Computer Assisted Mechanics and Engineering Sciences, **13**: 489–512, 2006.

Artificial neural networks in civil and structural engineering: Ten years of research in Poland

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(Received in the final form December 4, 2006)

This state-of-the-art paper reports the last ten year results, obtained by an informal research group completed of participants of some Polish universities at the Institute of Computer Methods in Civil Engineering (now Institute of Computational Civil Engineering) of the Cracow University of Technology, and supervised by the author of the paper. After a short introduction and brief discussion of ANNs basic ideas, the activities in five areas are described: i) ANNs as a new independent computational tool for the analysis of C&SE problems, ii) neural networks in FEM/ANN hybrid systems developed for the C&SE problems analysis, iii) various problems analyzed by ANNs, iv) modifications of BPNNs (Back-Propagation Neural Networks) and new learning methods, as well as other ANNs than those applied in problems mentioned above, v) promotion of ANNs. The representative six selected study cases are discussed: 1) concrete fatigue failure, 2) buckling of cylindrical shells with geometrical imperfections, 3) acceleration response spectra, 4) reliability of a plane frame, 5) hybrid updating of a thin-walled beam FE model, 6) hybrid identification of equivalent material in a perforated strip. Some general conclusions on prospects of ANNs applications in C&SE are given at the end of the paper.

Keywords, acronyms and abbreviations: artificial neural networks (ANNs), back-propagation neural network (BPNN), finite element method (FEM), hybrid FEM/ANN system, civil and structural engineering (C&SE), Cracow University of Technology (CUT), standing Seminar on Applications of ANNs in C&SE (the Seminar)

1. In lieu of Introduction

Biologically inspired methods of information processing have drawn attention of many scientists, researchers and engineers for over 60 years. Artificial neural networks (ANNs), fuzzy inference systems and genetic algorithms belong to so-called intelligent systems of soft computational methods [1, 2]. ANNs seem to play a special role as a new computational tool, clearly related to artificial intelligence with machine learning context [3].

The development of ANNs, initiated by the pioneering paper by McCulloch and Pitts in 1943, proceeded meandered up to publishing of the 1986 two-volume book by Rumelhart and McLelland, cf. references to historical notes in [2]. Since then a modern renaissance began with exploring of ANNs benefits. A tremendous growth in interest in the application of neurocomputing, i.e. computer simulations of NNs in civil and structural engineering (C&SE), started in 1989 when the corresponding first papers were presented at the ASCE'89 Congress and the first paper [4] was published in an archival journal. In Europe the first papers on ANNs in C&SE were presented at the Civil-Comp conference in 1991 [5]. Soon ANNs were introduced in nearly each C&SE discipline [6]. Besides many general books on ANNs, cf. e.g. [1, 2, 7], also books on ANNs applications in C&SE were published in 1990s, cf. [8–11].

The lecture [12], delivered at the 41st Polish Engineering Conference, Krynica, 1995, initiated Polish research on ANNs applications in C&SE. The extended activity in this area has been developed at Institute of Computer Methods in Civil Engineering of the Cracow University of Technology (CUT). The standing Seminar on ANNs in C&SE, called in short the Seminar, attracted many young researchers and students not only from the CUT Faculty of Civil Engineering but also from other Polish universities (Rzeszów and Zielona Góra UTs, Pedagogical University Cracow, then Silesian and Wrocław UTs, recently Białystok and Łódź UTs). This made it possible to arrange inter-institute and inter-university, multidisciplinary small research teams to start with the Seminar participants learning and investigating various C&SE topics. The Seminar participants started to attend many Polish and international conferences. The results of their research were reported in many papers and were quoted in many published general lectures and invited papers, cf. e.g. [13–22], then in books and monographs [11, 23–29].

The organization of the CISM Advanced School on ANNs in structural and material mechanics in Udine, Italy in 1998 is worth mentioning, cf. book [11]. The research has been supported by several grants of the Polish Committee for Scientific Research. In 2001, the author was awarded the Professors' Subsidy of the Polish Foundation for Science, which made it possible to offer financial support to young researchers in the frame of [30].

The main research has been focused on the regression type problems which fit well many problems of C&SE. That is why the main attention was drawn to the Back-Propagation Neural Networks (BPNNs) which are especially suitable for the regression analysis both of direct and inverse problems. Complementary features of ANNs to the standard computational methods (especially FEM) prompted us to develop very prospective hybrid FEM/ANN programs. Some main issues are discussed below on the basis of problems selected from many subjects we have investigated in the last ten years.

Below, the reader can also find some basic ideas on ANNs discussed using the example of BPNNs. In order to illustrate the research activity six representative case studies are discussed at the end of the paper. Some prospects of future applications of ANNs in C&SE are given in the final remarks.

Because of the scope of the paper only selected references are cited. In case of more detailed topics the references to quoted books or state-of-the art papers are given.

2. Some basic ideas on ANNs

ANNs have been applied especially successfully in the analysis of regression and classification problems. In the recent ten years our research has mainly concerned regression problems, i.e. mapping of real values of input data into real outputs. Depending on the formulation of input/output variables the mapping corresponds to direct or inverse analysis. More precisely, this is related to the analysis of prediction problems classified as simulation, identification and assessment problems, cf. [31]. This classification well fits problems analyzed in C&SE.

In our research we have chiefly based on the application of the feed-forward, multi-layered, error back propagation network. This network is called in literature Multi-Layer Perceptron [2], Feed-Forward NN [32], or Back Propagation Neural Network (BPNN) [11]. The acronym BPNN is used in the present paper since this network well fits the regression analysis problems and the paradigm of the NN error minimization.

A standard BPNN is shown in Fig. 1a on an example of a three layer network, composed of two hidden layers and an output layer. In Fig. 1b an individual hidden neuron is depicted, and in Fig. 1c commonly used activation functions are shown.

BPNN parameters correspond to the weights of connections w_{hj}^l and neuron biases $b_h^l = w_{h0}^l$. The values of these parameters are components of the generalized weight vector

$$\mathbf{w} = \{w_i\}_{i=1}^{W} \equiv \{w_{hj}^l \,|\, h, j = 0, 1, \ldots, H^l;\ l = 1, \ldots, \text{output}\} \in R^W \tag{1}$$

where: W – parameter space dimension.

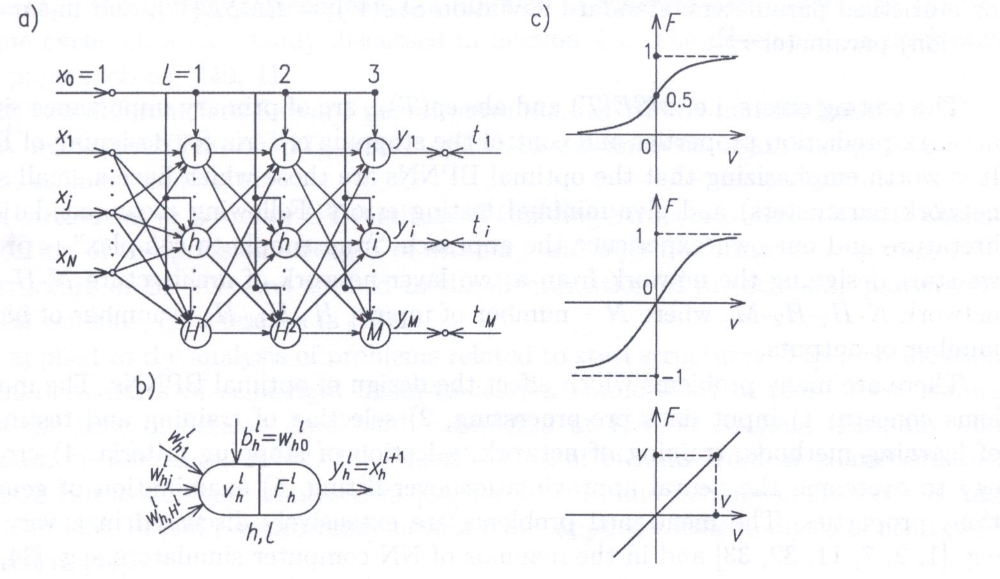

Fig. 1. a) Three-layer BPNN, b) Single neuron h in layer l, c) Binary sigmoid, bipolar sigmoid and identity activation functions, respectively

BPNN parameters are computed by means of the training (learning) process. This process explores a known set of input/output pair vector

$$\mathcal{L} = \{(\mathbf{x}, \mathbf{t})^p\}_{p=1}^{L}, \tag{2}$$

where $\mathbf{x} \in \mathcal{R}^N$, $\mathbf{t} \in \mathcal{R}^M$.

Computed values of outputs y_i are compared with known (target) values of outputs t_i , cf. Fig 1a, and they serve for the network error estimation. This error is used for tuning the network parameters by means of so-called learning methods. They are related to execution of a number of epochs, where the epoch s corresponds to the presentation of all the patterns $p = 1, \ldots, L$ (forward pass) and computation of all the network parameters is performed (error back-propagation pass). The number of epochs is controlled by a stopping criterion.

After the network is trained, its prediction (generalization) properties can be evaluated by the testing set of known patterns

$$\mathcal{T} = \{(\mathbf{x}, \mathbf{t})^p\}_{p=1}^{T}, \tag{3}$$

where the testing input/output vectors $\mathbf{x}^p/\mathbf{t}^p$ are different from those used in the training process.

The training and testing processes can be estimated by different error measures:

a) Mean-Squared-Error (MSE) and Root MES (RMSE),

$$MSE(V) = \frac{1}{VM} \sum_{p=1}^{V} \sum_{i=1}^{M} (t_i^p - y_i^p)^2, \qquad RMSE(V) = \sqrt{MSE(V)}, \tag{4}$$

where $V = L, T$ – numbers of the training and testing patterns, respectively;

b) relative average error

$$\text{avr } ep(V)_i = \frac{1}{V} \sum_{p=1}^{V} ep_i \qquad \text{where} \qquad ep_i = \left| 1 - \frac{y_i^p}{t_i^p} \right| \times 100\%; \tag{5}$$

c) statistical parameters: standard deviation $\mathrm{St}\,\varepsilon(V)_i = RMSE(V)_i$ and linear regression (correlation) parameter r_i^V.

