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Application of soft computing in uncertainty analysis carried out within structural dynamics

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The paper deals with the application of soft computing used in uncertainty analysis in the field of structural dynamics. Employing Genetic Algorithms, fuzzy sets theory as well as interval algebra authors show quite useful extension of well known approaches of solving eigenproblems considering assumed model uncertainties. During performed calculation, ranges of the first natural frequency of a simple FE model are found and then compared to those ones obtained with Monte Carlo simulation. As input uncertain parameters some of material properties are taken into account. The main objective of the work is to highlight possible advantages of the application in terms of reducing computation time meant for uncertainty analyses.

1. Introduction

Nowadays, uncertainty analyses have become an important field within structural dynamics. Different kinds of existing uncertainties should be considered in order to get to know as much as possible about all changes of dynamic behaviour of mechanical structures. Therefore, obtained ranges of natural frequencies as well as eigenvectors in terms of assumed uncertainties are supposed to be of engineers' concern. Mainly, two kinds of uncertainties can be distinguished [7, 12]: irreducible (sometimes also called variabilities or aleatory or stochastic uncertainties) and reducible (also referred to as epistemic or subjective uncertainties). The first type corresponds to the fact that successive items of same product are not of the same characteristics. They differ one from another regarding mainly geometric properties and other producing factors. It is so because manufacturing processes are not ideal and can not offer customer an infinite product's repeatibility. Product's quality depends on used tools and employed measurements techniques. As stated above this kind of uncertainty can not be reduced since non-zero manufacturing tolerances appear and they can change when time passes. The second group of uncertainties expresses a lack of knowledge of designers trying to launch a new product. It can be a matter of unknown loading, ageing, material properties, used models etc. A collection of possible product concepts can also be considered as subjective uncertainty. The important thing is that this kind of uncertainty can be reduced as engineer collects all necessary data on product characteristic. Moreover, subjective uncertainty may not even occur in some cases. Additionally, one should know that there is one more source of differences in results and caused by errors imposed by blunders, wrong models, incorrect descriptions of analyzed structures but these parameters are out of scope of presented paper and are not referred to in the following.

As objects of uncertainty analyses FE models are often used [1, 2]. They enable engineers to assess changes in both static and dynamic behaviour of mechanical structure with introduced uncertainties of its geometrical and material characteristics and applied loads and constraints. As a computational technique making possible to carry out uncertainty analyses a probabilistic Monte Carlo Simulation (MCS) may be used [7, 16]. Although it is well known and widely applied MCS features some disadvantages. Large number of simulation experiments is required to obtain reliable results for practitioner engineer. Moreover, probability density functions should be known or correctly assumed.

In this context, soft computing seems to be very interesting tool being able to deal with mentioned above kind of analyses very effectively. Hence, the intention of this paper is to present possible advantages of an application of this computing techniques considered as a combination of Genetic Algorithms, fuzzy sets and interval algebra theories.

In present paper authors show and discuss results of uncertainty analysis obtained performing an example of soft computing application. As a subject of analysis a simple mechanical structure modelled with FEM has been considered. Subjective uncertainties have been taken into account i.e. these ones related to material properties. Changes of the first natural frequency of the structure in terms of given uncertainties are studied. All applied theories are also briefly described.

2. FUZZY SETS, FUZZY FEM, FUZZY FEA

The theory of fuzzy sets was introduced by Zadeh in 1965 as an extension of classical set theory [3, 6, 12, 17]. In classical set theory, membership of element in a set is either 0 (not a member of the set) or 1 (member of the set). Zadeh extended the Boolean membership values of a set to real numbers between 0 and 1 by introducing fuzzy sets. Each element in a fuzzy set can be assigned by a membership value between 0 and 1. For a fuzzy set $\tilde{x}$, the membership function $\mu_{\tilde{x}}(x)$ for all x contained within the domain X is defined as follows,

$$\tilde{x} = \{(x, \mu_{\tilde{x}}(x)) \mid (x \in X)\,(\mu_{\tilde{x}}(x) \in [0,1])\} \tag{1}$$

Element x, for which $\mu_{\tilde{x}}(x) = 1$, is definitely a member of the set $\tilde{x}$. Element x, for which $\mu_{\tilde{x}}(x) = 0$, is definitely not a member of the set $\tilde{x}$. Element x, for which $0 < \mu_{\tilde{x}}(x) < 1$, is a member of the set $\tilde{x}$ in a certain degree (Fig. 1).

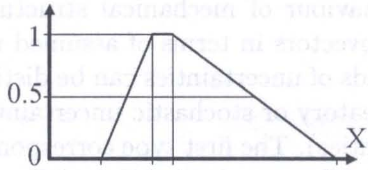

Fig. 1. An example of trapezoidal membership function

The shape of membership functions is derived from experimental data or expert knowledge. Usually triangular, Gaussian and trapezoidal shapes are used for the membership functions.

