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# Forging preform shape optimization using surrogate models

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**Forging of practical products from simple billet shapes is a complex and nonlinear process due to the multi-disciplinary phenomenon of material flow and processing conditions. General forgings are usually produced in a number of stages in order to avoid defects such as underfill, extra flash, voids, and folds. In spite of advancements in analysis techniques, forging process simulations do not provide function sensitivity information. Hence, the research focuses on exploring efficient non-gradient based preform shape optimization methods. In this research, an attempt is made to develop a preform shape design technique based on interpolative surrogate models, namely Kriging. These surrogate models yield insight into the relationship between output responses and input variables and they facilitate the integration of discipline-dependent analysis codes. Furthermore, error analysis and a comparison between Kriging and other approximation models (response surface and multi-point approximations) are presented. A discussion about what the results mean to a designer is provided. A case study of an automotive component preform shape design is presented for demonstration.**

**Keywords: preform shape optimization, surrogate models, Kriging, response surface model, multi-point approximation model**

## 1. INTRODUCTION

This research begins by exploring the possible non-gradient techniques for preform shape design in the hot forging process. A brief description of the various approximate and interpolative surrogate model techniques is provided. The preform shape design methodology that uses an interpolative surrogate model, namely Kriging, is developed. A comparison between interpolative and other approximation models is presented. The effectiveness of the proposed methodology is demonstrated with applications to automotive components.

Forging is a manufacturing process that produces many complex industrial and military components, as well as consumer goods. Metal forging processes offer potential savings in energy and material – especially in medium and large production quantities – for which tool costs can be easily amortized. Additionally, forged products exhibit better mechanical and metallurgical properties and reliability than other products that are manufactured by casting and machining.

In forging, a simple cylindrical shape is plastically deformed by applying compressive forces exerted by two or more dies [5]. The starting billet shape for most forging operations is simple: a bar with a round, square, or rectangular cross-section. If the final component shape is complex and intricate, the billet cannot be deformed to the final shape in a single operation. In order to avoid problems like improper material flow, folds, excessive die forces, localized deformation, and incomplete die fill, the workpiece is deformed through several intermediate stages of dies before the final product-shaped die is used. These workpiece intermediate shapes are called preform shapes. Generally, these preform shapes are designed through extensive empirical or trial-and-error methods. The preform shapes obtained through the physical build-and-test approaches are adequate for delivering the final part, but may not be the optimal shapes for cost and quality. Consequently, the most significant objective of any forging engineer is to design robust preform shapes and sequences.

The developed optimized preform shapes produce sound forging products in shorter design cycle times, thereby decreasing the total manufacturing cost. However, without knowledge of the influence of such variables as friction conditions, material properties, and process conditions on preform design, it would not be possible to design the optimum preform, or to predict and prevent the occurrence of defects. Therefore, the method of analysis should be capable of accurately determining not only the overall quantities involved in metal forging processes, such as forming loads, but also stresses and strain distributions under various deformation process conditions. Today, sophisticated *Finite Element Method* (*FEM*)-based software packages, such as DEFORM, SUPERFORGE, and ABAQUS, provide localized information of the deformation process to assist the process design.

The preform shape optimization problem is characterized by continually changing contact boundary conditions, large displacements, and nonlinear material behavior. Moreover, when the preform shape optimization problem involves a complex geometry, the forging process analysis becomes computationally more expensive due to frequent remeshing, smaller step sizes, and die penetration. Therefore, the total number of finite element analyses is one of the most important indexes to assess the efficiency of shape optimization methods. Building approximations by utilizing the available function information is the key in minimizing this computational cost. Another important consideration is the gradient computation (also known as sensitivity information) of cost and constraint functions in which the derivations of nonlinear finite element equations are involved, which is a difficult task when using commercial software packages, as none of the packages come with function sensitivity information. Rather, non-gradient techniques bridge the gap between analysis simulations and optimization with minimized computational cost.

Therefore, this paper focuses on non-gradient based optimization, which uses global surrogate models. In the literature, several approximation-based response surface models are developed for shape optimization. However, none concentrate on interpolative surrogate models for preform shape optimization. Therefore, an attempt is made to develop a shape optimum technique based on an interpolative surrogate model, namely Kriging. Furthermore, this research explores the applicability of various surrogate models (response surface model and multi-point approximation) especially for preform shape optimization in forging.

The selection of the surrogate model is a key issue in complex multidisciplinary preform shape design optimization. Higher quality approximations increase the region of their validity and reduce the repetitive cost of finite element analysis. There is a strong requirement for the designer to provide the relative advantages and recommendations of various models. The comparison between Kriging and other approximation models has been thoroughly explored in structural optimization [4, 10]. However, there is no literature presented for the application of interpolative surrogate models for preform shape optimization in the forging manufacturing process. Hence, this research also focuses on a comparison between various approximation and interpolative surrogate models that are utilized for shape optimization. Recommendations are given about the choosing of the appropriate surrogate models in order to increase the robustness in optimization and product quality. Relevant details of the surrogate models are presented in the following section.

