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Constructal design: A survey

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This paper gives a review of the new Bejan's constructal theory and its various applications to design and optimization. According to Bejan, the objective and constraints principle used in engineering is the same mechanism from which the geometry in natural flow systems emerges. This observation is the basis of the new constructal theory. The topics covered in this review are: mechanical structure, thermal structure, heat trees, and structure in transportation and economics. In the conclusion, remarks on possibility of coupling this approach with computational mechanics are given.

Keywords: optimization, constructal theory, constructal design, review

1. INTRODUCTION

The objective of this paper is to make a review of the new Bejan's constructal theory and its applications to design and optimization. The basic reference is the Bejan's book [11]. According to the author, shape and structure spring from the struggle for better performance in both engineering and nature; the objective and constraints principle used in engineering is the same mechanism from which the geometry in natural flow systems emerges. From heat exchangers to river channels, the book draws many parallels between the engineered and the natural world. The topics covered in the book are: mechanical structure, thermal structure, heat trees, ducts and rivers, turbulent structure, and structure in transportation and economics. In the conclusion, we shall give remarks on possibility of coupling this approach with computational mechanics, which is new, because most of the applications of the constructal theory are developed in fluids mechanics, in particular for the optimization of flows.

2. THE CONSTRUCTAL THEORY

There are similarities in the geometry of flow systems in engineering and nature. For example, tree shaped flows are everywhere: in computers, lungs, dendritic crystals, urban street patterns, and communication links. Bejan's book [11] starts with the design and optimization of engineering systems and discovers a deterministic principle for the generation of geometric form in natural systems. Shape and structure spring from the struggle for better performance in both engineering and nature. This observation is the basis of the new constructal theory: the objective and constraints principle used in engineering is the same mechanism from which the geometry in natural flow systems emerges. Optimal distribution of imperfection is destined to remain imperfect. The system works best when its imperfections are spread around so that more and more internal points are stressed a much as the hardest working parts. Seemingly universal geometric forms unite the flow systems of engineering and nature. The book [11] advances a new theory whose author unabashedly hints that his law is in the same league as the second law of thermodynamics, because a simple law is purported to predict the geometric form of anything alive on earth.

The constructal theory of optimization is a new theory of total and macroscopic optimization. It is opposed to the fractal theory. This one is a local and microscopic theory. The constructal theory results from the following report: the natural laws lead to optimal solutions. For example, one can show that the human bronchi are overall optimal; this with regard to the flow and the absorption of flows of air; and this taking into account the natural limitations on the capacity of the rib cage.

We are going to see that many applications of the constructal theory were developed in fluids mechanics, in particular for the optimization of flows. On the other hand, there exists, to our knowledge, little examples of applications in solids or structures mechanics. In the conclusion, we shall give remarks on possibility of coupling this approach with computational mechanic. The constructal theory rests on the assumption that all creations of nature are overall optimal compared to the laws which control the evolution and the adaptation of the natural systems. The constructal principle consists in distributing the imperfections as well as possible, starting from the small scales to the largest. The constructal theory works with the total macroscopic structure starting from the assembly of elementary structures, by complying with the natural rules of optimal distribution of the imperfections. The objective is the research of lower cost. The constructal theory applies perfectly to all the systems subjected to flows.

We give now some computational aspects about constructal design. The theoretical development of the constructal principle relies on the analysis of natural dynamic systems with flows, currents and gradients that, internally, are not at thermodynamic equilibrium. The constructal principle, as compared to the fractal alternative, produces architectures that are based on geometric structures emergent by optimization and not by conjectural assumptions. Spatial and temporal structures observed in nature are the results of certain processes of global optimization subject to specific local and global constraints. This principle demands that a finite size heterogeneous system undergoes changes in shape and structure such as to provide for an as easy as possible access of its internal currents. Optimization principles such as minimum travel time, minimum flow resistance, minimum power consumption are frequently and constantly invoked and used in constructal design, and it is remarkable that these deterministic principles are independent, and do not result from other, known laws. These principles are used in constructal design to produce new structures for multifunctional materials. Constructal design starts by a clear understanding of the objective: the system mission, purpose, function and performance. This concept implies also optimization, because the designed system is expected to perform the best possible. Possible means that the designer recognizes the local and global limitations that the system may have. Some of the very common global constraints are the system mass and volume. The local constraints are more subtle, but equally important. The local constraints are then gathered in an objective function defined at the system level. The geometry is the unknown. The external shape and the internal structure emerge then by successive designs that comply with the global and local constraints, in the objective pursuit. The initial position that this engineering point of view occupies is essential to understand in a deterministic sense the emergence of shape and structure in natural systems, and in the constructal principle. We shall give an example of constructal design at the end of this part.

