Computer Assisted Mechanics and Engineering Sciences, **13**: 557–564, 2006.

Optimizing the shape of the prosthetic aortic leaflet valve

Piotr Konderla
Institute of Civil Engineering, Wrocław University of Technology
Wyb. Wyspiańskiego 27, 50-370 Wrocław, Poland

Krzysztof Patralski
Lower Silesian Centre of Heart Diseases MEDINET
ul. Kamieńskiego 73a, 51-124 Wrocław, Poland

(Received in the final form November 15, 2006)

The object under study reported on in this paper was an aortic valve based on a natural aortic valve as the original. Simulations were carried out to examine the functioning of a valve which was loaded with varying pressure until a buckling of the leaflets and a full opening of the valve were observed. The aim of the study was the optimal choice of the geometric and mechanical parameters for the class of construction assumed for analysis.

1. INTRODUCTION

In mechanical terms the human heart is a kind of pump. During pulsation, the heart pumps blood around the body through the circulatory system. The direction of blood flows and the mode of blood inflow into the ventricles are controlled by four valves which act as non-return valves. The object under study is an aortic valve, located in the aortic annulus. In case of degenerative failure (cusp tissue rupture), the aortic valve can not function properly and must be replaced with a prosthesis. Normally, use is made of biological or mechanical prostheses build of animal tissues and biomaterials, respectively. A natural aortic valve consists of three spherical leaflets connected to a common ring (Fig. 1). The structure of the prosthetic (biological) heart valve is similar to the natural one; that of the mechanical valves being usually simpler. Recently, use has been made of mechanical valves based on the geometry of the natural valve.

In mechanical terms the valve should display the following properties [4]:

- critical buckling pressure at the valve opening, less than 10 mm Hg (1.33 kPa),
- minimal stress concentrations in the leaflet structure and at the points of cusp attachment to the supporting frame of the stent,
- the cusp has to be shaped so as to provide smooth washout upon valve closure,
- the surface contact between leaflets has to be minimal so as to minimize hemolysis and prevent the adjacent leaflets from sticking to each other,
- adequate longevity, so that the patients need no reoperation due to valve failure,
- a small height of the valve so as not to stop blood inflow into the coronary vessels.

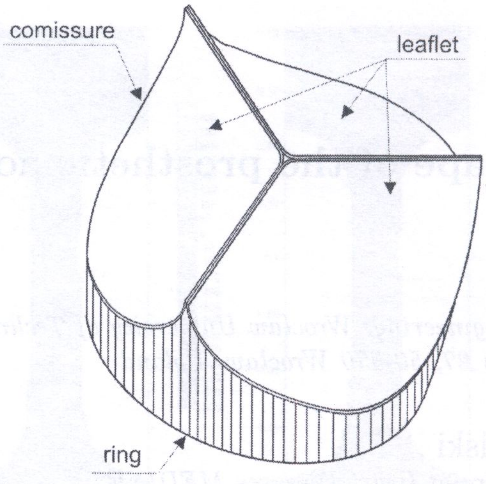

Fig. 1. Scheme of the native aortic valve

The properties itemized above are influenced by:

- valve leaflet geometry,
- material and structure of valve cusps,
- flexibility of valve stent.

2. FORMULATION OF THE OPTIMIZATION PROBLEM

In a previous paper [6] authors have presented the results of their research on the modelling of a natural material for the valve on the basis of physical data obtained by laboratory investigations. The study reported in the present paper is a furtherance of that research and involve the geometrical parameters of the bioprosthesis. Other investigators [1–4] have specified some geometry classes for valve leaflets that have the properties desired. The shape class of the leaflet shown in Fig. 2 was further analyzed. It has been assumed that the leaflet is a segment of the ellipsoid of revolution described by the equation [4].

