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Finite element models updating by means of a global approach

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Analytical models are often used to analyse behaviour of structures, particularly in the field of structural dynamics. The application of such models demands that they must predict the effects of structural modifications with a reasonable accuracy. Unfortunately, lacks of correlation between initial analytical predictions and experimental results are usually observed so that the analytical model needs to be updated with respect to an experimental reference. Many updating methods, involving two main categories of techniques, have been developed in recent years. In the first group of methods, the model to be adjusted is modified by means of correcting parameters associated with the regions containing dominant errors in modelling. The techniques require a localization of modelling errors and are essentially iterative. The second category involves one-step algorithms to globally correct the model in terms of its representative mass and stiffness matrices. These methods have come to be called direct or global methods. Each class of methods presents advantages and disadvantages. The main disadvantage of the iterative methods is the errors localization phase that may require an extensive amount of computational efforts. In addition, the convergence is not ensured for all iterative algorithms. The present paper deals with a direct approach to correct the whole mass and stiffness matrices of a derived finite element model. A modal analysis and a quantitative study of matrix changes are performed to evaluate the capability of the proposed algorithm and to investigate its potential usefulness in model updating.

Notations and symbols

$\mathbf{M}, \mathbf{K}, \mathbf{D}$ – mass, stiffness and damping matrices of any real system
$\mathbf{M}^\circ, \mathbf{K}^\circ$ – mass and stiffness matrices of the original analytical model
$\mathbf{M}^*, \mathbf{K}^*$ – updated mass and stiffness matrices
$\mathbf{I}$ – identity matrix
$\mathbf{y}$ – displacements vector
$\dot{\mathbf{y}}$ – velocities vector
$\ddot{\mathbf{y}}$ – accelerations vector
n – number of degrees of freedom of the analytical model
p – number of computed modes using the analytical model
m – number of measured modes
q – number of experiment points
c – dashpot damping value
ϕ_i – i-th eigenmode of the conservative (analytical) model
ω_i – i-th circular frequency of the conservative model
ψ_r – r-th complex eigenmode of the damped system
λ_r – r-th complex eigenvalue of the damped system

ζ_r – damping ratio of the r-th complex mode
ERD – Element Relative Deviation
REF – Relative Error in Frequency
$[\,]^T$ – transpose of matrix

1. Introduction

Finite element models are usually employed to analyse and predict dynamic behaviour of structures. Unfortunately, it is often observed that the initial finite element model is not an accurate reflection of the structure under study. Such inaccuracy arises because of a number of simplifying assumptions and idealizations that have to be made in modelling. Although the discrepancies can be traced to experimental data also, it is usually agreed that the test data should be considered as closer to the true representation of the structure. Thus, the analytical model is corrected such that agreement between predictions and test results is improved. The purpose of any updating process is to provide a reliable model in order to substitute for experimental tests with more confidence in subsequent analysis. The updating methods developed in recent years may be broadly grouped into local and global techniques. Both categories of methods present advantages and disadvantages. In local methods, the model to be adjusted is modified by means of correcting parameters associated with the regions containing dominant errors in modelling. The corrections are physically meaningful but a critical disadvantage is encountered. Indeed, the selection of the regions presenting the dominant errors and the computation of the correcting parameters are performed by using iterative algorithms. The calculations are often very long and the convergence is not ensured necessarily. Among local methods, Zhang *et al.* [16] proposed an eigenvectors sensitivity approach while Visser and Imregun [13] formulated a technique based on the frequency response functions properties. More recently, Sinha *et al.* [12] presented a gradient-based sensitivity method. On the other side, the global procedures are essentially one step or direct algorithms [4–6]. The system matrices are entirely modified and no localization phase is needed. Hence, the numerical disadvantage encountered in local methods is avoided. Nevertheless, the updated model may be influenced by the emergence of some unrealistic terms. An important survey of updating methods can be found in reference [11] including detailed appreciations about the various techniques. The present paper deals with a global algorithm to correct mass and stiffness matrices of finite element models. A particular care is taken to preserve matrix properties, like symmetry, in order to use the updated model in further finite element analysis [1, 2]. The capability of the proposed algorithm is investigated by performing a modal analysis and a quantitative study of matrix modifications effects. The potential combination of the algorithm with local techniques, especially in localizing dominant modelling errors, is discussed too.