In [28], a review of engineering developments in thermodynamic optimization, which shed light on a universal design principle that accounts for macroscopic organization in nature, is given. It is the principle of the constructal approach to predict organization in nature. It is shown that the optimal performance of a finite size system is always characterized by the equipartition of driving forces or the optimal allocation of material subject to overall constraints. Examples are drawn from natural inanimate systems and animate systems. It is shown that this principle also governs the architecture of tree networks. Tree networks can be obtained in purely deterministic fashion by minimizing the flow resistance between one point and a finite area or a finite volume. The shape of each volume element can be optimized for minimal flow resistance. The network is constructed by assembling the shape optimized building blocks, and proceeding in time from the smallest volume element toward larger constructs. In constructal theory small size and shapeless flow come first, and larger sizes and geometrical form come later.

In [32], the authors show that the time needed to discharge a volume to a concentrated sink can be minimized by making appropriate changes in the geometry of the flow path. The time dependent flow of heat between a volume and one point is chosen for illustration, however, the constructal principle holds for other transport processes. There are two classes of geometric degrees of freedom in designing the flow path: the external shape of the volume, and the distribution of high conductivity inserts that facilitate the volumetric collection of the discharge. The optimization of flow path geometry is executed in a sequence of steps that starts with the smallest volume elements and proceeds toward larger and more complex volume sizes. The high conductivity inserts come together into a tree network pattern which is the result of a completely deterministic principle. The paper concludes with a discussion of the relevance on this constructal law to predicting structure in natural flow, and to understanding why the geometry of nature is not fractal.

In [9], the starting point is the question of how to optimize the access between one point and a finite volume. The optimal access solution for the total volume is obtained by optimizing volume shape at every length scale, in a sequence that begins with the smallest building block, and proceeds toward larger building blocks. The solution is constructed, hence the constructal name of the associated theory. In [10], the paper reviews a relatively recent body of heat transfer work that bases on a constructal principle the occurrence of geometric form in systems with internal flows. In [42], a direct route to the construction of effective tree shaped flow structures is outlined. Dendritic flow structures dominate the design of natural and engineered flow systems, especially in thermal and fluid systems. The starting point is the optimization of the shape of each elemental area or volume, such that the length of the flow path housed by the element is minimized. Proceeding toward larger and more complex structures, the paper develops tree shaped flow structures. In [12], the fundamentals of the methods of exergy analysis and entropy generation minimization are outlined.

As a detailed example, we use [8]. The exact formulation of the problem solved in [8] is the following: consider a finite size volume in which heat is being generated at every point, and which is cooled through a small patch located on its boundary; a finite amount of high conductivity (k_p) material is available (k_p is a high thermal conductivity); determine the optimal distribution of (k_p) material through the given volume such that the highest temperature is minimized. The purpose of any portion of the conducting path (k_p) is to be in touch with the material that generates heat volumetrically. This material fills the volume (V), and its thermal conductivity is low (k_0) (k_0 is a low thermal conductivity). The optimal access problem reduces to the geometric problem of allocating conducting path length to volume of (k_0) material. The allocation can not be made at infinitesimally small scales throughout (V), because the (k_p) paths must be of finite length so that they can be interconnected to channel the total heat current q to the heat sink point. There is thus only one option: to optimize the allocation of conductive path to one sub system at a time; and to optimize the manner in which the volume elements are assembled and their (k_p) paths are connected.

The result developed in [8] is purely geometric: any finite size portion of the heat generating volume can have its shape optimized such that its overall thermal resistance is minimal. Optimized volume elements are then assembled into a larger volume the shape of which is also optimized (constructal design). This assembly and geometric optimization sequence is repeated in steps, from the smallest volume element to the largest assembly, until the given volume is covered. It is the principle of the constructal design. One of the features of the structure that emerges is a network of (k_p) paths that is shaped as a tree. All the features of the structure of the (k_0, k_p) composite material that covers the volume, and all the features of the associate tree of (k_p) paths are the result of a purely theoretical, deterministic process guided by a single principle.