## 2. Overview of surrogate models

### 2.1. Response surface model

A *Response Surface Model* (*RSM*) [8] is a method of constructing global approximations by conducting experiments in a design space. A *Design of Experiments* (*DOE*) approach is used to construct polynomial approximations of system performances that are obtained by using complex analysis codes. RSM is used to obtain an empirical relationship between a specified process response $y$ and a number of input shape parameters $x_i$. RSM generates a polynomial to system response $y$ via least squares regression fitting [1, 2]. Since experimental random error is present in all experiments, the response $y(x)$ is written as follows,

$$y(x) = f(x) + \varepsilon, \tag{1}$$

where $y(x)$ is the unknown function of interest, $f(x)$ is a known polynomial function of the design variables $x_i$, and $\varepsilon$ is random error, which is assumed to be normally distributed with mean zero and standard deviation $\sigma$. The most widely used response surface approximation functions are low order polynomials (i.e. linear, or quadratic). A quadratic polynomial is utilized further in this paper. The general mathematical representation of the quadratic polynomial can be written as

$$\hat{y} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k} \sum_{j>i}^{k} \beta_{ij} x_i x_j \tag{2}$$

where $\beta_i$, $\beta_{ii}$, and $\beta_{ij}$ are regression coefficients for linear, quadratic, and cross-product terms, respectively. The *Analysis of Variance* (*ANOVA*) [2] is done to test the significance of the coefficients estimated in the response-fitted model.

## 2.2. Multi-point approximation

*Multi-Point Approximation* (*MPA*) is constructed by combining weighting functions with local function approximations [11, 12]. With function and sensitivity (local design sensitivities that are computed through the finite difference method) information available at a series of points, a local approximation, *Two-point Adaptive Non-linear Approximation-2 (TANA-2)* is built at each point. All local approximations are then integrated into a multi-point approximation using a weighting function selected such that the approximation reproduces function and gradient information at the known data points. Mathematically, MPA in terms of local approximations can be represented as follows,

$$\tilde{F}(x) = \sum_{k=1}^{K} W_k(x) \tilde{F}_k(x) \tag{3}$$

where $\tilde{F}(x)$ is the unknown function value, $\tilde{F}_k(x)$is the $k$-th local approximation value, and $W_k$ is a normalized weighting function.

Polynomial modeling methods produce smooth approximation models of response data that have been contaminated with the random error found in typical physical experiments instead of computer experiments. Many computer analysis codes are deterministic and are not sensitive to random errors. In that case, the usual measures of uncertainty derived from least-squares residuals in approximation models have no obvious meaning. Therefore, for simulation-based design techniques, interpolation models are used to approximate the response data [9]. The interpolation model, Kriging, is considered further in this research. Unlike the other approximation models, the Kriging function passes through all of the data points.

## 2.3. Kriging approach

Kriging is an interpolative surrogate model based on an exponentially weighted sum of the sample data [6]. It is a statistically accurate and consistent method for interpolating deterministic computer experiments. It postulates a combination of a global model plus deviations, as follows,

$$y(x) = f(x) + Z(x), \tag{4}$$

where $y(x)$ is an unknown function of interest, $f(x)$ is a known function of design variables $x_i$, and $Z(x)$ is a realization of a stochastic process with mean zero, standard deviation $\sigma$, and a nonzero covariance. While $f(x)$ "globally" interpolates the design space, $Z(x)$ creates "localized" deviations so that the Kriging model interpolates the $n$ sampled data points. In this case, $f(x)$ is taken to be a constant $\beta$. Thus, $y(x)$ can be rewritten as,

$$y = \beta + r^T R^{-1}(y - f\beta) \tag{5}$$

where $f$ is a column vector of length $n$ filled with ones (because $f(x)$ is taken as constant), $r^T$ is the correlation vector between an untried value $x$ and sampled data points $(x_1, x_2, \ldots, x_n)$, and $R$ is the correlation function between untried $x$ and sample point $x_i$. Kriging is extremely flexible due to the wide range of correlation functions $R$ that may be chosen. Exponential, Gaussian, and cubic correlation functions can be chosen depending upon the problem characteristics. Each correlation function is explored in this paper and the appropriate one is utilized further in the preform shape optimization problem.

## 3. Preform shape design methodology

The preform shape design is an important aspect for improving the product quality and for decreasing the production cost in forging. In this paper, the preform shape is defined using Bézier curves. Thus, Bézier control points are considered as design variables. An optimization problem is formulated to minimize production costs by minimizing the total forging energy. Constraints are placed on underfill and on flash volume to ensure complete die fill with minimum material wastage. Cost and constraint functions are obtained by using surrogate models. Thus, a high quality product with no defects is produced while reducing the manufacturing costs.

Preform shape design methodology consists of the following steps: identifying and screening critical shape parameters, exploring the design space by DOE, constructing interpolative approximation models, and performing design optimization.

The preform design methodology starts by identifying the critical design parameters that affect the various forging process responses, such as forging energy, underfill, and flash volume. The forging product quality and reliability depends on preform shape parameters, such as shape coordinates, control points of Bézier curves, or geometry dimensions. Among all of the shape control parameters, some of the parameters are critical. Factorial design techniques are used to screen the critical parameters [7]. Among all of the factorial methods, the *Central Composite Design* (*CCD*) method offers a satisfactory alternative to a full factorial design. A CCD contains an embedded factorial or fractional factorial design with center points that are augmented with a group of "star points," which allow estimation of the curvature behavior of the system performance, and the quadratic terms are efficiently estimated through the axial points. Hence, this method is used for selecting the design points in this study. The total number of simulations required for the CCD method is $2^k + 2k + N$, where $k$ is the number of parameters and $N$ is the number of center points.

The responses of the forging process, such as energy, underfill, and flash, are obtained at different design points. An optimum correlation function and Kriging parameters are obtained by evaluating different correlation functions. Then, an interpolation model, i.e. Kriging, is constructed with the obtained responses at design points. Various other approximation models, such as RSM and MPA, are also constructed by fitting to the same responses as in Kriging. The accuracy or the error analysis of these surrogate models is performed. Then, preform shape optimization is performed by using generated surrogate models. The optimum results largely depend on the choice of surrogate models. An appropriate choice of surrogate models will make the design process more accurate and applicable over a wider region of the design space. The comparison between the different surrogate models for the system performance gives insight for choosing the appropriate model, and thereby increases the efficiency of optimization.