$$\frac{x^2}{a_x^2} + \frac{y^2}{a_y^2} + \frac{z^2}{a_z^2} = 1, \tag{1}$$

where

$$a_x = a_y = R_L\,; \qquad a_z = \frac{R_L R_2 \sin\Theta_2}{\sqrt{R_L^2 - (e-R)^2}}\,.$$

The radius of the leaflet's top edge R_L related to the aortic radius R is selected by virtue of the condition of leaflet sticking when the valve is closed. A single leaflet of the valve is obtained by intersecting the ellipsoid with a circular cylinder (aorta) of the equation

$$\frac{(x-e)^2}{R^2} + \frac{y^2}{R^2} = 1, \tag{2}$$

and with the plane $z = 0$. The optimization problem was formulated as follows:

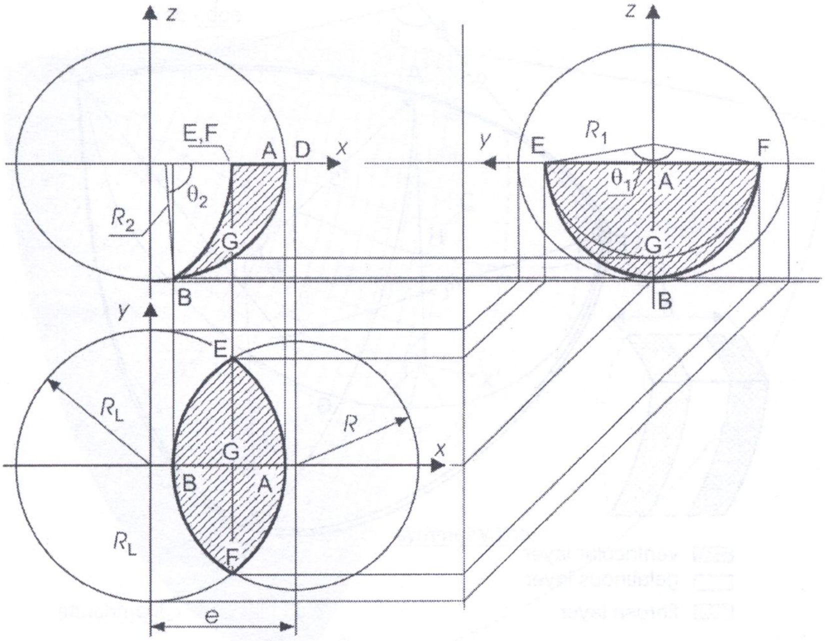

Fig. 2. The detailed engineering drawing showing the 3 views of valve design

a) We analyze a valve prosthesis which is treated as the model of a natural valve. The valve consists of three leaflets connected to a common ring. The following physical models were used:

- model 1: Kirchoff–Love thin shell, constant thickness $h = 0.6\,\mathrm{mm}$, isotropic, linear, elastic material: $E = 4.979\,\mathrm{MPa}$, $\nu = 0.49$,
- model 2: Kirchoff–Love thin shell, constant thickness $h = 0.6\,\mathrm{mm}$, isotropic, Mooney–Rivlin hyperelastic material,
- model 3: Mindlin–Reissner trilaminar thin shell (external layers are rigid $E = 5.875\,\mathrm{MPa}$, $\nu = 0.49$, internal layer is soft $E = 0.5\,\mathrm{MPa}$), constant thickness of each layer (external layers $h = 0.25\,\mathrm{mm}$, internal layer $h = 0.1\,\mathrm{mm}$), isotropic material.

b) In the material models of leaflet shell use was made of:

- linear model: Hooke model $\sigma_{ij} = C_{ijkl}\varepsilon_{ij}$, where material constants C_{ijkl} are determined by Young modulus E and Poisson coefficient ν.
- hyperelastic Mooney–Rivlin model: the strain energy function is defined

$$W = c_{10}(I_1 - 3) + c_{01}(I_2 - 3) + c_{20}(I_1 - 3)^2 + c_{11}(I_1 - 3)(I_2 - 3) + c_{02}(I_2 - 3)^2 + \frac{1}{d}(J - 1)$$

where I_1, I_2 are first and second invariants of Green's deformation tensor, J is the determinant of deformation gradient and d is the material incompressibility parameter; coefficients c_{ij} are material constants obtained after physical standard tensile strength tests.