Fuzzy sets allows for introducing fuzzy FEM (FFEM) [11]. This computational technique employs fuzzy sets to express all given input uncertainties for analyses of FE models and as expected also searches for assumed output parameters following fuzzy formulation. It should be also highlighted that well-known α-cut strategy seems to be the most effective and suitable for FFEM applications [12] i.e. when fuzzy FEA (FFEA) are needed. The main idea of this strategy is shown in Fig. 2.

Using the α-cut strategy all input fuzzy sets can be approximated by a number of intervals. For these input intervals an interval analysis is then performed at each α-level using FEM. The output intervals at each α-cut are assembled and result in fuzzy output. Fig. 2 shows this procedure for input parameters characterized by trapezoidal membership functions. However, one can also consider more specified case when only triangular shapes of membership functions are taken into account. In that case interval analysis carried out for cut α_4 becomes a deterministic one. The following section shows how to deal with uncertainty analysis when the α-cut strategy within FFEM has been chosen.

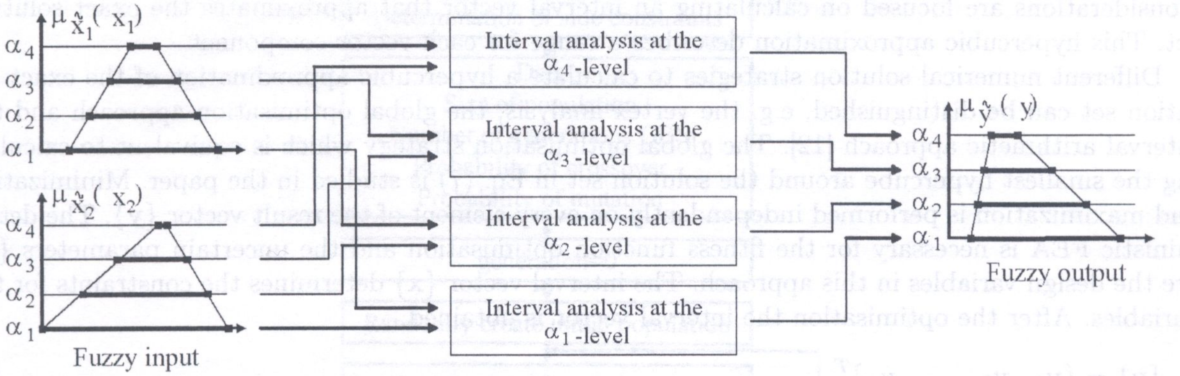

Fig. 2. α-cut strategy for a case with two input trapezoidal fuzzy parameters [12]

3. Interval algebra, interval FEM, interval FEA

As mentioned above problem of finding of output fuzzy parameters can be transformed into a number of tasks connected to interval computations dealing with successive degrees of membership. Therefore, FFEM is now replaced by interval FEM (IFEM) and similarly FFEA becomes a set of an interval FEA (IFEA) [11].

Essential assumptions and formulas concerning interval numbers and interval analysis can be found in [13]. According to the definition, an interval scalar consists of a single continuous domain in the domain of real numbers **R**. The domain of interval scalars is denoted by **IR**. The interval scalar is denoted by a boldface variable **x**. A real closed interval scalar is defined as

$$\mathbf{x} = \{x \mid (x \in \mathbf{R})\ (\underline{x} \le x \le \overline{x})\} \qquad \text{or} \qquad \mathbf{x} = [\underline{x}, \overline{x}] \tag{2}$$

where $\underline{x}$, $\overline{x}$ are respectively the lower and upper bounds of interval scalar. The set scalar is denoted by $\langle x \rangle$ and defined as:

$$\langle x \rangle = \bigcup_{i=1,\ldots,n} \mathbf{x}_i \,. \tag{3}$$

The interval vector is denoted by $\{\mathbf{x}\} \in \mathbf{IR}^n$. It describes the set of all vectors for which each vector component x_i belongs to its corresponding interval scalar $\mathbf{x}_i$,

$$\{\mathbf{x}\} = \{\{x\} \mid x_i \in \mathbf{x}_i\} \,. \tag{4}$$

The interval matrix $[\mathbf{X}] \in \mathbf{IR}^{n \times m}$ describes the set of all matrices for which each matrix component x_{ij} is contained within its corresponding interval scalar $\mathbf{x}_{ij}$,

$$[\mathbf{X}] = \{[X] \ x_{ij} \in \mathbf{x}_{ij}\} \,. \tag{5}$$

The set matrix $\langle [X] \rangle$ describes the set of all possible matrices for which each matrix component x_{ij} is contained within its corresponding set scalar $\langle x_{ij} \rangle$:

$$\langle [X] \rangle = \{[X] \mid x_{ij} \in \langle x_{ij} \rangle\} \tag{6}$$

The above interval algebra rules are applied within IFEA. Using them, let us consider then the FEA as a black-box function $f(\{x\})$ of non-deterministic model collected in a parameter vector $\{x\}$ and resulting in output vector $\{y\}$. The input parameter vector is contained within an interval vector $\{\mathbf{x}\}$. The IFEA procedure is numerically equivalent to looking for the following result set,

$$\langle \{y\} \rangle = \{\{y\} \mid (\{x\} \in \{\mathbf{x}\})\ (\{y\} = f(\{x\}))\} \,. \tag{7}$$

Considerations are focused on calculating an interval vector that approximates the exact solution set. This hypercubic approximation describes a range for each vector component.

