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Gaussian mixture model for time series-based structural damage detection

Marek Słoński
Cracow University of Technology, Institute for Computational Civil Engineering
Warszawska 24, 31-155 Kraków, Poland
e-mail: mslonski@L5.pk.edu.pl

In this paper, a time series-based damage detection algorithm is proposed using Gaussian mixture model (GMM) and expectation maximization (EM) framework. The vibration time series from the structure are modelled as the autoregressive (AR) processes. The first AR coefficients are used as a feature vector for novelty detection. To test the efficacy of the damage detection algorithm, it has been tested on the pseudo-experimental data obtained from the FEM model of the ASCE benchmark frame structure. Results suggest that the presented approach is able to detect mainly major and moderate damage patterns.

Keywords: dynamics, inverse problems, structural monitoring, damage detection, mixture model, novelty detection.

1. Introduction

Vibration-based damage detection is supported by the premise that structural damage causes changes in measured vibration signals. It uses dynamic data from a monitored structure to detect abnormal vibration patterns which may correspond to damage states in the structure [4]. There are two types of vibration-based methods. The most popular methods, which are called model-based, are supported by global dynamic analysis of vibration data and FE model updating for detecting changes in dynamic parameters of the monitored structure [5]. Other methods are non-model or model-free and are based on time-series analysis for novelty detection in vibration data. For example, the acceleration signals from sensors are modelled with time series models and the coefficients are used as damage-sensitive features [6].

Various algorithms and techniques developed in soft computing and machine learning communities are being used in the field of structural health monitoring (SHM), see for example [10, 22]. In particular, feed-forward layered neural networks (FLNNs) have proved to be very useful for the purpose of vibration-based structural damage detection and localization [15, 16, 21, 23].

This work examines the use of a Gaussian mixture model (GMM) for solving damage detection problem on the basis of pseudo-experimental acceleration time series obtained with the finite element model of the IAPR-ASCE SHM Task Group benchmark structure [8]. Most previous approaches used different machine learning or soft computing methods [7, 17, 18].

2. Gaussian mixture model for novelty detection

Novelty detection (also known as anomaly detection, outlier detection or one-class classification) is a process of identification of novel or abnormal patterns using, for example, statistical models like mixture models, built with a large number of normal data [19]. There are various approaches to describe normal data. The classical approach uses density-based modeling, where the form of the density distribution is assumed in advance and the parameters of the distribution are estimated applying maximum likelihood method and normal data. For multi-modal forms of data distribution mixture models can be used such as Gaussian mixture model (GMM) and the expectation maximization (EM) algorithm for GMM parameters estimation. GMM is also applied for clustering and classification. In this work, Gaussian mixture model (GMM) has been applied because the feature vectors form two distinct clusters.

GMM had previously been applied for structural damage detection. For example, Martin used GMM-based method for anomaly detection in Space Shuttle Main Engine (SSME) [11]. Nair showed in [13] that GMM together with the Mahalanobis distance can be useful in solving damage detection problems.

A Gaussian mixture model is defined as a superposition of K Gaussian densities and has the following form [1]:

$$p(\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k). \tag{1}$$

Each Gaussian component of the mixture $\mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$ has its own mean $\boldsymbol{\mu}_k$ and covariance $\boldsymbol{\Sigma}_k$. The parameters π_k are called mixing coefficients satisfying $\sum\limits_{j=1}^{K} \pi_k = 1$ and $0 \leq \pi_k \leq 1$. Therefore, these parameters satisfy the requirements to be probabilities.

One way to set the values of the Gaussian mixture distribution is to use maximum likelihood approach, maximizing the log of the likelihood function. It can be done with iterative optimization techniques like conjugate gradient method or alternatively by using a powerful framework called expectation maximization (EM) [3]. The log likelihood function is given by

$$\ln p(\mathbf{X}|\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \sum_{n=1}^{N} \ln \sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k), \tag{2}$$

where $\mathbf{X}$ represents learning dataset. It is an $N \times D$ matrix in which the n^{th} row is given by $\mathbf{x}_n^{\mathrm{T}}$.

Expectation-maximization algorithm for GMM is given by the following steps [1]:

1. Initialize parameters of GMM (means $\boldsymbol{\mu}_k$, covariances $\boldsymbol{\Sigma}_k$ and mixing coefficients π_k),

2. E step. Evaluate the responsibilities $\gamma(z_{nk})$ using the most recent values of GMM parameters

$$\gamma(z_{nk}) = \frac{\pi_k \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)}{\sum_{j=1}^{K} \pi_j \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)}, \tag{3}$$

   where z_{nk} is an element of K-dimensional binary random variable $\mathbf{z}$ for n^{th} learning pattern. This random variable has a 1-of-K representation, in which a particular element z_k is equal to 1 and all other elements are equal to 0.

