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An example of application of soft computing in experimental modal analysis

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The paper deals with application of AI tools in experimental modal analysis. The example of Stabilization Diagram processing, that is an intermediate stage of modal parameter estimation procedure, was selected. In order to automate decision-making carried out during Stabilization Diagram processing a set of tools employing: fuzzy reasoning and artificial neural nets was applied. The application of these tools enabled to ease and shorten execution time of Stabilization Diagram processing. Additionally, the result of processing has become operator-independent.

1. Introduction

Soft Computing as a computational method of Artificial Intelligence is the most often being applied in numerical analysis of structures and systems. In mechanical engineering the most intense use of various numerical analysis techniques takes place in the so-called virtual prototyping [16] process aiming at minimizing of time and cost effort involved in development of new products. Though in practice virtual prototyping is gaining more and more importance, there is at least a couple of reasons for application of testing in the product development. The most important one is verification of numerical analyses' results. Other reasons for use of testing in case of structural dynamics investigation is discussed in [16].

Increasing complexity of engineering problems that are being solved during product development causes higher and higher demands concerning testing. This in turn makes testing engineers to perform more and more complex tests and use more and more sophisticated model identification techniques. Usually, such the enhancement of experiments results in considerable increase of amount of data to be analyzed. Additionally, the demanded testing and parameter estimation time should be as short as short as possible and the most objective model identification results should be delivered. The relevant way to accomplish the stated demands is to automate the procedure of testing (including analysis of results). The following sections present formulation and report an example of automation of a crucial part of system identification procedure in case of experimental modal analysis.

2. Experimental modal analysis – background and problems of automation

Structural dynamics properties of mechanical systems determine behaviour of these systems when they are subjected to dynamic loads (forces and torques). When elastic range of deformation amplitude and discrete parameter spatial distribution of solids are concerned usually Finite Element method [11] is used for modelling vibration of a system under consideration. Assumptions of linearity (superposition principle), reciprocity and time invariance of modelled systems are typically made during analysis [4]. In such case the theory of linear vibration determines the following form of the structural model of a dynamic system [11],

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$ — mass matrix, $\mathbf{C}$ — damping coefficient matrix, $\mathbf{K}$ — stiffness coefficient matrix, $\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}$ — vibration displacement, velocity and acceleration vectors, respectively, $\mathbf{f}$ — excitation force vector, t — time variable.

The structural dynamics properties of a system described by Eq. (1) might be extracted by formulating and solving of eigenproblem [11], what results in determination of the modal model which is a set of n natural frequencies f_r (or circular frequencies ω_r), modal damping coefficients ξ_r, and mode shapes ψ_r $(r = 1, 2, \ldots, n)$. Modal displacements q_r compose a set of independent generalized coordinates that do not possess straight-forward physical interpretation. A set of equations (1) after introduction of modal coordinates

$$\mathbf{x}(t) = \mathbf{\Psi}^{\mathrm{T}}\mathbf{q}(t) \tag{2}$$

where: $\mathbf{q}$ is a modal displacement vector, $\mathbf{\Psi}$ — matrix of mode shapes, and applying appropriate orthogonality conditions [11], takes the following form,

$$\mathbf{M}_q\ddot{\mathbf{q}}(t) + \mathbf{C}_q\dot{\mathbf{q}}(t) + \mathbf{K}_q\mathbf{q}(t) = \mathbf{\Psi}^{\mathrm{T}}\mathbf{f}(t), \tag{3}$$

where $\mathbf{M}_q$ — modal mass matrix, $\mathbf{C}_q$ — modal damping coefficient matrix, $\mathbf{K}_q$ — modal stiffness coefficient matrix.

Modal matrices are diagonal, thus the set (3) is a set of independent equations.

While the structural type of dynamic model, in form of Eq. (1) or Eq. (3) is usually applied in numerical simulation of response to assumed excitation the description of modal parameters is investigated for assumption of stationarity of input and output with the use of input/output type of model. Application of Laplace or Fourier transform to Eq. (1) results in evaluation of the transfer or frequency response functions symmetrical matrix $\mathbf{H}(s)$ or $\mathbf{H}(\omega)$ for N points (stations) of a system under consideration,

$$\mathbf{H}(\omega) = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) & \cdots & H_{1N}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) & \cdots & H_{2N}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ H_{N1}(\omega) & H_{N2}(\omega) & \cdots & H_{NN}(\omega) \end{bmatrix}. \tag{4}$$

Each element H_{ik} of matrix $\mathbf{H}(\omega)$ is a complex value dynamic compliance for circular frequency value of ω, input (excitation) and output (response) stations (locations) i and k. The dynamic compliance might be expressed by the modal parameters in the following way [4],

$$H_{ik}(\omega) = \sum_{r=1}^{n} \left(\frac{Q_r\psi_{ri}\psi_{rk}}{j\omega - \lambda_r} + \frac{Q_r^*\psi_{ri}^*\psi_{rk}^*}{j\omega - \lambda_r^*} \right) \tag{5}$$

where Q_r — r-th scaling coefficient, λ_r — r-th eigenvalue, ψ_{ri} — element of r-th eigenvector (mode shape) at station i, $j = \sqrt{-1}$, $(.)^*$ — complex conjugate.

The above formula became a base for experimental modal analysis, since the dynamic compliance $H_{ik}(\omega)$ may be estimated with use of measurement results of excitation force and response signals at stations i and k [4]. Usually, one column or row of $\mathbf{H}(\omega)$ matrix is estimated basing on experimental data. Experimental modal analysis, as a system identification technique, is composed of general four steps: formulation of experiment plan, experiment, parameter estimation and model validation. The following steps of modal model identification differ from each other considerably. In the result the experimental model analysis procedure is composed of a set of subprocedures corresponding to the mentioned system identification four steps.

Effective application of experimental modal analysis requires the concurrent use of a variety of fast, reliable and objective tools. This may be achieved only by automation of the model identification procedure. The automation of experimental modal analysis should comprise automation of

- calculations like: assessment of location of excitation point, processing of measurement records, parameter estimation procedure,
- data and results management (including results' reporting),
- visualization of data and results,
- analyses results quality assessment.

The last of the listed above steps is very complex and difficult to be achieved nowadays as it usually requires physical interpretation of results which typically is based on a human operator experience. For all the mentioned steps there exist a problem of automation of decision-making. Decision-making is commonly understood as 'selection of alternatives' [5]. Decision-making bases on some objective function that relates the alternatives and their selection outcome. The most straightforward approach to decision-making is to formulate some deterministic objective function and use optimization to make decisions. When formulation of an appropriate objective function is difficult the probability of the decision outcome may be determined statistically and the decision-making is carried out by means of the statistical hypotheses testing [12]. Nevertheless, when the uncertainty of the decision-making is high instead of probability the membership functions are evaluated and fuzzy reasoning is being applied [5]. Apart from possibility to apply the fuzzy sets theory to decision-making in case of considerable uncertainty there is also possibility to use Artificial Neural Networks [15] or combined neuro-fuzzy techniques, which enable introduction of learning or adaptation of algorithms to specific data properties into decision-making.