The testing errors, i.e. $MSE(T)$ and abs $ep(T)_i$, are of primary importance since they reflect the network prediction properties and control the stopping criteria for designing of BPNN architecture. It is worth emphasizing that the optimal BPNNs are those which have a small size (low number of network parameters) and give minimal testing errors. Following recommendations in the existing literature and our own experience the approach "from simple to complex" is preferred and usually we start designing the network from a two layer network of architecture N-H-M to a three layer network N-H_1-H_2-M, where N – number of inputs, H, H_1, H_2 – number of hidden neurons, M – number of outputs.

There are many problems which affect the design of optimal BPNNs. The most important problems concern: 1) input data pre-processing, 2) selecting of training and testing sets, 3) selection of learning methods, training of network, selection of stopping criteria, 4) cross-validation strategy to overcome the neural approximation over-fitting, 5) examination of generalization (prediction) properties. The mentioned problems are extensively discussed in a very rich literature, cf. e.g. [1, 2, 7, 11, 32, 33] and in the manuals of NN computer simulators, e.g. [34, 35].

3. Research subjects

The developed research subjects have in general been related to the following four groups of problems:

1. Applications of ANNs as independent tools for the analysis of regression type problems of C&SE. The majority of analyzed problems were based on experimental evidence related to tests on laboratory models or measurements on structures in natural scale;

2. Hybrid systems FEM/ANN in which ANNs have been applied as efficient procedures or computer programs;.

3. Other simulation, identification and assessment problems, different from those discussed in groups I and II;

4. Modifications of ANNs, especially concerning their architecture and learning methods.

3.1. Applications of ANNs as independent tools

Feed-forward ANNs, discussed in Section 2, are general approximators, especially suitable in the analysis of regression problems, related to prediction of different output variables as functions of known inputs. ANNs, and especially BPNNs, have been explored to implicit modelling of various physical relationships. This means that in the mapping $f : \mathbf{x} \to \mathbf{y}$ any form of the regression function f is assumed and the trained networks are able to explore the knowledge hidden in data. That is why ANNs are called in literature "model free, data dependent" tools.

Starting with the activity on applications of ANNs in C&SE we focused on implicit neural modelling of materials relationships, cf. [36]. The first paper was devoted to the application of BPNNs to prediction of fracture toughness of dense concretes, cf. reference in [13], basing on laboratory tests carried out by Z. Rawicki in the CUT Institute of Building Materials and Structures. The nature of investigated phenomena was then better analyzed by means of neuro-fuzzy networks in [37, 38].

Great attention was paid to the analysis of fatigue failure of ordinary concretes. An extended experimental evidence of laboratory tests performed in many laboratories was collected by K. Furtak in [39]. A BPNN was successfully applied to the prediction of the number of fatigue cycles N

corresponding to damage of laboratory specimens as a function of concrete parameters and characteristic of the fatigue cycle, cf. a case study discussed in Section 4.1. The developed research was reported in several papers, cf. e.g. [40, 41].

In the paper [42], two different feed-forward neural networks, i.e. BPNN and Radial Basis NNs, and the neuro-fuzzy system ANFIS, cf. [27], were applied to the analysis of shrinkage strains in thick plates made of ordinary concretes.

Laboratory tests carried out in Wrocław UT Institute of Building Structures were a base to design BPNNs for identification of compression strength of ordinary and high-performance concretes [43].

A multi-stage structure of BPNNs was applied to the identification of pre-stressing parameters in external segmental tendons, cf. reference in [13].

ANNs were also applied to the analysis of problems related to steel structures. The identification of parameters of characteristics of semi-rigid beam-to-column connections of plane steel frames were performed on the base of Sericon data bank completed at RWTH Aachen, Germany and Białystok UT, Poland, cf. references in [44, 45]. Identification of bi- and trilinear characteristics, recommended by EC3 Eurocode, were analyzed in [44] for I beam and column cross-sections and for tube connections in [45]. In [46] a neuro-fuzzy network was applied to the analysis of semi-rigid connections presented in [44].

BPNNs were also applied to the prediction of buckling loads of axially compressed cylindrical shells with manufacturing imperfections, corresponding to the inclination of shell midsurfaces from the perfect cylindrical surfaces. The analysis was based on tests performed at the Aircraft Faculty of TU Delft, The Netherlands, and described in the report [47]. The application of ANNs to the identification of shell buckling loads was difficult because of a great number of imperfection parameters. The successful results were obtained in [48] due to the input data compression, carried out by the replicators formulated as the autoassociated BPNNs. This problem is shortly discussed in Section 4.2.

A great effort was devoted to applications of ANNs in the analysis of structural dynamics problems. Extensive research has been developed in Rzeszów University of Technology (RUT), Poland, under supervision of Professor L. Ziemiański. The research has been developed on the base of laboratory experiments carried out in Laboratory of RUT Chair of Structural Mechanics. The measured dynamic responses in time domain or responses transformed to spectral spaces were efficiently used to many identification problems discussed in the monograph [24] and in Chapter 9 of the book [28]. Identification of damage in beams was analyzed in W. Łakota's monograph [23]. An interesting application of the proposed method of identification of loads applied to elastoplastic beams was performed due to the use of measured dynamic structural responses as NNs inputs [49]. The identification of an additional mass placement was considered in a study case, taken from the Ph.D. dissertation by G. Piątkowski [50], discussed briefly as a study case in [21, 27]. Other problems of damage identification were analyzed in books [23, 24, 28] and several papers, cf. e.g. [51].

An extensive research has been developed by K. Kuźniar from the Pedagogical University of Cracow on applications of ANNs to the analysis of vibration problems of buildings subjected to paraseismic excitations. The research based on records of vibrations measured on real buildings in regions of mining exploitation (Upper Silesian Coalfield and Legnica-Głogów Copperfield), supervised and coordinated by Professors R. Ciesielski and E. Maciąg from the CUT Institute of Structural Mechanics, cf. references in [26]. From among many problems considered in K. Kuźniar's monograph [26] and in Chapter 11 of the book [29] the prediction of fundamental periods of vibrations of prefabricated buildings, displacement and response spectra are worth emphasizing. The results of research were published in highly evaluated post-conference proceedings, cf. references in [26, 29] and in reference journals, cf. e.g. [52, 53].

Generating of response spectra, also related to the soil-structure interaction, cf. case studies discussed in [21, 22] were especially valuable for the engineering practice. In Section 4.3, a study case concerns the neural prediction of acceleration response spectra (ARS) at the ground level from paraseismic excitations related to the mining tremors. Acceleration response spectra caused by the traffic excitations were also analyzed, cf. reference in [13].

3.2. Hybrid systems FEM/ANN

Hybrid systems, combining various computational methods, were highlighted in [54] as prospective new approaches in so-called Computational Structures Technology. ANNs have complementary features to FEM. These features are related to very great numerical efficiency of trained ANNs and easiness in analyzing nonlinearities at the implicit modelling of physical relationships. ANNs are also suitable for the inverse analysis and investigation of unilateral constraints.

A great operational efficiency of BPNNs was explored in the hybrid Monte Carlo method. This approach, suggested in [55], lies in the application of FEM for computing the training and testing patterns which are then used to design BPNNs formulated to generate the MC samples. The hybrid approach has been developed by J. Kaliszuk from the Zielona Góra University, Poland, in her Ph.D. dissertation [56] and applied to the reliability analysis of structures. It was proved on examples of plane frames analysis, steel girders and cylindrical panels, cf. study cases in [21, 22], that the hybrid MC approach is much less numerically costly than the application of FEM only for generating the MC samples, cf. study case discussed in Section 4.4.

BPNNs were used as a part of hybrid systems for updating FE models, cf. [4]. In the frame of a four stage algorithm, cf. Section 4.5, a BPNN is applied to the analysis of an inverse problem of computation of values of control parameters, needed for the updating of an initial FE model. Experimental data (results of tests on laboratory models of measurements on real structures) are used for the calibration of the control parameters by means of the trained BPNN. The numerical efficiency of the proposed hybrid MC approach was examined by B. Miller in his Ph.D. dissertation [57] and related to various study cases analyzed in several papers, e.g. [27, 51, 58].

A great deal of attention was paid to formulation and implementation of hybrid FEM/ANN programs for the analysis of boundary value problems of solids with nonlinear constitutive equations. In [59] a great BPNN was applied to the formulation of a procedure to analyze the Return Mapping Algorithm (RMA) in the analysis of equations of elastoplastic material with HMH yield surface and isotropic strain-hardening. The BPNN was trained and tested off line and then applied in [59] to the analysis of plane stress problems. A generalized RMA was developed in [60] for the analysis of bending of elastoplastic plates. The numerical efficiency of different procedures applied in the plate bending analysis was examined in [61].

The above mentioned problems have recently been developed by E. Pabisek from the CUT Institute of Computer Methods in Civil Engineering. The BPNNs applications to identification of a simple equivalent (homogenized) material models for real structures are especially worth emphasizing. This problem, discussed in [62, 63], is related to the formulation of a Neural Network based Constitutive Model (NNCM) as a BPNN trained on patterns generated on line, taking into account measurements of structural displacements in selected control points, cf. a study case discussed in Section 4.6. The algorithms examined in [64] were applied to the identification of NNCMs in simple structures, cf. [62]. Some new results concerning the BPNNs training on patterns generated by the hybrid FEM/NNCM programs are presented in Section 4.6.

A hybrid FEM/BPNN program was analyzed in [65] for the simulation of waves transmission in elastic solids using BPNNs to formulate artificial (transparent) boundary conditions.

Besides BPNNs also the Hopfield–Tank recurrent network (HTNN), cf. [2, 11], was applied in hybrid FEM/HTNN programs to analyze elastic and elastoplastic plane stress problems with unilateral constraints, cf. reference in [14]. The Panagiotopoulos approach was explored, i.e. FEM matrices were substituted into HTNN to formulate evolutionary equations as a differential analogue of the FEM algebraic equations, cf. [11]. This approach was also applied in [66] to formulate interfaces at boundaries of FE systems in order to consider unilateral constraints with Coulomb and non-monotonic models of friction.

3.3. Applications of ANNs in the analysis of various problems

The Seminar participants were also involved in the analysis of other problems than those listed in Sections 4.1 and 4.2. The FWNN was applied to prediction of water absorption in the sealing process in a dam ground curtain [67]. BPNNs were explored in the inverse analysis for health assessment of concrete dams [68]. In [69] BPNNs were applied to the identification of dynamic deformation modulus for non-cohesive soils.

