Computer Assisted Mechanics and Engineering Sciences, 14: 43–51, 2007.

Genetic optimal shape design of thin axisymmetric shells and axisymmetric structures

Jean Luc Marcelin
Laboratoire Sols Solides Structures 3S, UMR CNRS C5521
Domaine Universitaire, BP No. 53, 38041 Grenoble Cedex 9, France

(Received December 20, 2004)

This work gives some applications of genetic algorithms for shape optimization of thin axisymmetric shells and axisymmetric structures. Calculations are relatively fast for thin axisymmetric shells. For general axisymmetric structures, the concept of mobile or fixed substructures is used and associated to an automatic mesh generator, so calculations are also relatively fast for axisymmetric structures. The limitations or the optimization constraints are included in the chromosomes coding. Three applications are presented; the first one deals with the optimization of the shape of a drop of water, the second one deals with the optimization of the shape of a bottle, and the third one deals with the optimization of the shape of a hydraulic hammer's rear bearing.

Keywords: shape optimization, axisymmetric shells, axisymmetric structures, genetic algorithms

1. INTRODUCTION

There are many recent papers devoted to shape optimization of axisymmetric shells and axisymmetric structures.

In [14], the design of axisymmetric thin-walled structures and parts that can support specified loads with minimum material is described. The result is an optimum shape and thickness distribution in the final part when the strength of the material is assumed to be uniform. The membrane approximation is used to determine the shapes and thickness distributions for these axisymmetric shell structures. A Lagrange multiplier technique is employed. In [6], shape optimization study is presented for umbrella-shaped axisymmetric shells of variable thickness with the self-weight as the dominant load. The importance of the proper selection of the design variables is highlighted. A shape algorithm developed for the particular application is described, which generates the shapes and the representative finite element meshes, while excluding the unacceptable shapes from the design space. In [5], the main aspects of the design and construction of cooling towers are outlined. Special considerations includes the realistic non-axisymmetric distribution of soil characteristics, wind action due to interference effects, optimization of the shell shape to improve structural and dynamic behaviour. In [15], the application of ideal forming theory to design sheet stretching processes that can produce the optimum shapes and thickness distributions from flat sheets of uniform thickness is demonstrated. Specific designs are achieved for producing minimum weight shell structures that will support a specified uniform pressure. In [1], the load carrying capacity, of externally pressurised and optimally shaped metallic shell, has been increased. The optimal geometry has been sought within a class of generalised ellipses by the application of simulated annealing algorithm. In [7], a survey is given about optimal structural design of shells. [18] lists more than 600 cases of shell optimization. Many of them are axisymmetric shells.

There are also many recent papers devoted to optimal shape design of axisymmetric structures, but only a few deals with the use of genetic algorithms. In [13], GAs are used for the first time in the shape optimization of airship axisymmetric hulls. In [3], shape optimization of acoustic scattering bodies is carried out using GA coupled to a boundary element method for exterior acoustics. The particular problem of interest considers an incident wave approaching an axisymmetric shaped body. The objective is to arrive at a geometric configuration that minimizes the acoustic intensity captured by a receiver located at a distance from the scattering body.

The present paper is concerned with the optimal shape design of constant thickness axisymmetric shells and general axisymmetric structures, by use of genetic algorithms. Application of GAs to optimal design of axisymmetric shells of constant wall thickness is not new. The papers [8, 9] rely on variants of Gas for structural optimization of shells. The paper [9] aims to optimize the design of filament-wound multilayer-sandwich submersible pressure hulls, taking into consideration the shell buckling strength constraint, the angle-ply laminated facing failure strength constraint and the low-density isotropic core yielding strength constraint under hydrostatic pressure using the hybrid genetic algorithm (HGA). The objective of [8] is to investigate the optimal design of a fiber-reinforced composite cylindrical skirt subjected to a buckling strength constraint and an overstressing strength constraint under aerodynamic torque and axial thrust. The optimal design problem of [8] involves in determining the best laminate configuration to minimize the weight of the cylindrical skirt. To find the optimal solution accurately and quickly, the HGA is employed in this work. In [11], two examples of application of stochastic techniques for the optimization of stiffened plates or shells are given. This paper [11] (written by the author of the present paper) is not concerned with shape optimization, but genetic algorithms are used. In [11], the research strategy consists in substituting, for finite-element calculations in the optimization process, an approximate response of a neural network, or an approximate response from the Ritz method. More precisely, this paper [11] describes the use of a back-propagation neural network or the Ritz method in creating function approximations for use in computationally intensive design optimization based on genetic algorithms. Two examples of applications are presented; the first one deals with the optimization of stiffeners on a plate by varying their positions, while having well-defined dimensions; the second example deals with the optimization of a thin shell subject to buckling.