Application of fuzzy-reasoning, artificial neural networks or classical comparison of some indicator values with arbitrary selected threshold values might be classified as techniques of heuristic choice which are used to introduce artificial intelligence into some analysis procedure.

Experimental model analysis is a multi-step procedure of system identification. The main problem of automation of experimental modal analysis is decision-making which is typically carried out subjectively by human operator. Within each step of the procedure decision-making mainly deals with selection of the best results of an analyze performed in the step. Such a decision-making is usually suitable for automation. The decision-making carried out between the subsequent steps is more difficult as it is usually based on assessment of results of various analyses. In such case the decision made often depends on data specific properties and it should correspond to the model identification purpose.

Since the decision-making step in experimental modal analysis is still far from comprehensive automation the current practice is to use the so-called autonomous parameter estimation procedures. Selected examples of the following procedures of modal parameter estimation comprise:

- iterative application of ERA/DC algorithm [13] supported by Genetic Algorithm aided estimation parameters selection followed by model consolidation [1, 2],
- iterative application of SMAC algorithm based on modal filtering idea [9] followed by model consolidation [10],
- statistical frequency domain Maximum Likelihood algorithm consisting in application of LSCF (Least Squares Complex Frequency) parameter estimation algorithm followed by the Maximum Likelihood parameter estimation algorithm with automated pole selection procedure [17],
- three-step procedure imitating an experienced analyst action in range of: parameter estimation planning, automated pole selection (stabilization diagram processing) and model consolidation [6, 7] with use of statistical indicators or fuzzy reasoning.

The authors understand the autonomous experimental modal analysis as a set of automatic subprocedures that include internal decision-making routines. These decisions determine the way of further execution of the autonomous procedure. The procedure is supervised by an operator who interprets partial results of the procedure and decides about: interruption of the procedure, repeating of some subprocedures, selection of algorithms to be used in further analysis.

The following description deals with the mentioned above three-step autonomous parameter estimation procedure. From the point of view of objectivity of analysis, in practice, the crucial step of the parameter estimation procedure is the stabilization diagram processing.

3. Example algorithms of processing of stabilization diagrams

The stabilization diagram is a tool that is commonly used in practice of experimental modal analysis. It is intended for system physical (true) poles selection. It presents the frequency location of estimated system poles for models of increasing model order (left part of Fig. 1) or location of poles for models of various order in frequency-damping coefficient (f–ξ) plane (right part of Fig. 1).

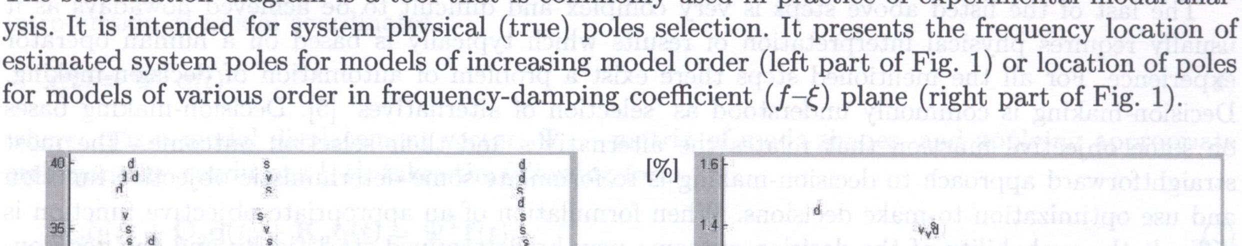


Fig. 1. Example of the stabilization diagram: two ways of presentation

The use of stabilization diagrams is a result of lack of effective algorithms of the tested system model order estimation in case of real, complex objects. The differences between values of natural frequency and damping coefficients of the poles belonging to models of the subsequent model order are calculated and, if small enough, the poles belonging to the model of higher order are concerned to be stabilised. The calculation proceeds from the lowest to the highest model order. Usually, mode shapes are also used for poles' stabilisation assessment. When a pole was found to stabilise for a couple of model orders the pole is considered to be a physical pole, otherwise it is rejected as a spurious (a computational) pole. Usage of the stabilization diagram is usually a subject of complaints of testing engineers due to necessity of decision-making. Additionally, multiple possibility of selection of poles from the diagram limits considerably parameter estimation objectivity. That is why much detail research was done on the stabilization diagram processing. The formulated algorithms of stabilization diagram processing are either heuristic or they base on the state-space model of a dynamic system. The automated heuristic stabilization diagram processing algorithms consist usually of two steps:

- decomposition of the diagram into clusters of poles corresponding to a single structural mode
- choice of the representative pole for each extracted cluster of poles.

The decomposition of the diagram, as a task of classification, is usually carried out with use of: some statistic procedure [14], fuzzy clustering [18] or fuzzy reasoning [6]. Also classification algorithms that use Support Vector Machines [3] were used for the stabilization diagram processing. The model-based methods instead of the physical pole direct selection usually consist in spurious poles rejection with use of: pole-zero cancellation [14], truncation of balanced representation [14] or backward normalisation [1].

The simplicity and effectiveness of use of the stabilization diagram caused its wide application in practice. In principle, the poles selected from the stabilization diagram by a human operator or by a heuristic autonomous algorithm are not proved to be the optimally chosen. Introduction of more sophisticated model-based autonomous algorithms like the one reported in [17], that use optimization, will allow in the future to get rid of such an inconsistent tool like the stabilization diagram and decrease the uncertainty of physical poles selection related to its use. Nevertheless, at the moment stabilization diagrams are widely used in practice of modal parameter estimation and generally the heuristic algorithms of the stabilization diagram processing perform better than the currently available model based ones. This proves relevancy of AI approach in case of considerable uncertainty.

In this section there is presented one algorithm of decomposition of the diagram into lines of poles followed by four algorithms of selection of the representative poles from a line of poles. The difference between a line of poles and a cluster of poles consists in information about the model order to which a pole belongs which is disregarded in case of a clusters of poles. The formulation of the mentioned algorithms was preceded by analysis of properties of stabilization diagrams.

The first investigation dealt with determination if there exists some convergence of location in f–ξ coordinates of poles belonging to a single line of poles to the last pole (of this line) that corresponds to the highest considered model order. In Fig. 2a an example of line of poles estimated for simulated data is presented. For the presented line of poles the considered convergence does exist. In such a case the only reasonable way of selection of the representative poles is just to collect the poles corresponding to the model of the highest order for all the discriminated lines of poles. However when real data is concerned this convergence usually does not exist like for a line shown in Fig. 2b.