## 4. Case Study

Preform shape optimization of a plane-strain rail section is considered as the case study. The rail section consists of two cavities; one is deeper than the other. The complexity of the forging process of the rail section increases with the height-to-width ratio of the cavity. In order to have practical significance, the deeper cavity ratio is taken as 2.0, and the shallow cavity ratio as 1.0. A horizontal symmetry is assumed; therefore, a half model and only the top die are considered for further analysis. The three-dimensional rail section and the finite element model are shown in Fig. 1.

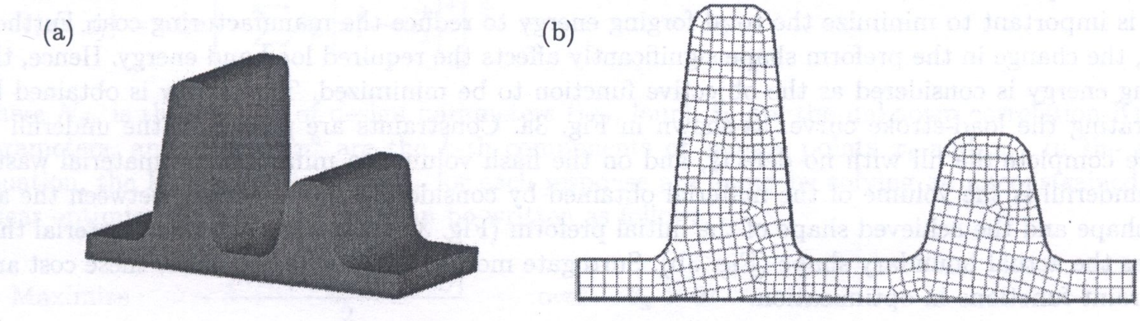


**Fig. 1.** (a) Three-dimensional rail section; (b) Finite element model

### 4.1. Problem definition

The preform shape for the rail section's bottom surface will remain horizontal and flat. Likewise, the outer rim of the disc shape will remain a straight vertical line. The top surface is defined with a Bézier curve. It is formed with five Bézier control points, as shown in Fig. 2. Among all ten coordinates of the points, the vertical coordinates of points 2, 3, and 4 and the horizontal coordinate of point 4 are determined as critical design parameters through DOE screening methods [7]. The remaining control points, 1 and 5, are fixed to their mean values. In order to avoid numerical singularities, the design parameters are normalized over the range of [−1, 1] and are used for constructing surrogate models.

A plane-strain rail section is forged using a mechanical press. A FEM software package, DEFORM 2D [3], is used to simulate the hot forging process and to predict the forging loads, metal flow, and

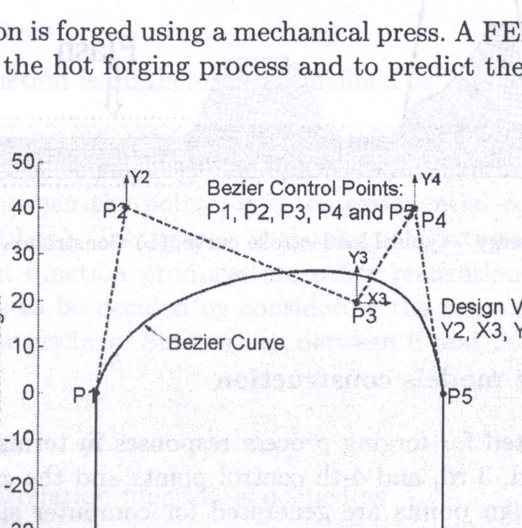


**Fig. 2.** Preform shape definition

deformation patterns. The rail section bottom die is considered stationary and flat, whereas the top die is given at 80 mm/sec velocity. The process is assumed as isothermal and AISI 4340 steel is used as the billet material. In hot forging, the initial billet is heated to 1000°C, and the forging dies are usually heated to temperatures as high as 250–400°C to reduce the die chilling effect.

One of the other important criteria in the forging process is lubrication. A graphite-based lubricant is used with a 0.3 shear friction factor between the die and the workpiece contact surfaces. The distance between the top die and the bottom die (10 mm) is taken as the stopping criterion for the simulation to provide corresponding stroke length/top die movement for different heights of preform shapes.

It is important to minimize the total forging energy to reduce the manufacturing cost. Furthermore, the change in the preform shape significantly affects the required load and energy. Hence, the forging energy is considered as the objective function to be minimized. The energy is obtained by integrating the load-stroke curve, as shown in Fig. 3a. Constraints are placed on the underfill to ensure complete die fill with no defects, and on the flash volume to minimize the material waste. The underfill is the volume of the material obtained by considering the difference between the actual shape and the achieved shape of the initial preform (Fig. 3b). Flash is the excess material that crosses the actual boundary shape (Fig. 3b). Surrogate models are used to construct these cost and constraint functions in optimization.