Different numerical solution strategies to calculate a hypercubic approximation of the exact solution set can be distinguished, e.g. the vertex analysis, the global optimisation approach and the interval arithmetic approach [12]. The global optimisation strategy which is equivalent to calculating the smallest hypercube around the solution set in Eq. (7) is studied in the paper. Minimization and maximization is performed independently on every element of the result vector $\{\mathbf{y}\}$. The deterministic FEA is necessary for the fitness function optimisation and the uncertain parameters $\{\mathbf{x}\}$ are the design variables in this approach. The interval vector $\{\mathbf{x}\}$ determines the constraints for the variables. After the optimisation the interval vector is obtained,

$$\{\mathbf{y}\} = \{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n\}^T \tag{8}$$

where

$$\mathbf{y}_i = [\underline{y_i}, \overline{y_i}] \tag{9}$$

with:

$$\underline{y_i} = \min_{\{x\}\in\{\mathbf{x}\}} f_i(\{\mathbf{x}\}) \qquad i = 1, \ldots, n, \tag{10}$$

$$\overline{y_i} = \max_{\{x\}\in\{\mathbf{x}\}} f_i(\{\mathbf{x}\}) \qquad i = 1, \ldots, n. \tag{11}$$

In the work, Genetic Algorithms are chosen to calculate the output interval vector boundaries to estimate variability of selected natural frequency of the mechanical system.

4. Genetic algorithms

Genetic Algorithms (GA) [5, 9] are stochastic global search method that mimics the metaphor of natural biological evolution. GA operate on a population of potential solutions applying the principle of survival of the fittest to produce better and better approximations to a solution. Unlike classical methods, GA search feasible domain of decision variables, starting not from one point but from certain population of points. GA do not use deterministic but probabilistic rules of choice that imitate natural processes of evolution and adaptation. GA manage well with difficult optimisation problems formulated for discontinuous, discrete fitness functions even those of very complicated topology. The structure of the GA procedure is presented in Fig. 3.

Firstly, the following characteristics are determined: the side constraints describing mutual dependencies and the ranges in which input parameters can vary as well as the size of used population, number of generations, probabilities of performed crossover and mutation. Then GA procedure creates an initial population of members (individuals) considered as first solution proposals which means that every member (actually a vector of the decision variables) corresponds to a point in a search space and represents a feasible solution. Within every generation, evaluation of each member performance based on a problem dependent fitness function is carried out. This process assigns selection probabilities to each design (member). New generation is then created by selection of designs for further processing and matching them into pairs. Crossover operator exchanges properties of each pair with the crossover probability. Mutation operator mutates some properties of a design. Having the next population GA may continue calculations within the loop. Constant improvement of the solution can be observed while the generations pass. GA application finishes when the assumed number of generations is achieved. Yielded results are considered as global optimum of the optimization problem.

GA may be therefore used effectively as the optimization tool within the area of FFEA/IFEA [10]. Applying this methodology the bounds of various parameters describing both static and dynamic properties of mechanical structures can be found.

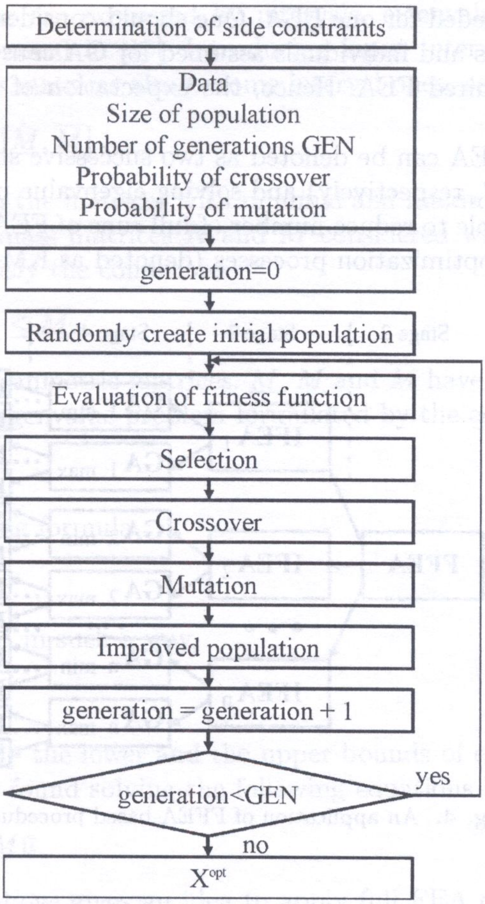

Fig. 3. The structure of GA

5. Formulation of the method based on FFEA

Theory presented in previous sections can be expressed by the scheme shown in Fig. 4. It represents whole computational procedure based on FFEA. This procedure can be disassembled and its particular components can be then separately analysed.