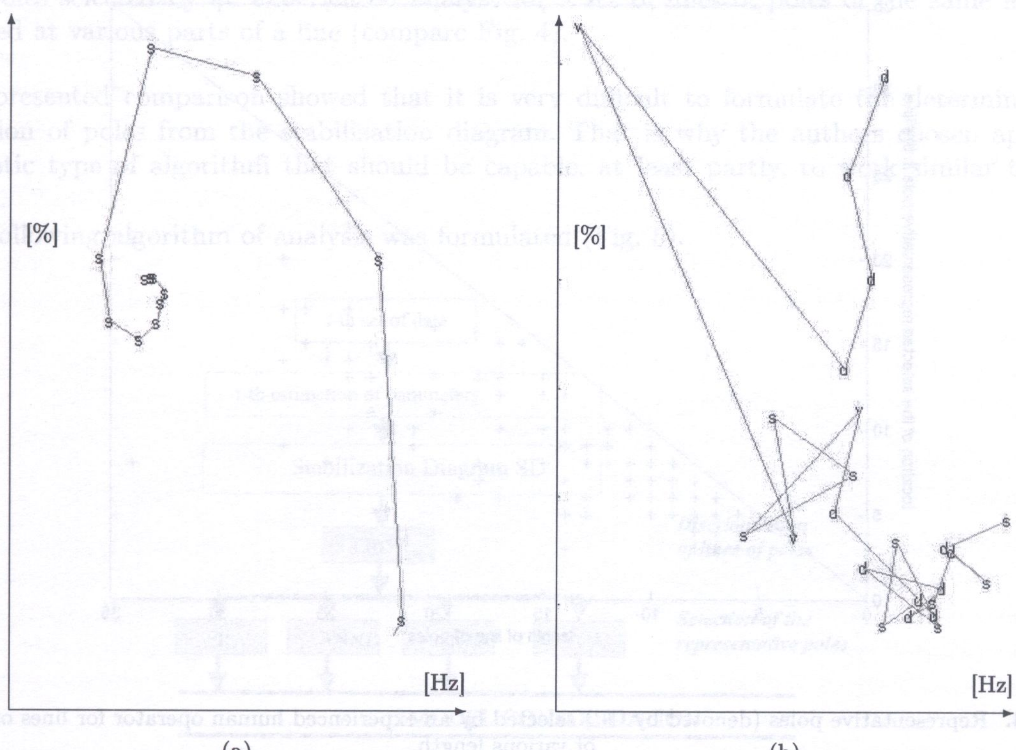


Fig. 2. Example of a line of poles in f–ξ plane: (a) — for simulated data, (b) — for real data

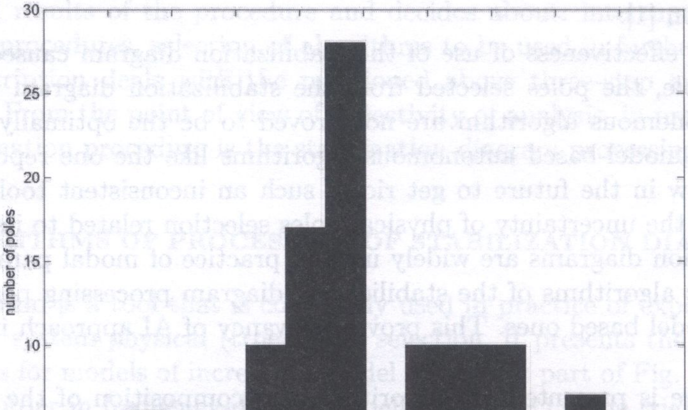


Fig. 3. Position in line of the closest pole to the poles cluster centre

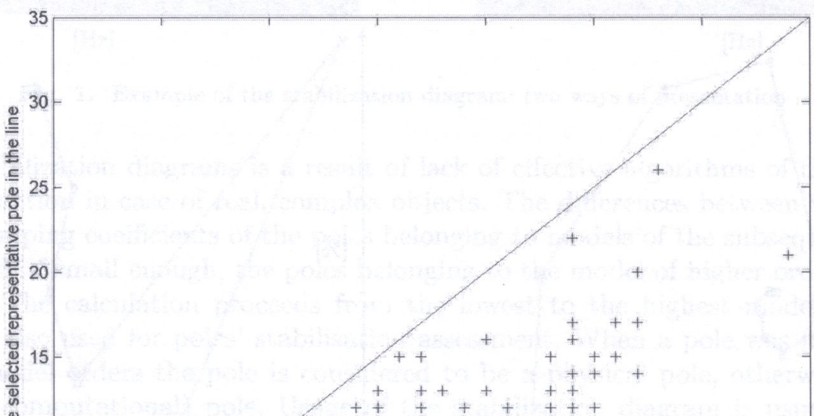


Fig. 4. Representative poles (denoted by '+') selected by an experienced human operator for lines of poles of various length

In Fig. 3 a histogram of the position of a pole in a line of poles that is the closest to the centre of the cluster of poles in f–ξ coordinates is presented for 125 lines of poles corresponding to true experimental data. The histogram proves that generally there is no convergence of the lines of poles to the limit location in f–ξ coordinates.

When the fact that usually less accurate estimates of modal parameters are obtained for the lowest values of model order is taken into consideration the histogram indicates that the representative pole for a line should be looked for somewhere close to the middle of a line and further toward the end, but rather not at the same end of a line of poles.

In Fig. 4 locations of a representative pole selected by an experienced analyst are showed for line of poles of various length.

The initial investigation carried out by the authors led to determination of the following properties for 125 lines of poles extracted by an autonomous heuristic algorithm:

- majority of the extracted lines of poles are rather short — composed of 5–8 poles (compare Fig. 4)
- if the assumed maximum model order does not exceed 40 the longest lines of poles are composed of approximately 27–33 poles
- for approximately 70% of the considered lines of poles the natural frequency variation does not exceed approximately 0.8 Hz and damping coefficient variation does not exceed approximately 0.8%
- the centre of a cluster of poles corresponding to a line of poles might be located close to any pole of the line (except the poles lying in the first 10 % of the length of a line — compare Fig. 3), but the most probable location of the centre is between 40% and 50% of a line length (this actually corresponds well to part B of Fig. 2)
- the poles selected by an experienced analyst for a set of lines of poles of the same length are located at various parts of a line (compare Fig. 4).

The presented comparison showed that it is very difficult to formulate the deterministic rule of selection of poles from the stabilization diagram. That is why the authors chosen application of heuristic type of algorithm that should be capable, at least partly, to work similar to human operator.

The following algorithm of analysis was formulated (Fig. 5).

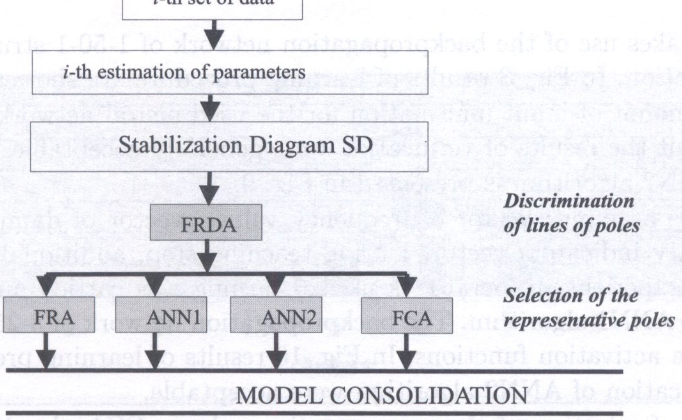


Fig. 5. Formulated algorithm of processing of the Stabilization Diagram corresponding to i-th parameter estimation

The first stage of the analysis is carried out by FRDA algorithm. To complete the second stage four algorithms were formulated. All the described in this paper algorithms were implemented in Matlab with use of Fuzzy Logic toolbox and Neural Network toolbox.