2. Theoretical background

2.1. Model assumptions

In structural dynamics, analytical models are often derived by means of the finite element method and are generally conservative since damping modelling is neither evident nor easy. Hence, the data derived from these models are real while those of experiments (eigenmodes, frequency response functions) are complex. The two sets of data may be presented as (see notations and symbols)

(i) Analytical data:

- a mass matrix $\mathbf{M}^\circ \in R^{n,n}$ and a stiffness matrix $\mathbf{K}^\circ \in R^{n,n}$,
- A modal matrix $\mathbf{\Phi} \in R^{n,p}$ and a spectral matrix $\mathbf{\Omega} \in R^{p,p}$,

(ii) Experimental data:

- an eigenvector matrix $\mathbf{\Psi} \in C^{q,m}$ and an eigenvalue matrix $\mathbf{\Lambda} \in C^{m,m}$,
- damping ratios ζ_r $(r = 1, 2, \ldots, m)$.

The correction of an analytical model using experimental results requires the comparison of both data sets. This comparison is possible if

- the data belonging to the two sets have the type (complex or real).
- the number of measured points agrees with that of analytical degrees of freedom $(q = n)$.

Usually, the first condition is satisfied by transforming complex modes into normal modes as explained in Section 2.2. However, the second condition is rarely verified in practice. Indeed, due to various limitations related to experiments and modelling, the number of measured points is substantially less than the degrees of freedom of the analytical model so that

$$n > p > q > m \quad \text{with} \quad n \gg q.$$

Thus, to satisfy this constraint and not alter strongly the two sets of data, we propose to reduce the size of the analytical modal matrix so that $n = p$ and to expand the number q so that $q = p$. Under these two considerations the analytical and experimental data would be represented by reasonable medium size sets as [1]

- $\mathbf{M}^\circ \in R^{n,n}$, $\mathbf{K}^\circ \in R^{n,n}$, $\mathbf{\Phi} \in R^{p,p}$, $\mathbf{\Omega} \in R^{p,p}$ – for the analytical set,
- $\mathbf{\Psi} \in C^{p,m}$, $\mathbf{\Lambda} \in C^{m,m}$, ζ_r $(r = 1, 2, \ldots, m)$ – for the experimental set.

2.2. Governing equations

The general equation of a damped free vibrating system is given by

$$\mathbf{M}\,\ddot{\mathbf{y}}(t) + \mathbf{D}\,\dot{\mathbf{y}}(t) + \mathbf{K}\,\mathbf{y}(t) = 0. \tag{1}$$

It must be satisfied by experimental data as well as by analytical ones. Thus, assuming solutions in the form

$$\mathbf{y}_r(t) = \boldsymbol{\psi}_r\, e^{\lambda_r t}, \qquad r = 1, 2, \ldots, m, \tag{2}$$

we may write Eq. (1) in the state space as [1, 8–10]

$$\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\psi}_r \\ \lambda_r \boldsymbol{\psi}_r \end{Bmatrix} = \lambda_r \begin{Bmatrix} \boldsymbol{\psi}_r \\ \lambda_r \boldsymbol{\psi}_r \end{Bmatrix}, \qquad r = 1, 2, \ldots, m, \tag{3}$$

and therefore

$$\begin{bmatrix} \mathbf{M}^{-1}\mathbf{K} & \mathbf{M}^{-1}\mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\psi}_r \\ \lambda_r \boldsymbol{\psi}_r \end{Bmatrix} = -\left\{ \lambda_r^2 \boldsymbol{\psi}_r \right\}, \qquad r = 1, 2, \ldots, m. \tag{4}$$

Equation (4) is very important in the proposed algorithm since it will be used to convert complex modes to real ones as suggested above. This operation is performed after computing the submatrix $[\mathbf{M}^{-1}\mathbf{K}]$ and resolving its associated eigenvalue problem [1].