The simplest example of shape optimization is provided by the two dimensional volume element represented by the rectangle in Fig. 1. The area of the rectangle is fixed but its shape may vary. The amount of (k_p) material allocated to the rectangle is fixed. The heat current generated by this volume element is collected by a blade of high conductivity material (the black area on Fig. 1), and taken out of the volume through the point M (origin of x axis). The rest of the rectangle boundary is adiabatic. The volumetric heat generation rate is uniform. The hot spot occurs at the point K,

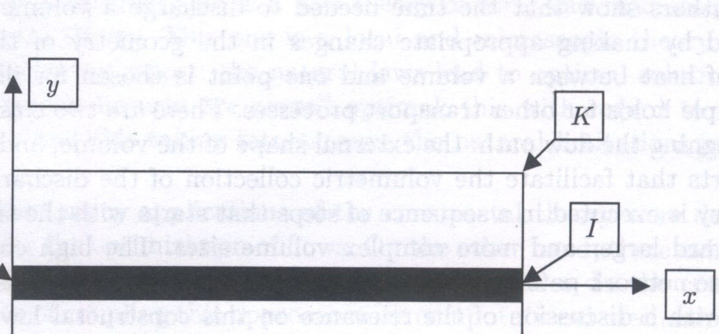

Fig. 1. 2-D elemental volume with uniform volumetric heat generation rate and high conductivity insert along its axis of symmetry

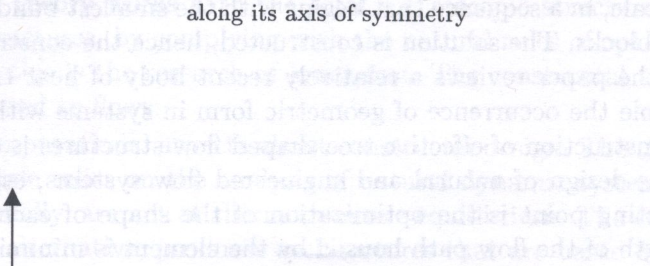

Fig. 2. The first construct: a large number of elemental volumes connected to a central high conductivity path

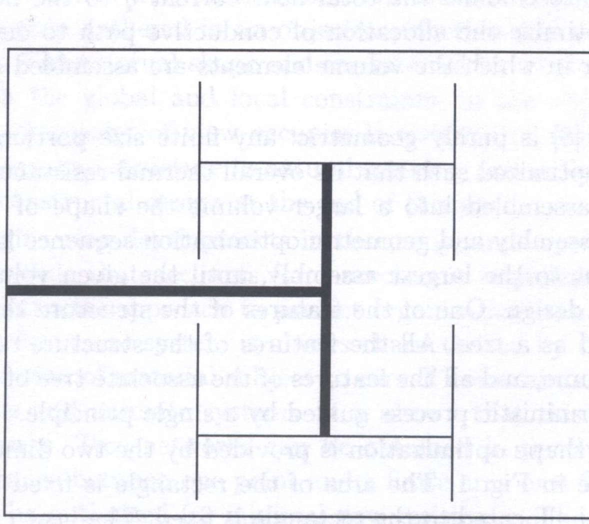

Fig. 3. The optimized fourth construct

which is the farthest from the heat sink M. Since the heat current is fixed, the minimization of the thermal resistance of the volume element is equivalent to the minimization of the peak excess temperature, the temperature drop from K to M. The conduction through the (k_0) material is practically parallel to y, while the conduction through the (k_p) material is along the x axis. The conductivity $(k0)$ of the material is constant throughout the rectangle. In this limit, the volume to point thermal resistance is the sum of two contributions, one for (k_0) conduction from the corner, and the other for (k_p) conduction from I to the origin M of x axis. This quantity can be minimized geometrically by varying the shape of the system.

The same geometric optimization principle applies at larger scales. The next larger volume is an assembly (a construct in constructal design terminology) of optimized elemental volumes (Fig. 2). When the total volume of (k_p) material contained by the first construct is fixed, in addition to an optimal shape, there is an optimal way to allocate the (k_p) material. At the second construct level, the optimal external aspect ratio can be found. Numerical optimizations of the elemental system, first construct and second construct are exhibited in [8]. In Fig. 3, the optimized fourth construct and its distribution of (k_p) material are shown Every single geometric feature of the cooling scheme shown in Fig. 3 is the result of analysis. The geometrically optimal construction started in Figs. 1 to 3 can be continued to a higher order of assembly, until the structured composite $(k_0,\ k_p)$ covers the given space.