With regard to the papers on shape optimization of general axisymmetric structures, they are so numerous that we cannot quote them here. Let us only say that the author already used a long time ago the substructure technique for the optimization of axisymmetric structures, but with traditional optimization techniques [12].

We are going to demonstrate in the present paper that the approximations used in [11] are not necessary for the genetic optimal shape design of thin axisymmetric shells, because the calculations are sufficiently fast. We are also going to demonstrate that the substructure technique for the optimization of general axisymmetric structures is very efficient when GA are used. So, the structural model used in both cases for re-analyses is finite element method (with substructure technique for the optimization of general axisymmetric structures).

In the present paper, the design variables are selected to describe a large variety of axisymmetric shells and axisymmetric structures using the variables identified by most designers and taking into account most of the technological limitations. The objective is to make a reference stress minimum or uniform along one, several or all parts of the boundary. Three test examples are presented.

2. THE METHODS USED

The geometry of the axisymmetric shell or the axisymmetric structure is defined by a generating line. This line is described by successive straight or circular segments described in a given sense and defined by input data of master point coordinates and radius values. The data is a set of nodal points connected by straight segments. Each nodal point is identified by its two cylindrical coordinates (r, z), and a real R which represents the radius of the circle tangent to the two straight segments intersecting at the point. The other computer calculations give the coordinates of any boundary point and especially the tangent points necessary to define the circular arc lengths. The design variables are the master point coordinates and radius values. The side constraints are established in such a way that only small changes in geometry are allowed. They are included in the coding of the design variables.

For axisymmetric shells, analysis is performed by the finite element method with three-node parabolic elements using the classical Love-Kirchoff shell theory [17]. An automatic mesh generator creates the finite element mesh of each straight or circular segment considered as a macro finite element.

For axisymmetric structures, analysis is performed by the finite element method in which the special character of a GA optimization process has been considered, to ease the calculations, and to save computer time. First, because just a few parts of the structure must often be modified, the substructure concept is used to separate the "fixed" and the "mobile" parts. The fixed parts are calculated twice: once at the beginning and also at the end of the optimization process. Only the reduced stiffness matrices of these substructures are added to the matrices of the mobile parts. Related to this division, an automatic generator creates the finite element mesh of each substructure considered as a macro finite element. These macro elements are either triangular (six nodes) or quadrilateral (eight nodes). Following a well-known technique, the same subdivision is used in the parent space to obtain the mesh itself, which is obviously made out of the same types of elements. During the optimization process this mesh is controlled and a new discretization can be chosen if necessary.

The objective is to obtain shapes giving rise to uniform or minimum reference stress σ along the variable boundary. For shells, the optimization is carried out both for internal and external stresses. σ may be selected as σ_s (meridional stress), σ_t (circumferential stress) or σ_{vm} (von Mises stress). See the examples for the choice of the cost function which is different for each example.

More precisely, the general optimization problem is the following:

Objective function. To minimize a reference stress σ along the variable boundary. σ may be selected as σ_s meridional stress, σ_t circumferential stress or σ_{vm} von Mises stress.

Design variables. The design variables are the master point coordinates and radius values which describe the "mobile" contour of the "mobile" parts.

Constraints. The side constraints are established in such a way that only small changes in geometry are allowed. They are included in the coding of the design variables.

