

In general, a project is defined as a set of activities needed to create a unique product or services. According to Brilman [6] over 25% of all professional activity can be managed as a project, therefore project management is a branch of organizing and managing resources.

Three necessary conditions: the scope, the schedule and the budget are required for the project satisfaction. The scope sets a minimum standard for the project result. The budget sets a maximum cost. The schedule sets the maximum time for the project. This three conditions are interdependent (e.g. the longer a project takes, the more it costs, the longer project takes the more opportunities exist to change the scope) [12].

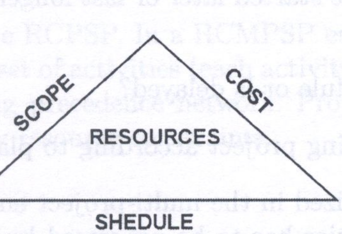


**Fig. 1.** Three necessary conditions [12]

A constrained project is a set of activities which must be performed according to some precedence constraints requiring that some activities cannot start before some others are completed. In the case when resource constrains are not taken into account, a project can be represented by an acyclic directed graph. In case of necessity project scheduling there are two different type of representation the graph: AON – *Activity On Node*, where the nodes stand for activities and the arcs for precedence relation, and the second type AOA – *Activity On Arc*, where the arcs stand for activities and the nodes for precedence relation [10].

The paper is organized in a following way: in Section 2 the problem of project management is stated, Section 3 contains a review of resources constrained project scheduling problem. Section 4 describes Ant Colony Optimization, Section 5 contains numerical example.

## 2. PROJECT SCHEDULING

Project management is the application of knowledge, skills, tools and techniques to a broad range of activities in order to meet the requirements of the particular project.

In case of necessity project scheduling there are two different type of representation the graph: AON – Activity On Node, where the nodes stand for activities and the arcs for precedence relation, and the second type AOA – Activity On Arc, where the arcs stand for activities and the nodes for precedence relation [21].

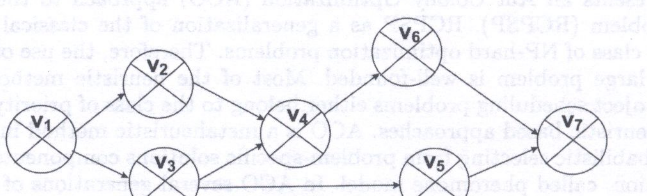


**Fig. 2.** The AON network

The network techniques is the most applied methods of project planning and controlling realization.

Popular project scheduling methods, such as Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT) are applied for single project management. Generally CPM and PERT allow either for the minimization the project completion time, or the minimization of project completion cost through crashing, or shortening particular activities. This methods allow to answer following questions [14, 21]:

- When project will be completed?
- Which activities come into critical path (critical path-sequences of activities that determine the minimal project duration, is the one from the start of project to finish of project where the slack times are all zeros)?
- Which activities or tasks can be started later or last longer without project delays — finding the non-critical activities?
- Is the project realized on schedule or is delayed?
- What is the probability of ending project according to plan (PERT method)?

About 90% of projects are realized in the multi-project environment. Scheduling problems arise in situations where a set of activities has to be processed by a limited number of resources during a limited period of time. The scheduling problem consists of resources allocation and resources scheduling — ordering of activities on each resource.

A project management problem typically consists of planning and scheduling decisions. Planning requirements scheduling involves the allocation of the given resources to project to determine the start and the completion times of activities. Scheduling and sequencing is concerned with the optimal allocation of resources to activities over time. Activities in project management (commonly referred to as job) consist of one or more operations, which do have processing requirements and might be considered to have release dates, due dates and weights. Jobs may have precedence relations, and may or may not be interrupted [23].

## 2.1. Resource-constrained scheduling problem

In general, scheduling problems are NP-hard, there are no know algorithms for finding optimal solutions in polynomial time. In most general form the resource-constrained scheduling problem (RCSP) is defined as follows.