3. APPLICATIONS: HEAT FLOW, INCLUDING "TREES"

In [47], constructal theory is used to predict the formation of geometric shape and structure in finite size fluid systems subjected to heating from below. In [22], it is shown that the geometry of the heat flow path between a volume and one point can be optimized in two fundamentally different ways. In the original constructal theory the structure is optimized starting from the smallest volume element of fixed size. In the design method the overall volume is fixed, and the designer works by optimizing the internal features of the heat flow path. It is shown analytically that the two methods produce comparable geometric results. In [34], a numerical study of the geometric minimization of the resistance to Darcy flow between a finite size volume and one point is reported. The volume is two dimensional and contains materials with several permeabilities. The optimization starts with the smallest volume subsystem, and proceeds toward larger subsystems until the given volume is covered. In [19], constructal optimization of T-shaped fin assemblies, where the objective is to maximize the global thermal conductance of the assembly, is reported. In [5], the constructal optimization method is extended to cylindrical assemblies of pin fins. The assembly is arranged as a tree with one stem and many radial branches. The optimization consists of maximizing the global conductance subject to fixed total volume and amount of fin material. In [41], the fundamental problem of optimizing the internal structure of a vertical wall that must meet two requirements is addressed. The two requirements are: thermal insulation and mechanical strength. The wall is a composite of solid material and parallel air caverns with varying thickness and number. It is shown that the internal structure of the wall can be optimized so that the overall thermal resistance of the wall is maximal, while the mechanical stiffness of the wall is fixed. In [48], it is shown that the sizes of heat and fluid flow systems that function on board vehicles such as aircraft can be derived from the maximization of overall performance. In [36], the constructal theory of generation of shape and structure in flow systems connecting one point to a finite size area is presented. A cost minimization model is used in optimization of point to area or area to point flow problems. In [49], it is shown analytically and numerically how an originally uniform flow structure transforms itself into a nonuniform one when the objective is to minimize global flow losses. In [50], the temporal and spatial structure of adsorption desorption processes are optimized for maximal packing of mass transfer into a fixed space, and for minimal overall pumping power.

As a detailed example, we give the example of references [19, 23, 24]. In this example, the more complex configurations where the flow of heat between a finite volume and one point is aided by the flow of a fluid, are studied. The resulting structures are trees in which convection plays an important role. In every elemental volume convection is coupled with pure conduction, in a phenomenon of conjugate heat transfer. The method to developing the optimal flow architecture, from the smallest elemental volumes to larger and larger constructs, is to find this optimal balance between convection and conduction. Convective flow architectures are of two types, depending on which portions of the structure are reserved for convection.

In the first type the interstices are occupied by solid that generates heat at every point, and conducts the heat by diffusion in the manner of the (k_0) material analyzed in Fig. 1. Convection is located in the branches of a tree formed by ducts filled with flowing fluid. The ramifications of two trees of this type visit each elemental volume. One tree delivers cold fluid to each element. The other tree collects the fluid heated by the element, and reconstitutes it into a single stream that eventually leaves the volume [24].

In the convective tree of the second type the spaces occupied by conduction and convection are reversed. Convection is in the interstices, and is coupled with pure conduction in solid parts, which form trees. Every interstitial space serves as source for the current that passes through the root of the tree. Numerous applications for this flow structure are found in the design of heat transfer enhanced surfaces for heat exchangers, and for cooling electronics. In this case, the tree structures are better known as fin trees.

The constructal fin tree problem statement is now reviewed, this time with reference to the general geometry shown in Fig. 4. The volume of frontal area A and fixed length W is considered, where W is aligned with the free stream. The problem consists of distributing optimally through this volume a fixed amount of high conductivity (k_p) material, which takes heat from one spot on the boundary and discharges it throughout the volume. The boundary spot may be taken as the external surface of an electronic module that must be cooled. In this case the volume AW is the space that is allocated for the purpose of cooling the module by forced convection. As in the pure conduction applications of the constructal method (Fig. 1), the space filling optimization sequence is started from the smallest finite size scale. The smallest system in constructal terminology consists

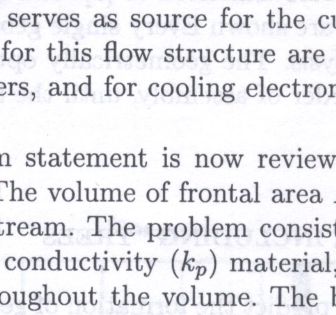

Fig. 4. The volume AW that serves as convective heat sink for the concentrated heat current q; A is the frontal area, W is the side length

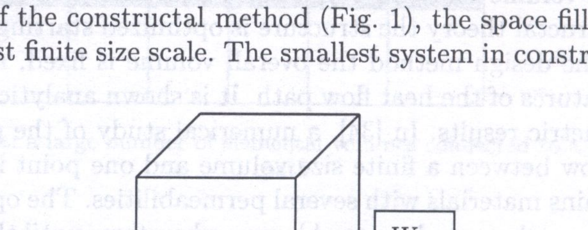

Fig. 5. The smallest volume element defined by a single plate fin (it is a front view of the elementary system)