c) We search for the optimal valve leaflet geometry construction which is determined by the design variables $\mathbf{b} = (b_1, b_2)^T$. The variable b_1 is a geometric variable equal to the radius R_2. And this is the variable that determines unambiguously the shape of the leaflet for the given aortic radius R. The variable b_2 is the shell kind of material parameter (model 1,3: elastic, model 2: hyperelastic).

d) The natural leaflet has the structure of a tree-layer shell (Fig. 3). Each layer consists of a nonlinear elastic material [6]. The valve model we analyzed, the material is assumed to be an isotropic

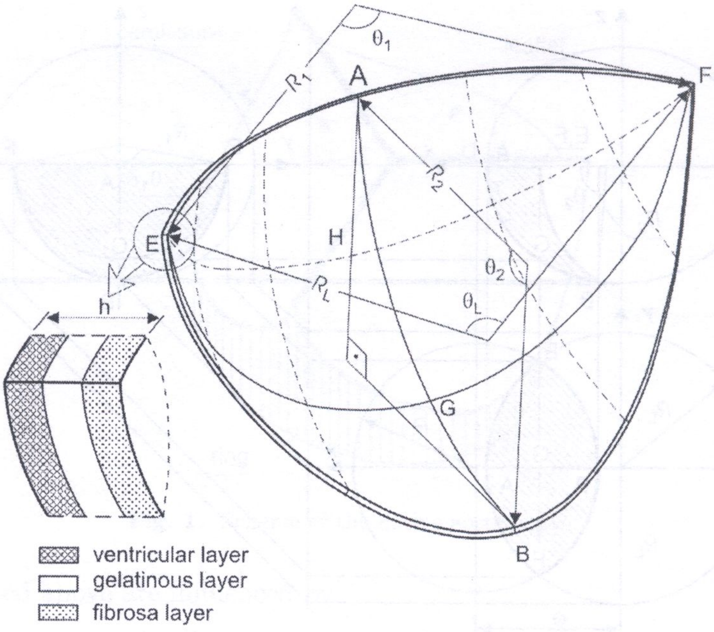

Fig. 3. The model of valve leaflet shell geometry

body which displays elastic properties obtained by averaging the properties of the natural shell layers.

e) The other geometric and mechanical parameters of the model are assumed as fixed.

f) The opening of the valves is induced by the buckling of the leaflets as a result of the pressure difference on the limit surfaces of the leaflet.

g) One objective function is considered in this study. The critical pressure for the valve leaflet, $F(\mathbf{b}) = p_b$, has been defined.

3. NUMERICAL CALCULATIONS

The optimization algorithm was constructed on the basis of the sensitivity theory [5]. The optimization problem is solved numerically by using FEM. The computations were performed using the Ansys system. Analysis was carried out with a single leaflet loaded by varying pressure applied to its surface. The shell was attached on the EB arc $u_{x'} = u_{y'} = u_{z'} = 0$. The following boundary condition was used on the symmetry line $\varphi_{z'} = u_{x'} = 0$ in the local coordinate system $\{x', y', z'\}$. In the discrete model was used the four-node shell elements with 6 degrees freedom in each node (Fig. 4).

The elements used permit us to model and analyze thin shells by taking into account geometric and material nonlinearity. Nonlinear geometric analysis was performed by controlling the path parameter.

Numerical analysis was carried out under the following assumptions:

- full nonlinear analysis of stability (buckling),
- the arc-length method is used,
- equilibrium path for nodal parameters at points A and C is analyzed,
- in general there are limit points on the equilibrium path only, the bifurcation points are lacking.