The first formulated FRDA algorithm, that carries out discrimination of lines of poles, was originally described in [6]. It uses Mamdani type fuzzy reasoning [5] for extraction of lines of poles. The algorithm starts from determination of the most similar poles out of model of the higher order to each pole of the model of the current order. For similarity assessment, like during the stabilization diagram formulation, differences of the natural frequency values and modal damping values as well as MAC (a form of the scalar product between eigenvectors) [4] values are used. Replacement of comparison with threshold values used during the stabilization diagram formulation with membership functions and reasoning rules enables to balance importance of the used poles similarity indicators. The used membership function are of triangular or trapezoidal type. As a result of defuzzyfication the similar poles are indicated. Processing of all poles of one diagram leads to determination of lines of the similar poles. A set of poles belonging to a single line composes a cluster in the natural frequency-modal damping coordinates (f–ξ plane) but FRDA algorithm allows retaining information about the sequence of the poles in a line of similar poles of each cluster.

The second step of the stabilization diagram processing (representative poles selection) is carried out with the use of the following four algorithms.

The first of them — FRA algorithm uses fuzzy reasoning. Three indicators are used: distance of each pole from the centre of the cluster in f–ξ plane, location of each pole in the line of poles corresponding to the cluster and, finally, the quality of stabilization of each pole (property presented in the stabilization diagram by 's', 'v', 'd', 'f' and 'o' symbols). The formulated membership functions (of Gaussian, sigmoid or triangular type) and reasoning rules used in the reported algorithm are presented in Fig. 6.

The next two algorithms base on application of artificial neural nets (ANN). These algorithms were preliminary reported in [8]. Figure 4 is a good justification of application of ANN as it shows that it is extremely difficult to formulate a function that selects the representative pole like an experienced human operator for a line o poles of some length. Generally, ANN are used to such tasks as predictions, classifications, recognition and data grouping [15]. The reported algorithms employ a backpropagation neural network. Architecture of such a type of the network allows for the learning process with teacher, and that is why during teaching the information got from experienced engineers, who can properly select poles from the stabilization diagram, may be used.

ANN1 algorithm uses as input only information how long is a line of poles. The learning procedure was carried out with use of 125 lines of poles extracted from stabilization diagrams formulated for real object testing results. The larger part of data (116 lines) was used in the learning process, and the remaining 9 lines were used in the algorithm verification process. The diagram of the algorithm is showed in Fig. 7.

ANN1 algorithm makes use of the backpropagation network of 1-50-1 structure and Tansig and Purelin activation function. In Fig. 8 results of learning procedure are showed.

Because of small amount of input information for the used neural network, the learning process proved to be slower, but the results of verification were generally acceptable.

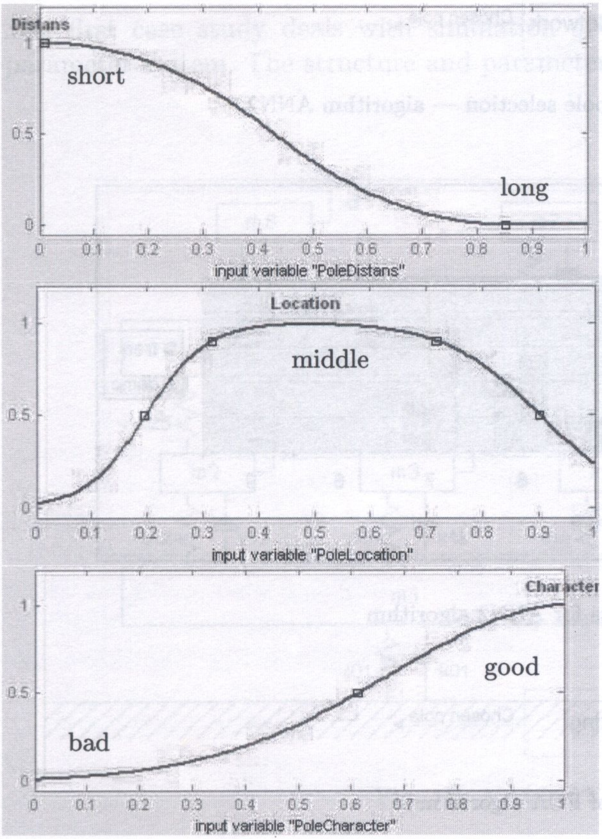
The diagram of ANN2 algorithm is presented in Fig. 9.

The algorithms uses as input: vector of frequency values, vector of damping coefficient values and stabilization quality indicators vector. During teaching step, additionally, information about a pole selected by an experienced operator is used. Learning was carried out with the use of the same data as in case of ANN1 algorithm. The backpropagation network of 3-25-1 structure was used with tansig and purelin activation functions. In Fig. 10 results of learning procedure are showed.

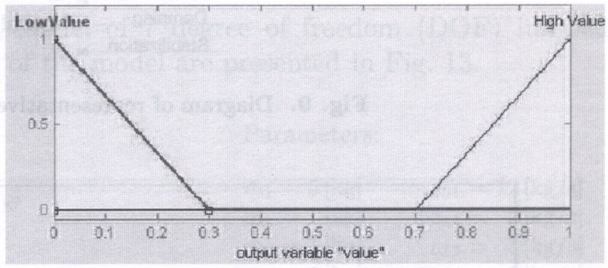
The results of verification of ANN2 algorithm were acceptable.

The last algorithm of selection of the representative pole — FCA algorithm, unlike the three described above algorithms, uses as input a cluster of poles. It is a typical fuzzy clustering algorithm. The diagram of the formulated algorithm is showed in Fig. 11.

Input MF



Output MF



Rules

distance from a cluster centre	location in line of poles	quality of stabilization	rank
short	middle	good	high
long			low
	ends		low
		bad	low