With regard to the optimization procedure, a classical and standard genetic algorithm (as described in [4]) is used because the finite element calculations with our assumptions are relatively fast. The author often used the genetic algorithms in the past for various problems of mechanical structures optimization [10, 11].

3. Examples

3.1. Optimization of the shape of a drop of water

The first test example is the optimization of the shape of a drop of water (Fig. 1). In [2], the problem of axisymmetric modal analysis of liquid-storage tanks considering compressibility effects is addressed. In the present paper, the problem is considered to be equivalent to an equal resistance tank calculated by the membrane theory. Nature's optimum design is a solution of the following equations [16],

$$N_\phi \left(\sin\phi/r + \frac{\mathrm{d}\sin\phi}{\mathrm{d}r} \right) = p_0 + p_z$$

with $\tan\phi = \frac{\mathrm{d}z}{\mathrm{d}r}$, $N_\phi = N_\theta = \text{const.}$

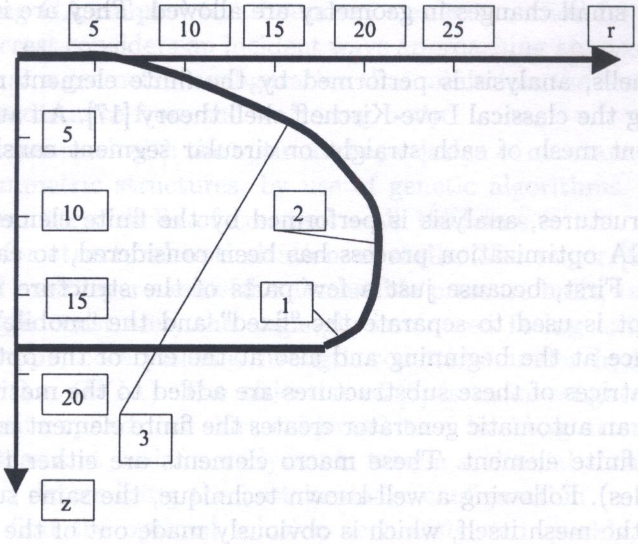

Fig. 1. Optimization of the shape of a drop of water

The design of the drop of water is described by three arcs of circles as indicated in Fig. 1. Only three circular segments are used for the drop for this demonstration example, but it is not a problem if one wants to use more circular segments.

The objective is to obtain a shape for the drop of water giving an equal resistance tank. With regard to the optimization procedure, a classical genetic algorithm (as described in [17]) is used.

The design of the drop of water is described by three arcs of circle (Fig. 1). Their centers and radius are the design variables. So, there are 9 design variables: r_1, z_1, R_1 for the circle 1, r_2, z_2, R_2 for the circle 2, and r_3, z_3, R_3 for the circle 3. In the genetic algorithm, each of these design variables is coded by three binary digits.

To summarize, the optimization problem is the following:

Objective function. The cost function is to obtain uniform circumferential stress σ_t: $\min(\sigma_t - \sigma_{\text{ref}})$, where σ_{ref} is the value of Timoshenko [16].

Design variables. Nine design variables are retained: r_1, z_1, R_1 for the circle 1, r_2, z_2, R_2 for the circle 2, and r_3, z_3, R_3 for the circle 3.

Constraints. The side constraints are established in such a way that only small changes in geometry are allowed. They are included in the coding of the design variables.