3. M step. Re-estimate the GMM parameters with the up-to-date values of responsibilities $\gamma(z_{nk})$

$$\boldsymbol{\mu}_k^{\text{new}} = \frac{1}{N_k} \sum_{n=1}^{N} \gamma(z_{nk}) \mathbf{x}_n \tag{4}$$

$$\boldsymbol{\Sigma}_k^{\text{new}} = \frac{1}{N_k} \sum_{n=1}^{N} \gamma(z_{nk}) (\mathbf{x}_n - \boldsymbol{\mu}_k^{\text{new}})(\mathbf{x}_n - \boldsymbol{\mu}_k^{\text{new}})^{\mathrm{T}} \tag{5}$$

$$\pi_k^{\text{new}} = \frac{N_k}{N}, \tag{6}$$

   where $N_k = \sum\limits_{n=1}^{N} \gamma(z_{nk})$ and N is the number of learning patterns.

4. Evaluate the log likelihood (2) and monitor convergence of either GMM parameters or the log likelihood. If the stopping criterion is not fulfilled return to step 2.

For better convergence of the EM algorithm, the optimization process is often initialized with the parameter estimates of GMM obtained from the K-means algorithm. Useful implementation of EM algorithm for GMM can be found in Netlab toolbox for MATLAB [12].

3. Application of gaussian mixture model to damage detection in frame

To illustrate the concept of novelty detection as the basis of a practical system for damage detection, we consider a specific application of novelty detection in structural health monitoring (SHM) concerning the determination of presence of damage in the 3D frame benchmark structure [8]. The presented algorithm is validated by using data generated from the FEM model of the ASCE benchmark steel frame. This benchmark is a standardized simulation tool for development and comparison of algorithms for SHM and the analysis is limited to the linear stationary signals only.

3.1. Description of benchmark structure

A modular 4-storey, 2-bay by 2-bay, steel-frame scale-model structure has been designed and built by the Earthquake Engineering Research Laboratory at the University of British Columbia (UBC), see the diagram of the structure on the left side of Fig. 1. It is approximately 3.6 m tall with a total width of 2.5 m. Each floor is 0.9 m high and each bay is 1.25 m wide. The support conditions of the benchmark structure are regarded as fully rigid. The members are hot rolled grade 300W steel with a nominal yield stress 300 MPa. The columns are all oriented to be stronger bending toward the x-direction (i.e., about the y-axis). The floor beams are oriented to be stronger bending vertically, i.e., about the y-axis (x-axis) for those oriented with longitudinal axis parallel to the x-axis (y-axis). The braces have no bending stiffness, so their orientation is irrelevant. There is one floor slab per bay per floor: four 800 kg slabs at the first level, four 600 kg slabs at each of the second and third levels, and, on the fourth floor, either four 400 kg slabs or three 400 kg and one 550 kg to create some asymmetry.