Fig. 6. Example of a set of membership functions and rules used in FRA algorithm

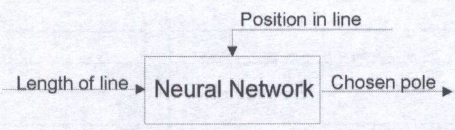


Fig. 7. Diagram of representative pole selection — algorithm ANN1

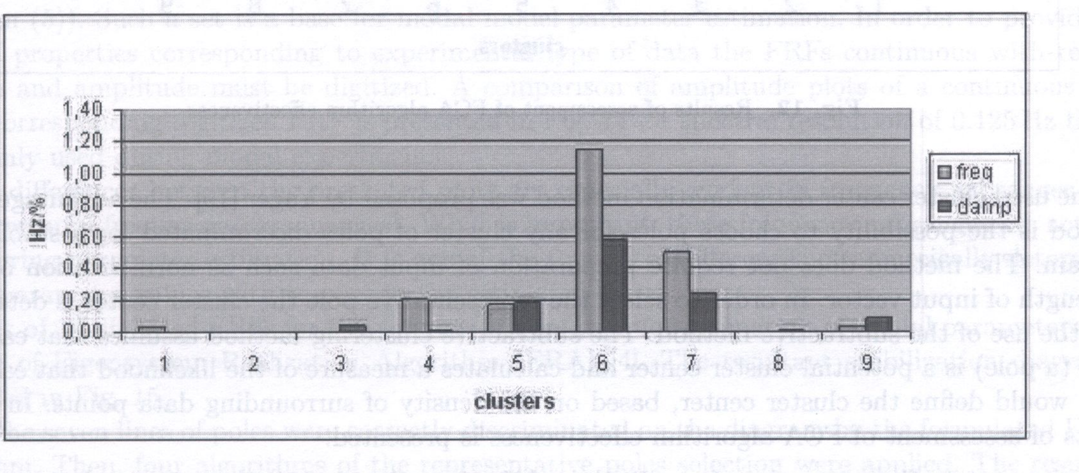


Fig. 8. Learning results for ANN1 algorithm

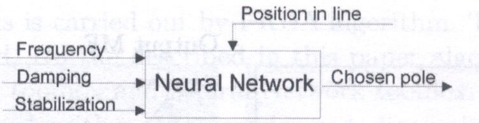


Fig. 9. Diagram of representative pole selection — algorithm ANN2

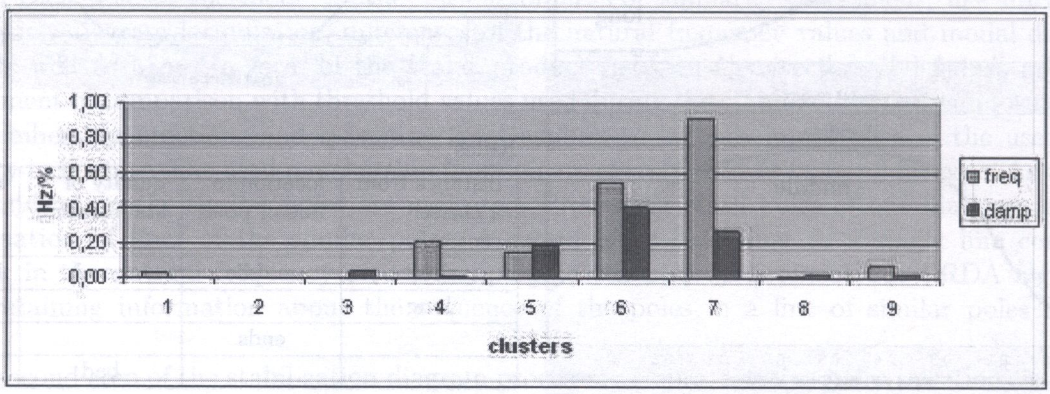


Fig. 10. Learning results for ANN2 algorithm

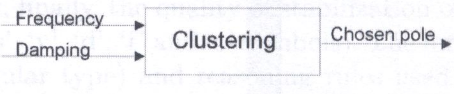


Fig. 11. Diagram of FCA algorithm

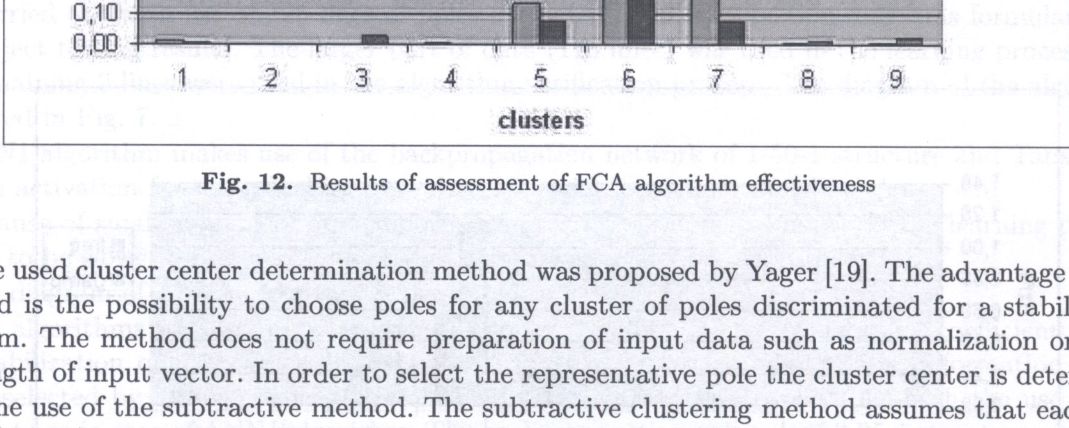


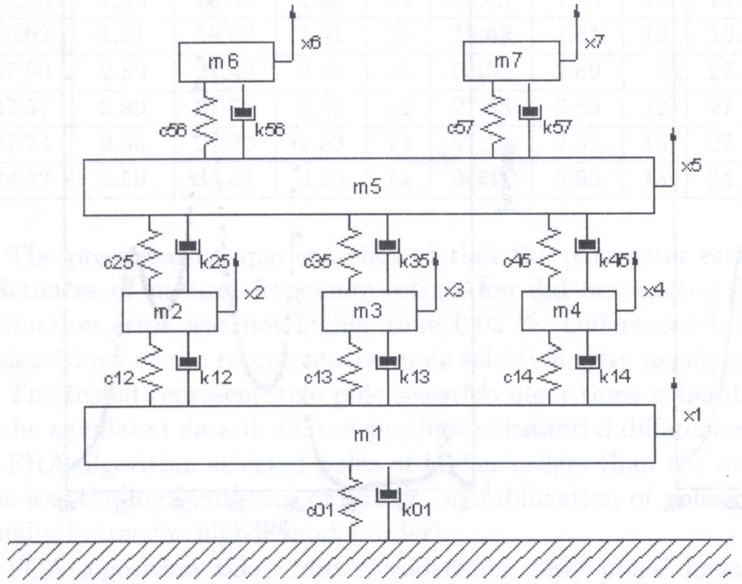
Fig. 12. Results of assessment of FCA algorithm effectiveness

The used cluster center determination method was proposed by Yager [19]. The advantage of that method is the possibility to choose poles for any cluster of poles discriminated for a stabilization diagram. The method does not require preparation of input data such as normalization or fixing the length of input vector. In order to select the representative pole the cluster center is determined with the use of the subtractive method. The subtractive clustering method assumes that each data point (a pole) is a potential cluster center and calculates a measure of the likelihood that each data point would define the cluster center, based on the density of surrounding data points. In Fig. 12 results of assessment of FCA algorithm effectiveness is presented.

In the next section examples of application of the formulated algorithms to simulation an measurement data are reported.

4. Case studies

The first case study deals with simulation of a model of 7 degree of freedom (DOF) lumped-parameter system. The structure and parameters of the model are presented in Fig. 13.