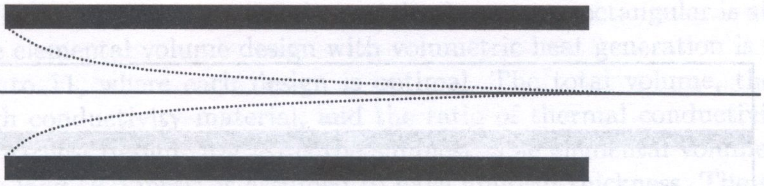

Fig. 6. Merging boundary layers (side view), the flow comes from the left

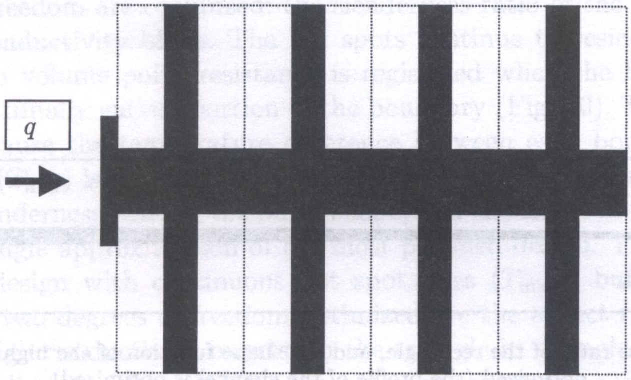

Fig. 7. First assembly (construct) consisting of a large number of elemental one fin volumes

of a two dimensional volume in which there is only one blade of (k_p) material (Fig. 5). Heat is transferred from one boundary spot to the entire elemental volume.

Unlike in Fig. 1, where it was possible to optimize the shape of the elemental volume, in the present problem the thickness of the elemental volume is fixed because it is the same as the optimal spacing between two successive plate fins. The spacing is optimal when the laminar boundary layers that develop over the length W become thick enough to touch at the trailing edge of each plate fin. The spacing is optimal when the time of fluid travel along W matches the thermal diffusion time across the channel (Fig. 6). The optimal spacing is determined uniquely by the length W and the pressure difference maintained across the volume.

Larger scales are now studied. The next volume is the first assembly (or construct in constructal design terminology) which is shown in Fig. 7. The shape of this volume is free to vary. The assembly is defined by a central blade, which is connected to all the elemental volumes that are needed to fill the AW volume. The blade connects the roots of all the fins. When the number of elemental volumes in this assembly is large, the cooling effect provided by the fins is distributed almost uniformly along the stem. The geometry of the construct has two degrees of freedom. In the numerical work detailed in [23], the two optimized features are the external shape and the internal ratio.

4. APPLICATIONS: HEAT CONDUCTION AND FLUID FLOW COUPLED WITH HEAT TRANSFER

As a detailed example, we give references [44] and [45]. In [44], it is shown that the global thermal resistance to flow between a volume and one point can be reduced to unprecedented levels by optimizing the shape of the external boundary of each volume element. This degree of freedom is optimized, next to internal features such as the shape and volume fraction of the high conductivity channels. The volume is covered in a sequence of optimization and assembly steps that proceeds toward larger sizes. The resulting architecture is a leaf like tree structure with high conductivity nerves and low conductivity leaf material. And in [45], the constructal method of minimizing geometrically the thermal resistance between a heat generating volume and one point is extended to three dimensional heat flow.

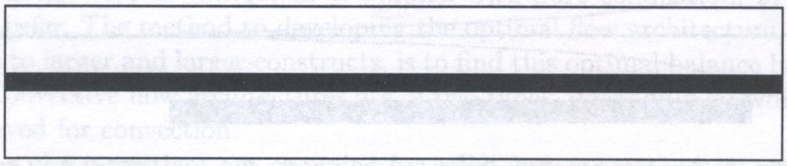

Fig. 8. The elemental volume is rectangular, and the insert has uniform thickness

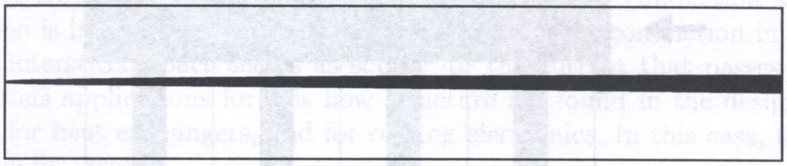

Fig. 9. The slenderness ratio of the rectangle, and the shape function of the high conductivity blade are optimized (the profile of the channel is optimized)