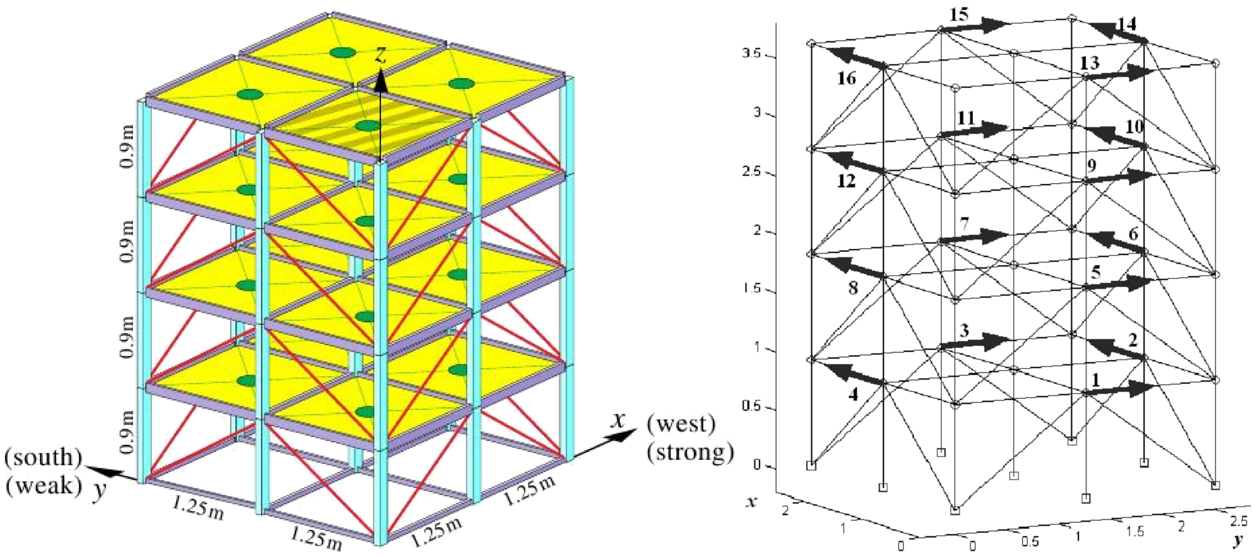

Fig. 1. Diagram of the ASCE benchmark structure [8] (left); acceleration sensors location and direction of measured signals [8] (right).

As a main part of the benchmark problem, six damage cases were defined in addition to the undamage case (D0). Structural damage of the benchmark structure is simulated mainly by reducing the stiffnesses in the braces to zero. The damage cases are defined as follows:

- D1: removal of all braces on the first floor,
- D2: removal of all braces on the first and third floors,
- D3: removal of one brace on the first floor,
- D4: removal of one brace on the first and third floors,
- D5: case 4 + unscrew the left end of element 18,
- D6: area of one brace on the first floor reduced to 2/3.

The first and the second damage cases are treated as severe damage scenarios and the rest of damage cases are examples of minor damage. For more information about the benchmark frame and related projects using this frame, see references [2, 20].

3.2. Data generation

A finite element model with 120 DOFs based on this structure was developed to generate the simulated response data. This model only requires that floor nodes have the same horizontal translation and in-plane rotation. The columns and floor beams were modeled as Euler-Bernoulli beams. The braces are bars with no bending stiffness [9]. A diagram of the analytical model is shown on the right side of the Fig. 1. Note that x-direction (i.e., bending about the y-axis) is the strong direction due to the orientation of the columns. Moreover, to be consistent with the axes used in later experimental tests, the compass directions associated with the axes are South for the positive y (weak) direction, and West for the positive x (strong) direction. The data generation scripts written in MATLAB were available on the web at http://mase.wustl.edu/asce.shm.

The simulated structure's responses are measured using 16 virtual accelerometers. Locations of sensors and directions of measured accelerogram signals are shown on the right side of the Fig. 1. The frame can be excited in different ways but this paper uses only acceleration responses which were obtained by simulating an electrodynamic shaker placed at the center of one of the four bays on the top level of the structure. In the top plot of Fig. 2, an example of time series of simulated exciting force in y-direction is presented together with the corresponding accelerograms time series in undamage case in y-direction measured by two sensors placed on the first and the fourth floor of the structure (the bottom plot of Fig. 2).

3.3. Preprocessing and feature extraction

In this paper, the autoregressive (AR) model of order p is used. The acceleration signal $x_{\mathrm{acc},i}(t)$ from sensor i^{th} is modelled by

$$x_{\mathrm{acc},i}(t) = \sum_{k=1}^{p} \alpha_{ik} x_{\mathrm{acc},i}(t-k) + \epsilon(t), \tag{7}$$

where α_{ik} is k^{th} AR coefficient and $\epsilon(t)$ is the residual term.

The AR coefficients contain information about the dynamic characteristics of the structure (modal natural frequencies and damping ratios). Thus, changes to a structure stiffness matrix as result of permanent damage will change the AR coefficients. It turns out that it is sufficient to use only the first AR coefficient to pick up changes in structural stiffness resulting from damage [14].