Considering now the new sets of data presented in Section 2.1, Eq. (4) may be written in an alternative form as

$$\mathbf{H}\mathbf{X} = \mathbf{Z} \tag{5}$$

where $\mathbf{H}$ and $\mathbf{Z}$ are two known matrices given by

$$\mathrm{H}_{(m\times 2p)} = \begin{bmatrix} \mathbf{\Psi}^T & (\mathbf{\Psi\Lambda})^T \end{bmatrix} \quad \text{and} \quad \mathbf{Z}_{(m\times p)} = -\mathbf{\Lambda}^2\mathbf{\Psi}^T.$$

$\mathbf{X}$ is an unknown matrix containing submatrices $[\mathbf{M}^{-1}\mathbf{K}]$ and $[\mathbf{M}^{-1}\mathbf{D}]$ such that

$$\mathbf{X}_{(2p\times p)} = \begin{bmatrix} (\mathbf{M}^{-1}\mathbf{K})^T \\ (\mathbf{M}^{-1}\mathbf{D})^T \end{bmatrix}.$$

The system (5) represents a set of mp complex equations or $2mp$ real equations for $2p^2$ real unknowns. Since $m < p$, this system cannot be resolved and needs $p(p-m)$ other complex equations or $2p(p-m)$ real equations. The required equations may be obtained using analytical data as suggested in reference [1, 8]. Indeed, since these data satisfy also Eq. (1), the system (5) is completed by introducing $(p-m)$ pairs of higher modes computed from the analytical model and that do not correspond to the m measured modes, namely $(\omega_i\,,\,\phi_i\,:\; i=m+1, m+2, \ldots, p)$. Hence, according to the assumed data presented above, a set of p^2 complex equations may be constructed and presented in the form

$$\mathbf{AX} = \mathbf{B} \tag{6}$$

with

$$\mathbf{A} = \begin{bmatrix} \mathbf{\Psi}^T & (\mathbf{\Psi\Lambda})^T \\ \mathbf{\Phi}^T & (\mathbf{\Phi\Omega})^T \end{bmatrix} \in C^{p,2p}, \qquad \mathbf{B} = \begin{bmatrix} (\mathbf{\Psi\Lambda}^2)^T \\ (\mathbf{\Phi\Omega}^2)^T \end{bmatrix} \in C^{p,p}.$$

The complex system (6) can now be transformed into a real one by introducing new matrices $\mathbf{P}$ and $\mathbf{Q}$ defined by [1]

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \end{bmatrix} \in R^{2p,2p}, \qquad \text{and} \qquad \mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix} \in R^{2p,p},$$

where $\mathbf{P}_1$ and $\mathbf{Q}_1$ are submatrices representing real parts of matrix $\mathbf{A}$ and matrix $\mathbf{B}$, respectively, while $\mathbf{P}_2$ and $\mathbf{Q}_2$ are the imaginary parts of both matrices. After rearranging terms, a set of $2p^2$ real and linear equations is obtained as

$$\mathbf{PX} = \mathbf{Q}. \tag{7}$$

Resolving the system (7) by means of any classical method, the matrix $\mathbf{X}$ can be obtained and, therefore, submatrices $[\mathbf{M}^{-1}\mathbf{K}]$ and $[\mathbf{M}^{-1}\mathbf{D}]$ are determined. Of particular interest is the submatrix $[\mathbf{M}^{-1}\mathbf{K}]$ which will be used to compute real modes according to the following eigenvalue problem,

$$[\mathbf{M}^{-1}\mathbf{K}]\,\psi = \omega^2\psi. \tag{8}$$

The p pairs of real eigenvalues and associated eigenvectors determined from Eq. (8) will be used to correct the analytical model in terms of its representative mass and stiffness matrices.