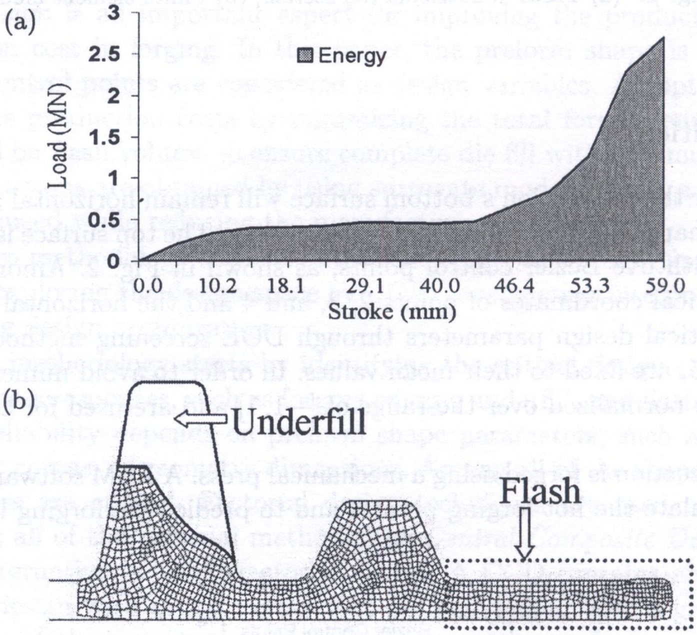


**Fig. 3.** (a) Energy – typical load–stroke curve; (b) Constraints definition

## 4.2. Interpolative surrogate models construction

Surrogate models are constructed for forging process responses in terms of four critical shape parameters ($y$ coordinates of 2-nd, 3-rd, and 4-th control points and the $x$coordinate of 4-th control point). By using CCD, 25 design points are generated for computer simulations. By performing forging simulations at these design points, forging process responses, i.e. energy, underfill, and flash, are obtained. Using this response data, an interpolative Kriging model is constructed.

The Kriging model is built using the 25 sample DOE points that are generated for the four critical design parameters. The accuracy of the Kriging model depends on the correlation $R$between an untried value $x$ and sampled data points $(x_1, x_2, \ldots, x_n)$. The selection of the correlation function greatly depends on the process response characteristics and determines the accuracy of the model. Exponential, Gaussian, and cubic correlation functions are generally used to represent correlations between the sample points in the construction of the Kriging model. All three correlation functions are evaluated to obtain the best suited correlation function for this example.

*Exponential correlation function*

An exponential correlation function $R$ can be mathematically represented as

$$R(x_i, x_j) = \exp\left[-\sum_{k=1}^{N_{dv}} \theta_k \left|x_i^k - x_j^k\right|^2\right] \tag{6}$$

where $N_{dv}$ is the number of design parameters (i.e., four), $\theta_k$ are the unknown correlation Kriging parameters, and $x_i^k$ and $x_j^k$ are the $k$-th components of sample points $x_i$ and $x_j$. In the above equation, the Kriging parameters $\theta_k$ for each response are found by solving an unconstrained non-linear optimization problem, and can be written as follows,

$$\text{Maximize} \quad \frac{[n\ln(\sigma^2) + \ln(|R|)]}{2} \quad \text{over} \quad \theta_k > 0 \tag{7}$$

where $\sigma^2$ is the estimated variance between the global model $\beta$ and actual response $\hat{y}$. The resulting Kriging parameters $\theta_k$ for each response and design parameters, (i.e. energy, underfill, and flash) are summarized in Table 1. With these optimum parameters, an exponential correlation function $R$ is computed for each of the responses.

**Table 1.** Kriging optimum parameters (exponential correlation)

| Response | $\theta_{x1}$ | $\theta_{x2}$ | $\theta_{x3}$ | $\theta_{x4}$ |
|---|---|---|---|---|
| Energy | 0.1814 | 0.1873 | 0.7192 | 0.5711 |
| Underfill | 0.2993 | 0.1000 | 0.1000 | 0.1056 |
| Flash volume | 0.1000 | 0.1000 | 0.2272 | 0.5593 |

*Gaussian correlation functions*

A Gaussian correlation function is mathematically defined as

$$R(d) = \exp(-\theta|d|^q) \tag{8}$$

where $d$ is the distance between the points, $q$ is the exponential coefficient, $0 < q \leq 2$, and $\theta$ is a Gaussian function $\theta \in (0,\infty)$. Taking $q = 1$ recovers the exponential correlation function. As $q$ increases, the correlation function produces smoother realizations. The Gaussian function $\theta$ is assumed as two, which has to be decided by considering the response distribution. The quality of the model differs with changes in $q$. Since $q$ lies between 0 and 2, different values of $q$ (1.5, 1.8, and 2.0) are considered.

*Cubic correlation functions*

Mathematically, a cubic correlation function is defined as

$$R(d) = 1 - \frac{3(1-\rho)}{2+\gamma}\, d^2 + \frac{(1-\rho)(1-\gamma)}{2+\gamma}\, |d|^3 \tag{9}$$

where $\rho$ is the correlation between end point observations, $\gamma$ is the correlation between endpoints of the derivative process, and $\rho = 0.125$ and $\gamma = 0.503$ are used in the cubic function.

As explained in Section 2.3, Kriging models consist of two terms, one is a constant $\beta$ and the other one is a local deviation term. The corresponding constant terms are calculated for each model and are tabulated in Table 2. Each model has distinct $\beta$ values and therefore has different local deviations. The $\beta$ values for cubic correlation functions are significantly different from Gaussian and exponential correlation functions for all of the responses.

**Table 2.** $\beta$ values for various correlation functions

| Response/ Correlations | (Exponential) $q = 1.0$ | (Gaussian) $q = 1.5$ | (Gaussian) $q = 1.8$ | (Gaussian) $q = 2.0$ | Cubic |
|---|---|---|---|---|---|
| Energy | 0.4542 | 0.4537 | 0.4537 | 0.4536 | 0.4175 |
| Underfill | 0.4220 | 0.4216 | 0.4216 | 0.4215 | 0.3805 |
| Flash volume | 0.3133 | 0.3127 | 0.3126 | 0.3126 | 0.2479 |

The accuracy of the models with different correlation functions is checked through *average error* and *Root Mean Square Error* (*RMSE*) values. Low values of average error and RMSE are considered to be a measure of good fit. The resulting average error and RMSE values for each correlation function are given in Table 3. Here, average error is defined as the ratio of the total sum of errors to the number of sample points, and RMSE is defined as

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}} \tag{10}$$

where $n$ is the number of sample points, $y_i$ is the actual response, and $\hat{y}_i$ is the predicted value.