Given is:

- a set of executed activities,
- a set of renewable (e.g. labor) or non-renewable resources (e.g. raw materials, capital),
- a set of constraints which must be satisfied,
- a set of objectives.

Activities have duration, cost and resources capacities. Activities may be preemptive or non-preemptive (preemption means that some activities can be interrupted). Constraints define a feasibility of a schedule. Objective define the optimality of a schedule. Objective should be satisfied, constraints must be satisfied [23].

## 2.2. Project scheduling and multi-project scheduling

The resource-constrained project scheduling problem (RCPSP) and resource-constrained multi-project scheduling problem (RCMPSP) can be characterized by the objective function, resources requirements, precedence relations and preemptive conditions.

RCPSP and RCMPSP involve assigning jobs to a resources (or a set of resources) with limited capacity in order to meet some predefine objective. RCPSP is the problem of determining starting times of each activity of a project satisfying precedence and resource constraints in order to minimize the total project duration. The precedence constraints impose that an activity can start after the completion of all its predecessor activities. The execution of an activity cannot be interrupted and requires, for each period of its duration, constant amounts of a subset of renewable resources. Many different objectives are possible and these depend on the goals of the decision maker. The most common objective is to find the minimum makespan (i.e. minimization of project duration). in the RCPSP, each activity has a single execution mode: both the activity duration and its requirements for a set of resources are assumed to be fixed, and only one execution mode is available for any activity [23, 17].

RCMPSP is a generalization of the RCPSP. In a RCMPSP environment a company has several concurrent projects, each consist of a set of activities (each activity has several associated attributes) and each project has a corresponding precedence network. Projects depend on a common set of resources and are therefore related by resource constraints.

## 2.3. Problem formulation

The resource-constrained project scheduling problem is a general scheduling problem that contains the job-shop, flow-shop, and open-shop problem as special cases. RCPSP is optimization problem to schedule the activities of a project to meet the predefined objective (e.g. minimizing the makespan with given constraints). The RCPSP can be formulated as follows. The project consist of a set of activities. Given is a set of resources, a set of constraint and a set of objective. The $J$ denotes the set of activities, $J = \{0, \ldots, n+1\}$, where activity 0 stands for start activity and have no predecessor and $n+1$ stand for end activity and have no successor. The activities are interrelated by two kinds of constraints. The precedence constraints are given, such as some activities cannot start before some others are finished. The second constraint is availability of resources with limited capacities. $Q$ is a set of $q$ resource types. $R_i > 0$ is the resource capacity for resources of type $i \in Q$. Each activity $j \in J$ characterize duration $p_j$ and resources capacities requirements $r_{j,1}, \ldots, r_{j,q}$ where $r_{j,i}$ is the capacity requirement of $j$-th resource per time unit for scheduled $j$-th activity. Start and end activity have zero duration time. A schedule can be represented by the vector of start time of activities $(s_o, s_1, \ldots, s_{n+1})$ where the $s_j$ is the start time of activity $j \in J$. The finish time for activity $j$ is $f_j = s_j + p_j$. For a schedule the start time is the minimum star time $\min\{s_j \mid j \in J\}$ of all activities, and the finish time is the maximum finish time $\max\{s_j + p_j \mid j \in J\}$ of all activities. The classical RCPSP can be formulated as follows,

$$\min\{\max s_j + p_j \mid j \in J\}, \tag{1}$$

$$s_j \geq s_i + p_i \quad \text{for every} \quad p_i \in P_i \quad \text{(the set of predecessors)}, \tag{2}$$

$$\sum_{j \in J} r_{j,i} \leq R_i \,. \tag{3}$$

Equation (1) represent the objective function, the minimal makespan, Eq. (2) represent the precedence constraints and Eq. (3) represent the resources constraints which have to be satisfied, for every time unit, the sum of the resources requirements of all scheduled activities does not exceed the resources capacities [16].

For RCMPSP the problem can be formulated very similar, but multi-project scheduling consist of a set of project. Each project contain a set of activities. The general model for RCMPSP can be represented with three similar equations.