In the frame of cooperation with the Catholic University Leuven, Belgium ANNs were applied in a biomechanics problem. In [70] BPNNs and the neuro-fuzzy system ANSYS were explored to predict the proximal femur strength.

The feed-forward ANNs (BPNNs and Radial Basis Functions) were applied in [71] for the prediction of the lie land of the town Zielona Góra in Poland.

Radial Basis Functions were used in [72] for prediction of the assessment of the technical state of old flat buildings in Zielona Góra, Poland. A problem of valuation of building lots in Cracow, Poland was analyzed by BPNNs in [73].

3.4. Modifications of ANNs and methods of their learning

For the analysis of the problems described above the standard ANN computer simulators were applied – mostly SNNS (Stuttgart Neural Network Simulator [34]) and the MATLAB Neural Network Toolbox [35]. Our own software was related first of all to programs for interaction with the mentioned simulators in the hybrid FEM/ANN systems and programs described in Section 3.2.

What is worth emphasizing is the modification of BPNN for computation of membership functions in the network FWNN (Fuzzy Weight Neural Network) parameters, cf. [38]. This network enables us to obtain predictions in intervals, instead of crisp outputs computed by standard BPNNs. This approach was applied in the analysis of problems of concrete mechanics, cf. Section 4.1, and semi-rigid steel connections.

In order to improve the accuracy of neural approximation the Kalman filters, applied in the control theory, have recently been used as an advanced method of BPNNs learning. The application of Kalman filtering algorithms, formulated in [32], enables an increase of the accuracy of response spectra prediction in [74], as discussed briefly in Section 4.3.

An important problem of data pre-processing was analyzed in some papers quoted in the present paper, cf. e.g. [27, 48, 52, 53]. Besides application of a replicator for the input data compression, cf. Section 4.2, also the Principle Component Analysis was used in [75, 76].

Besides the feed-forward ANNs and Hopfield-Tank recurrent neural networks the neuro-fuzzy system ANFIS (Adaptive Neuro-Fuzzy Inference System) was also used to the analysis of some problems of experimental mechanics and biomechanics [77]. This system is in fact a feed-forwards network since it performs the mapping $\mathbf{x} \to y$ of crisp data.

Quite recently we have turned attention to probabilistic neural networks, namely to Bayesian NNs, cf. [3, 33]. The advantages of this approach are associated with the computation of both mean values and the probability density distribution of outputs. Another advantage is related to the penalization of the over-fitting phenomena. The application of Bayesian inference approach was demonstrated in [78] where the analysis of a soil-structure interaction problem related to the prediction of response spectra was made. The Gaussian Process Analysis (GPA) was efficiently applied to predict fatigue failure of concretes in paper [79], published in this CAMES special issue. A similar approach was applied in [76], where GPA was used for the identification of characteristic length of microstructure in heterogeneous material.

3.5. Promotion of ANNs

A real achievement related to the development of research on applications of ANNs in C&SE was promotion of ANNs at the Polish universities. A visible result of scientific activity was also presentation of ANNs at various scientific conferences in Poland and abroad. An increasing number of papers on ANNs applications in various disciplines of civil engineering was presented at the Polish Conferences on Civil Engineering in Krynica, Poland, at the Polish Conferences on Computational Methods in Mechanics and at international congresses of Computational Mechanics (CM). An important promotion of ANNs was made due to invited lecturers and organization of mini-symposia and special sessions, see e.g. the 2nd and 3rd European ECCOMAS Conferences on CM, Cracow 2001, Lisbon 2006; Asian-Pacific Congress of CM, Sydney 2001; V World IACM Congress, Vienna 2002; MIT Conferences on Computational Fluid and Solid Mechanics in 2003 and 2005. The International Symposium on Neural Networks and Soft Computing in Structural Engineering was also organized in Cracow in 2005 in the frame of ECCOMAS Thematic Conferences (a special issue of CAMES is in preparation).

We should also mention the organization and delivering of lectures at the CISM Advanced Schools in Udine, Italy, devoted to ANNs applications in mechanics of structures and materials, cf. chapters in the corresponding books [11, 27]. The same concerns the course organized on soft computing by the Polish Association for CM in Rzeszów in 1999 [58].

ANNs were introduced in the syllabus for graduate and Ph.D. students at the Faculty of Civil Engineering of CUT in the frame of special one semester courses. Lectures on ANN engineering applications were delivered at the invitation of various Polish and foreign universities, including a series of lectures for Ph.D. students of the Heriot-Watt University, Edinburgh, UK, and for graduate students of the Budapest UTE and University of Florence in the frame of the CEPUS and Socrates programs, respectively.

4. Study cases

4.1. Concrete fatigue failure

Fatigue failure is defined as the number of load cycles N causing fatigue damage of plain concrete specimens. In [39] there was collected evidence corresponding to over 400 cubic or cylindrical specimens, tested in many laboratories in the years 1934–80. All the specimens were subjected to compressive loads within cycles at fixed frequencies.

The fatigue failure N can be related to the mechanical properties of concrete and to the characteristics of the load cycle. A relationship between N and four input parameters x_j was derived in [39] as an empirical formula $F(N, x_j) = 0$,

$$\log N = \frac{1}{A}\left[\log\left(1.16\,\frac{C_f}{\chi}\right) + \log\left(1 + BR\log N\right)\right], \tag{6}$$

where five basic variables were used: N – number of cycles associated with fatigue damage, $R = \sigma_{\min}/\sigma_{\max}$ – ratio of minimal and maximal stresses in a loading cycle, f [Hz] – cycle frequency, $\chi = f_{cN}/f_c$ – ratio of fatigue strength of concrete f_{cN} and strength of concrete in compression f_c. The main variables C_f and B are functions of basic variables.

Laboratory results with crisp variables, correspond to $P = 218$ concrete specimens. This set was randomly split into $L = 118$ and $T = 100$ training and testing patterns, respectively. The BPNN: 4-5-4-1 was designed for the input vector $\mathbf{x} = \{f_c, \chi, R, f\}$ and output scalar variable $y = \log N$. The sigmoid bipolar neurons in the hidden layers and linear output were used, cf. Fig. 1.

In Table 1 selected computational errors, taken from [41] are shown. Besides the errors obtained by Eq. (6) errors for the network BPNN are shown. Then the errors for the neuro-fuzzy network FWNN (Fuzzy Weight NN) are written, obtained for the value $\mu_y = 1.0$ of the output membership function (it corresponds to the α-cut $\alpha = 1.0$ of triangular membership function, cf. [38]). In

Table 1. Errors of fatigue failure predictions

Simulator	avr $ep(V)$ [%]		$r(V)$		$\mathrm{St}\varepsilon(V) = RMSE(V)$	
	L	T	L	T	L	T
Formula (6)	17.7	26.3	0.843	0.843	0.873	0.991
BPNN: 4-5-1	13.6	20.5	0.871	0.855	0.701	0.777
FWNN: 4-5-5 for $\alpha = 1.0$	14.0	19.7	0.879	0.861	0.700	0.772
ML GPR: 1-15-1	12.9	20.4	0.868	0.879	0.724	0.681

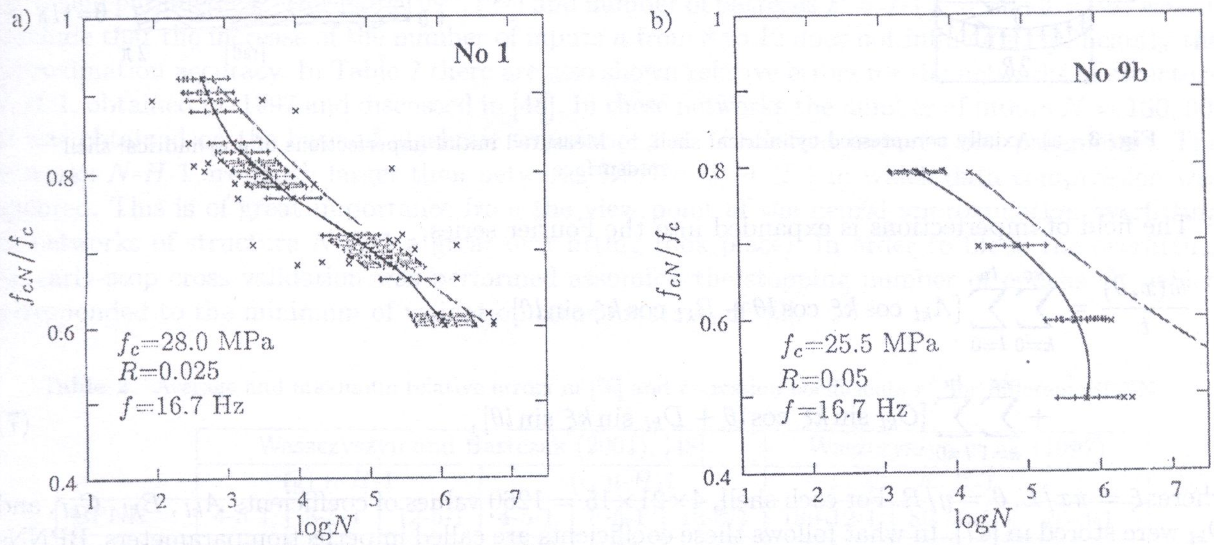

Fig. 2. Neural crisp and interval predictions (for $\alpha = 1.0$, 0.9, 0.75) of relation $\chi = f_{cN}/f_c - \log N$ for data taken from data banks: a) No. 1, b) No. 9b

Table 1 there are also shown errors obtained for the Bayesian neural network ML GPR: 4-15-1, taken from [79].

The errors obtained for Eq. (6) and different neural networks seem not to be too distant from each others (this concerns especially the linear regression coefficients r^V). The applied ANNs enable us to obtain distributions of predicted variables fitting much better experimental results than those calculated by means of the empirical formula (6). In Fig. 2 , the relationships $f_{cN}/f_c - N$ are shown for the data banks Nos. 1 and 9b taken from [39]. In the same figures there are shown results obtained by the network FWNN for different α-cuts. The used intervals cover the majority of laboratory tests.

The neuro-fuzzy network FWNN was also used in [41] for the prediction of concrete fatigue failure for the experimental evidence completed in [39] for intervals of the input variables f_c and f.

4.2. Buckling of cylindrical shells with geometrical imperfections

Axially compressed cylindrical shells are very sensitive to geometrical imperfections. In [48] the geometrical imperfections were analyzed as related to an inclination of the shell midsurface from the perfect cylindrical surface. Such imperfections are an unavoidable effect of manufacturing process and they significantly decrease buckling load of axially compressed shells.