Parameters:

$m_1 = 5$ [kg] $c_{01} = 12$ [kg/s]
$m_2 = 1$ [kg] $c_{12} = 5$ [kg/s]
$m_3 = 1$ [kg] $c_{13} = 5$ [kg/s]
$m_4 = 1$ [kg] $c_{14} = 5$ [kg/s]
$m_5 = 4$ [kg] $c_{25} = 5$ [kg/s]
$m_6 = 2$ [kg] $c_{35} = 5$ [kg/s]
$m_7 = 2$ [kg] $c_{45} = 5$ [kg/s]
$c_{56} = 9$ [kg/s]
$c_{57} = 9$ [kg/s]

$k_{01} = 80000$ [N/m]
$k_{12} = 15000$ [N/m]
$k_{13} = 15000$ [N/m]
$k_{14} = 15000$ [N/m]
$k_{25} = 14800$ [N/m]
$k_{35} = 15000$ [N/m]
$k_{45} = 15000$ [N/m]
$k_{56} = 28000$ [N/m]
$k_{57} = 28000$ [N/m]

Fig. 13. Model of 7 degree of freedom system

Values of the structural parameters: masses m_i, damping coefficients c_{ij} and stiffness coefficients k_{ij} were selected in such a way that two groups of the so-called close modes (modes of close natural frequency values) are present in the modal model of the considered system. The study is intended to investigate how the formulated FRDA algorithm of line of poles discrimination copes with discrimination of close modes and to compare results of application of a set of the representative pole selection algorithms.

The modal model corresponding to the considered system might be determined by solution of the eigenvalue problem for a set of dynamic equation of motion (a model of structural type in the form (1)) formulated for the considered system. On the other hand, for the set of dynamic equations of motion an equivalent set of transfer functions might be determined (a set of FRFs in the form (5)). Such a set is a base for modal model parameter estimation. In order to provide the data of properties corresponding to experimental type of data the FRFs continuous with respect to time and amplitude must be digitized. A comparison of amplitude plots of a continuous FRF and a corresponding digitized FRF is presented in Fig. 14 for spectral resolution of 0.125 Hz that is commonly used during modal experiments.

The differences between the presented plots are especially evident in frequency subranges comprising natural frequencies (resonances). What is important, these frequency subranges are actually used during parameter estimation. It is one of the sources of differences of numerically determined modal parameter values with respect to the estimated ones.

A set of 14 digitized FRFs for two references were used for estimation of modal parameters with the use of Eigensystem Realization Algorithm (ERA) [4]. The resultant stabilization diagram is presented in Fig. 15.

All the seven lines of poles were correctly discriminated on the diagram by the formulated FRDA algorithm. Then, four algorithms of the representative poles selection were applied. The results of parameter estimation are listed in Table 1 and compared with accurate parameter's values obtained by numerical eigenvalue problem solution for the considered system.