Within the procedure 5 stages can be distinguished, each being responsible for particular scope of tasks. At stage 1, the most general one, a need of uncertainty analysis is formulated and all uncertain parameters are recognised and assumed. At this stage a decision about used analysis method is also made up. Stage 2, as FFEM is chosen, covers processes of description of input uncertain parameters using fuzzy set theory and preparing FEM model of an examined mechanical structure. At this stage it is assumed that further computations are carried out applying α-cut strategy so as mentioned previously FFEA is transformed to a set of related IFEA. Stage 3 means interval calculations performed within successive IFEA for evaluated bounds of input uncertain parameters. GA are here chosen as a tool being capable of finding extremes of output interval parameter. Stage 4 is referred to as optimization procedures used for searching both minimal and maximal values of output data. Finally, at stage 5 FEA are performed to give the results for objective function described for GA. Considering above scheme the computation time of FFEA t_{FFEA} equals

$$t_{FFEA} = \sum_{i=1}^{n} \left(\sum_{j=1}^{m_{n,\min}} t_{FEA} + \sum_{k=1}^{m_{n,\max}} t_{FEA} \right) \tag{12}$$

where t_{FEA} means the time needed for one FEA. One should consider then that the grater number of α-cuts in FFEA, generations and individuals assumed for GA causes larger computational effort needed for performing all required FEA. Hence, the expectation of reducing required calculation time grows significantly.

For structural dynamics, FEA can be denoted as two successive steps: global stiffness and mass matrices evaluation (K and M, respectively) and solving eigenvalue problem. At this point an idea appears that it is maybe possible to reduce number of full runs of FEA and perform only evaluation of matrices K and M during optimization processes (denoted as KM in Fig. 5).

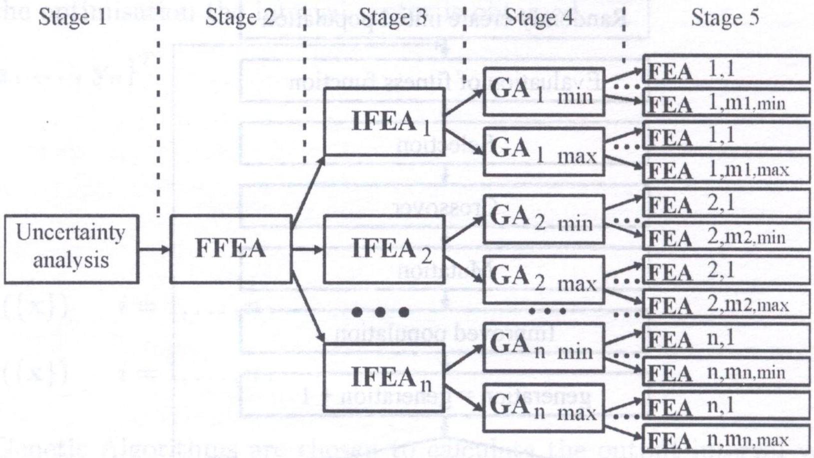

Fig. 4. An application of FFEA-based procedure

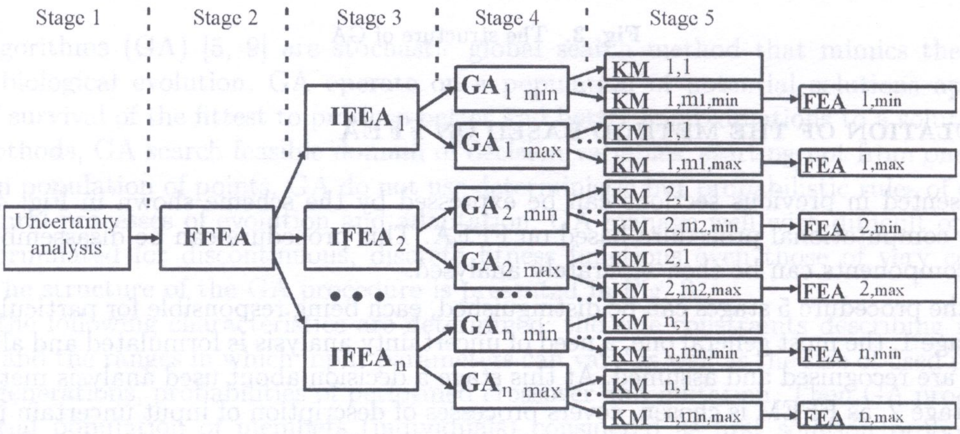

Fig. 5. Modification of FFEA-based procedure

In such case, the only necessity is to apply a methodology enabling us to find such matrices K and M for which extremes of chosen natural frequency can be found. Therefore, only two full FEA are required at the end of optimizing processes. Savings of computation time $t_{SAVINGS}$ may be expressed as

$$t_{SAVINGS} = \sum_{i=1}^{n} \left(\sum_{j=1}^{m_{n,\min}} t_{EIG} + \sum_{k=1}^{m_{n,\max}} t_{EIG} - 2t_{FEA} \right) \tag{13}$$

where t_{EIG} denotes time needed for solving eigenvalue problem. In the following an example of such soft computing application is presented.