The tables of coding-decoding are the following

– for r_1:

000	001	010	011	100	101	110	111
17.7	17.8	17.9	18	18.1	18.2	18.3	18.4

– for z_1:

000	001	010	011	100	101	110	111
16.4	16.6	16.8	17	17.2	17.4	17.6	17.8

– for R_1:

000	001	010	011	100	101	110	111
−0.062	−0.063	−0.064	−0.065	−0.066	−0.067	−0.068	−0.069

– for r_2:

000	001	010	011	100	101	110	111
13.6	13.65	13.7	13.75	13.8	13.85	13.9	13.95

– for z_2:

000	001	010	011	100	101	110	111
11.75	12	12.25	12.5	12.75	12.8	12.85	12.9

– for R_2:

000	001	010	011	100	101	110	111
−7.66	−7.68	−7.7	−7.72	−7.74	−7.76	−7.78	−7.8

– for r_3:

000	001	010	011	100	101	110	111
3.9	3.95	4	4.05	4.1	4.15	4.20	4.25

– for z_3:

000	001	010	011	100	101	110	111
21.3	21.35	21.4	21.45	21.5	21.55	21.6	21.65

– for R_3:

000	001	010	011	100	101	110	111
−20.8	−20.85	−20.9	−20.95	−21	−21.05	−21.1	−21.15

All these binary digits are put end to end to form a chromosome length 27 binary digits.

The standard genetic algorithm of [4] is run for a population of 40 individuals, a number of generations of 60, a probability of crossing of 0.6, and a probability of mutation of 0.05.

The optimal solution corresponds to the chromosome

100 011 010 100 010 011 011 011 011

which gives the solution of Fig. 1, for which

$r_1 = 18.1$, $z_1 = 17$, and $R_1 = -0.064$;

$r_2 = 13.8$, $z_2 = 12.25$, and $R_2 = -7.72$;

$r_3 = 4.05$, $z_3 = 21.45$, and $R_3 = -20.95$.

It is very close to the nature's optimal solution for the shape of a drop of water. It is also very close to the Timoshenko's solution [16].

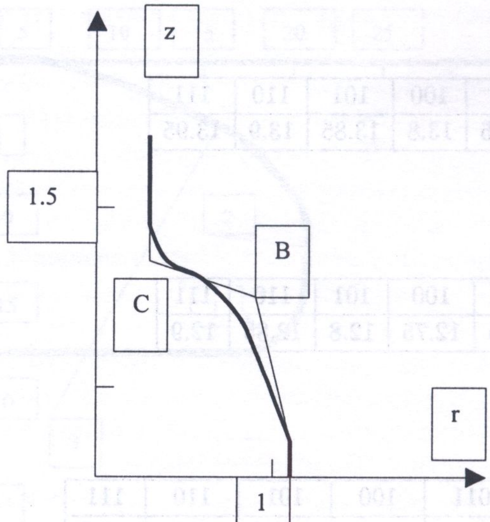

Fig. 2. Optimization of the shape of a bottle

3.2. Optimum shape of a bottle

Some results for the optimum shape of a bottle are now presented. This example is shown in Fig. 2. Concerning the forces acting on the shell, gravity is neglected and only the hydrostatic pressure of the water filling the bottle is considered. The design variables are r_B, z_B, R_B, r_C, z_C and R_C. Here only the circumferential stress σ_t is minimized because the meridional stress is about 20% of σ_t.

In the genetic algorithm, each of these design variables is coded by three binary digits. We must notice that the limitations or the optimization constraints are included in the coding of the chromosomes used in genetic algorithms.

To summarize, the optimization problem is the following:

Objective function. Minimization of the circumferential stress σ_t.

Design variables. The design variables are r_B, z_B, R_B, r_C, z_C and R_C (Fig. 2).

Constraints. The side constraints are established in such a way that only small changes in geometry are allowed. They are included in the coding of the design variables.

The tables of coding-decoding are the following:

– for r_B:

000	001	010	011	100	101	110	111
0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00

– for z_B:

000	001	010	011	100	101	110	111
0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02

– for R_B:

000	001	010	011	100	101	110	111
0.88	0.89	0.90	0.91	0.92	0.93	0.94	0.95

– for r_C:

000	001	010	011	100	101	110	111
0.286	0.287	0.288	0.289	0.290	0.291	0.292	0.293

– for z_C:

000	001	010	011	100	101	110	111
1.100	1.125	1.150	1.175	1.200	1.225	1.250	1.275

– for R_C:

000	001	010	011	100	101	110	111
−0.075	−0.080	−0.085	−0.090	−0.095	−0.100	−0.105	−0.110

All these binary digits are put end to end to form a chromosome length 18 binary digits.