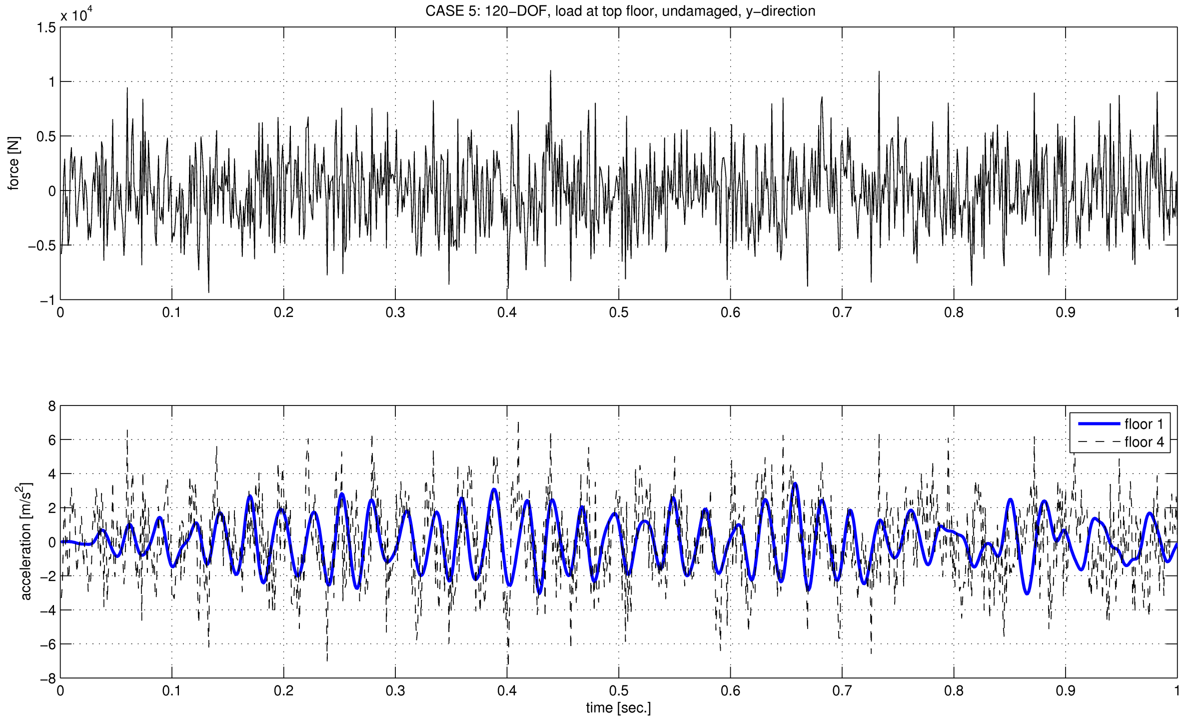

Fig. 2. An example of simulated time series for benchmark structure. The top plot shows the exciting force time series generated by an electrodynamic shaker placed on the top level of the structure. The bottom plot shows the accelerograms for two sensors placed on the first and the fourth floor in undamage case.

Using time series modelling of the structure acceleration responses and the autoregressive (AR) coefficients α_{i1} as a features vector $\mathbf{x} = \{\alpha_{11}, \alpha_{21}, ..., \alpha_{I1}\}$, it is possible to build a one-class classifier which is able to detect the damage in the structure. The coefficients computed for the undamaged structure form a statistical model of normality. After training, this model is subsequently applied to damage detection. In this work, only the first coefficient of the AR model for each sensor is used.

3.4. Data visualization

To visualize the feature vectors for all seven damage cases, we subtract the data mean and project the data onto the principal component subspaces of dimensionality $M = 2$ and $M = 3$ obtained from PCA, respectively. The left-hand plot of Fig. 3 shows the eigenvalues arranged in decreasing

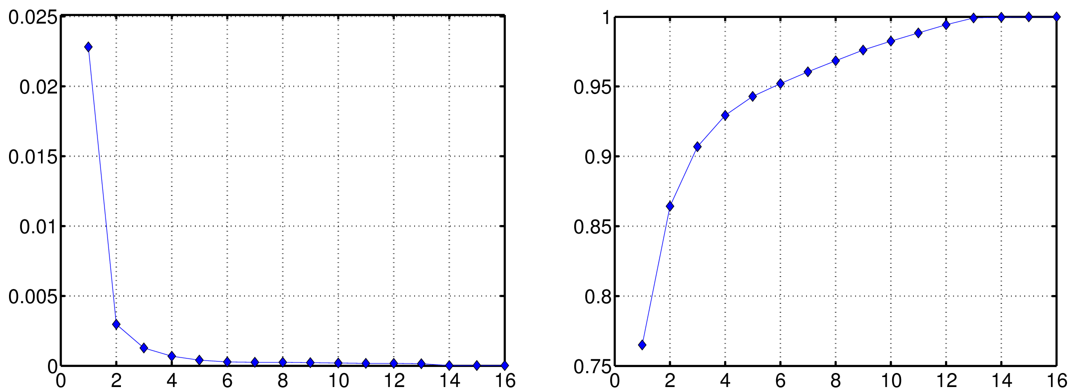

Fig. 3. Plots of the eigenvalues from PCA decomposition of dataset. The left plot shows the complete spectrum of eigenvalues sorted into decreasing order and the right plot shows the accumulative sum of the eigenvalues, expressed as a fraction of the whole sum.

order. The cumulative sum of the eigenvalues, presented as a fraction of the entire sum is shown on the right-hand plot of Fig. 3. They show that there is one dominating principal component with the corresponding largest eigenvalue λ_1 and that the last four eigenvalues are nearly zero.