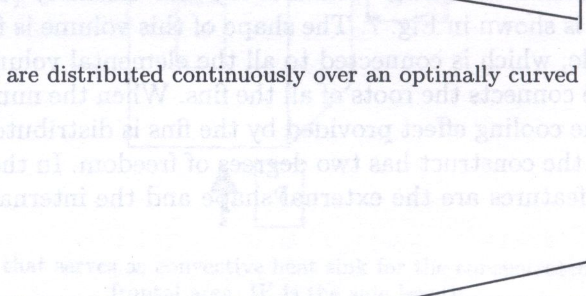

Fig. 10. The hot spots are distributed continuously over an optimally curved portion of the boundary

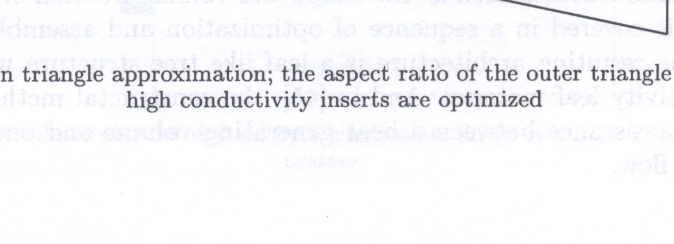

Fig. 11. A triangle in triangle approximation; the aspect ratio of the outer triangle and the shape of the high conductivity inserts are optimized

More precisely, The assumption that the elemental volumes are rectangular is abandoned [44, 45]. The evolution of the elemental volume design with volumetric heat generation is first studied. This is shown in Figs. 8 to 11, where each design is optimal. The total volume, the volume fraction occupied by the high conductivity material, and the ratio of thermal conductivities are the same in each design. The earliest design (Fig. 8) is the simplest. The elemental volume is assumed to be rectangular, and the long (k_p) insert is assumed to have uniform thickness. The thermal resistance is minimum when the large rectangle has a certain shape. The hot spots are concentrated in the two corners $(T_{\max})$ that are situated the farthest relative to the heat sink. In the design shown in Fig. 9, two degrees of freedom are optimized: the slenderness ratio of the rectangle, and the shape function of the high conductivity blade. The hot spots continue to reside in two points. A more substantial reduction in volume point resistance is registered when the hot spots are distributed continuously over an optimally curved portion of the boundary (Fig. 10). The design is said to have constant resistance because the temperature difference between each boundary point $(T_{\max})$ and the common heat sink $(T_{\min})$ is constant [44]. Three degrees of freedom are optimized in this case: the outer shape, the slenderness ratio of the outer shape, and the shape of the (k_p) insert. Figure 11 shows a triangle in triangle approximation of the most polished design. This approximation is also a constant resistance design with continuous hot spot lines $(T_{\max})$, but the external triangular shape is assumed. The two degrees of freedom optimized are the aspect ratio of the outer triangle and the shape of the high conductivity inserts. In the triangle in triangle structure, the formation of continuous hot spot lines has its origin in the optimization of the aspect ratio of the external triangle. When the external triangle is too slender, the hot spot is concentrated in one point: the sharp tip of the triangle. In the opposite extreme, the hot spots are located in the two corners on the side with the heat sink. The optimal slenderness is in between, when the hot spot jumps from the tip of the triangle to the two base corners. In this manner, the hot spot traces with $(T_{\max})$ the two long sides of the triangle, and the design acquires its constant resistance. When these elemental designs are assembled into larger constructs, they cover the inner most scales of the tree structure. Designs based on elements of Figs. 10 and 11 cover their allotted space incompletely.

Figure 12 shows the optimized first construct that results from using the best element of Fig. 10. In this design the volume fraction of (k_p) material in the elemental volumes and the volume fraction averaged over the entire first construct satisfy the optimized proportion. Second constructs can be perfected in a similar way, for optimal external shape of the occupied territory, and for optimal shape of the new central (k_p) blade [44]. Similar progress can be made in three dimensions [45].