## 3. Research Review

The research literature for the RCPSP is quite large. A great number of exact methods to solve the RCPSP are proposed in the literature. The exact methods applied to the RCPSP can be classified into three categories: dynamic programming, zero-one programming and implicit enumeration with branch and bound. Blazewicz *et al.* in [4] showed that the RCPSP as a generalization of the classical job shop scheduling problem belongs to the class of NP-hard optimization problems. Therefore, the use of heuristic solution procedures when solving large problem is well-founded. Most of the heuristics methods used for solving resource-constrained project scheduling problems either belong to the class of priority rule based methods or to the class of metaheuristic based approaches, e.g. [9]. While exact solution methods are able to solve smaller problems, heuristic and metaheuristic approaches are needed for larger problem instances. Many metaheuristic methods, such as genetic algorithms (GA), simulated annealing (SA), tabu search (TS), and ant colonies (AC), have been applied to solve the RCPSP. Metaheuristics based on GA are the most common, e.g. [1, 13]. Simulated annealing algorithms in resource constrained project scheduling problem are presented in [2, 3]. Tabu search based on metaheuristics are proposed in e.g. [19]. Ant Colony approach to the RCPSP is proposed in [16].

Pritsker *et al.* in [20] proposed a zero–one programming approach for the multi-project scheduling. Most of the heuristic method used for solving RCMPSP belong to the class of priority rule based methods. Several approaches of this class have been proposed in the literature, e.g. [11, 15, 22, 24].

Heuristic methods typically require less time and/or space than exact methods. The heuristics specify how to make a decision given a particular situation (heuristics are rules for deciding which action to take). Heuristics may be deterministic — they end up with the same result every time — or they may be stochastic — each time they are run they may produce a different result. Scheduling heuristics operate on a set of tasks and determine when each task should be executed. If a task may be executed in more than one execution mode or on any one of a set of resources, the heuristic must also determine which resources and/or execution mode to use [17, 23]. The metaheuristic based on ACO is proposed. In ACO several generations of artificial ants search for good solution. Every ant build up a solution and mark their paths by putting some amount of the pheromone. In next steps ants follow this trail with a very high degree of probability and enhance the trial by depositing its own pheromone.

## 4. SWARM INTELLIGENCE — ANT ALGORITHM

The resource-constrained project scheduling problem (RCPSP) is a general scheduling problem that contains the job-shop, flow-shop, and open-shop problem as special cases. A Swarm Algorithm approach is proposed to the RCPSC as an alternative way of scheduling. The concept of swarm intelligence originates from the observations of animals and insects in nature. The term "swarm intelligence" refers to the complex behaviors exhibited by a group, referred to as a swarm, of autonomous individuals, referred to as agents, performing simple actions within an environment. Swarm algorithms have gained popularity recently because of their design and ability to solve hard problems. Their popularity has led to these algorithms being applied to many fields of computer science. One way to develop and test a swarm algorithm is simulation [8, 18].

### 4.1. Ant Colony Optimization

Ant Colony Optimization is modeled on the foraging behavior or Argentine ants. ACO is a metaheuristic method in which artificial ants build solutions by probabilistic selecting from problem-specific solutions components influenced by a parameterized model of solution, called pheromone model. Real ants are capable of finding shortest path from a food source to the nest (Fig. 3a). In ACO several generations of artificial ants search for good solution. Every ant builds a solution step by step going through several probabilistic decisions. If ant find a good solution mark their paths by putting some amount of pheromone (which is guided by some problem specific heuristic) on the edges of the path. The following ants are attracted by the pheromone so that they search in the solution space near previous good solutions. If the obstacle has appeared, those ants which are just in front of the obstacle cannot continue to follow the pheromone trail and therefore they have to choose between turning right or left (Fig. 3b). In this situation half of the ants choose to turn right and the other half to turn left (the same situation can be found on the other side of the obstacle,