**Table 3.** Kriging models comparison

| Correlation | Response | | | | | |
|---|---|---|---|---|---|---|
| | Energy | | Flash Volume | | Underfill | |
| | Avg. Err. | RMSE | Avg. Err. | RMSE | Avg. Err. | RMSE |
| Exponential | 4.420 | 0.950 | 1.660 | 0.450 | 1.300 | 0.240 |
| Gaussian $q = 1$ | 4.420 | 0.951 | 1.660 | 0.450 | 1.300 | 0.242 |
| Gaussian $q = 1.5$ | 5.904 | 1.956 | 1.726 | 0.626 | 1.438 | 0.245 |
| Gaussian $q = 1.8$ | 5.922 | 1.958 | 1.776 | 0.625 | 1.440 | 0.245 |
| Gaussian $q = 2.0$ | 5.937 | 1.959 | 1.781 | 0.625 | 1.442 | 0.245 |
| Cubic | 6.184 | 1.990 | 1.696 | 0.594 | 1.346 | 0.255 |

For the energy model, Kriging with a Gaussian correlation for $q = 1$ gives the same results as Kriging with an exponential correlation. It is obvious theoretically, as $q = 1$ represents the exponential correlation function. Among all other Kriging models (Gaussian with $q = 1.5, 1.8, 2.0$, and cubic), the cubic correlation function give the least accuracy. Kriging with exponential correlation gives a lesser RMSE compared to other models, which illustrates that the Kriging exponential is the appropriate fit for response energy. For the flash volume, the exponential and cubic correlations give a reasonable average error and RMSE. For all other Gaussian correlation functions, the average error percentage increases as the $q$ value increases. However, the RMSE is approximately the same for all of the $q$ values. Kriging with the exponential correlation function gives the minimum error percentage of (1.3%) and RMSE of 0.24 for underfill volume. For all other Kriging models, the RMSE has the same value, whereas the error percentage increases as $q$ increases.

In total, Kriging models with different correlation functions have distinct predictability. Their accuracy differs from response to response. From the comparison, a Kriging model with an exponential correlation function is recommended for the selected design problem and considered further in the optimization problem. The developed interpolative Kriging model is further compared with the other approximation models, RSM and MPA.

### 4.3. Comparison of surrogate models

For the three forging responses – energy, underfill, and flash volume – interpolative Kriging models are generated with exponential correlation functions, and these are compared with RSM and MPA models.

The second-order quadratic RSM is used for energy, underfill, and flash volume. The RSM model consists of one constant, four linear, four quadratic, and six interaction terms. The models are generated for normalized response values.

MPA models use TANA-2 local approximations. If the distance between sampled design points is less than 20% of the maximum distance, then local TANA-2 approximations are generated. This distance condition avoids the effect of distant local approximation and facilitates the capture of variance locally. Using a total of 25 sample design points, 73 TANA-2 approximations are constructed.