The standard genetic algorithm of [4] is run for a population of 30 individuals, a number of generations of 50, a probability of crossing of 0.5, and a probability of mutation of 0.06.

The optimal solution corresponds to the chromosome

001 101 110 100 110 011

which gives the solution of Fig. 2, for which

$r_B = 0.94$, $z_B = 1.0$, and $R_B = 0.94$;

$r_C = 0.29$, $z_C = 1.25$, and $R_C = -0.09$.

3.3. Example of an axisymmetric structure

In this part, the very localized optimization of the rear bearing of a hydraulic hammer is presented. The bearing in question (Fig. 3) breaks after relatively few cycles of operation. The objective is the minimization of the maximum value of the *von Mises* equivalent stress along the mobile contour, whilst taking into account some technological constraints.

To summarize, the optimization problem is the following:

Objective function. Minimization of the maximum value of the *von Mises* equivalent stress along the mobile contour

Design variables . The design variables are radius r and width X near the radius (Fig. 3).

Constraints. The side constraints are established in such a way that only small changes in geometry are allowed. They take into account the technological constraints. They are included in the coding of the design variables.

The tables of coding-decoding are the following:

– for r:

0000	0001	0010	0011	0100	0101	0110	0111
1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65
1000	1001	1010	1011	1100	1101	1110	1111
1.70	1.75	1.80	1.85	1.90	1.95	2.00	2.05

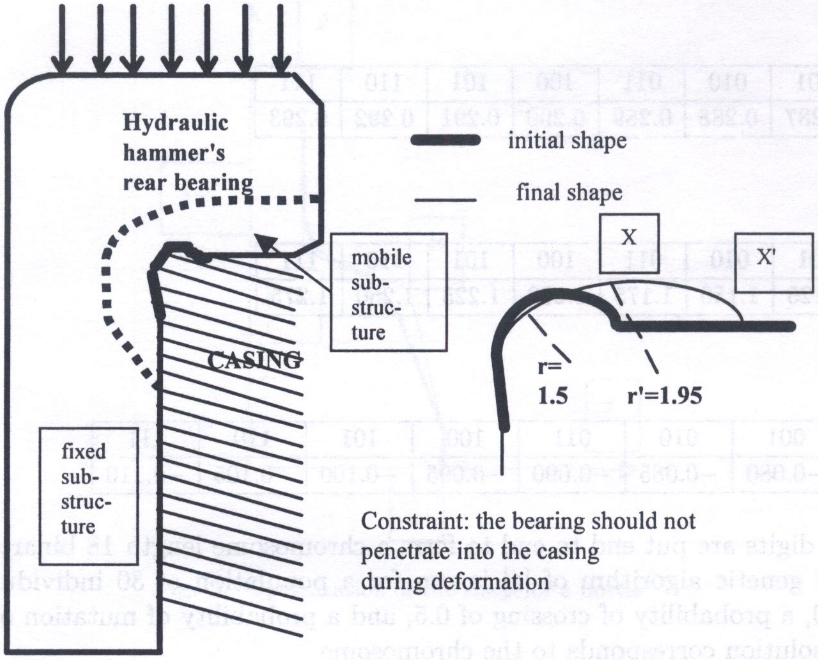

Fig. 3. Optimisation of the shape of a hydraulic hammer's rear bearing

– for X:

0000	0001	0010	0011	0100	0101	0110	0111
2.8	3.2	3.6	4.0	4.4	4.8	5.2	5.6
1000	1001	1010	1011	1100	1101	1110	1111
6.0	6.4	6.8	7.2	7.6	8.0	8.4	8.8

All these binary digits are put end to end to form a chromosome length 8 binary digits.

The standard genetic algorithm of [4] is run for a population of 12 individuals, a number of generations of 30, a probability of crossing of 0.5, and a probability of mutation of 0.06.