Results of buckling tests on laboratory specimens of cylindrical shells subjected to axial compression were stored in the initial imperfection data bank arranged at the Faculty of Aerospace Engineering of TU Delft, the Netherlands [47]. Geometrical and material data of a group of 33 specimens are shown in Fig. 3a, and an example of the imperfect shell midsurface is shown in Fig 3b.

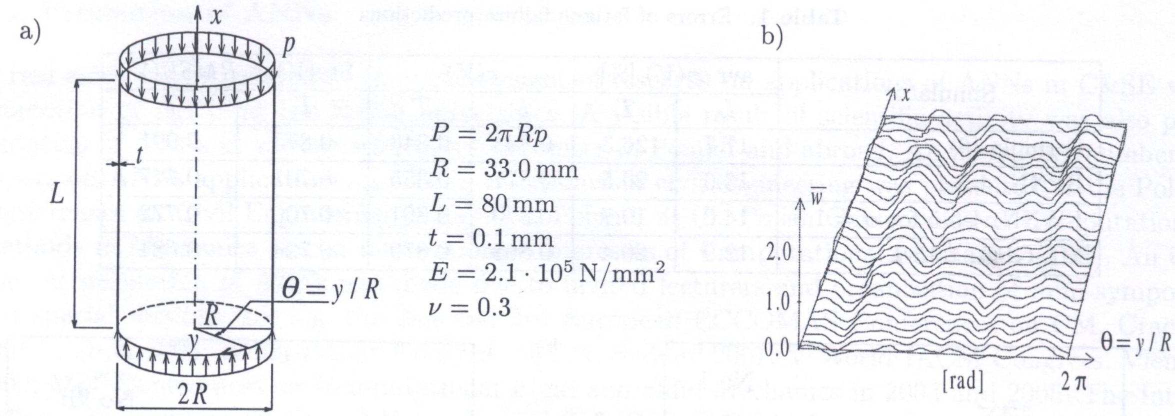

Fig. 3. a) Axially compressed cylindrical shell, b) Measured initial imperfections of a cylindrical shell midsurface

The field of imperfections is expanded into the Fourier series,

$$\frac{\bar{w}(x,\theta)}{t} = \sum_{k=0}^{kc}\sum_{l=0}^{lu}[A_{kl}\cos k\xi\,\cos l\theta + B_{kl}\cos k\xi\,\sin l\theta] + \sum_{k=1}^{ks}\sum_{l=0}^{lu}[C_{kl}\sin k\xi\,\cos l\theta + D_{kl}\sin k\xi\,\sin l\theta], \tag{7}$$

where: $\xi = \pi x/L, \theta = y/R$. For each shell, $4\times 21\times 15 = 1260$ values of coefficients A_{kl}, B_{kl}, C_{kl}, and D_{kl} were stored in [47]. In what follows these coefficients are called imperfection parameters. BPNNs were used for the mapping of condensed imperfection parameters into the dimensionless buckling load $y = P_{\mathrm{exp}}/P_{\mathrm{cl}}$, where $P_{\mathrm{cl}} = 2\pi E t^2/(3-\nu^2)^{1/2}$. In order to diminish the number of inputs, the data compression procedure was applied. For this purpose a BPNN is used as a replicator , cf. [2]. In Fig. 4 the replicator is shown as an autoassociative network with one hidden layer composed of $n < N$ sigmoid neurons, where N, n – initial (not compressed) and compressed number of inputs. After training the replicator can be split into two BPNNs which can be used as the data compressor and decompressor.

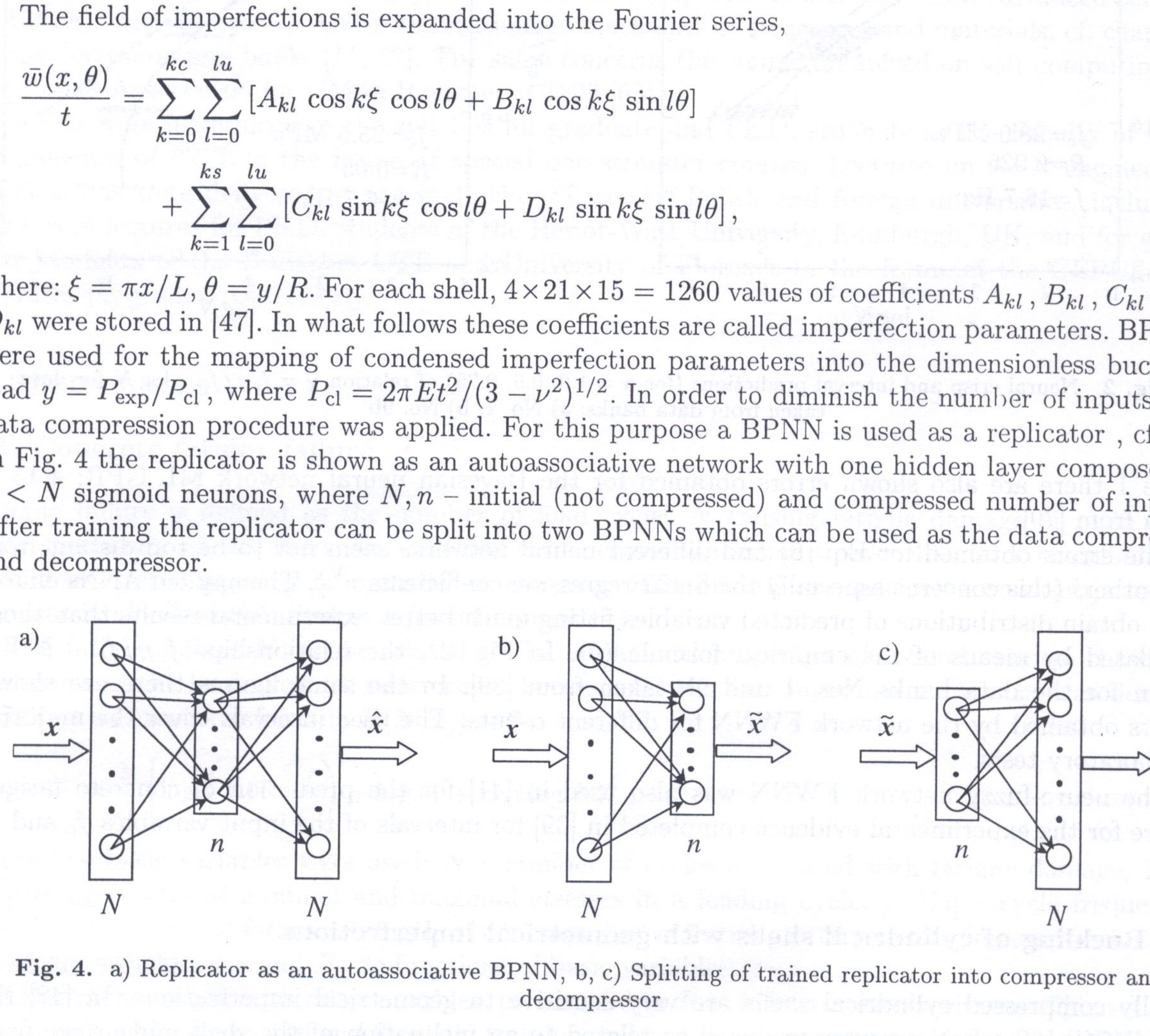

Fig. 4. a) Replicator as an autoassociative BPNN, b, c) Splitting of trained replicator into compressor and decompressor

In the paper [48] a replicator was formulated as BPNN: 360-n-360, where the number of inputs $N = 360$ corresponds to 90 coefficients A_{kl}, B_{kl}, C_{kl}, and D_{kl} in the Fourier series (7) for $k = 0, 1, \ldots, 5$ and $l = 0, 1, \ldots, 14$. The number $n = 4, 8, 12$ was considered for the data compressed values. Two types of replicators were trained. The general replicator (g) was trained by means of 30 specimens and tested by 3 specimens selected, as in paper from 1997, referred in [13]. The individual replicator (i) was initially trained by means of all specimens to have initial parameters of network parameters. Then the individual replicator training was ended for one specimen only. In this way input data were compressed for each specimen individually.

The compressed data were used as inputs in BP neural networks of structures (g) n-H-1 and (i) n-H-1, where $n = 4, 8, 12$ – number of input data compressed by general and individual compressors, respectively. These BPNNs were formulated as networks with sigmoid neurons in the hidden layer and with linear neuron in the output layer. Then they were trained and tested by means of the same patterns as those described in [13].

In Table 2 relative errors are put together for BPNNs of structure (g) n-5-1 and (i) n-5–1. It is clear that the network BPNN for (i) $n = 8$ gives satisfactory approximation. Looking at the regression parameter r^P (for pairs $(y^{(p)}, t^{(p)})$ and number of patterns $P = L+T = 30+3 = 33$) we can conclude that the increase of the number of inputs n from 8 to 12 does not influence significantly the approximation accuracy. In Table 2 there are also shown relative errors for the networks of structure N-H-1, obtained in 1997 and discussed in [48]. In these networks the number of inputs $N = 180$, 90, 20 was obtained on the basis of algebraic transformation of selected imperfection parameters. The networks N-H-1 are much larger than networks BPNN: (i) n-H-1 in which data compression was explored. This is of great importance from the view point of the neural approximation overfitting (in networks of structure N-H-1 a great over-fitting took place). In order to block the overfitting the early-stop cross validation was performed assuming the stopping number of epochs S^*, which corresponded to the minimum of validation error function, cf. [2].