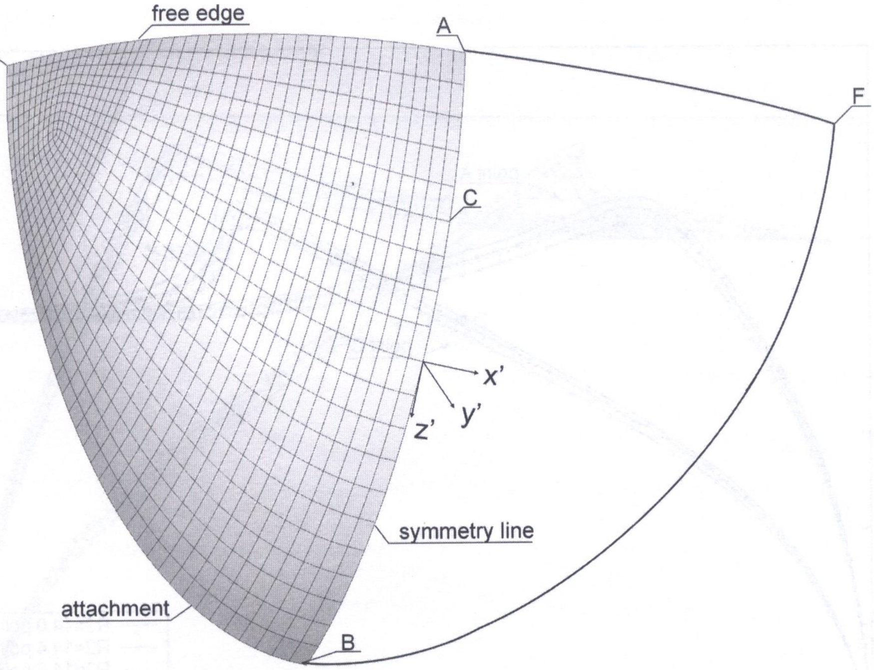

Fig. 4. Discrete model

4. RESULTS AND DISCUSSION

The result of our analysis is the set of the design parameters for which the buckling pressure of the valve leaflets is minimal. The calculations performed within this study enabled a comprehensive analysis of such a valve opening where the buckling effect is the initial fragment of the process. It is clear that the course of the process and the level of the critical pressure depend on the construction of the valve leaflet model. In Figs. 5–7 authors presented the equilibrium paths obtained for each model of the valve leaflet in two characteristic points A and C. The displacement denoted by u_x is perpendicular to the shell of the leaflet.

Model 1: The equilibrium path has the critical point in 1.1 kPa. We observe a sudden pressure drop after transition through the critical point. This is the typical overshoot effect observed during the buckling process of the spherical elastic shell (Fig. 5).

Model 2: The first critical point has been observed on the equilibrium path near 0.3 kPa. Than after the small displacement, the leaflet shell has been stiffening during the raise of the pressure (Fig. 6). The character of the equilibrium path is similar to the behavior of the plastic material with hardening.

Model 3: The equilibrium path presented in Fig. 7. is similar to this one obtained for the first model. The critical buckling pressure is close to 1.0 kPa.

On Fig. 8 authors presented the following stages of the displacing of the valve leaflet through the critical point. The models of the valve leaflet construction differ each other in to elements: (a) model of the material, (b) shell structure. The analysis of the results shows that the model of the material has the main influence on differentiation of the valve leaflet model behavior. It is comprehensible that the behavior of the second model differ from two remain models. For elastic models of the materials the level of the critical buckling pressure is high. This situation bring about the overshoot effect with a little displace of the structure. (small opening of the valve). This situation may induce the turbulent flow through the valve in the real conditions of the dynamic flow in the circulatory system. The equilibrium path obtained after computations of the second model displays another pressure