3. Correction of system matrices

3.1. Matrix norm minimization

A convenient way to find a pair of matrices $(\mathbf{M}^*, \mathbf{K}^*)$ which give minimum changes to the original and known pair of corresponding matrices $(\mathbf{M}^\circ, \mathbf{K}^\circ)$ is to minimize the distance between both pairs. Two complementary approaches may be used and lead to similar results: the Lagrange multipliers method (LMM) [4–7, 14] and the matrix transformation method (MTM) [15]. The LMM, which is used here, consists of two steps:

(i) Introduce matrix norms in the form

$$\varepsilon_1 = \|\mathbf{W}_1^{-1}\,\Delta\mathbf{M}\,\mathbf{W}_1^{-1}\|, \qquad \varepsilon_2 = \|\mathbf{W}_2^{-1}\,\Delta\mathbf{K}\,\mathbf{W}_2^{-1}\| \tag{9}$$

where

$$\Delta\mathbf{M} = \mathbf{M}^* - \mathbf{M}^\circ, \qquad \Delta\mathbf{K} = \mathbf{K}^* - \mathbf{K}^\circ, \tag{10}$$

and $\mathbf{W}_1$ and $\mathbf{W}_2$ are weighting matrices related to mass and stiffness, respectively.

(ii) Minimize objective functions, using the matrix norms above and Lagrange multipliers, according to matrix properties considered as constraints [1, 4–7, 14].

3.2. Mass matrix correction

Assuming little changes between original and updated models and considering that modelling errors concern the stiffness essentially, the original mass matrix being relatively exact, the weighting matrix may be introduced as [6]

$$\mathbf{W}_1 = \sqrt{\mathbf{M}^\circ}\,. \tag{11}$$

The corrected matrix has to satisfy the orthogonality constraint

$$\mathbf{\Phi}^T\mathbf{M}^*\mathbf{\Phi} = \mathbf{I} \tag{12}$$

while the original mass matrix verifies the analogous condition

$$\mathbf{\Phi}^T\mathbf{M}^\circ\mathbf{\Phi} = \mathbf{m}_g \tag{13}$$

where $\mathbf{\Phi}_{(p\times m)}$ contains the m first modes computed from Eq. (8). $\mathbf{I}_{(m\times m)}$ is the identity matrix and $\mathbf{m}_{g\,(m\times m)}$ represents the original generalized mass matrix which is not necessary diagonal.

The associated objective function is then [1, 6, 11]

$$\mathbf{J}_1 = \varepsilon_1 + \sum_{i=1}^{m}\sum_{j=1}^{m}\mu_{ij}\left(\mathbf{\Phi}^T\mathbf{M}^*\mathbf{\Phi} - \mathbf{I}\right) = \varepsilon_1 + \sum_{i=1}^{m}\sum_{j=1}^{m}\mu_{ij}\left(\mathbf{\Phi}^T\,\Delta\mathbf{M}\,\mathbf{\Phi} + \mathbf{m}_g - \mathbf{I}\right) \tag{14}$$

where μ_{ij} are Lagrange multipliers used to enforce constraint (12).

Minimizing the objective function with respect to each element (ΔM_{ij}) of matrix $\Delta\mathbf{M}$ leads to

$$\Delta\mathbf{M} = \mathbf{M}^\circ\mathbf{\Phi}\mathbf{m}_g^{-1}(\mathbf{I} - \mathbf{m}_g)\mathbf{m}_g^{-1}\mathbf{\Phi}^T\mathbf{M}^\circ, \tag{15}$$

and then

$$\mathbf{M}^* = \mathbf{M}^\circ + \Delta\mathbf{M} = \mathbf{M}^\circ\left[\mathbf{I} + \mathbf{\Phi}\mathbf{m}_g^{-1}(\mathbf{I} - \mathbf{m}_g)\mathbf{m}_g^{-1}\mathbf{\Phi}^T\mathbf{M}^\circ\right]. \tag{16}$$

It appears from Eq. (16) that the updated matrix $\mathbf{M}^*$ is symmetrical since $\mathbf{M}^\circ$ is symmetrical too. This result gives prominence to the representativity of the new mass matrix that will be used to correct the stiffness matrix below.