In case when uncertainties are expressed by intervals, eigenvalue problem may be solved by computational technique proposed in [15]. Introducing input interval parameters causes the fact that global stiffness and mass matrices also become interval ones, as follows,

$$K^I = [\underline{K}, \overline{K}], \qquad M^I = [\underline{M}, \overline{M}], \tag{14}$$

where $\underline{K}$, $\overline{K}$, $\underline{M}$ and $\overline{M}$ mean the matrices with minimal and maximal values of their elements. All possible global stiffness and mass matrices K and M considered within interval matrices K^I and M^I are supposed then to satisfy the conditions

$$\underline{K} \leq K \leq \overline{K}, \qquad \underline{M} \leq M \leq \overline{M}. \tag{15}$$

$\underline{K}$, K and $\overline{K}$ have to be real symmetric matrices. $\underline{M}$, M and $\overline{M}$ have to be additionally real positive definite. In this connection, eigenvalue problem formulated by the equation

$$Ku = \lambda M u \tag{16}$$

may be related to the following formula,

$$K^I u = \lambda^I M^I u. \tag{17}$$

Interval vector λ^I is described in such a way,

$$\lambda^I = [\underline{\lambda}, \overline{\lambda}], \tag{18}$$

where $\underline{\lambda}$ and $\overline{\lambda}$ are respectively the lower and the upper bounds of eigenvalues.

These two vectors may be found solving the following equations [15]:

$$\underline{K}\underline{u} = \underline{\lambda}\overline{M}\underline{u}, \qquad \overline{K}\overline{u} = \overline{\lambda}\underline{M}\overline{u}. \tag{19}$$

The approach presented above gives an idea to apply full FEA only for two cases expressed in Eq. (19). It is then expected that optimization tool (GA) is capable of finding matrices $\underline{K}$, $\overline{K}$, $\underline{M}$ and $\overline{M}$ in terms of assumed uncertain input interval parameters attached to all considered α-cuts. As presented in case described later during optimisation of the values of global stiffness and mass matrices one has opportunity to find two pairs of these matrices $\underline{K}^*$, $\overline{M}^*$ and $\overline{K}^*$, $\underline{M}^*$ which can be considered as $\underline{K}$, $\overline{M}$ and $\overline{K}$, $\underline{M}$, respectively. First pair may be found during minimizing values of global stiffness matrix K and maximizing values of global mass matrix M done simultaneously, as presented in Eq. (20). The second one can be obtained in the similar way, while maximizing values of elements of K and minimizing values of elements of M, which is described in Eq. (21). The problem can be then defined by the formulas

$$\exists\,(K = \underline{K}^*,\ M = \overline{M}^*) : \min_{\text{GA}} \left(\sum_{i,j} k_{ij} - \sum_{i,j} m_{ij} \right) = \sum_{i,j} \underline{k_{ij}^*} - \sum_{i,j} \overline{m_{ij}^*}, \tag{20}$$

$$\exists\,(K = \overline{K}^*,\ M = \underline{M}^*) : \max_{\text{GA}} \left(\sum_{i,j} k_{ij} - \sum_{i,j} m_{ij} \right) = \sum_{i,j} \overline{k_{ij}^*} - \sum_{i,j} \underline{m_{ij}^*}, \tag{21}$$

where

$$\underline{K}^* = \left[\underline{k_{ij}^*}\right], \qquad \overline{K}^* = \left[\overline{k_{ij}^*}\right], \qquad \underline{M}^* = \left[\underline{m_{ij}^*}\right], \qquad \overline{M}^* = \left[\overline{m_{ij}^*}\right]. \tag{22}$$

Finally, after finding $\underline{K}^*, \overline{M}^*$ and $\overline{K}^*, \underline{M}^*$, two FEA connected to expressions (19) are performed and lower and upper limits of eigenvalues ($\underline{\lambda}^*$ and $\overline{\lambda}^*$) can be found,

$$\underline{K}^*\underline{u}^* = \underline{\lambda}^*\overline{M}^*\underline{u}^*, \qquad \overline{K}^*\overline{u}^* = \overline{\lambda}^*\underline{M}^*\overline{u}^*. \tag{23}$$

The procedure is presented graphically in Fig. 6.