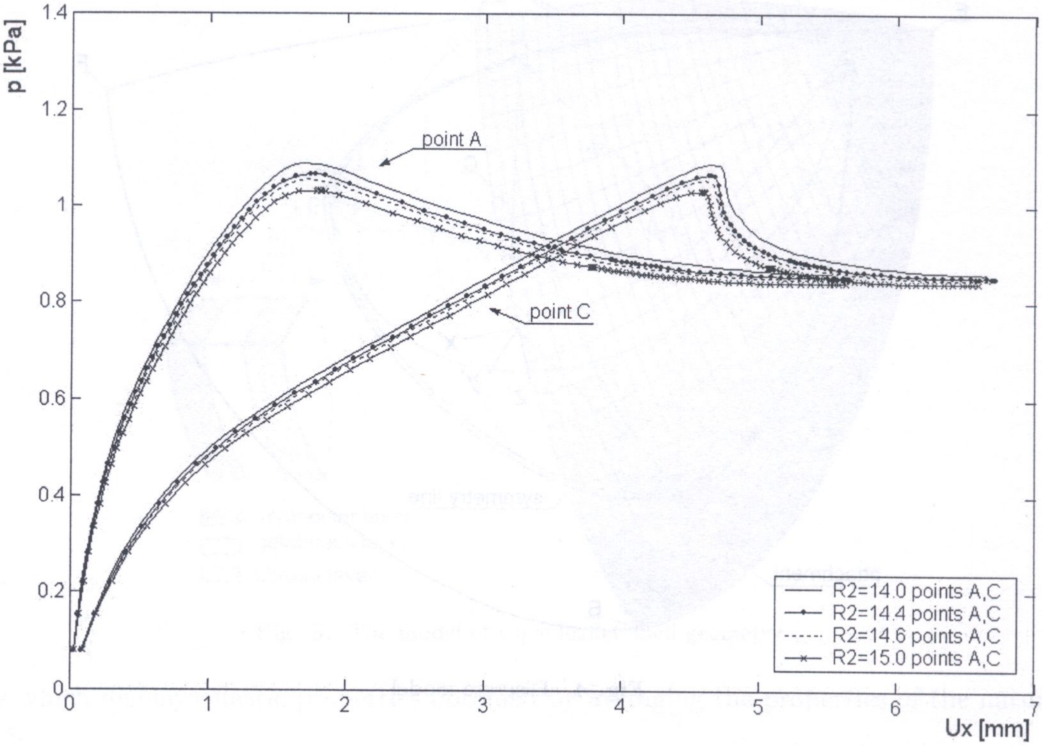

Fig. 5. Equilibrium paths of valve leaflet for model 1

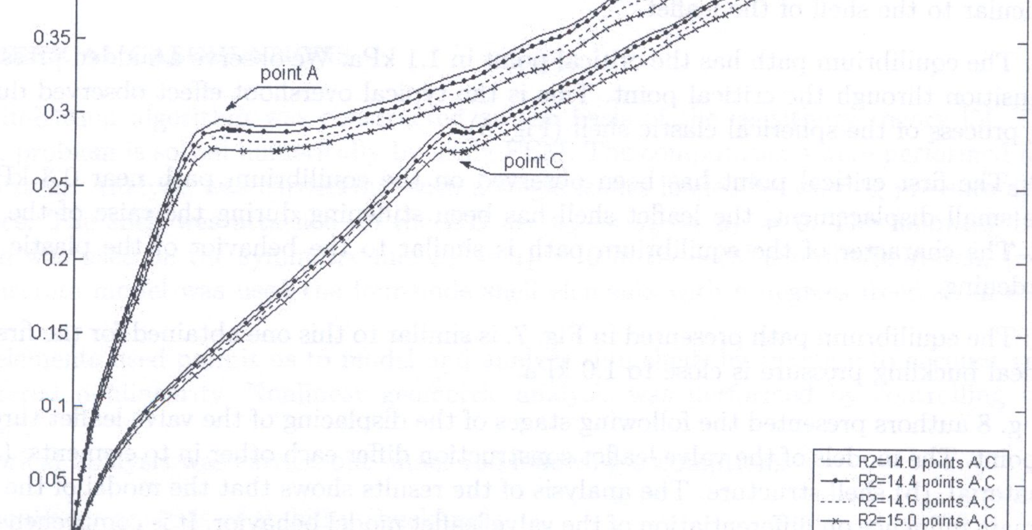

Fig. 6. Equilibrium paths of valve leaflet for model 2

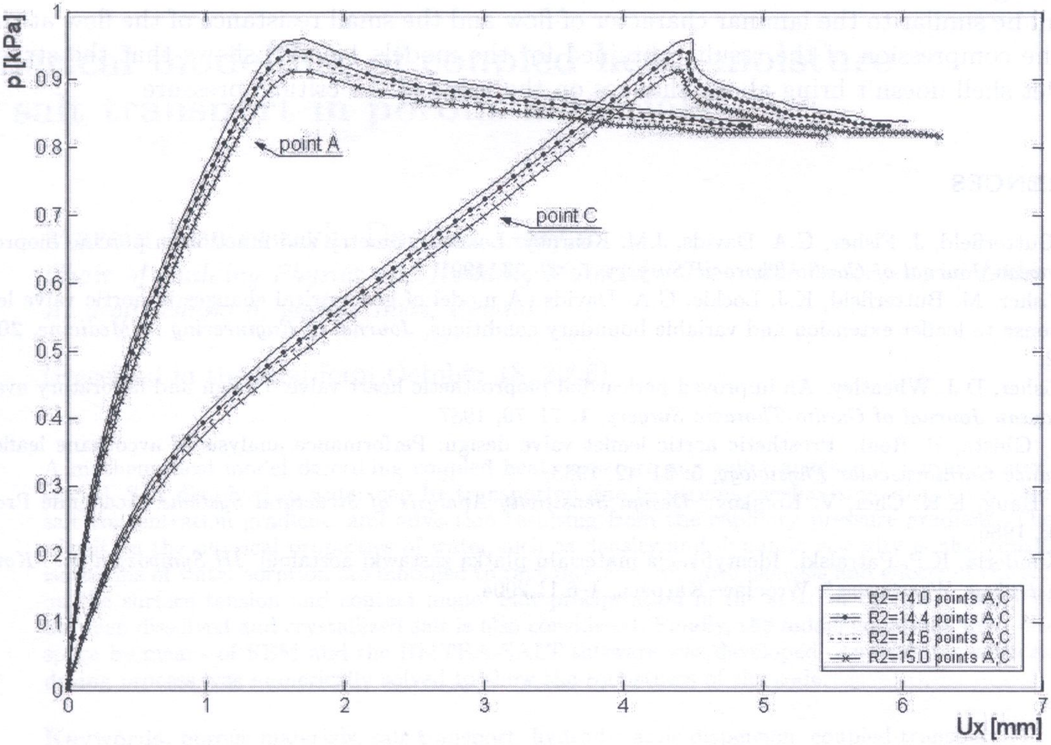

Fig. 7. Equilibrium paths of valve leaflet for model 3

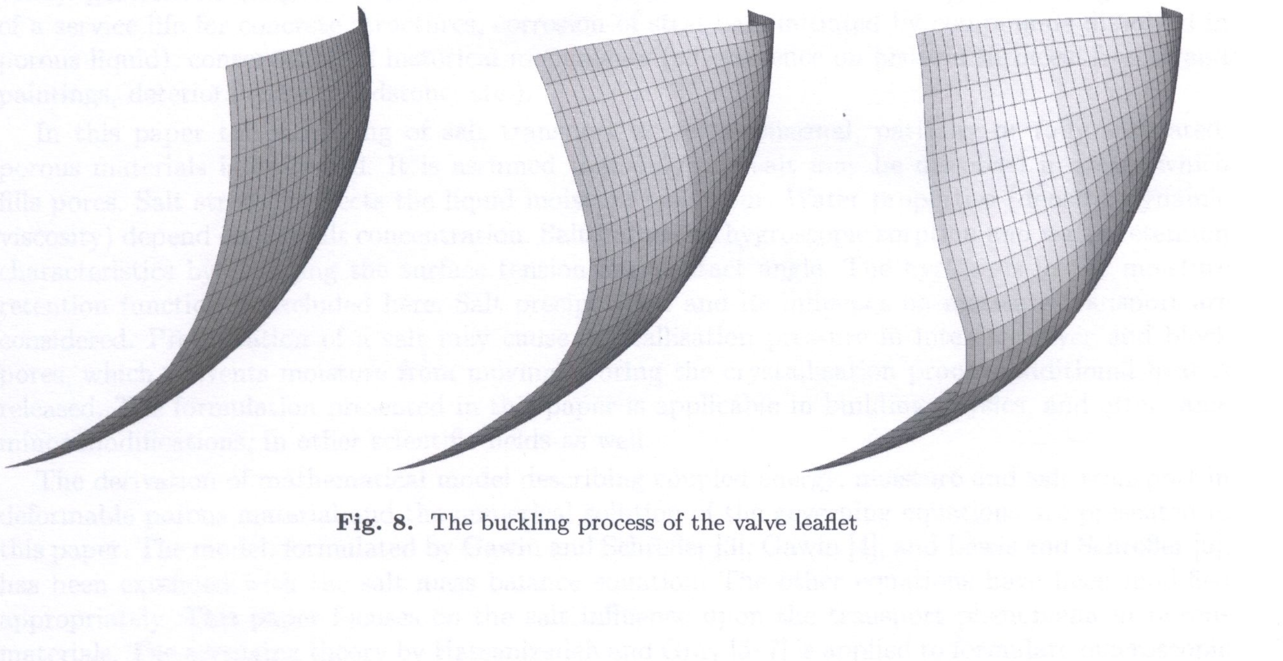

Fig. 8. The buckling process of the valve leaflet

distribution. The first buckling effect has been already noticed for the critical pressure equals near 30% of the parameters obtained for the models 1 and 3. Than the stiffness of the construction has been raising a little. In this situation, in the natural conditions the flow of the blood through