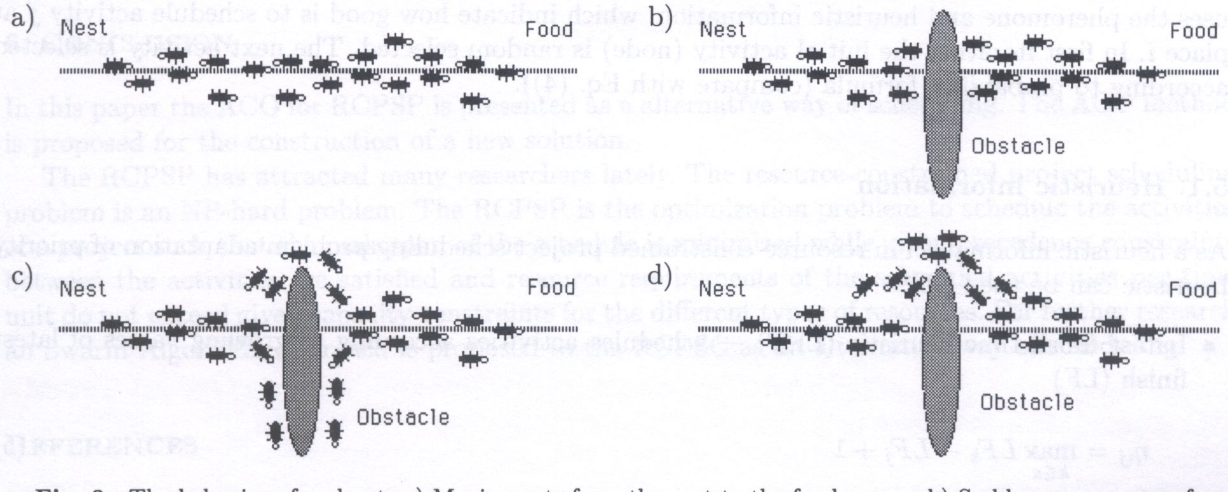


**Fig. 3.** The behavior of real ants a) Moving ants form the nest to the food source; b) Sudden appearance of an unexpected obstacle; c) Finding the new path; d) New shortest path form the food source to the nest [7]

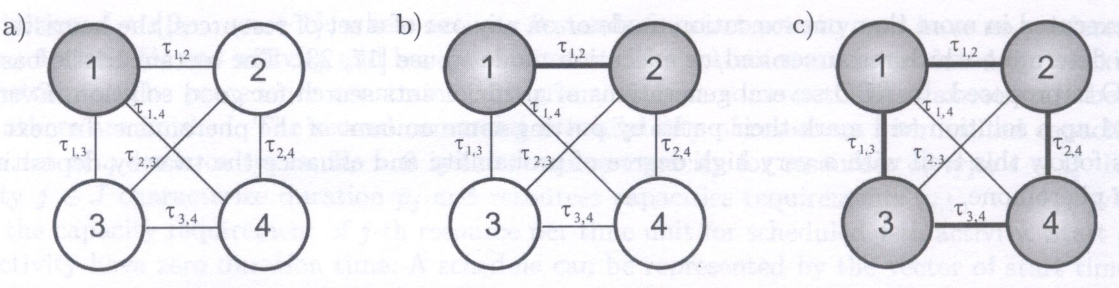


**Fig. 4.** a) Random selection of initial node; b) Move to another city b the probabilistic formula (the initial node goes to the tabu list); c) Optimal solution [5]

Fig. 3c). The shorter path chosen by chance around the obstacle will receive a higher amount of pheromone (Fig. 3d) [7].

The first Ant System (AS) was proposed by Dorigo *et al.* (1996). The AS was applied to Travel Salesman Problem (TPS). The TPS is the problem of finding a minimal length closed tour that visits each cities once.