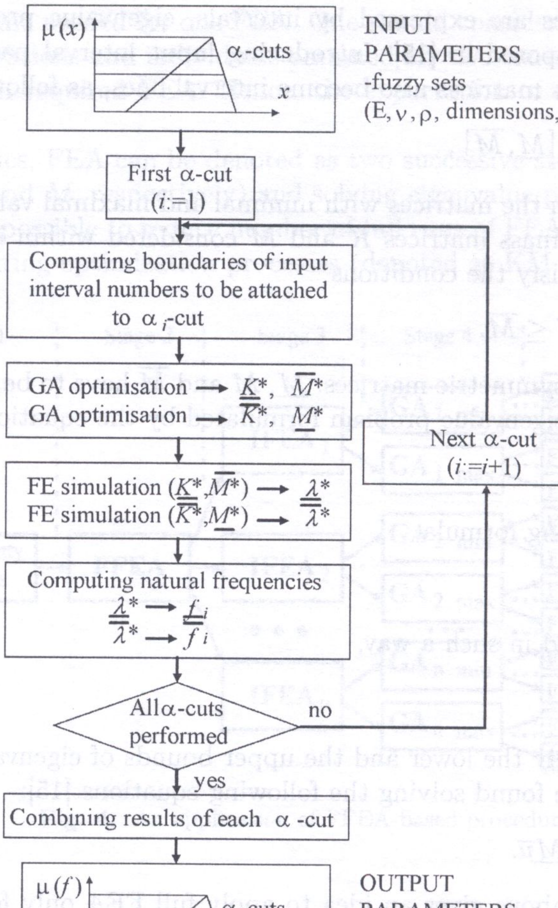

Fig. 6. Examined application of soft computing

This application gives no conservative results because it does not loose any dependency between elements of matrices K and M. It is so because pairs of matrices $\underline{K}^*, \overline{M}^*$ and $\overline{K}^*, \underline{M}^*$ are found simultaneously at two successive optimization procedures. The application have been implemented in MATHWORKS/MATLAB and applied for a model of a simple mechanical structure. FEA have been carried out using MSC/NASTRAN [14].

Presented above application should be considered as an example of soft computing within the area of structural dynamics and some extensions of it may be proposed [8]. In general, some other application having similar scheme (as presented in Fig. 5) may by introduced e.g. considering global stiffness and mass matrices decomposition techniques [4] or even replacing GA with other optimization tool. One can also take into account possible use of described approaches also for static FFEA (considering only matrix K) but these kinds of analyses do not seem to be as difficult to deal with as eigenvalue problems solutions existing in structural dynamics.

6. Case study

As a case example FE model of structure shown in Fig. 7 has been created using MSC.PATRAN. It consists of 6 components and is fixed using three groups of displacements. Six kinds of materials have been defined as well. There are 18 parameters defined as uncertain i.e. material properties such a Young's modulus, Poisson's ratio and mass density. The changes of the first natural frequency of the structure under introduced uncertainties are studied.

Membership function in a form of triangle (Fig. 8) defines fuzzy uncertainties of chosen model parameters. Fuzzy set theory is employed to describe the parameter uncertainties. It is assumed 10% level of uncertainty of material parameters for α_1-cut. It means that intervals concerning this level are limited within the range of 0.9 to 1.1 of nominal values of material properties. α_5-cut is related directly to nominal value of each uncertain parameter and represents deterministic case. Nominal values of material properties are as follows: $E = 2.1 \cdot 10^{11}$ [Pa], $\nu = 0.3$ [-] and $\rho = 7860$ [kg/m^3]. The task is to find the fuzzy output set of the first natural frequency f_1.

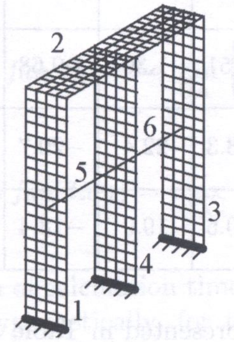

Fig. 7. A simple structure used for simulations

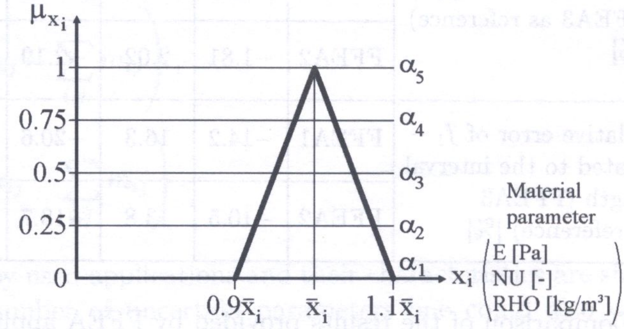

Fig. 8. Fuzzy sets used for describing uncertain parameters

Simulations have covered searching minimal and maximal values of the first natural frequency f_1 of given structure for each of five levels of the membership function assumed for FFEA. To obtain the results the following techniques have been used:

- FFEA in the form presented in Fig. 4 — ranges of output parameter are found applying GA operating directly on the results of FEA (denoted below as FFEA1),
- FFEA in the form presented in Fig. 6 — ranges of output parameter are found applying GA as a tool for optimizing global stiffness and mass matrices (denoted below as FFEA2),
- FFEA with employed MCS to produce referential results (denoted below as FFEA3).