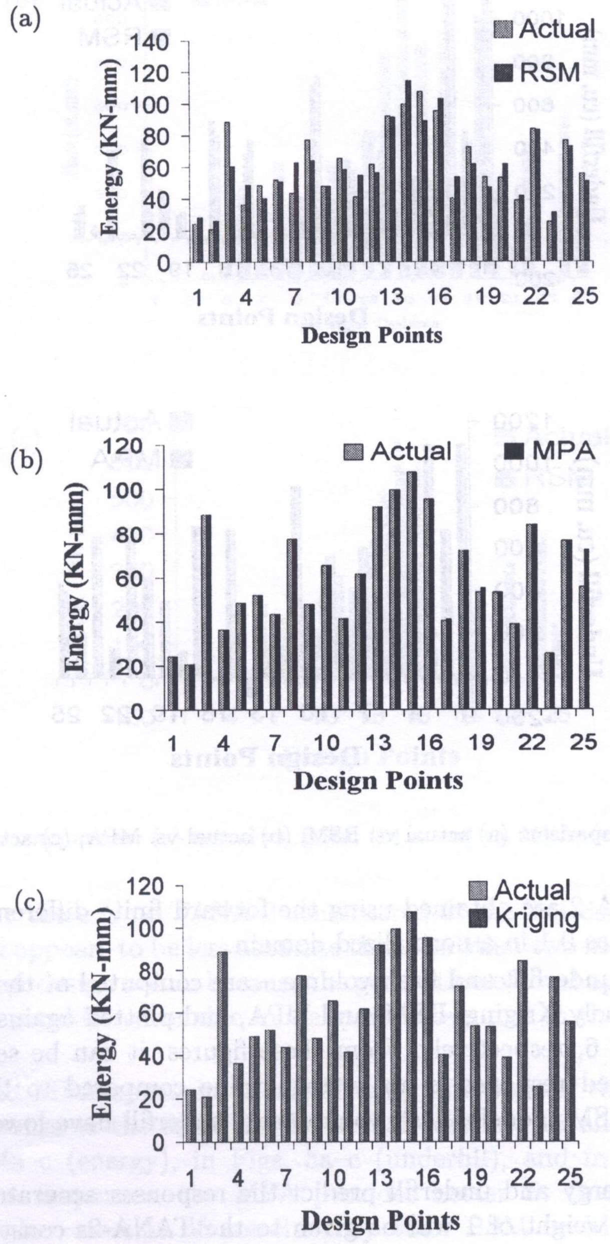


**Fig. 4.** Energy comparison: (a) actual vs. RSM; (b) actual vs. MPA; (c) actual vs. Kriging

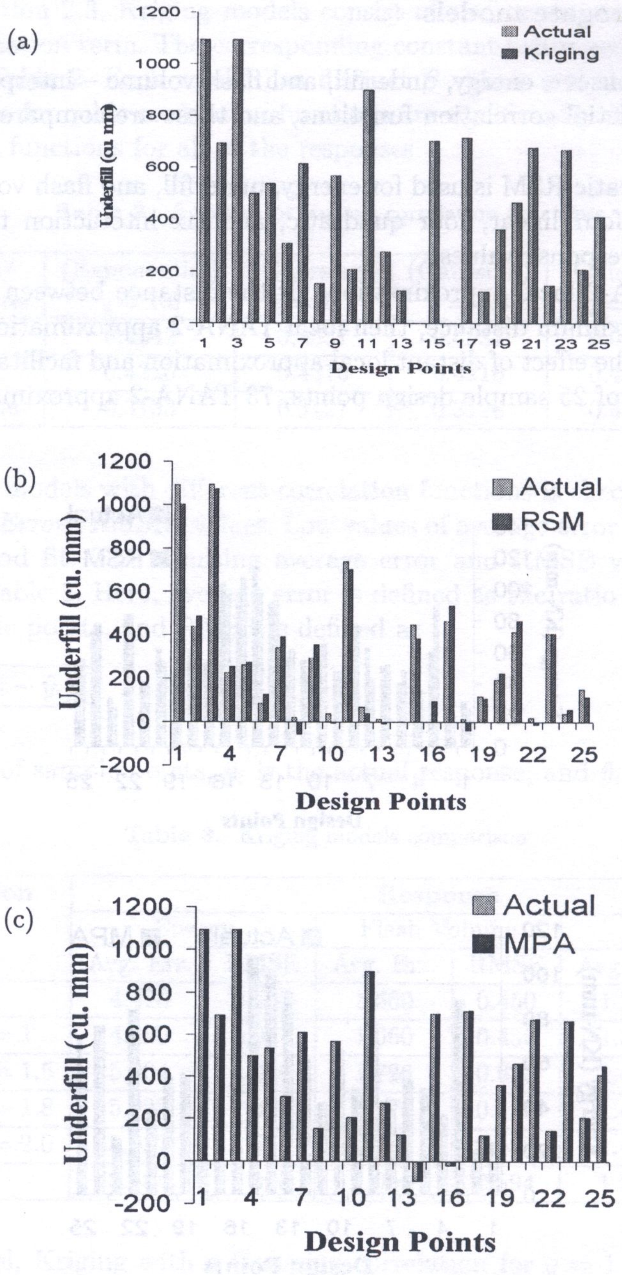


**Fig. 5.** Underfill comparison: (a) actual vs. RSM; (b) actual vs. MPA; (c) actual vs. Kriging

The gradients used in TANA-2 are obtained using the forward finite difference method. The finite difference step size is taken as 0.1 in a normalized domain.

The responses – energy, underfill, and flash volume – are computed at the sample design points using the three models, namely Kriging, RSM, and MPA, and plotted against the actual response, as shown in Figs. 4, 5, and 6, respectively. From these figures, it can be seen that for all of the responses, the RSM-predicted response is an average value compared to the MPA and Kriging models. Among all of the RSM models, flash volume and underfill have lower error and fit better than energy.

The MPA models for energy and underfill predict the responses accurately at sampled design points because a maximum weight of 1 will be given to the TANA-2s constructed at that design point. Hence, the predicted response exactly matches the actual response and fit is better than the traditional polynomial RSM. However, the responses at the last design point don't match the