The TPS problem consist of $m$ ants and $n$ cities (Fig. 4). Number of ants is equal to number of cities. The salesman can travel from any city to any other city, but must visit each city once and only once. An agent in city move to another city by the probabilistic formula (4) (the next city is the city with the highest value of probability).

$$p_{ij} = \frac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l \in N}[\tau_{il}]^{\alpha}[\eta_{il}]^{\beta}}. \tag{4}$$

The pheromone information, denoted by $\tau_{ij}$, and the heuristic information, denoted by $\eta_{ij}$, are indicators of how good is seems to visit a $j$ city. $\alpha$ and $\beta$ are constants that determine the relative influence of the pheromone values and the heuristic values on the decision of the ant.

In order to satisfy the constraint that ant visit all different cities each ant have a internal memory, called tabu list, in which saves the solutions (the already visit cities). After each iteration the pheromone trail evaporate in order to no to have too strong influence of old pheromone trail on the future [7, 16].

## 5. ACO FOR RCPSP

The ant algorithm procedure is proposed to make a project schedule. The idea of this system is very similar to TSP. In every iteration each ant construct one solution. For selection the activities ant uses the pheromone and heuristic information, which indicate how good is to schedule activity $j$ at place $i$. In first iteration the initial activity (node) is random selected. The next activity is selected according to probability formula (compare with Eq. (4)).

### 5.1. Heuristic information

As a heuristic information in resource-constrained project scheduling problem adaptation of priority heuristic can be used [16]:

- Latest finish time heuristic (LFT) — schedules activities according to growing values of latest finish (LF)

$$\eta_{ij} = \max_{k \in \varepsilon} LF_k - LF_j + 1 \tag{5}$$

where $\varepsilon$ — set of eligible activities,

- Latest start time (LTS) heuristic-schedules activities according to growing values of latest start (LS)

$$\eta_{ij} = \max_{k \in \epsilon} LS_k - LS_j + 1, \tag{6}$$

- Minimum slack time (MST)

$$\eta_{ij} = \max_{k \in \epsilon}(LS_k - ES_k) - (LS_j - ES_j) + 1 \tag{7}$$

where $ES$ — earliest time of activity $j$.

### 5.2. Framework for an algorithm

**Algorithm 1: ACO**
**Input:** $\tau_{ij}$ , $\eta_{ij}$ , $s_0$ , $f_0$
**Output:** Schedule
**1. Initialize** $\tau_{ij}$ , $\eta_{ij}$ , initial activity is random selected
**2. for each** ant $k$ **do**
    choose next activity according to probability formula
    append the chosen move to the tabu list
**3. for** $k = 1, m$ **do**
    **repeat**
      compute $\eta_{ij}$
      choose next activity according to probability formula
      append the chosen move to the tabu list
    **until** $k$ has completed its solution
    carry the solution to its local optimum
  **end for**
  **for each** ant move
    compute $\tau_{ij}$ , update pheromone trial
  **end**
  **If not end go to step 2**

## 6. Conclusion

In this paper the ACO for RCPSP is presented as a alternative way of scheduling. The ACO method is proposed for the construction of a new solution.

The RCPSP has attracted many researchers lately. The resource-constrained project scheduling problem is an NP-hard problem. The RCPSP is the optimization problem to schedule the activities of a project such that the makespan of the schedule is minimized while given precedence constraints between the activities are satisfied and resource requirements of the scheduled activities per time unit do not exceed given capacity constraints for the different types of resources. For further research an Swarm Algorithm approach is proposed to the RCPSC as an alternative way of scheduling.

## References

[1] J. Alcaraz, C. Maroto. A robust genetic algorithm for resource allocation in project scheduling. *Annals of Operations Research*, **102**: 83–109, 2001.

[2] F.F. Boctor. An adaptation of the simulated annealing algorithm for solving resource-constrained project scheduling problems. *International Journal of Production Research*, **34**: 2335–2351, 1996.

[3] K. Bouleimen, H. Lecocq. A new efficient simulated annealing algorithm for the resource-constrained project scheduling problem and its multiple mode version. *European Journal of Operational Research*, **149**: 268–281, 2003.

[4] J. Blazewicz, J.K. Lenstra, K.A.H.G. Rinnooy. Scheduling projects to resource constraints: classification and complexity. *Discrete Applied Mathematics*, **5**: 11–24, 1983.