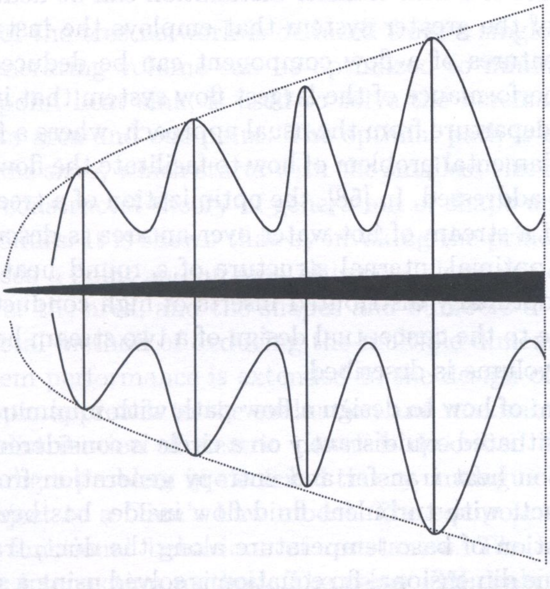

Fig. 12. Optimal external and internal features of the first construct with constant thermal resistance

In [39], the fundamental problem of how to connect a heat generating volume to a point heat sink by using a finite amount of high conductivity material is studied. The problem is one of minimizing the thermal resistance between a finite size volume and one point. The solution is constructed by covering the volume with a sequence of building blocks, which proceeds toward larger sizes. In [26], the constructal theory of the origin of geometrical form in natural flow systems is used to predict the formation of crack patterns in solids subjected to volumetric cooling by convection. The approach is purely deterministic, because it starts from the principle of geometric minimization of resistance to flow, and leads to the existence of optimal distances between successive cracks and, consequently, optimal crack widths. In [1, 38], the geometric constructal method of minimizing the overall thermal resistance between a finite size volume and a small heat sink is extended to three dimensional heat conduction and to convective heat transfer. In [3], two methods of improving the performance of volume to point tree networks for two dimensional heat conduction are given. These improvements are offered relative to the design produced by the constructal method, in which optimized volume elements are presented and grouped into larger constructs, which are also optimized.

The fundamental problem of maximizing the thermal contact between fin entire heat generating volume and a pulsating stream of coolant that bathes the volume is studied in [51]. In [54], it is shown that the internal geometric configuration of a component can be deduced by optimizing the global performance of the installation that uses the component. The example chosen is the counterflow heat exchanger that serves as condenser in a vapor compression cycle refrigeration system for environmental control of aircraft. The optimization of global performance is achieved by minimizing the total power requirement or the total entropy generation rate. In [18], the solution to the problem of maximizing the extraction of exergy from a stream of hot gas show that the hot stream must be cooled in a counterflow heat exchanger with optimal imbalance of capacity rates. This paper outlines the first few steps toward making this solution practical, by combining the optimized counterflow with conventional components for compressing and expanding the cold stream.

The optimal geometric layout of schemes for distributing hot water uniformly over an area is studied in [57]. In [4], constructal tree designs are used for optimization of nonuniformly distributed tree shaped flow structures for conduction. In past work, the structure was optimized as a sequence of building blocks, which started with the smallest size and continued toward larger and more complex assemblies. The resulting structure had a uniform distribution of interstitial spaces, because the size of the elemental volume was fixed. In this paper [4], the elemental size constraint is relaxed, and it is shown that the added design freedom leads to significant improvements in global performance. In [2], it is shown that many features of a heat transfer installation can be deduced from the maximization of the global performance of the greater system that employs the installation. In [55], it is shown that the main geometric features of a flow component can be deduced from the thermodynamic optimization of the global performance of the largest flow system that incorporates the component. This approach represents a departure from the usual approach, where a flow component is optimized in isolation. In [46], the fundamental problem of how to facilitate the flow of heat across a conducting slab heated from one side is addressed. In [58], the optimization of a tree shaped system of insulated pipes for the distribution of a stream of hot water over an area is described. In [52], a hierarchical strategy to developing the optimal internal structure of a round heat generating body cooled at its center with the help of optimally distributed inserts of high conductivity material is described. In [13], the constructal route to the conceptual design of a two stream heat exchanger with maximal heat transfer rate per unit volume is described.

The fundamental problem of how to design a flow path with minimum overall resistance between one point and many points situated equidistantly on a circle is considered in [59]. In [53], a numerical analysis of natural convection heat transfer and entropy generation from an array of vertical fins, standing on a horizontal duct, with turbulent fluid flow inside, has been carried out. The analysis takes into account the variation of base temperature along the duct, traditionally ignored by most studies on such problems. One dimensional fin equation is solved using a second order finite difference scheme for each of the fins in the system and this, in conjunction with the use of turbulent flow correlations for duct, is used to obtain the temperature distribution along the duct. In [56], the fundamental problem of optimizing the geometry of the interface between two conductive bodies is considered, with the objective of minimizing the thermal resistance. The interface geometry is free to change. In [14], attention is drawn to constructal theory and design, which relies on global maximization of performance in the pursuit of flow system architecture. Exergy analysis establishes the theoretical performance limit. Thermodynamic optimization brings the design as closely as permissible to the theoretical limit. The design is destined to remain imperfect because of constraints. Improvements are registered by spreading the imperfection. In [15], a series of recent results based on the geometric minimization of the resistance to flow between one point and a volume or an area are reviewed. In [29], some propositions are made to design flow structures with maximal heat transfer rate per unit volume, by shaping each duct so that it fits optimally on the body of the convective flow.