Table 2. Average and maximum relative errors in [%] and regression coefficients r^P for different BPNNs

	Waszczyszyn and Bartczak (2001), [48]						Waszczyszyn et al. (1997)		
	(g) n-H-1			(i) n-H-1			N-H-1		
BPNN	4-5-1	8-5-1	12-5-1	4-5-1	8-5-1	12-5-1	180-12-1	80-50-1	20-3-1
avr $ep(P)$	6.66	3.59	4.43	5.27	2.38	1.18	0.64	3.05	5.07
r^P	0.595	0.897	0.828	0.699	0.943	0.979	–*)	–*)	–*)
S^*	2030	800	280	120	280	480	200	64	200

*) regression coefficients r^P were not computed in paper published in 1997, cf. reference in [48]

4.3. Acceleration response spectra

Response spectra are often applied in structural design as for determining resistance of existing buildings, cf. references in [26]. The response spectrum is computed on the basis of the equilibrium of motion of a 1DOF oscillator assuming angular frequency $\omega_i = 2\pi/T_i$, damping fraction ξ and kinematic excitation corresponding to ground acceleration $a_g(t)$. In what follows, we consider only the Acceleration Response Spectrum (ARS) using the definition

$$\beta(T_i) = \frac{S_a(T_i)}{a_{g\max}}, \qquad \text{for} \quad S_a(T_i) = \max_j |a(t_j; T_i, \xi) + a_g(t_j; E, r_e)|. \tag{8}$$

The accelerograms of surface waves, caused by mining tremors of two parameters, i.e. the tremor energy $E \in [2 \times 10^4, 4 \times 10^6]$ J and epicenter distance $r_e \in [0, 1200]$ m, were recorded on the soil level at monitored buildings of the Upper Silesian Coalfield region, Poland. A set of 145 ARS was randomly selected and then computed in [81] in discretized form $\beta(T_k)$ for pseudo-time parameters $k = 1, 2, \ldots, 198$. This made a set of $P = 145 \times 198 = 28710$ patterns. The 145 sets were randomly split into $ARSL = 113$ and $ARST = 32$ training and testing sets, respectively.

The following input vector and output scalar variables were adopted in [81],

$$\mathbf{x} = \{E, r_e, T_k\}, \qquad y = \beta_{k+1} = \beta(T_{k+1}). \tag{9}$$

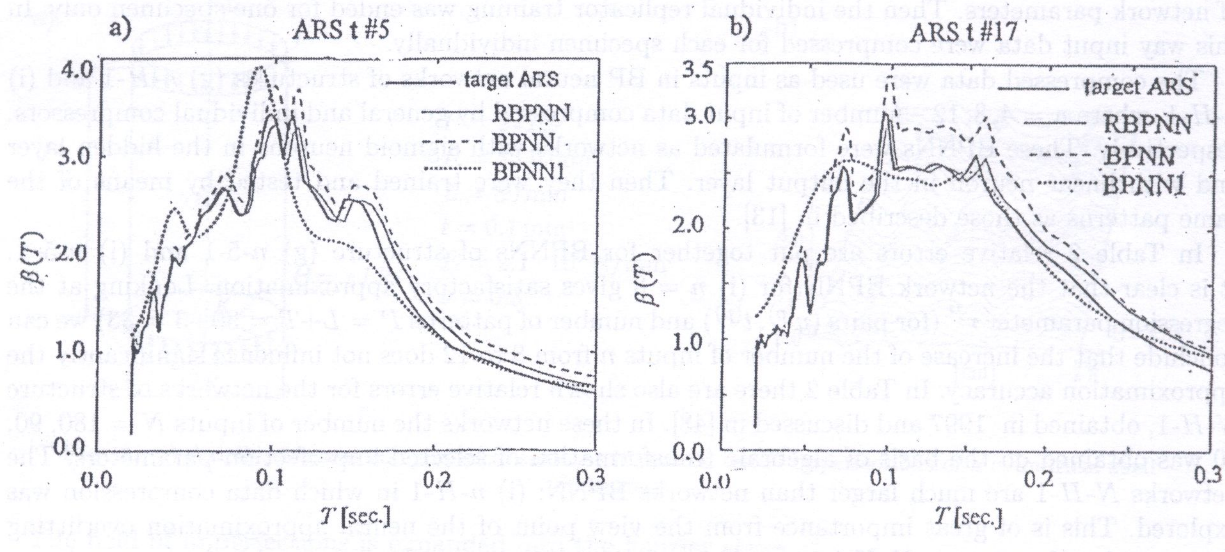

Fig. 5. Target and neurally computed spectra for selected testing accelerograms Nos. 5 and 17

From among BPNNs designed in [81] the smallest network BPNN1 had the architecture 3-12-6-1 corresponding to $NNP = 133$. The network was trained by the standard learning method Rprop (Resilient-Propagation), cf. [35]. After $S = 10000$ epochs the training and testing errors $MSEL = 6.0 \times 10^{-3}$ and $MSET = 7.8 \times 10^{-3}$, respectively, were obtained. In Fig. 5 two testing spectra ARS t # 5 and ARS t # 17 are shown. Great differences are visible between target and predicted ARS, especially for low values of vibration periods.

In the paper [74] the time delay variable β_k was used instead of T_k, so the following input vector was formulated,

$$\mathbf{x} = \{E, r_e, \beta_k\}, \tag{10}$$

applying the output variable $y = \beta_{k+1}$ as in [74]. Using theses inputs and output variables the network was significantly diminished to BPNN: 3-15-1 with $NNP = 76$. Applying the Rprop learning method and $S = 10000$ epochs the errors were diminished to $MSEL = 0.40 \times 10^{-3}$ and $MSET = 0.56 \times 10^{-3}$. In the paper [74] the standard network BPNN was modified to the Recurrent BPNN (RBPNN) shown in Fig. 6.

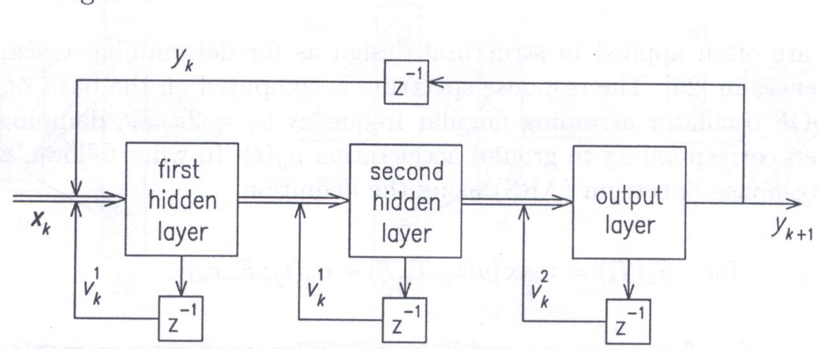

Fig. 6. Recurrent Back-Propagation Neural Network (RBPNN) with internal time-delay connections and autoregressive input

This modification lies in the introduction of internal time-delay connections (feedback) corresponding the neuron potential, similarly as that applied in the Elman NN, cf. [82]. In case the network RBPNN: 3-15-1 was trained by the Rprop method during $S = 10000$ epochs the errors were decreased to $MSEL = 0.40 \times 10^{-3}$ and $MSET = 0.45 \times 10^{-3}$. The error graphics related to the application of BPNN and RBPNN are shown in Fig. 5.

The paper [74] was devoted to application of Kalman filtering as an advanced method for the BPNNs learning. This approach is supported on the equations of discrete stochastic processes and the computation of 'a posteriori' values of state variables $\mathbf{w}(k)$ and $y(k)$ applying the following recurrent equations, cf. [32],

$$\{\mathbf{w}_i(k+1), v_i(k+1)\} = \{\mathbf{w}_i(k), v_i(k)\} + \boldsymbol{\omega}(k), \qquad y(k) = h(\mathbf{w}(k), \mathbf{v}(k), \mathbf{x}(k)) + \nu(k), \tag{11}$$

where k – discrete pseudo-time parameter, i – the number of neuron in ANN; $\mathbf{w}(k) = \{\mathbf{w}_i(k), v_i(k) \mid i = 1, 2, \ldots, n\}$ – state vector (one-column matrix) corresponding to the set of vectors $\mathbf{w}$ of synaptic weights and biases, and neuron recurrent outputs $\mathbf{v}$ for n neurons of NN; h – nonlinear function of input/output relation; $\mathbf{x}, y$ – input vector and output variable; $\boldsymbol{\omega}(k)$, $\nu(k)$ – Gaussian process and measurement noises.

In [74] the RDEKF (Recurrent Decoupled Kalman Filter) algorithm was applied in RBPNN: 3-15-1. The use of RDEKF as a learning method led to errors $MSEL = 0.41 \times 10^{-3}$ and $MSET = 0.37 \times 10^{-3}$ after $S = 100$ epochs. Unfortunately, such great acceleration of the training procedure caused about 37% increase of CPU time than the application of Rprop learning method and $S = 10000$ epochs.

4.4. Reliability of a plane frame

The probabilistic analysis has to be applied in the assessment of a structural system, cf. references in [56]. The Monte Carlo (MC) methods are usually explored and MC samples are simulated on the basis of the Finite Element Method (FEM). Thousands of needed simulations make the FEM analysis very costly. That is why in [55] BPNNs for computing MC samples were suggested. In such a hybrid approach FEM is used only to compute patterns for the BPNN training and testing.

A stationary type structural problem is analyzed with the probability of reliability Q defined for a fixed time by the following relationship,

$$Q = \text{Prob}\,\{G(R, S) > 0\} \equiv \text{Prob}\,\{R > S\} = \int_{G(X)>0} f(X)\,\mathrm{d}X, \tag{12}$$

where: R – resistance of structure, S – actions (loads) applied to structure, $\mathbf{X} = [\mathbf{X}^R, \mathbf{X}^S]$ – vector of random variables.

The Monte Carlo simulation corresponds to computation of integer in Eq. (12). Following the law of large numbers the Classical Monte Carlo (CMC) estimator of the probability of reliability is

$$\bar{Q} = \frac{1}{NMC} \sum_{i=1}^{NMC} I(X_i), \tag{13}$$

where NMC – the number of CMC samples.

A simple plane frame, called calibrating FRAME I, with data taken from [83], was analyzed in [84]. The geometrical and material data are shown in Fig. 7.

Yield points of the frame beams and columns, R_b and R_c, respectively, as well as the global inclination ψ_0 were adopted as random variables. The following mean values and standard deviations were adopted in [84]: $\bar{R} = \sigma_0 = 300\,\text{MPa}$, $\sigma_R = 30\,\text{MPa}$, $\bar{\psi}_0 = 0$, $\sigma_\psi = 0.00394$.

The single parameter load vector $\mathbf{P}_{\text{ult}} = \lambda_{\text{ult}}\mathbf{P}^*$ was assumed, where the reference vector $\mathbf{P}^* = \{p_1, p_2, H_1, H_2\} = \{68.89, 44.28, 28.68, 14.35\}$ was calibrated in [84] to have $\lambda_{\text{ult}} = 1$ for $\psi_0 = 0$. In this way the load parameter λ_{ult} was adopted as the function $I(X_i)$ in Eq. (13).