It should be noted also that the calculation of $\Delta\mathbf{M}$ requires inversion of $\mathbf{m}_{g\,(m\times m)}$ only. This does not cause much problem since the number of measured modes (m) is not very large in practice. Moreover, the process is not iterative. Therefore, a moderate amount of computational effort is invested presenting an advantage of the proposed algorithm.

3.3. Stiffness matrix correction

The corrected mass matrix being determined now, the updated stiffness matrix is calculated similarly by introducing 3 associated constraints:

– the eigenvalue equation,

$$\mathbf{K}^*\mathbf{\Phi} = \mathbf{M}^*\mathbf{\Phi}\Omega^2, \tag{17}$$

– the orthogonality condition,

$$\mathbf{\Phi}^T\mathbf{K}^*\mathbf{\Phi} = \Omega^2, \tag{18}$$

– and the symmetry of $\mathbf{K}^*$,

$$\mathbf{K}^* = (\mathbf{K}^*)^T \tag{19}$$

Using the corrected mass matrix as weighting matrix, i.e.

$$\mathbf{W}_2 = \sqrt{\mathbf{M}^*}, \tag{20}$$

the corresponding objective function would be

$$J_2 = \varepsilon_2 + \sum_{i=1}^{p}\sum_{j=1}^{m} \mu_{1\,ij}\left(K^*\Phi - M^*\Phi\Omega^2\right)_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{m} \mu_{2\,ij}\left(\Phi^T K^*\Phi - \Omega^2\right)_{ij} + \sum_{i=1}^{p}\sum_{j=1}^{p} \mu_{3\,ij}\left(K^* - (K^*)^T\right)_{ij}. \tag{21}$$

The Lagrange multipliers $\mu_{k\,ij}$ are used to enforce constraints (17)–(19) respectively and the minimizing process leads to a global expression of $\mathbf{K}^*$ given by [1, 6, 11]

$$\mathbf{K}^* = \mathbf{K}^\circ + \mathbf{\Delta} + \mathbf{\Delta}^T \tag{22}$$

with

$$\mathbf{\Delta} = \left[\frac{1}{2}\mathbf{M}^*\mathbf{\Phi}\left(\mathbf{\Phi}^T\mathbf{K}^\circ\mathbf{\Phi} + \Omega^2\right) - \mathbf{K}^\circ\mathbf{\Phi}\right]\mathbf{\Phi}^T\mathbf{M}^*. \tag{23}$$

It is important to note that although matrix $\mathbf{K}^*$ depends upon $\mathbf{M}^*$, its computation is not difficult since expression (23) is a simple matrix product and no iterative process is needed here too. On another hand $\mathbf{K}^*$ is also symmetrical and gives prominence to the representative pair $(\mathbf{M}^*, \mathbf{K}^*)$ of the updated model in order to use it in further finite element analysis.

4. Application: Results and comments

4.1. Presentation of models

Simulated studies on a simply-clamped beam are conducted for evaluating the updating algorithm developed above and for analyzing the effects of matrix changes. Thus, two analytical models corresponding to the same structure are used as shown in Figs. 1 and 2. The first model is conservative and represents the original undamped model to be adjusted. The second one, which is dissipative and presents slight changes in geometry and physical properties, is considered as representing experimental model and hence the reference. A modal analysis and a quantitative study of matrix changes are performed.