Figure4 shows projected feature vectors for all damage cases using the first two and first three principal components. It can be seen that the principal components projection separate the most severe damaged case D2 well (removal of all braces on the first and third floors) and partially well the damage case D1 (removal of all braces on the first floor). The applied projection does not separate the remaining damage cases well (D3-D6) and undamage case D0 as well.

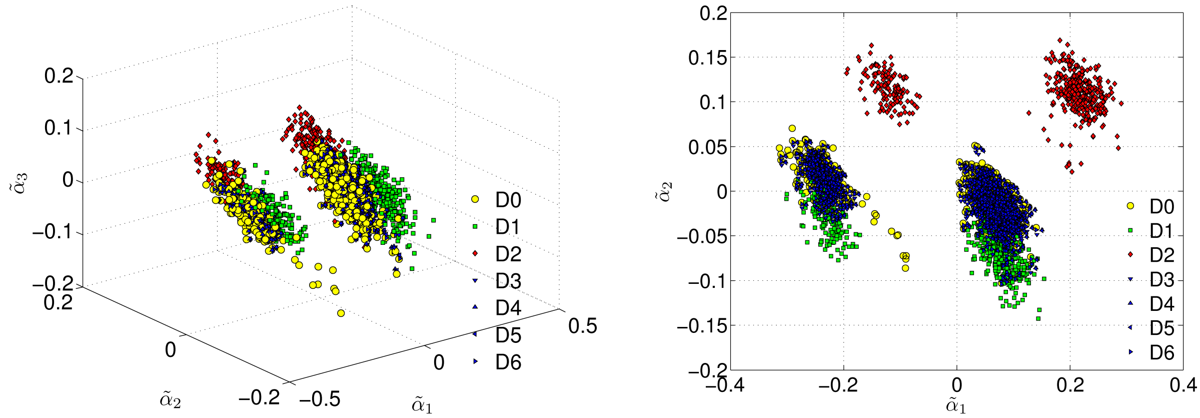

Fig. 4. Visualization of the feature vectors obtained by projecting the data onto principal components. Seven damage cases are shown: undamaged (D0) and damaged (D1, D2, ..., D6). The plot on the left shows projected feature vectors in three-dimensional space and the plot on the right shows projected feature vectors in two-dimensional space.

4. Experiments and results

In numerical experiments, 580 patterns (16-dimensional feature vectors) were generated from undamaged structure and 2940 corresponding patterns for all six damage cases. At first, all patterns were transformed by standardization (zero mean and unit variance). Patterns from undamaged frame (normal data) were modeled using Gaussian mixture model with two components and trained with EM algorithm using Netlab toolbox for MATLAB [12]. For novelty detection, novelty threshold was estimated by applying 10-fold cross-validation and taking into account also patterns computed for all six damage scenarios (two severe and four minor cases). Finally, the presented algorithm was checked by using testing patterns for all seven cases. The results for structural damage detection with 16-dimensional feature vectors are presented in the first row of Table 1. In case of severe damage scenarios, the presented algorithm gives almost perfect result, detecting correctly 98% of the patterns. But this result was obtained at the expense of rather large number of misclassifications of minor damage and undamage cases.

For comparison, feature vectors from 2D and 3D spaces, computed by PCA, were also considered and the corresponding two GMMs were trained and applied for novelty detection. The results are shown in the last two rows of Table 1. It is interesting to note that the coefficient of success in case

Table 1. Results as coefficients of success in % in detection of severe and minor damage scenarios. Results of testing the proposed algorithm for damage detection for various number of input variables (16, 3 and 2 (after applying PCA)). In the last column the coefficients of success are presented for detecting undamage case.

Number of inputs	severe damage[%]	minor damage[%]	undamaged[%]
16 (all)	98	67	60
3 (PCA)	98	60	53
2 (PCA)	95	54	43

of severe damage cases using three-dimensional feature vector is the same as with full 16D feature vector.

5. Final remarks

In this work, the Gaussian mixture model (GMM) has been used for novelty detection in acceleration time series data represented by the first coefficients of auto-regressive (AR) model as the feature vector. GMM optimal parameters were estimated by employing expectation-maximization (EM) framework and the feature vectors obtained from the undamaged structure. Application of the proposed algorithm for damage detection in the ASCE benchmark structure in case of simulated data, demonstrates that it is simple to implement and very effective in recognizing moderate and major damage scenarios. These results are very similar to those presented in two papers by Nair [13, 14]. However, the proposed algorithm is currently ineffective in detecting minor damage cases. This probably is caused by the inefficiency of current features in representing acceleration time series for minor damage cases and this issue is now under study.

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