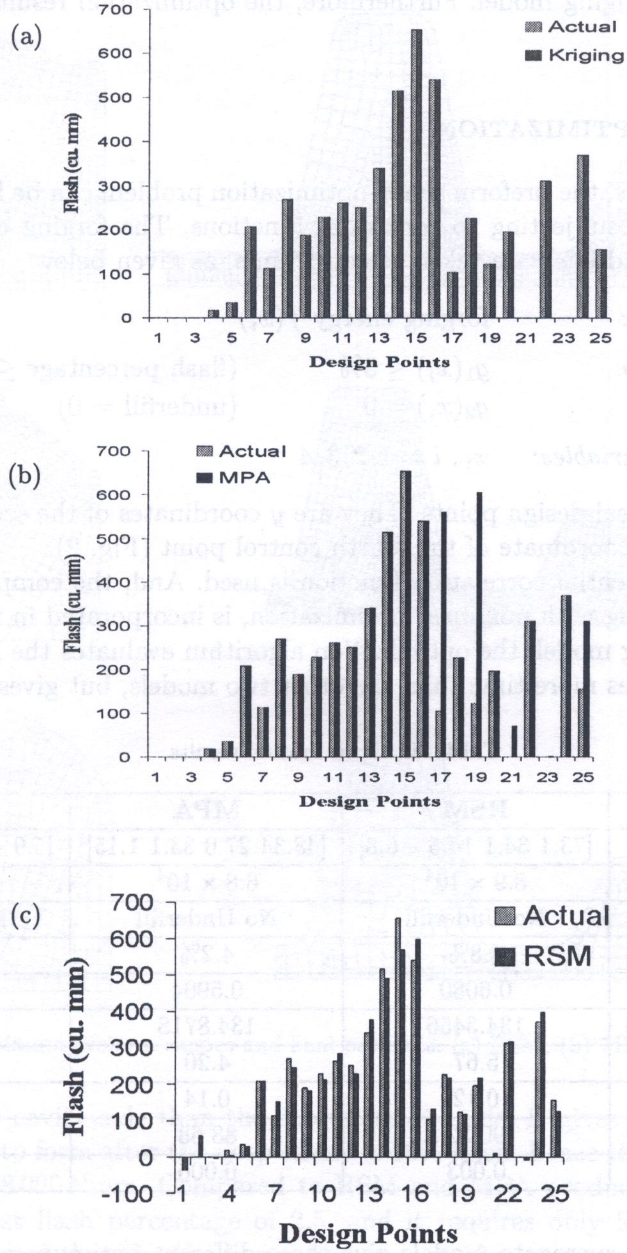


**Fig. 6.** Flash comparison: (a) actual vs. RSM; (b) actual vs. MPA; (c) actual vs. Kriging

actual response because there is no TANA-2 constructed at the last design point. Among all MPA models, the flash model appears to be less accurate than the other two models and gives an erroneous response at a total of four design points because the generation of local approximations has failed and the TANA-2s couldn't provide reasonable model parameters due to their non-optimal distribution of sampling points.

Unlike the RSM and MPA approximations, Kriging interpolates the responses. Therefore, the predicted Kriging response is the same as the actual response in all three cases, which can be clearly seen in Figs. 4a–c (energy), in Figs. 5a–c (underfill), and in Figs. 6a–c (flash volume). In particular, for these responses, the MPA provides better fit than RSM. Kriging interpolation can capture a higher amount of non-linearity than RSM or MPA, and it follows the exact path as the actual response. Therefore, it is recommended that the interpolative surrogate model, i.e. Kriging, gives better fit than any other global approximation model. Then, optimization is performed using the Kriging model. Furthermore, the optimization results of all the models are compared.

## 5. Preform shape optimization

Using the surrogate models, the preform shape optimization problem can be formulated to minimize the objective function by subjecting to constraint functions. The forging energy is considered as objective, and underfill and flash are taken as constraints, as given below:

| | | | |
|---|---|---|---|
| *Objective:* | *Minimize:* | forging energy $f(x_i)$ | |
| | *Subject to:* | $g_1(x_i) \leq 5\%$ | (flash percentage $\leq 5\%$ |
| | | $g_2(x_i) = 0$ | (underfill $= 0$) |
| | *Design variables:* | $x_i\,,\ i = 1, 2, 3, 4$ | |

where $x_i$ are the four critical design points. They are $y$ coordinates of the second, third, and fourth control points, and the $x$ coordinate of the fourth control point (Fig. 2).

For Kriging, the exponential correlation function is used. And, the computation of Kriging parameters $\theta_k$ (Eq. (7)), along with nonlinear optimization, is incorporated in the shape optimization algorithm. For the Kriging model, the optimization algorithm evaluates the Kriging parameters for every iteration, which takes more time than the other two models, but gives effective results.

**Table 4.** Optimization results

| Optimum design | RSM | MPA | Kriging |
|---|---|---|---|
| Design point | [73.1 34.1 14.5 −6.8] | [48.34 27.0 33.1 1.15] | [5.9 −25.8 33.4 35.0] |
| Objective function | $8.9 \times 10^4$ | $6.8 \times 10^4$ | $5.7 \times 10^4$ |
| Underfill | No Underfill | No Underfill | No Underfill |
| Flash | 4.8% | 4.2% | 2.5% |
| Strain variance | 0.6080 | 0.5904 | 0.5575 |
| Strain-rate variance | 134.3456 | 134.8718 | 65.1148 |
| Maximum strain | 5.67 | 4.20 | 3.44 |
| Minimum strain | 0.12 | 0.14 | 0.03 |
| Max strain-rate | 90.32 | 88.68 | 73.63 |
| Min strain-rate | 0.003 | 0.006 | 0.036 |

Interestingly, all of the surrogate models give three different optimum preform shapes, though all of them satisfy constraints. The optimum design variables, objective, constraint functions, and simulation results for all of these models are summarized in Table 4. The optimum preform shapes are generated using the optimum control points. RSM gives higher vertical distance for the first design parameter (i.e. $x_1$) than MPA and the Kriging optimum. The $x$ coordinate of the third control point (i.e. $x_2$) has a positive value for both RSM and MPA. But Kriging gives negative distance, which moves a lot of material towards the deep cavity. Yet, the fourth optimum design parameter in Kriging is positive, whereas RSM and MPA optimums have negative and small positive values. This positive number is driving the curve towards the right. The interaction of both a negative x-coordinate and a high positive height gives a distinct optimum shape. Forging simulations are performed using these three optimum shapes. The three optimum preform shapes and the resulting final products from these preforms are shown in Fig. 7.

The preform shape from RSM has more material than the other preforms. Hence, it produces the greatest flash percentage of 4.8%, and the flash forms on both sides (Fig. 7a). Therefore, the required forging energy is greater compared to the other preforms, and the total energy required to forge this preform is 89 000 Nmm. The MPA preform forging final shape is given in Fig. 7b. Less