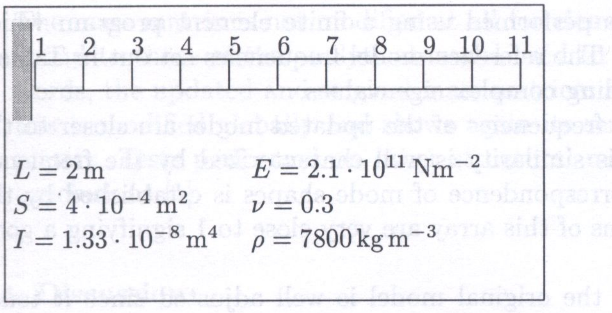

Fig. 1. Model to be adjusted

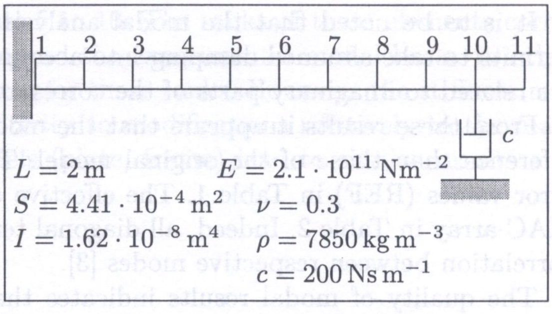

Fig. 2. Reference model

4.2. Modal analysis

As a preliminary step, it is assumed that the condition requiring the correspondence of both numbers of analytical degrees of freedom and experimental points is already satisfied as mentioned in Section 2.1. Thus, the incompleteness addressed here concerns the measured modes only. Considering two models with 10 degrees of freedom as shown in Figs. 1 and 2, we use

- an analytical modal matrix of 10 modes ($p = 10$).
- an experimental modal basis of 4 modes only ($m = 4$). The associated matrix will be extended by adding the 6 higher analytical modes according to the expansion procedure suggested in Section 2.2.

Once the updated matrices $\mathbf{M}^*$ and $\mathbf{K}^*$ are calculated, a modal analysis is numerically performed on original, reference and updated models respectively. A convenient way to compare the different results is to present them in terms of

(i) mode frequencies as shown in Table 1.

(ii) mode shapes consistency check using the modal assurance criterion (MAC) presented in Table 2.

Table 1. Mode frequencies comparison

Modes	Mode Frequencies (Hz) Original Model	Reference Model	Updated Model	Relative Errors in Frequency (%) Original/Reference Models	Updated/Reference Models
1	64.89	67.86	67.93	4.37	0.10
2	195.52	204.50	204.68	4.39	0.08
3	328.54	343.74	343.90	4.42	0.05
4	465.10	486.72	486.79	4.44	0.01

Table 2. Modal assurance criterion array

Updated Model	Reference Model 1	2	3	4
1	**0.990**	0.004	0.007	0.008
2	0.014	**0.990**	0.006	0.009
3	0.009	0.010	**0.990**	0.007
4	0.009	0.007	0.009	**0.990**

It is to be noted that the modal analysis is performed using a finite element program which permits to take assumed damping into account. The reference model frequencies set out in Table 1 are related to imaginary parts of the corresponding complex eigenvalues.

From these results it appears that the mode frequencies of the updated model are closer to the reference than those of the original model. This similarity is well characterized by the frequency error values (REF) in Table 1. The effective correspondence of mode shapes is established by the MAC array in Table 2. Indeed, all diagonal terms of this array are very close to 1 signifying a good correlation between respective modes [3].

The quality of modal results indicates that the original model is well adjusted since it tends towards the reference one. However, it should be interesting to quantify the changes of system matrices and, therefore, to analyse the contribution of the updating algorithm.

4.3. Matrix changes analysis

The difference between reference and original models concerns not only damping but also physical and geometrical characteristics. In order to analyse the updating process effects on system matrices, we propose to quantify and compare the matrix elements by evaluating the element relative deviation (ERD) between diagonal elements of original and updated matrices respectively. The analysis is performed according to the following scheme:

(i) In the first test, we consider distributed perturbations of mass and stiffness as shown in Figs. 1 and 2. These perturbations are about 11% for mass and 22% for stiffness.