The optimal solution corresponds to the chromosome

1101 1000

which gives the solution of Fig. 3, for which

$$r = 1.95, \; X = 6.0$$

The automatic optimization of the shape of this product has, simply by a small modification of shape, which is difficult to predict other than by calculation (increased radius, decreased width), considerably improved the mechanical durability of the bearing: the over-stress being reduced by 50%.

4. Conclusion

Genetic algorithms have been used to perform shape optimization of thin axisymmetric shells and axisymmetric structures (with a substructure approach for the mobile and the fixed parts). Genetic algorithms give very good results for our examples. The calculations are relatively fast. The method is easy to use. The limitations or the optimization constraints are included in the coding of the chromosomes used in genetic algorithms.

REFERENCES

[1] J. Blachut. Optimal barreling of steel shells via simulated annealing algorithm. *Comput. Struct.*, **81**(18–19): 1941–1956, 2003.

[2] J.R. Cho, K.W. Kim, J.K. Lee, T.H. Park, W.Y. Lee. Axisymmetric modal analysis of liquid-storage tanks considering compressibility effects. *Int. J. Num. Meth. Engrg.*, **55**(6): 733–752, 2002.

[3] E.A. Divo, A.J. Kassab, M.S. Ingber. Shape optimization of acoustic scattering bodies. *Engrg. Anal. Boundary Elem.*, **27**(7): 695–703., 2003.

[4] D.E. Goldberg. *Genetic Algorithm in Search, Optimization, and Machine Learning.* Addison–Wesley, 1989.

[5] R. Harte, W.B. Kratzig. Large-scale cooling towers as part of an efficient and cleaner energy generating technology. *Thin Walled Structures*, **40**(7–8): 651–664, 2002.

[6] M.H. Imam. Shape optimization of umbrella-shaped concrete shells subjected to self-weight as the dominant load. *Comput. Struct.*, **69**(4): 513–524, 1998.

[7] J. Krużelecki, M. Życzkowski. Optimal structural design of shells — a survey. *SM Archives*, **10**: 101–170, 1985.

[8] C.C. Liang, H.W. Chen. Optimum design of fiber-reinforced composite cylindrical skirts for solid rocket cases subjected to buckling and overstressing constraints *Composites, Part B: Engineering*, **34**(3): 273–284., 2003.

[9] C.C. Liang, H.W. Chenb, C.Y. Jenb. Optimum design of filament-wound multilayer-sandwich submersible pressure hulls. *Ocean Engrg.*, **30**(15): 1941–1967., 2003.

[10] J.L. Marcelin. Evolutionary optimization of mechanical structures. *Engrg. Optim.*, **31**(5): 571–588, 1999.

[11] J.L. Marcelin. Genetic optimization of stiffened plates and shells. *Int. J. Num. Meth. Engrg.*, **51**(9): 1079–1088, 2001.

[12] J.L. Marcelin, P. Trompette. On the choice of objectives in shape optimization. *Engrg. Optim.*, **11**: 89–102, 1987.

[13] V. Nejati, K. Matsuuchi. Aerodynamics design and genetic algorithms for optimization of airship bodies. *JSME Int. J., Series B - Fluids and Thermal Engrg.*, **46**(4): 610–617, 2003.

[14] O. Richmond, A. Azarkhin. Minimum weight axisymmetric shell structures. *Int. J. Mech. Sci.*, **42**(12): 2439–2453, 2000.

[15] O. Richmond, K. Chung. Ideal stretch forming for minimum weight axisymmetric shell structures. *Int. J. Mech. Sci.*, **42**(12): 2455–2468, 2000.

[16] S. Timoshenko, S. Woinowsky-Krieger. *Théorie des Plaques et Coques.* Librairie Polytechnique Ch. Béranger, Paris, 1961.

[17] O.C. Zienkiewicz. *The Finite Element Method in Engineering Science.* McGraw–Hill, London, 1971.

[18] M. Życzkowski. Recent advances in optimal structural design of shells. *Eur. J. Mech., A/Solids*, **11**: 5–24, 1992.