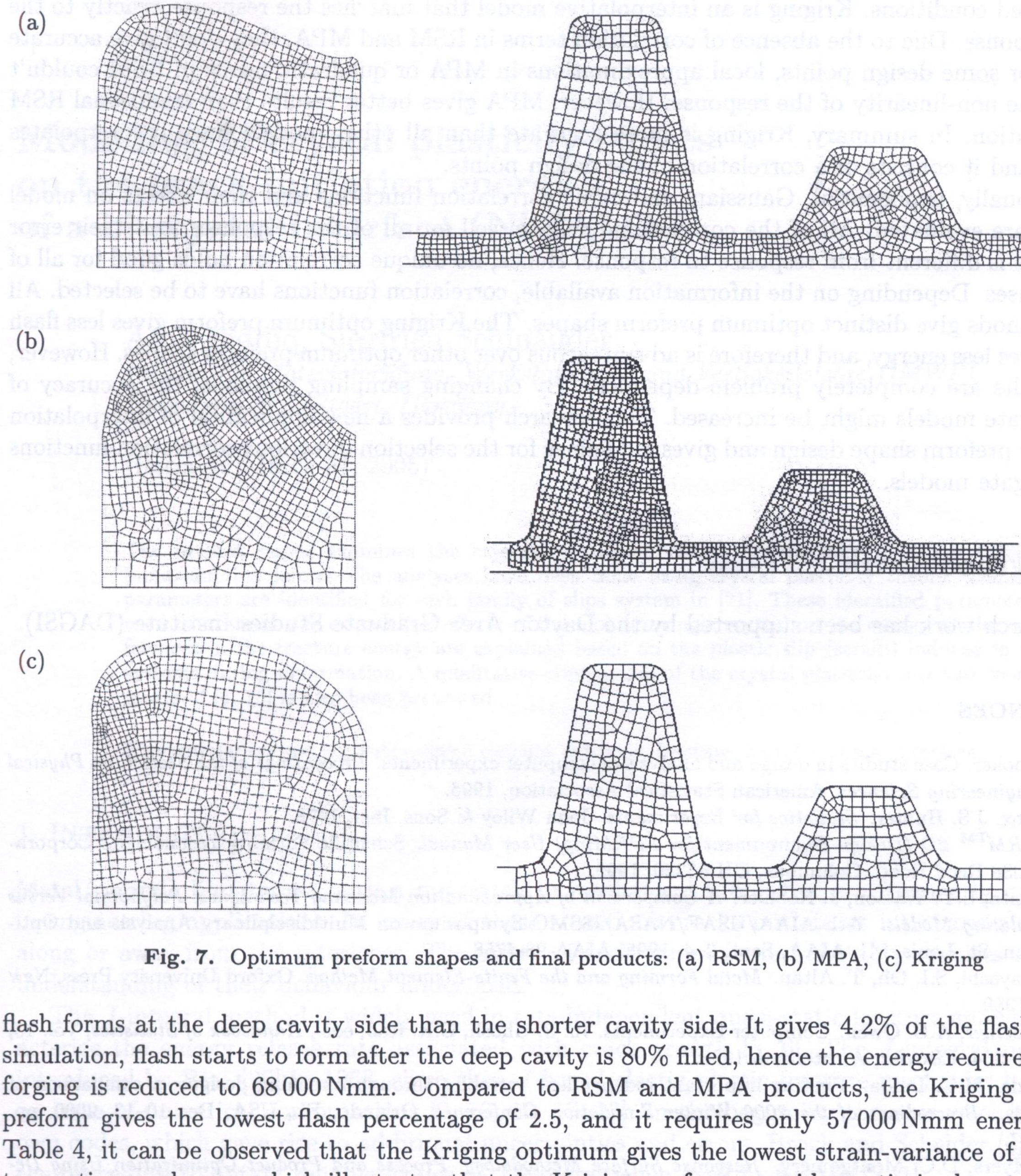


**Fig. 7.** Optimum preform shapes and final products: (a) RSM, (b) MPA, (c) Kriging

flash forms at the deep cavity side than the shorter cavity side. It gives 4.2% of the flash. In this simulation, flash starts to form after the deep cavity is 80% filled, hence the energy required for this forging is reduced to 68 000 Nmm. Compared to RSM and MPA products, the Kriging optimum preform gives the lowest flash percentage of 2.5, and it requires only 57 000 Nmm energy. From Table 4, it can be observed that the Kriging optimum gives the lowest strain-variance of the other two optimum shapes, which illustrates that the Kriging preform provides optimized material flow, thereby decreasing the probability of defects such as folds. The strain-rate variance for both the RSM and MPA are approximately the same and higher than the Kriging optimum shape. The difference between maximum, minimum values of strain, and strain-rate is least in Kriging than in the MPA and RSM optimum results. In summary, all of the optimum preforms satisfy the constraints, but the Kriging optimum preform has the advantages of less material waste and less energy utilization.

## 6. SUMMARY REMARKS

A new preform shape optimization method is explored by using an interpolative surrogate model, Kriging. Additionally, various approximation models, namely RSM and MPA, are also investigated for preform shape optimization. RSM is an approximation method in which the predicted response is not exact to the actual response, even at sampled points. MPA couldn't match the actual response at some sample points; one possible reason is that there is no local approximation constructed as per the specified conditions. Kriging is an interpolative model that matches the response exactly to the actual response. Due to the absence of correlation terms in RSM and MPA, they don't give accurate results. For some design points, local approximations in MPA or quadratic terms in RSM couldn't capture the non-linearity of the response. However, MPA gives better results than traditional RSM approximation. In summary, Kriging is more accurate than all other models since it interpolates the data and it consists of a correlation of the design points.

Additionally, exponential, Gaussian, and cubic correlation functions and their effects on model accuracy are evaluated. All of the correlations behave well for all of the responses, but their error percentage is different from response to response. Hence, no unique correlation holds good for all of the responses. Depending on the information available, correlation functions have to be selected. All of the methods give distinct optimum preform shapes. The Kriging optimum preform gives less flash and requires less energy, and therefore is advantageous over other optimum preform shapes. However, these results are completely problem-dependent. By changing sampling methods, the accuracy of the surrogate models might be increased. This research provides a new application of interpolation models for preform shape design and gives guidelines for the selection of various correlation functions and surrogate models.

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