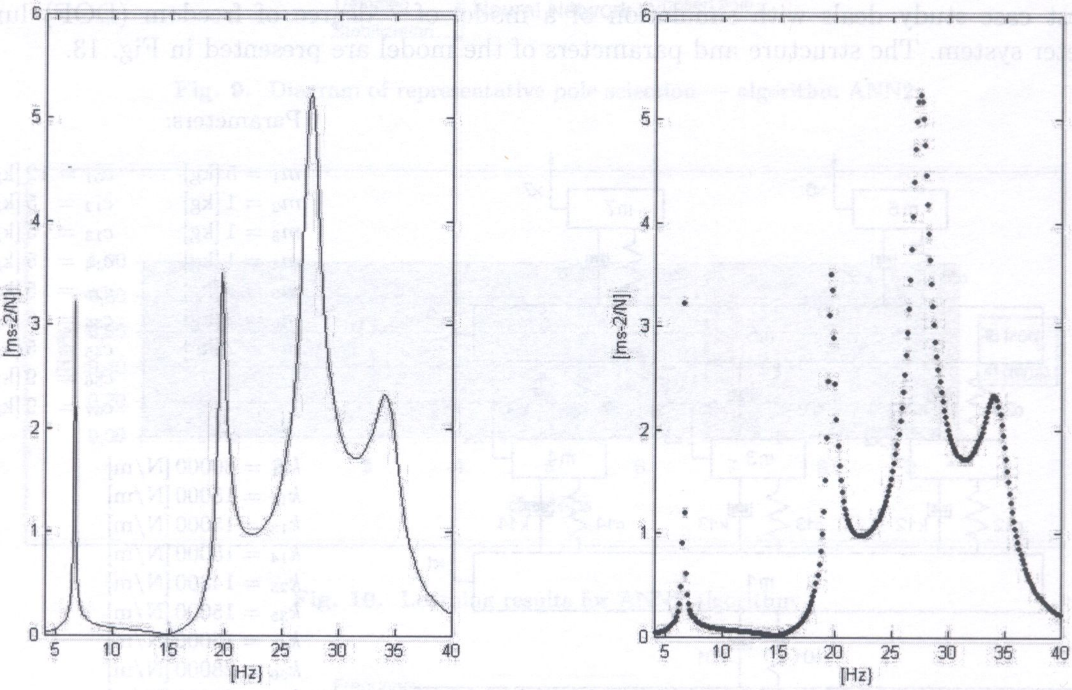


Fig. 14. Comparison of a continuous and a digitized FRF amplitude plot

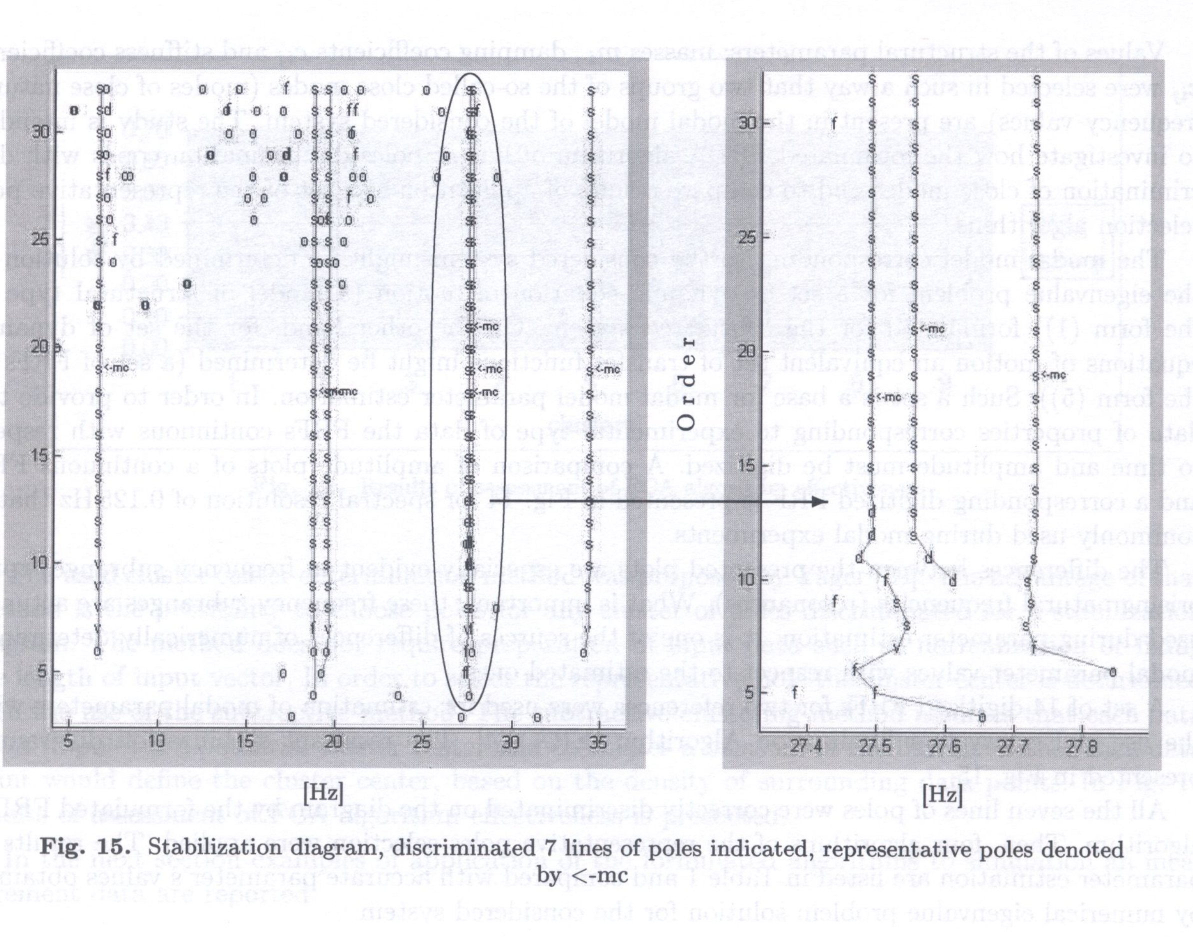


Fig. 15. Stabilization diagram, discriminated 7 lines of poles indicated, representative poles denoted by <-mc

Table 1. Results of selection of the representative poles from the discriminated lines of poles: f_n — natural frequency, ξ_n — modal damping coefficient, No. — number of a pole in a line of poles

Accurate values		FRDA – FRA			FRDA – ANN1			FRDA – ANN2			FRDA – FCA		
f_n [Hz]	ξ_n [%]	f_n [Hz]	ξ_n [%]	No.	f_n [Hz]	ξ_n [%]	No.	f_n [Hz]	ξ_n [%]	No.	f_n [Hz]	ξ_n [%]	No.
6.64	0.59	6.64	0.59	14	6.64	0.59	13	6.64	0.59	12	6.64	0.58	6
18.83	1.90	18.83	1.90	14	18.83	1.90	10	18.83	1.90	13	18.83	1.90	7
19.62	1.51	19.62	1.51	14	19.62	1.51	10	19.62	1.51	14	19.62	1.51	6
27.50	2.89	27.49	2.89	15	27.53	2.89	5	27.47	2.91	9	27.49	2.89	11
27.57	2.89	27.55	2.89	12	27.55	2.89	12	27.55	2.89	13	27.55	2.89	5
27.74	2.53	27.73	2.53	14	27.73	2.53	13	27.73	2.53	13	27.73	2.53	9
34.43	3.50	34.41	3.50	14	34.41	3.50	13	34.41	3.50	12	34.41	3.48	7

The presented comparison showed that the parameter estimation accuracy was very good, the differences of natural frequency estimation did not exceed 0.03 Hz and the damping coefficient estimation error was not higher than 0.02 %. Differences between mode shapes that result from 4 algorithms of the representative pole selection were negligible.

The tested representative pole selection algorithms generally selected different poles but in case of the simulated data it did not produce substantial differences of the estimated modal parameters.

FRA algorithm selected poles of higher orders than the other three algorithms. The reason for this was the high influence of quality of stabilization of poles on poles selection (the stabilization is usually better for higher model order).

FCA algorithm takes into consideration only poles' location in f–ξ plane. That is why, the algorithm selected poles corresponding to modal models of low order. Such the poles are often only rough approximations of actual poles and their location f–ξ plane might considerably differ from the location of physical poles.

Position of poles selected by ANN1 and ANN2 algorithms in lines of poles was close to the position of poles selected by FRA algorithm, but the model order was usually slightly lower.

Concluding the first case study it might be said that in case of the closed modes the algorithm of extraction of line of poles from the stabilization diagram proved to be effective. Additionally, the differences of results of application of the four formulated algorithms of selection of the representative pole from the line of poles were small and did not influence the parameter estimation accuracy.

The second case study deals with estimation of modal model for real object — a helicopter airframe. The aim of the analysis was the comparison of the four algorithms of selection of the representative poles from line of poles with results of the selection performed by experienced analyst. In the real object case the accurate values of modal parameters are unknown. The stabilization diagram obtained with use of ERA estimation method is showed in Fig. 16.

The presented diagram is less clear than that corresponding to simulation data (Fig. 15). On the above stabilization diagram seven distinct separate lines of poles might be recognized. There is also present a cluster of three lines of poles one on top of the others in the vicinity of frequency of 20 Hz. The cluster of lines corresponds to the local type of modes in which the end of tail boom with vertical and horizontal stabilizers vibrate the most. As the local type of mode shapes they are not well represented in the measurement data thus they are difficult to identify and properly interpret.

Comparison of results of the representative poles selection with use of the four formulated algorithms is presented in Table 2.

The presented comparison showed that FRA algorithm of the representative pole selection from a line of poles performed the best. For six out of ten selections the result of application of the algorithm is the same as the choice made by the experienced analyst. Other choices did not produce large differences in natural frequency estimation (difference less than 0.02 Hz), the difference of damping coefficient is also small (less than 0.08%), but one of the MAC values between mode shape selected by the analyst and by the FRA is low (only 44%). The remaining three algorithms performed worse. The differences of natural frequency value indicated by the analyst and by the algorithms

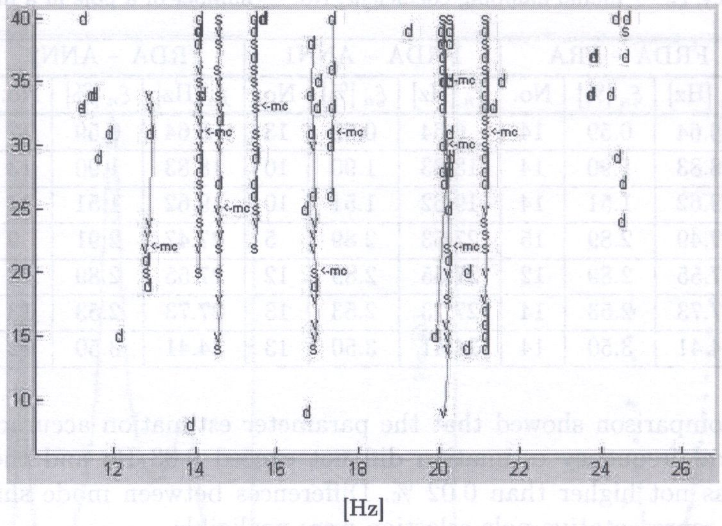


Fig. 16. **Stabilization diagram obtained for a helicopter airframe modal testing, discriminated 10 lines of poles indicated, representative poles denoted by <-mc**

Table 2. Results of modal model estimation for real data, bold face indicate the same selection as made by an experienced analyst