In order to compute the training and testing patterns the FE program ELPLAF-v1 was taken from [85]. The program is based on the II order nonlinear theory and small elastoplastic uniaxial stresses. The components of the cross-sectional consistent tangent matrix were computed by the Simpson formula, cf. Fig. 7b. The number of training patterns was computed by the formula $L = (n+1)^N$, where: N – dimension of input space, n – number of equal part division of the N-cube side.

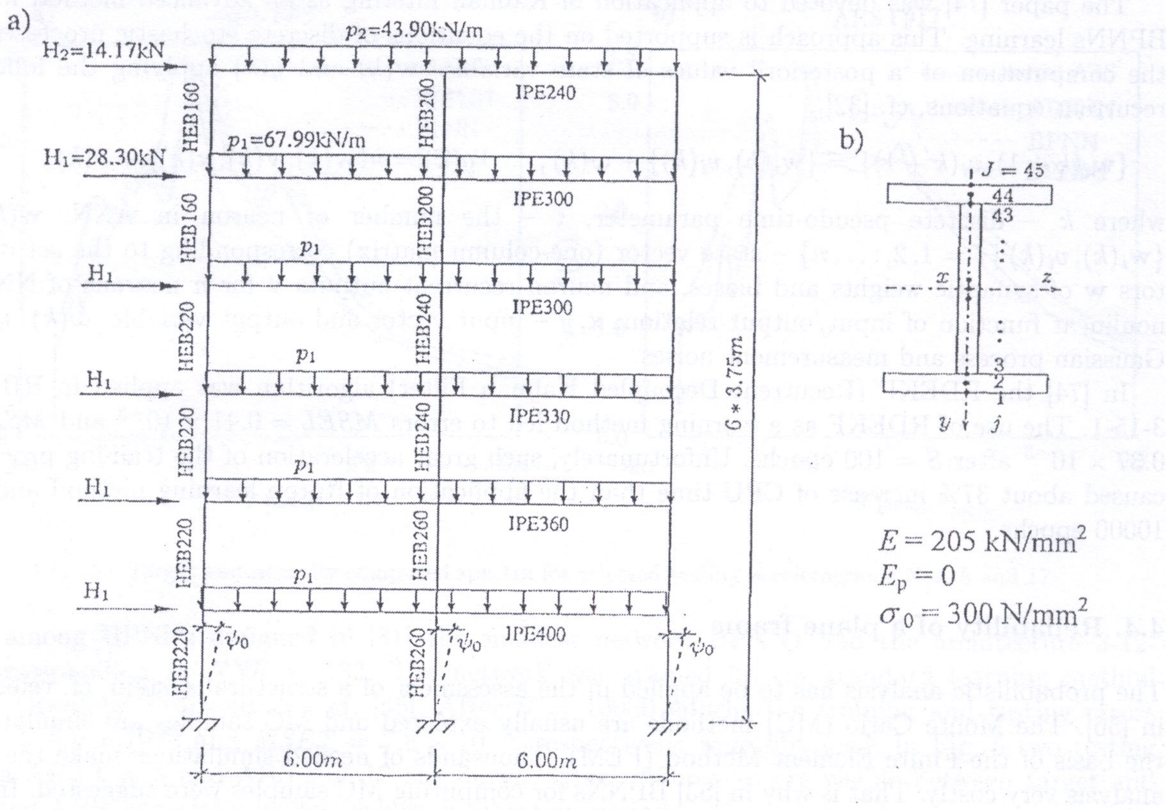

Fig. 7. a) Scheme of FRAME I, b) Simpson points $j = 1, \ldots, 45$ and mechanical data for I cross-section

The lengths of the cube sides correspond to the following ranges of intervals: $R \in [\bar{R} - 4\sigma_R\,,\ \bar{R} + 4\sigma_R]$, $\psi_0 \in [-2\bar{\psi}_0\,,\ 2\bar{\psi}_0]$. The testing patterns were randomly selected from the above listed interval assuming Gaussian pdf $N(\mu, \sigma^2)$.

Two numerical examples were considered with the scalar output $y = \lambda_{\text{ult}}$ and the following input vectors,

$$1)\ N = 2: \ \ \mathbf{x} = \{R_b\,,\ R_c\}, \qquad 2)\ N = 3: \ \mathbf{x} = \{R_b\,,\ R_c\,,\ \psi_0\}. \tag{14}$$

The number of adopted training patterns L is listed in Table 3. For these patterns and randomly selected $T = 2000$ patterns the values of ultimate load parameter λ_{ult} were computed by the program ELPLAF-v1. The average CPU time of the PC used (see detailed description in [84]) was circa 27 s per one computed pattern.

Table 3. Errors of neural approximation

N	n	L	BPNN:	$RMSE \times 10^2$		avr ep [%]	
				L	T	L	T
2	2	9	2-2-1	2.70	7.14	2.44	3.89
	4	25	2-5-1	1.39	1.73	1.39	1.39
	8	81	2-6-1	0.60	0.87	0.52	0.59
	32	1089	2-5-4-1	1.71	0.69	1.26	0.41
3	4	125	3-8-1	0.98	0.90	0.67	0.69
	16	4913	3-20-6-1	0.71	0.70	0.38	0.38

From among many examined networks only some selected networks are listed in Table 3 in order to show that even very small BPNN networks 2-6-1 and 3-8-1 give approximation quite satisfactory from the viewpoint of testing errors.

In Fig. 8 two reliability curves are shown, obtained for BPNN networks corresponding to $N = 2$ and 3 random input variables. These curves are very close to each other so the conclusion is that adding the third variable $X_3 = \psi_0$ does not affect significantly the reliability of the frame in question. In Fig. 8 the case $N = 2$ is investigated only applying small networks. It can be concluded that the network BPNN: 2-6-1 gives results accurate enough, comparable with those obtained by means of a great network BPNN: 2-5-4-1.

It was stated that the selection and computing 57 points at the reliability curves shown in Fig. 8 needed the CPU time circa 34 s and 58 s when the trained networks BPNN: 2-5-4-1 and 3-20-6-1 were applied, respectively. It was evaluated that the computation of 1×10^5 CMC samples by the considered BPNNs needed on average about 0.46 s or 1.10 s, correspondingly. The total CPU time written in Table 4 was needed for the computation of one reliability curve by the hybrid FEM/BPNN method and a hypothetical time corresponds to the computer simulation of CMC samples by the FE program only. It was evaluated that the application of hybrid approach enables a decrease of the CPU time of order 10 times in comparison with the FE program for simulation of 100000 CMC samples. In case we use $L = 125$ and $T = 200$ patterns and 100000 CMC simulations this time can be decreased 34 times.

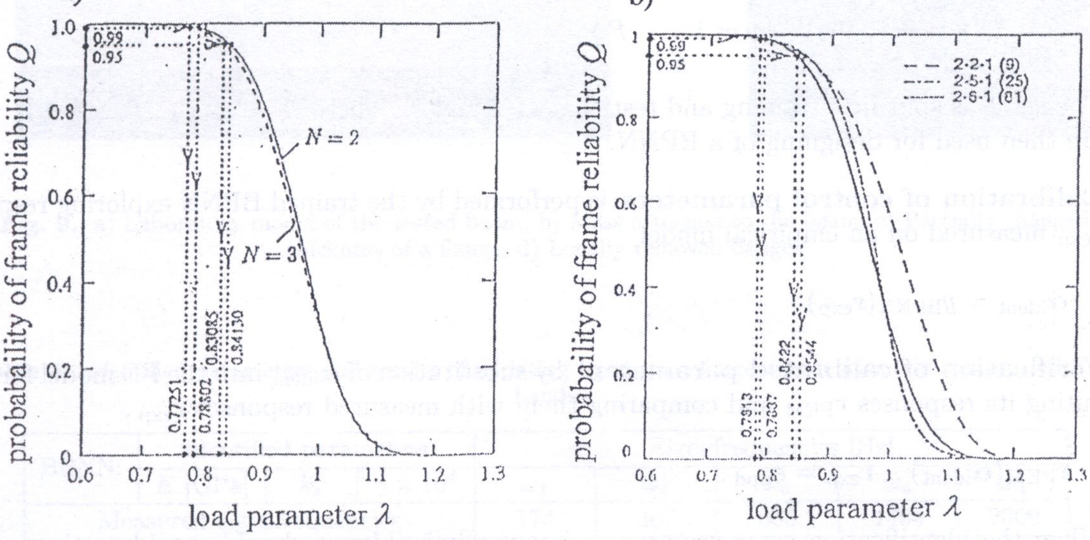

Fig. 8. Reliability curves $Q(\lambda)$ corresponding to simulation by networks: a) BPNN: 2-5-4-1 (1089) trained by means of $L = 1089$ patterns, BPNN: 3-20-6-1 (4913), b) BPNN: 2-2-1 (9), BPNN: 2-5-1 (25), BPNN: 2-6-1 (81)

Table 4. Comparison of CPU times for two numerical versions of CMC (Classical Monte Carlo method)

Simulation of CMC samples by BPNN: 3-20-6-1		Simulations of CMC trials by FEM program ELPLAF-V1	
Operations	CPU time [sec.]	Operations	CPU time [sec.]
Preparation of 6913 patterns by FEM, 6913 × 27 =	186651	Computation of one pattern	27
Training and testing of BPNN, about 20 hrs	72000	–	–
Simulation of 10^5 CMC trials	1.10	Hypothetical computations of 10^5 trials	$27{\times}10^5$
Total CPU time	**2.6×10^5 s**	**Total CPU time**	**2.7×10^6 s.**

4.5. Hybrid updating of a thin-walled beam FE model

Modelling of structures is a difficult task because of many uncertainties corresponding to material characteristics, parameters of applied loads, modelling of joints, boundary conditions, etc. FEM is usually applied as a numerical model suitable for computer simulations. The responses of an experimental model (laboratory models or natural scale structures) often do not agree with computed responses because of the above mentioned uncertainties. That is why the numerical model should be updated by means of certain control parameters, cf. [51, 57]. The updating process is related to the analysis of an inverse problem for minimizing differences between computed and measured structural responses, cf. [86].

A hybrid updating approach bases on the following algorithm:

I. Direct analysis of FE model is related to generating a set of patterns

$$\mathcal{P}' = \left\{ (\boldsymbol{\alpha}, \mathbf{r})^{(p)} \Big| \, p = 1, \ldots, P \right\}, \tag{15}$$

where $\boldsymbol{\alpha}$ – vector of control parameters, $\mathbf{r}$ – vector of FE model response (e.g. eigenfrequencies) as mapping $\boldsymbol{\alpha} \to \mathbf{r}$ for all the patterns p.