(ii) In the second test, the same perturbations are affected to elements 5 and 6 only.

The ERD analysis for both tests allows us to formulate the following statements:

- For distributed modifications (Test 1), the effect of the updating process is negligible for the mass matrix and relatively important for the stiffness matrix. Note that this effect is distributed also (Table 3).
- For local perturbations (Test 2), modifications of mass are more important than those in the previous case but remain very small compared to the introduced perturbations. Modifications of stiffness are more significant for perturbed elements (5 and 6). Thus, a certain localization of perturbations, hence errors modelling, is identified (Table 4).
- The stiffness matrix is more affected than the mass matrix. This result agrees with theory since the computation of $\mathbf{K}^*$ depends upon that of $\mathbf{M}^*$ and requires more constraints. Moreover, modelling errors are generally more significant for the stiffness matrix.

Table 3. Matrix element relative deviations for test 1

Diagonal Term		1	2	3	4	5	6	7	8	9	10
ERD	Mass	0.01	0.01	0.01	0.01	0	0	0	0	0	0
(%)	Stiffness	1.15	1.38	0.51	1.04	0.91	0.69	1.31	0.45	0.69	0.93

Table 4. Matrix element relative deviations for test 2

Diagonal Term		1	2	3	4	5	6	7	8	9	10
ERD	Mass	0.43	1.09	0.21	1.90	0	0	0	0	0	0
(%)	Stiffness	0.06	0.30	0.34	0.43	1.92	1.22	0.92	0.33	0.09	0.01

– The mass matrix is not modified at all for elements 5 to 10. The ranks of these elements correspond to those of the analytical model used to expand the experimental modal basis. In other words, the updated and original mass matrices agree for these ranks. However, the stiffness matrix is modified globally and shows again its sensitivity to modifications relatively to the mass matrix. Tests performed on other structural models (plate, frame) lead to similar results and conclusions [1].

4.4. Discussion

The results of all tests and analysis performed in the two last sections show that

(i) Mass and stiffness matrices remain symmetrical after updating. This result is very interesting in order to use the corrected model in further finite element analysis concerning other aspects like reanalysis of modified structures and forced response studies [1].

(ii) The analytical model is well adjusted on the basis of the reference model. This adjustment is expressed by the ERF values and the MAC array.

(iii) The updated model is not deviated strongly from the original model as indicated by the weak values of ERD. It should be noted that an updating procedure is valid if only it is applied on a representative model containing small errors with respect to the real structure. The Lagrange multipliers formalism, used here to optimally minimize errors, allows such results to be obtained.

(iv) The stiffness matrix is more sensitive to modifications comparatively to the mass matrix and confirms the fact that modelling errors are more significant for stiffness than mass as it is generally encountered in practice.

The results above agree closely with theory and globally with experience. The result concerning matrix errors localization, especially for the stiffness matrix, is particularly interesting and indicates that the process might be seen also as a useful tool to analyse structures. The algorithm contains certainly some drawbacks, as any other process, but it presents specific advantages [1]. Indeed, the real behaviour of systems is taken into account by considering damping. Real modes are computed from complex ones directly and the properties of original system matrices are preserved in order to use the updated system in further finite element analysis like those of forced response and modified systems. On another hand, the process is not iterative and does not require excessive amounts of calculation. Concerning disadvantages, the corrected model may be influenced by analytical data since a high number of analytical modes may be needed to complete the missing experimental modes. Another negative influence may result from the double operation of condensation–expansion mentioned in Section 2.1 leading to some unrealistic terms in updated matrices.