Obtained fuzzy output parameters are presented in Fig. 9. Solid line and circles describe fuzzy output parameter evaluated by the first application of FFEA (direct optimization of natural frequency). Dashed line and ×-marks denotes the result yielded be the second FFEA (optimization of global stiffness and mass matrices). Finally, dotted line with pluses represent the referential result given by MCS.

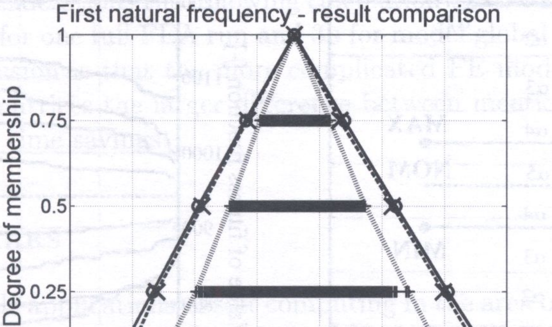

Fig. 9. Obtained results of carried out uncertainty analyses

Table 1. Comparison of the results obtained within applications of FFEA

Parameter		α_1-cut		α_2-cut		α_3-cut		α_4-cut		α_5-cut
		MIN	MAX	MIN	MAX	MIN	MAX	MIN	MAX	NOM
f_1 [Hz]	FFEA1	27.594	33.943	28.328	33.057	29.060	32.221	29.823	31.401	30.599
	FFEA2	27.772	33.822	28.394	32.988	29.119	32.167	29.848	31.331	30.599
	FFEA3	28.283	33.151	29.029	32.430	29.565	31.735	30.053	31.166	30.599
Relative error of f_1 (FFEA3 as reference) [%]	FFEA1	−2.44	2.39	−2.41	1.93	−1.71	1.53	−0.77	0.75	–
	FFEA2	−1.81	2.02	−2.19	1.72	−1.51	1.36	−0.68	0.53	–
Relative error of f_1 – related to the interval length (FFEA3 as reference) [%]	FFEA1	−14.2	16.3	−20.6	18.4	−23.3	22.4	−20.7	21.1	–
	FFEA2	−10.5	13.8	−18.7	16.4	−20.6	19.9	−18.4	14.8	–

Comparison of the results provided by FFEA applications is presented in Table 1.

Relative errors of f_1 are quite small i.e. all of them are placed within ±2.5% and as expected the narrower interval the smaller relative error. Significant differences in results, however, appear while relating them to the length of interval connected to particular α-cut. It is so because of small but still existing conservatism of tested approaches as well as probably not enough large number of performed iteration during MCS (10 000 iterations at each α-cut). Advantageously, the results also let us to conclude that no of global extremes existing within the domain of input parameters has been skipped. MCS has been intentionally chosen to check this phenomenon since assuming uniform probability density functions guaranties to cover uniformly input domain with generated samples. Nevertheless, it is taken into account that the quality of results should be also checked for cases with grater number of introduced uncertainties and more complicated FE models.

Convergence diagrams of applied GA are presented in Fig. 10. Figure on the left hand side is related to direct optimization of the first natural frequency (FFEA1) whereas the second diagram corresponds to the case in which global system matrices are optimized (FFEA2).

Solid lines represent changes of fitness function values and dashed lines set out the levels concerning mean values of the input uncertain parameters. Circles indicate the final values of the fitness function and are related to the individuals which are interpreted as the combinations of the input parameters for which extremes appear. In case when FFEA2 is performed, for mentioned above com-