II. Inverse analysis corresponds to the training and testing of a BPNN using a set of patterns (15) but with inverse input and output vectors

$$\mathcal{P} = \left\{ (\mathbf{x} = \mathbf{r}, \mathbf{t} = \boldsymbol{\alpha})^{(p)} \Big| \, p = 1, \ldots, P \right\}. \tag{16}$$

The set $\mathcal{P}$ is split into training and testing sets $\mathcal{L}$ and $\mathcal{T}$, where: $\mathcal{P} = \mathcal{L} \cup \mathcal{T}$, $\mathcal{L} \cap \mathcal{T} = \emptyset$, which are then used for designing of a BPNN.

III. Calibration of control parameters is performed by the trained BPNN exploring responses $\mathbf{r}_{\mathrm{exp}}$ measured on an empirical model

$$\boldsymbol{\alpha}_{\mathrm{ident}} = y_{\mathrm{BPNN}}(\mathbf{r}_{\mathrm{exp}}). \tag{17}$$

IV. Verification of calibrated parameters by substitution of $\boldsymbol{\alpha}_{\mathrm{ident}}$ into the FE model for computing its responses $\mathbf{r}_{\mathrm{FEM}}$ and comparing them with measured responses $\mathbf{r}_{\mathrm{exp}}$,

$$\mathbf{r}_{\mathrm{FEM}}(\boldsymbol{\alpha}_{\mathrm{ident}}) - \mathbf{r}_{\mathrm{exp}} = \boldsymbol{\varepsilon}_{\mathrm{upd}}. \tag{18}$$

When the identification error vector $\boldsymbol{\varepsilon}_{\mathrm{upd}}$ is not admissible we should consider other control parameters which could be introduced into the considered FE model and return to Stage I.

From among many applications analyzed in [57] the updating of thin-walled beam parameters is discussed. The laboratory model of a beam made of aluminium alloy of density 2743 kg/m^{3}was suspended on two elastic strings, Fig. 9. Poisson ratio was assumed $\nu = 0.33$ and Young modulus was identified in the range $E \in [66.6, 80.0]$ GPa.

Dynamic responses were measured and the vibrations of the tested model were excited using an impact hammer. The first two eigenfrequencies were omitted as corresponding to the rigid motion of the beam and the next five eigenfrequencies, numbered $\omega_1, \ldots, \omega_5$ (they are listed in the first row of Table 5) were used for the updating of control parameters.

In Fig. 10, the scheme of the beam is shown. 24 Timoshenko elements were used and points of the excitation application and measured accelerations correspond to FE nodes.

A set of patterns was computed for 16 values of Young modulus E and 21 values of shape ratio k_s. This gave $P = 336$ inputs to compute five eigenfrequencies ω_i. A testing set with $T = 34 \approx 0.1P$ patterns was randomly selected. After the cross-validation process the network BPNN: 5-10-2 was

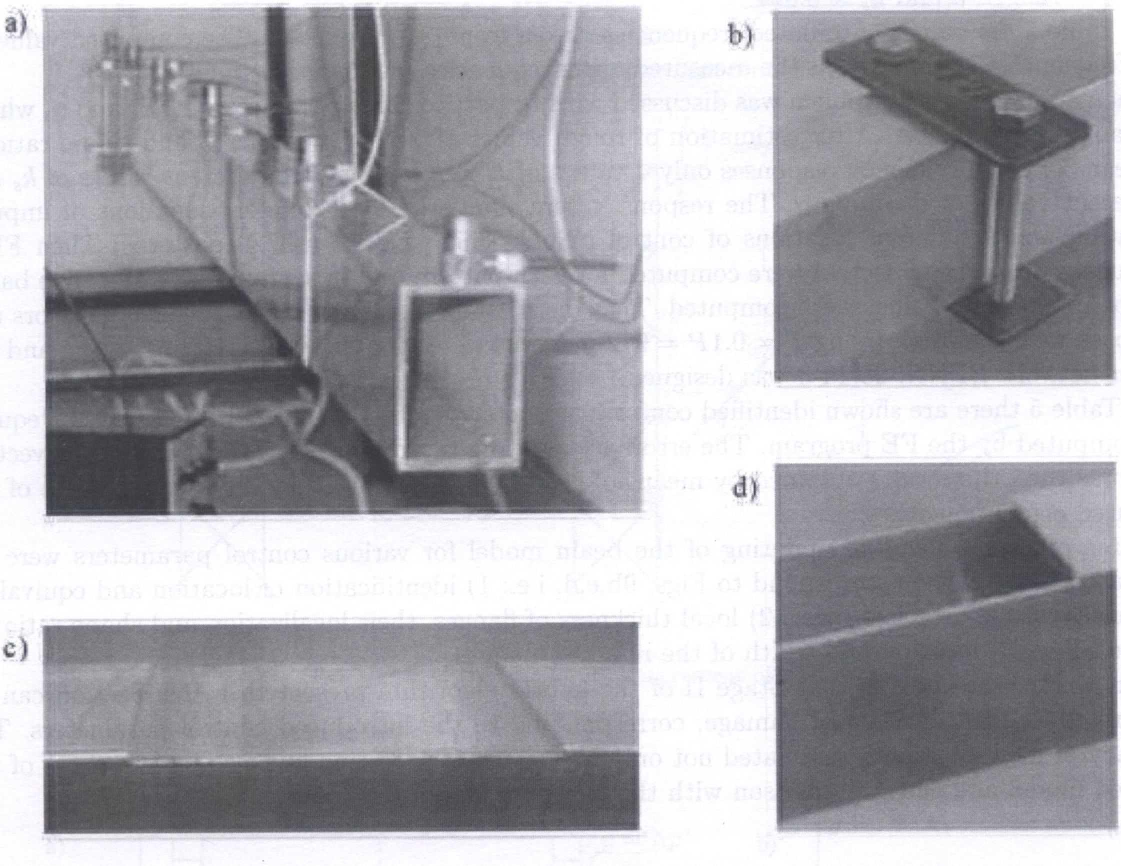

Fig. 9. a) Laboratory model of the tested beam, b) Mass attached to the beam, c) Partially changed thickness of a flange, d) Locally removed flanges

Table 5. Identified parameters and results of updating by means of eigenfrequencies and compressed FRS bands

BPNN:	Identified parameters			Eigenfrequencies [Hz]				
	E [GPa]	k_s	$\gamma \times 10^4$	ω_1	ω_2	ω_3	ω_4	ω_5
Measured eigenfrequencies				174	467	885	1404	2009
5-10-2	70.8	0.564	–	173.6 0.22%	467.1 0.02%	886.8 0.20%	1407.4 -0.24%	2004.3 -0.23%
25-14-3	71.6	0.551	3.75	175.2 −0.69%	470.2 −0.68%	889.5 −0.51%	1405 −0.07%	1992 0.84%

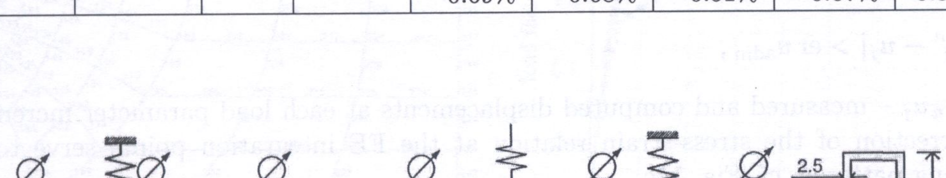

Fig. 10. Scheme of the tested beam

designed and after the network training the following values of control parameters were computed: $E_{\text{BPNN}} = 70.8\,\text{GPa}$ and $k_s = 0.564$.

In Table 5 the values of updated frequencies, taken from [27], are listed. The computed values of eigenfrequencies ω_i differ from the measured eigenfrequencies by not more than 0.3%.

Then a more general problem was discussed with respect to three parameters E, k_s and γ, where: γ – damping parameter. After estimation of rough values of Young modulus E and shape ratio k_s by means of eigenfrequency responses only 4 values of E were assumed, 5 different values of k_s and 6 different values of damping γ. The responses were simulated by 9 different locations of impulse excitation and 9 different locations of control points. This gave $P = 9720$ patterns. Then FRSs (Frequency Response Spectra) were computed for each pattern and in surroundings of ω_i five bands each of 100 discrete values were computed. They were compressed $100 \to 5$ by five compressors and 25 inputs were adopted. Using $T = 0.1P = 972$ patterns were randomly selected for testing and the master network BPNN: 25-14-3 was designed.

In Table 5 there are shown identified control parameters and verification of the updated frequencies computed by the FE program. The errors of updating by compressed FRS bands input vectors are worse than the results obtained by means of eigenfrequencies but they do not exceed 1% of the measured eigenfrequencies.

Three other problems of updating of the beam model for various control parameters were investigated in [57]. They correspond to Figs. 9b,c,d, i.e.: 1) identification of location and equivalent FE density of the attached mass, 2) local thickness of flanges, their localization and shape ratio for defected FEs, 3) location and width of the removed flanges.

It is worth emphasizing that Stage II of the hybrid algorithm presented in this Section can be interpreted as identification of damage, corresponding to the introduced control parameters. The accuracy of identification is estimated not only by the network testing but also by responses of the updated model and their comparison with the test on a laboratory model.

4.6. Hybrid identification of equivalent material in a perforated strip

A hybrid FEM/BPNN program can be formulated for the identification of a simple equivalent material of real structures, cf. [62, 63]. In the computer program a homogenized (equivalent) material is implicitly modelled by the Neural Network Constitutive Model (NNCM). NNCM is a BPNN whose parameters and input/output data enable computation of the consistent, tangent stiffness, material constitutive matrix, cf. [87]. The NNCM parameters are calibrated on the base of measured structural responses.

The main problems are the generation of patterns and NNCM training which are analyzed using the autoprogressive or cumulative algorithms, suggested in [62, 63] and developed in [64], cf. also references in [88]. The algorithms are based on a two stage modification of the Newton–Raphson method, as shown in Fig. 11. Stage I corresponds to the classical incremental FEM procedure and in Stage II the correction of the displacements at the control points j is made using the following error measure,

$$ {}_n d_j \equiv {}_n|u_j^m - u_j| > \text{er}\, u_{\text{adm}}\,, \tag{19}$$

where: ${}_n u_j^m$, ${}_n u_j$ – measured and computed displacements at each load parameter increment $\Delta_n \lambda$. Then the correction of the stress-strain relation at the FE integration points serve to generate NNCM training patterns, cf. Fig. 11c.