Although the tests are performed on simple models, the algorithm might be extended to larger models since it does not need a great effort of calculations. It might be considered also as a preprocessing tool in model updating field. Indeed, with a localization of the main matrix modifications, especially those concerning stiffness matrix, the proposed method would provide precious information concerning uncertainties of analytical models. If more accurate indications about physical modelling errors are needed, then an appropriate local approach will be applied to the localized region only. Note that the main disadvantage of local methods is the extensive effort of calculations since these procedures are often iterative, especially in localizing the dominant modelling errors.

5. Conclusion

The updating algorithm presented in this paper deals with a global correction of an entire model in terms of its mass and stiffness matrices. Results of modal analysis show that the original model is well adjusted and tends towards the reference one. The main result of the process is the preservation of the representativity of both matrices in order to use the updated model in subsequent analysis involving finite element models. The possibility of localizing the main matrix modifications, especially in the stiffness matrix, is an interesting result too in order to exploit the proposed algorithm, in combination with a suitable local technique, in structural dynamics analysis. Thus, the objective is not only to improve the global way of model adjustment but also to provide a pre-processing tool for local techniques which accuracy remains certainly greater.

REFERENCES

[1] A. Aiad. *Recalage de modèles éléments finis à partir des propriétés modales expérimentales. Proposition d'un algorithme global à signification physique enrichie.* Doctoral Thesis. University of Valenciennes, France, 1991.

[2] A. Aiad, P. Level, Y. Ravalard. A global updating procedure to improve finite element models. *Proceedings of the 10th International Modal Analysis Conference,* San Diego (USA), 1051–1056, 1992.

[3] R.J. Allemang, O.L. Brown. A correlation coefficient for modal vector analysis. *Proceedings of the 1st International Modal Analysis Conference* (USA), 110–116, 1982.

[4] M. Baruch. Optimal correction of mass and stiffness matrices using measured modes. *AIAA Journal,* **20**(11): 1623–1626, 1982.

[5] A. Berman. Mass matrix correction using an incomplete set of measured modes. *AIAA Journal,* **17**(10): 1147–1148, 1979.

[6] A. Berman, E.J. Nagy. Improvement of a large analytical model using test data. *AIAA Journal,* **21**(8): 1168–1173, 1983.

[7] P.D. Cha, W. Gu. Model updating using an incomplete set of experimental modes. *Journal of Sound and Vibration,* **233**(4): 587–600, 2000.

[8] L. Chih, S.R Ibrahim. The use of complex modes versus normal modes in structural model improvement. *Proceedings of the 2nd International Modal Analysis Conference,* Orlando (USA), 415–424, 1984.

[9] S.R. Ibrahim. Computation of normal modes from identified complex modes. *AIAA Journal,* **21**(3): 446–451, 1983.

[10] S.R. Ibrahim. Dynamic modelling of structures from measured complex modes. *AIAA Journal,* **21**(6): 898–901, 1983.

[11] J.E. Mottershead, M.I. Friswell. Model updating in structural dynamics: A survey. *Journal of Sound and Vibration,* **167**(2): 347–375, 1993.

[12] J.K. Sinha, A.R. Rao, R.K. Sinha. Advantage of the updated model of structures: A case study. *Nuclear Engineering and Design,* **232**: 1–6, 2004.

[13] W.J. Visser, M. Imregun. A technique to update finite element models using frequency response data. *Proceedings of the 9th International Modal Analysis Conference,* Florence (Italy), 462–468, 1991.

[14] F.S. Wei. Stiffness matrix correction from incomplete test data. *AIAA Journal,* **18**(10): 1274–1275, 1980.

[15] D.W. Zhang, L. Zhang. Matrix transformation method for updating dynamic model. *AIAA Journal,* **30**(5): 1440–1443, 1992.

[16] Q. Zhang, G. Lallement, R. Fillod, J. Piranda. A complete procedure for the adjustment of a mathematical model from the identified complex modes. *Proceedings of the 5th International Modal Analysis Conference,* London (G.B.), 1183–1190, 1987.