Paper [37] deals with the constructal theory based solution for conductive cooling of electronics. The problem falls in the category of a more general flow problem. In [61], the strong relation that exists between the changing architecture of a complex flow system and the maximization of global performance under constraints is documented. In [43], the fundamental problem of how to design a flow path with minimum overall resistance between one point and many points situated equidistantly on a circle is reviewed. The paper [16] is a review of a growing body of fundamental research that documents the opportunity for optimizing geometrically the cooling of spaces that generate heat volumetrically. In [31], the optimization approach based on the biological evolution principle is used to construct the heat transport paths for volume to point problem. The transport paths are constructed by inserting high conductivity materials in the heat conduction domain where uniform or nonuniform heat sources exist. In [25], a new concept for generating the multi scale structure of a finite size flow system that has maximum heat transfer density maximum heat transfer rate is presented. In [17], the basic rules and promise of two of the simplest methods for solving problems of convection in porous media are outlined. In [60], the optimal tree shaped flow paths for cooling a disc shaped body by convection is developed. In [30], the fundamental relation between the maximization of global performance and the maleable architecture of a flow system with global constraints is documented. In [33], an application of the constructal method to the discovery of the optimal distribution of discrete heat sources cooled by laminar natural convection is presented.

5. APPLICATIONS: STRUCTURES IN TRANSPORTATION AND ECONOMICS

In [27], the geometric form of the tree network is deduced from a single mechanism. The discovery that the shape of a heat generating volume can be optimized to minimize the thermal resistance between the volume and a point heat sink, is used to solve the kinematics problem of minimizing the time of travel between an area and one point. The optimal path is constructed by covering the area with a sequence of volume sizes, which starts with the smallest size and continues with stepwise larger sizes. In [20, 21], the constructal theory of generation of shape and structure in natural flow systems is extended to economics. It is shown that by invoking the principle of cost minimization in the transport of goods between a point and an area, it is possible to anticipate the dendritic pattern of transport routes that cover the area, and the shapes and numbers of the interstitial areas of the dendrite. In [6], the constructal method of deducing the multiple dimensions of a network from the maximization of global system performance is extended to the design of electric power distribution networks. In [7], a macroscopic approach to the evaluation and maximization of performance in the design of networks for the distribution of electric power is described. In [40], Bejan's constructal theory is explored for a paradigm problem in electrical theory, analogue of a wide range of transport problems involving flow subject to a Ohm's Law model. The equipotential optimisation is derived and generalised to further functional behaviour of resistances. The original case illustrates the power of Bejan's asymptotic intersection method. The paper [35] deals with the constructal theory of generation of shape and structure in flow systems connecting one point to a finite size area. The flow direction may be either from the point to the area or the area to the point. The formulation of the problem remains the same if the flow direction is reversed. Two models are used in optimization of the point to area or area to point flow problem: cost minimization and revenue maximization.

6. CONCLUSION

In conclusion, the constructal theory seems very promising for engineering design and optimization. These new ideas will also appeal to a broad range of scientists not only in engineering, but also in natural sciences, economics, and business.

Nevertheless, most of the applications of the constructal theory were developed in fluids mechanics, in particular for the optimization of flows. So, we are going to give now remarks on possibility of coupling this approach with computational mechanics. The question is: how one can apply the constructal theory to the optimization of mechanisms ? Our ideas are the following. We could start by introducing the concept of skeleton of a mechanism, i.e. a mechanism reduced to an assembly of machine elements simply represented by bars, and connected between them by standard connections. The constructal theory, which rests on a only one and single principle of minimization of energies in the connections, would make it possible to optimize the topology of this skeleton in a first time. In a second time, we could pass to the phase of optimizing the machine elements shapes. The constructal theory, which always rests on a only one and single principle (which, in this second phase, would be the minimization of total potential energy for each machine element, subjected to technological constraints or limitations), would make it possible to obtain a shape optimization of the skeleton "skin" (or shape) for each machine element.

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