The proposed hybrid approach was discussed in [21] on simple examples, i.e. identification of material in a plane frame taken from [62] and shown in Fig. 11, and elastoplastic plane stress model of material in a beam bending boundary value problem taken from [87].

Below new results concerning the formulation and design of NNCM are related to [59] where a network BPNN was applied for the modelling of the elastic-plastic material with HMH yield surface and isotropic strain-hardening. Figure 12 corresponds to the plane stress analysis of a perforated

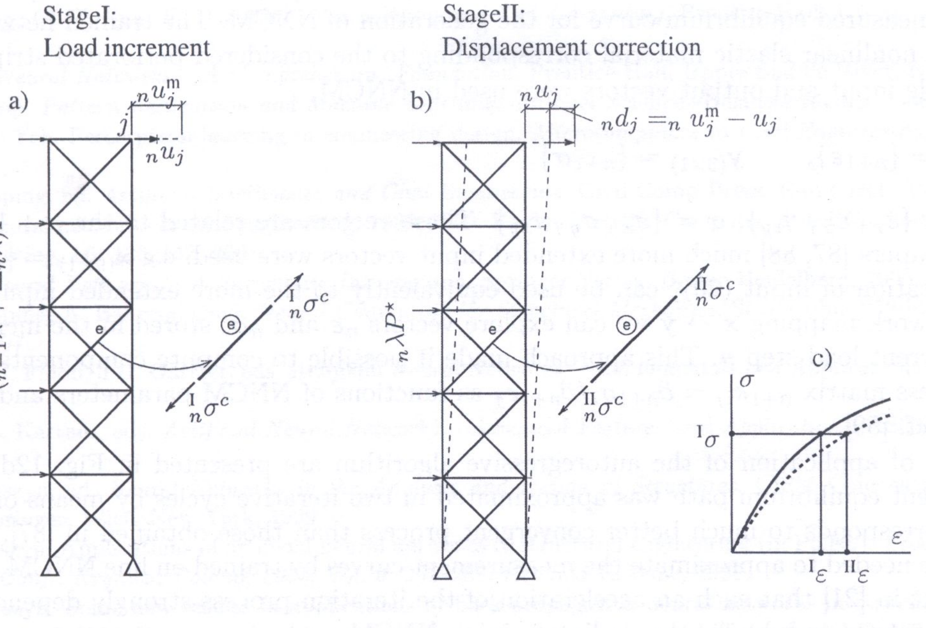

Fig. 11. Two stages of analysis for each increment of load parameter

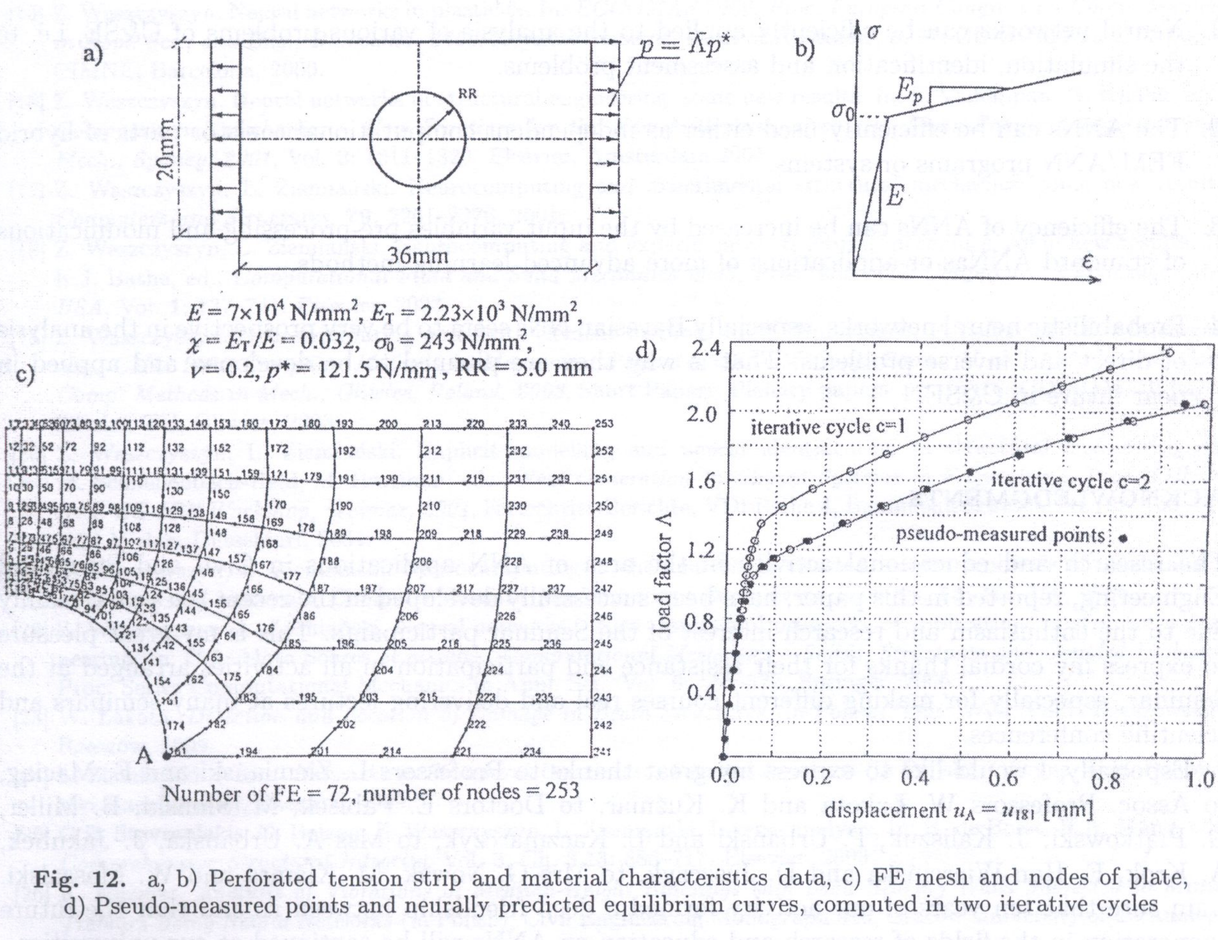

Fig. 12. a, b) Perforated tension strip and material characteristics data, c) FE mesh and nodes of plate, d) Pseudo-measured points and neurally predicted equilibrium curves, computed in two iterative cycles

strip by the hybrid FEM/BPNN program. The network for material modelling was trained and tested off line. The equilibrium curve $\Lambda - u_A$, computed in [59] and shown in Fig. 12, was used as a pseudo-measured equilibrium curve for the generation of NNCM. The trained network models an equivalent nonlinear elastic material corresponding to the considered perforated strip.

A simple input and output vectors were used in NNCM,

$$\mathbf{x}_{(3\times1)} = \{_{n+1}\boldsymbol{\varepsilon}\}, \qquad \mathbf{y}_{(3\times1)} = \{_{n+1}\boldsymbol{\sigma}\}, \tag{20}$$

where $\boldsymbol{\varepsilon} = \{\varepsilon_x\,, \varepsilon_y\,, \gamma_{xy}\}$, $\boldsymbol{\sigma} = \{\sigma_x\,, \sigma_y\,, \tau_{xy}\}$. These vectors are related to the $n+1$ load level. In previous papers [87, 88] much more extended input vectors were used, e.g $\mathbf{x}_{(9\times1)} = \{_n\boldsymbol{\varepsilon},\, _{n+1}\boldsymbol{\varepsilon},\, _n\boldsymbol{\sigma}\}$. The application of input $(20)_1$ can be used equivalently to the more extended inputs since in the neural network mapping $\mathbf{x} \to \mathbf{y}$ we can explore vectors $_n\boldsymbol{\varepsilon}$ and $_n\boldsymbol{\sigma}$, stored in the incremental FEM for the current load step n. This approach made it possible to compute components of the consistent stiffness matrix $_{n+1}k_{ij} = \partial_{n+1}\sigma_i/\partial_{n+1}\varepsilon_j$ as functions of NNCM parameters and input/output variables, cf. [59].

Results of application of the autoregressive algorithm are presented in Fig. 12d. The pseudo-measurement equilibrium path was approximated in two iterative cycles by means of BPNN: 3-15-15-3. It corresponds to much better convergent process than those obtained in [87], where several cycles were needed to approximate the measurement curves by trained on line NNCM models. It was pointed out in [21] that such an acceleration of the iteration process strongly depends on selection of patterns generated during the on line training NNCM.

5. Final conclusions

1. Neural networks can be efficiently applied to the analysis of various problems of C&SE, i.e. to the simulation, identification and assessment problems.
2. The ANNs can be efficiently used either as independent computational tools or parts of hybrid FEM/ANN programs or systems.
3. The efficiency of ANNs can be increased by the input variables pre-processing and modifications of standard ANNas or applications of more advanced learning methods.
4. Probabilistic neural networks, especially Bayesian NNs seem to be very prospective in the analysis of direct and inverse problems. That is why they are planned to be developed and applied in near future in C&SE.

Acknowledgments

The research and educational activity in the area of ANN applications in Civil and Structural Engineering, reported in this paper, have been successfully developed in the recent ten years, mainly due to the enthusiasm and research interest of the Seminar participants. This is my great pleasure to express my cordial thanks for their assistance and participation in all activities arranged at the Seminar, especially for making different courses real and delivering lectures at many seminars and scientific conferences.

Especially, I would like to express my great thanks to Professors L. Ziemiański and E. Maciąg, to Assoc. Professors W. Łakota and K. Kuźniar, to Doctors E. Pabisek, M. Słoński, B. Miller, G. Piątkowski, J. Kaliszuk, P. Urbański and Ł. Kaczmarczyk, to Mss A. Urbańska, J. Jakubek, A. Krok, E. Han-Wiercińska and D. Juszczyk, to Mrs G. Nowak, M. Klamra and W. Massalski. I am sure that ten years of our activity formed a deep friendship between us and that the future cooperation in the fields of research and education on ANNs will be continued at our universities.

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