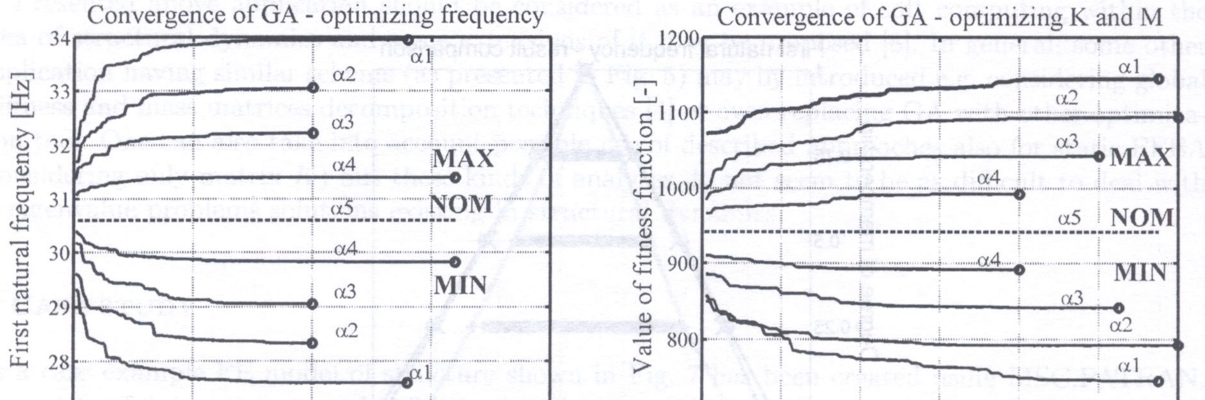

Fig. 10. Convergences of GA used within FFEA

binations of parameters the eigenvalue problem is then solved to obtain minimal or maximal value of the first natural frequency of the structure (as presented in Fig. 6). During computations different numbers of generations have been assumed and as observed, performing at least 80 generations at each of two FFEA can give reliable results.

The minimal and maximal values of the fitness function defined for FFEA2 have been obtained using formulas (24) and (25) where the sum of global stiffness matrix elements is divided by 10^{12}. For analysed FE model, this allows for having elements of both matrix K and matrix M normalized i.e. similar values (of the same order) of the sums of all elements of these matrices are achieved,

$$\min\left(\textit{fitness function}\right) = \min_{\mathrm{GA}}\left(\frac{1}{10^{12}}\sum_{i,j} k_{ij} - \sum_{i,j} m_{ij}\right), \tag{24}$$

$$\max\left(\textit{fitness function}\right) = \max_{\mathrm{GA}}\left(\frac{1}{10^{12}}\sum_{i,j} k_{ij} - \sum_{i,j} m_{ij}\right). \tag{25}$$

Comparison of calculation time required by used applications and their characteristics are shown in Table 2. Hypothetically, for the same number of uncertain parameters one could also apply some non-probabilistic approaches like: the vertex analysis [12] or its general form known as the transformation method [6]. In these cases calculation time would equal at least 896 532 seconds (approximately 249 hours). However for these cases, applications of sensitivity analyses carried out in order to reduce the number of uncertainties could be still the solution reducing computational effort. Nevertheless, savings of calculation time in case of FFEA2 are clearly seen.

Table 2. Characteristics of used FFEA and required calculation time

Application	Characteristics	Total number of full FEA runs	Number of stiffness mass matrices evaluations	Total calculation time [s]
FFEA1	25 individuals (80% of individuals (20) are changed per one generation)	20 000	–	68 400 (19 hours)
FFEA2	Different number of generations (as presented in Fig. 10)	8	16 700	50 127 (13.9 hours)
FFEA3	10 000 iterations per one α-cut	40 000	–	136 800 (38 hours)

For presented above considerations the following calculation times have been estimated and taken into account: 3.42 seconds for one full FEA run and 3 s for model global matrices assembling as used within FFEA2. The conclusion is that the more complicated FE model and in turn larger size of global stiffness and mass matrices the larger difference between mentioned computation times and finally the grater observed time savings.

7. Concluding remarks

The paper describes possible applications of soft computing in the area of structural dynamics. Using simple example of FE model uncertainty analysis has been carried out. FFEA has been employed to find the influence of considered model uncertainties on the change of the first natural frequency of the mechanical structure. As input uncertain parameters material properties like Young's modulus, Poisson ratio, and density have been introduced. In the work three possible examples of FFEA have been described and then applied i.e.: two of them considering GA as optimizing tool and the last one using MCS and giving referential results. Relative errors of FFEA1 and FFEA2 have been presented and discussed. For all cases, α-cut strategy and interval algebra have been used.

Using an application of FFEA in which global stiffness and mass matrices are optimized (denoted as FFEA2) one can see possible savings of calculation times. It can be so since under some conditions full FEA can be replaced by the process of assembly of system global matrices. Requirements of several applications in terms of computation time have been presented in order to show advantage of FFEA2. Additionally, needed calculation time for selected non-probabilistic approaches has been also estimated to have a bit wider overview on computational effort within the field of uncertainty analyses. It should be also noted that neither the vertex nor the transformation method guarantee receiving extremes of output parameter. Moreover, for described FE model the time required for the solving of eigenvalue problem is very short, about 0.4 seconds so it is natural then to formulate an expectation that the difference between computation times should be grater as complexity of model grows.

Presented FFEA1 and FFEA2 yield results which are not very conservative because applied procedures do not cause loosing mutual dependency between elements of stiffness and mass matrices during the search of optimum values. Additionally, as presented in previous section, these applications do not skip any of global extremes existing within the domain of input uncertain parameters.

The convergence diagrams prepared for FFEA1 and FFEA2 have been shown. Analysing obtained shapes of convergence curves, one can assume that for presented case study 80 generations for each optimization process carried out by GA should be enough to achieve reliable results. However, it should be also noted that every time GA are employed the need to find the balance between the number of generation and the number of individuals arises. Unfortunately, this problem seems to be case dependent.

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