Human Operator			FRDA – FRA				FRDA – ANN1			
f_n [Hz]	ξ_n [%]	No.	f_n [Hz]	ξ_n [%]	No.	MAC [%]	f_n [Hz]	ξ_n [%]	No.	MAC [%]
12.80	2.66	**4**	12.80	2.66	**4**	100	12.92	3.15	7	78
14.13	0.81	**15**	14.15	0.75	11	97	14.13	0.91	10	95
14.60	1.18	**4**	14.59	1.25	8	91	14.62	1.29	9	92
15.50	0.73	**11**	15.51	0.73	9	99	15.50	0.72	15	98
16.96	0.36	**6**	16.96	0.36	**6**	100	16,94	0.45	8	87
17.40	0.58	**4**	17.39	0.65	3	44	17.40	0.58	**4**	100
20.12	1.44	**4**	20.12	1.44	**4**	100	20.15	1.51	5	93
20.13	3.53	**4**	20.13	3.53	**4**	100	20.13	3.10	**4**	100
20.24	1.36	**8**	20.24	1.36	**8**	100	20.25	1.31	7	98
21.18	1.03	**15**	21.18	1.03	**15**	100	21.17	0.99	14	100

Human Operator			FRDA – ANN2				FRDA – FCA			
f_n [Hz]	ξ_n [%]	No.	f_n [Hz]	ξ_n [%]	No.	MAC [%]	f_n [Hz]	ξ_n [%]	No.	MAC [%]
12.80	2.66	**4**	12.92	3.15	7	78	12.87	2.43	1	75
14.13	0.81	**15**	14.13	0.91	10	95	14.13	0.75	13	99
14.60	1.18	**4**	14.59	1.25	8	92	14.61	1.17	19	90
15.50	0.73	**11**	15.49	0.69	16	98	15.50	0.73	4	98
16.96	0.36	6	16.94	0.38	10	87	16.96	0.36	4	90
17.40	0.58	**4**	17.39	0.65	3	100	17.40	0.5	**4**	100
20.12	1.44	**4**	20.15	1.51	5	93	20.30	1.43	3	84
20.13	3.53	**4**	20.20	2.48	5	98	20.13	3.53	3	51
20.24	1.36	**8**	20.24	1.36	**8**	100	20.24	1.36	**8**	100
21.18	1.03	**15**	21.17	1.02	17	89	21.18	1.03	**15**	100

were small and did not exceed 0.18 Hz — the value comparable with the modal experiment resolution of 0.125 Hz. The difference between damping coefficient values were higher (up to 1.05%). It should be noted that limited precision of modal damping coefficient estimation is a usual problem of modal parameter estimation.

The differences between selected mode shapes were generally acceptable for majority of the representative pole selections (MAC $\geq$ 72%) except for two selections for relatively short lines of poles (line No. 6 and No. 8).

5. Final conclusions

In this paper there was presented a set of example algorithms of the stabilization diagram processing, being a crucial part of modal parameter estimation procedure. These algorithms use fuzzy reasoning results or artificial neural networks classification results. The presented algorithms are of heuristic type so they are very sensitive to the assumed parameters of the procedures or to the selection of data used during learning. There is always some uncertainty in results of application of such the algorithms. To lower this uncertainty level it is advised to use always multi-criterion assessment and decision-making in heuristic type autonomous parameter estimation procedures.

The application of the presented autonomous parameter estimation procedures proved to ease considerably the modal model parameter estimation, as well as to shorten the modal model identification time. Thanks to the use of large variety of indicators application of the formulated procedures improves the identification results' objectivity and thus also their credibility. The application of the autonomous parameter estimation procedures should be nevertheless done with care. It is still not likely that such the procedures will soon eliminate the professional testing engineers from experimental modal analysis. An engineer should be very careful especially while applying autonomous parameter estimation procedures during identification of modal model for control synthesis purpose or whenever the accuracy of some modal parameter values estimation is crucial.

References

[1] B. Cauberghe. *Applied Frequency-domain System Identification in the Field of Experimental and Operational Modal Analysis*, PhD thesis. Vrije Universiteit Brussel, 2004

[2] K.S. Chhipwadia, D.C. Zimmerman, G.H. James III. Evolving autonomous modal parameter estimation. *Proc. of 17th IMAC*, SEM USA, 819–825, 1999.

[3] I. Goethals, B. Vanluyten, B. De Moor. Reliable spurious mode rejection using self learning algorithms. *Proc. of ISMA2004*, K.U. Leuven, 991–1003 , 2004

[4] W. Heylen, S. Lammens, P. Sas. *Modal Analysis Theory and Testing.* K.U. Leuven, Departement Werktuigkunde, Leuven, 1997

[5] G.J. Klir, B. Yuan. *Fuzzy Sets and Fuzzy Logic.* Prentice Hall. 1995.

[6] W. Lisowski. Selected problems of decision-making during modal parameter estimation. *Proc. of Int. Conf. on Structural Dynamics Modelling, Madeira Island, Portugal*, 501–508. 2002.

[7] W. Lisowski, P. Kurowski. Applicability of selected indicators and algorithms for decision-making in an autonomous modal parameter estimation procedure. *Proceedings of the ISMA2002*, K.U. Leuven, 1369–1378, 2002.

[8] W. Lisowski, T. Uhl, P. Kurowski, K. Mendrok, G. Góral, A. Klepka. Example of autonomous parameter estimation procedure and its application to results of modal testing of an airplane. *Proceedings of the ISMA2004*, K.U. Leuven, 1019–1031, 2004.

[9] R.L. Mayes, S.E. Klenke. Automation and other extensions of the SMAC modal parameter extraction package. *Proc. of 18th IMAC*, SEM USA, 799–785, 2000.

[10] R.L. Mayes, S.E. Klenke. Consolidation of modal parameters from several extraction sets. *Proc. of 19th IMAC*, SEM USA, 1023–1028, 2001.

[11] L. Meirovitch. *Elements of Vibration Analysis.* McGraw-Hill, 1975.

[12] H.G. Natke, C. Cempel. *Model-Aided Diagnosis of Mechanical Systems.* Springer, 1997.

[13] R.S. Pappa, S.E. Woodard, J.-N. Juang. A benchmark problem for development of autonomous structural modal identification. *Proc. of 15th IMAC*, SEM USA, 1071–1077, 1997.

[14] M. Scionti, J. Lanslot, I. Goethals, A. Vecchio, H. Van der Auweraer, B. Peeters, B. De Moor. Tools to improve detection of structural changes from in-flight flutter data. *Proc. of the Eight International Conference on Recent Advances in Structural Dynamics (ISVR), Southampton, UK*, 12p., 2003

[15] R. Tadeusiewicz. *Neural Network* (in Polish). AOW, Warszawa, 1983.

[16] H. Van der Auweraer. Requirements and opportunities for structural testing in view of hybrid and virtual modeling. *Proc. of ISMA 2002*, KU Leuven, 1687–1702, 2002.

[17] P. Verboven, E. Parloo, P. Guillaume, M. Van Overmeire. Autonomous modal parameter estimation based on a statistical frequency domain maximum likelihood approach. *Proc. of 19th IMAC*, SEM USA, 1511–1517, 2001

[18] P. Verboven, B. Cauberghe, P. Guillaume, S. Vanlanduit, E. Parloo. Modal parameter estimation and monitoring for online flight flutter analysis. *Mechanical Systems and Signal Processing*, **18**(3): 587–610, 2004.

[19] R. Yager, D. Filev. Generation of fuzzy rules by mountain clustering. *Journal of Intelligent and Fuzzy Systems*, **2**(3): 209–219, 1994.