[5] Ch. Blum. *Ant Colony Optimization: Introduction and Recent Trends.* Physics of live reviews, 2005.

[6] J. Brilman. *Nowoczesne Koncepcje i Metody Zarządzania.* PWE, Warszawa, 2002.

[7] M. Dorigo, V. Maniezzo, A. Colorni. *The Ant System: Optimization by a Colony of Cooperating Agents.* IEEE Systems, Man and Cybernetics, Italy, 1996.

[8] Ch. Hantak. *Comparison of Parallel Hardware Based and Graphics Hardware Based Platforms for Swarm Intelligence Simulations.* Integrative Paper, UNC-Chapel Hill, 2003.

[9] R. Kolisch, S. Hartmann. Heuristic algorithms for solving the resource-constrained project scheduling problem: Classification and computational analysis. In: J. Weglarz, ed., *Handbook on Recent Advances in Project Scheduling*, pp. 147–178. Kluwer, Dordrecht, 1999.

[10] A. Kostrubiec. *Harmonogramowanie realizacji projektów – przegląd modeli.* www.zie.pg.gda.pl/koipsp/4adamkostrubiec.pdf

[11] S.R. Lawrence, T.E. Morton. Resource-constrained multi-project scheduling with tardy costs: Comparing myopic bottleneck and resource pricing heuristics. *European Journal of Operational Research*, **64**: 168–187, 1993.

[12] L.P. Leach. *Critical Chain Project Management.* Artach Mouse, Boston 2000.

[13] J.K. Lee, Y.D. Kim. Search heuristics for resource-constrained project scheduling. *Journal of the Operational Research Society*, **47**: 678–689, 1996.

[14] D. Lock. *Podstawy zarządzania projektami.* Polskie Wydawnictwo Ekonomiczne, Warszawa, 2003.

[15] A. Lova, C. Maroto, P. Tormos. A multicriteria heuristic method to improve resource allocation in multiproject scheduling. *European Journal of Operational Research*, **127**: 408–424, 2000.

[16] D. Merkle, M. Middendorf, H. Schmeck. Ant colony optimization for resource-constrained project scheduling. *IEEE Transactions on Evolutionary Computation*, **6**: 333–346, 2002.

[17] A. Mingozzi, V. Maniezzo, S. Ricciardelli, L. Bianco. *An Exact Algorithm for the Resource Constrained Project Scheduling Problem Based on a New Mathematical Formulation*, www.csr.unibo.it/~maniezzo/pspaper.ps, 1995

[18] J. Montgomery, F. Fayad, S. Petrovic. *Solution Representation for Job Shop Scheduling Problems in Ant Colony Optimization*, www.asap.cs.nott.ac.uk/publications/pdf/ACO06Final.pdf

[19] E. Pinson, C. Prins, F. Rullier. Using tabu search for solving the resource-constrained project scheduling problem. *Proceedings of the 4th International Workshop on Project Management and Scheduling*, Leuven, Belgium, pp. 102–106, 1994.

[20] A. Pritsker, B. Allan, L.J. Watters, P.M. Wolfe. Multiproject scheduling with limited resources: A zero-one programming approach. *Management Science*, **16**: 93–108, 1969.

[21] M. Trocki, B. Grucza, K. Ogonek. *Zarządzanie Projektem.* Polskie Wydawnictwo Ekonomiczne, Warszawa, 2002.

[22] S. Tsubakitani, R.F. Deckro. A heuristic for multi-project scheduling with limited resources in the housing industry. *European Journal of Operational Research*, **49**: 80–91, 1990.

[23] M.B. Wall, *A Genetic Algorithm for Resource-Constrained Scheduling*, Doctoral Dissertation for Mechanical Engineering. Massachusetts Institute of Technology, 1996.

[24] V.D. Wiley, R.F. Deckro, J.A. Jackson. Optimization analysis for design and planning of multi-project programs. *European Journal of Operational Research*, **107**: 492–506, 1998.