the valve will be similar to the laminar character of flow and the small resistance of the flow at the same time. The compression of the results obtained for the models 1 and 3 shows that the structure of the leaflet shell doesn't bring about changes on the value of the critical pressure.

REFERENCES

[1] M. Butterfield, J. Fisher, G.A. Davids, J.M. Kearney. Leaflet geometry and function in porcine bioprostheses. *European Journal of Cardio-Thoracic Surgery*, **5**: 27–33, 1991.

[2] J. Fisher, M. Butterfield, K.J. Lockie, G.A. Davids. A model of geometrical changes in aortic valve leaflets in response to leaflet extension and variable boundary conditions. *Journal of Engineering in Medicine*, **206**: 7–14, 1992.

[3] J. Fisher, D.J. Wheatley. An improved pericardial bioprosthetic heart valve. Design and laboratory evaluation. *European Journal of Cardio-Thoracic Surgery*, **1**: 71–79, 1987.

[4] D.N. Ghista, H. Reul. Prosthetic aortic leaflet valve design: Performance analysis of avcothane leaflet valve. *Advance Cardiovascular Physiology*, **5**: 31–42, 1983.

[5] E.J. Haug, K.K. Choi, V. Komkov. *Design Sensitivity Analysis of Structural Systems*. Academic Press, New York, 1986.

[6] P. Konderla, K.P. Patralski. Identyfikacja materiału płatka zastawki aortalnej. *III Sympozjon nt. "Kompozyty, Konstrukcje Warstwowe"*, Wrocław–Karpacz, 4–6.11.2004.