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Structure optimization system based on the bone surface adaptation phenomenon

Michał Nowak, Marek Morzyński
Poznań University of Technology, Division of Machine Design Methods
ul. Piotrowo 3, 60-965 Poznań, Poland

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In the paper the structure optimization system based on the surface remodeling is presented. The base of algorithm formulation was the trabecular bone surface remodeling phenomenon leading to optimization of the trabecular net in the bone as well as the design with optimal stiffness principle. The closed system including Finite Element mesh generation, decision criterion for structure adaptation and Finite Element Analysis in parallel environment are presented. The issues concerning the use of the tool for the mechanical design are discussed. Some results of computations, using special prepared software are presented.

Keywords: biomechanics, structural optimization

1. INTRODUCTION

The Wolff's law, which has been stated in 19th century tells, that the bone is able to adapt to the mechanical stimulation. After many experiments it is clear, that the amount and organization of the beams in trabecular bone tend to mechanical optimum. The research of the remodeling of trabecular bone is in the main area of many medical research centers, but the phenomena of bone mass losing is also very important during the space exploration. The mechanical stimulation is one of the most important factors of the normal bone functionality. Although the trabecular bone remodeling simulation was a domain of strictly biomechanical research, the authors intend to repeat the biological phenomenon leading to structural adaptation in the virtual engineering space. The base of this research direction is the observation, that the trabecular bone remodeling models, used in computer simulations of the trabecular bone remodeling process and well known results of optimal design concern the strain energy in the structure. Thus the goal of the research is both to better understanding of the process of bone remodeling and use this knowledge in mechanical design.

2. REMODELING SIMULATION

There are many models of trabecular bone surface remodeling [4, 7, 11] used for the adaptation simulations of the bone, treated as a continuum material. The main idea behind that is to prepare the model of the bone adaptation as a material of specific properties. These properties are varying and depend on the load history. The progress in computer hardware technology and parallel computations enable now modeling of the bone adaptation process using the real topology of the trabecular bone with use of a linear model of the trabecula [1, 8]. The latter is justified by experimental investigations stating that on the trabecular level the bone can be treated as an linear material [6]. Such approach can be considered as very useful, especially when the details of mechanical stimuli are discussed.

The model of the bone remodeling process discussed in the paper is based on the regulatory model of Ruimerman and Huiskes [4, 11]. The main idea of the model consists of a regulatory mechanism on the bone surface only between bone resorption and formation. The processes of resorption and formation are coupled in basic multicellular units, regions smaller than the single trabecula where the sequence of successive bone tissue resorption and formation occurs. However, bone formation depends on mechanical stimulation. To maintain the right balance of bone remodeling, mechanical stimulation is needed from external loads. If there is no mechanical stimulation (as in the case of microgravity), the remodeling regulation model leads to a diminution of bone tissue. In this regulatory model the strain energy density (SED) is used as a measure of mechanical stimulation level in the remodeling criterion. It is assumed that surface evolution is a function of SED with a "lazy zone".

On the other hand the shape design with optimal stiffness investigations carried out by Wasiutyński [13], Dems and Mróz [2] and others lead to some theorems concerning the surface-design shape. Bearing in mind the design with optimal stiffness as presented by Pedersen [10], for the local design parameter h_e in the domain, that change the design in the domain only the following formula can be employed,

$$\frac{\mathrm{d}U_\epsilon}{\mathrm{d}h_e} = -\left(\frac{\delta((\bar{u}_\epsilon)_e V_e)}{\delta h_e}\right)_{\text{fixed strains}}, \tag{1}$$

where $\mathrm{d}U_\epsilon$ is the elastic energy, $\bar{u}_\epsilon$ is the mean value of the strain energy density and V_e is the domain volume.

If the shape is subject to variation using only two parameters h_i and h_j and the total volume of the structure is fixed, then

$$\Delta V = \frac{\mathrm{d}V}{\mathrm{d}h_i}\Delta h_i + \frac{\mathrm{d}V}{\mathrm{d}h_j}\Delta h_j = \frac{\mathrm{d}V_i}{\mathrm{d}h_i}\Delta h_i + \frac{\mathrm{d}V_j}{\mathrm{d}h_j}\Delta h_j = 0, \tag{2}$$

$$V_e = V_e(h_e) \quad \text{and} \quad \frac{\mathrm{d}V_e}{\mathrm{d}h_e} > 0, \tag{3}$$

$$\Delta U_\epsilon = -\left(\frac{\mathrm{d}U_\epsilon}{\mathrm{d}h_i}\Delta h_i + \frac{\mathrm{d}U_\epsilon}{\mathrm{d}h_j}\Delta h_j\right)_{\text{fixed strains}}, \tag{4}$$

$$\Delta U_\epsilon = -\left(u_{\epsilon_i}\frac{\mathrm{d}V_i}{\mathrm{d}h_i}\Delta h_i + u_{\epsilon_i}\frac{\mathrm{d}V_j}{\mathrm{d}h_j}\Delta h_j\right) \tag{5}$$

and inserting Eq. (2) into Eq. (5) we obtain

$$\Delta U_\epsilon = -(u_{\epsilon_i} - u_{\epsilon_j})\frac{\mathrm{d}V_i}{\mathrm{d}h_i}. \tag{6}$$

A necessary condition for optimality $\Delta U_\epsilon = 0$ with $\frac{\mathrm{d}V_i}{\mathrm{d}h_i} > 0$ is therefore

$$u_{\epsilon_i} = u_{\epsilon_j}. \tag{7}$$

With all design parameters, Eqs. (2) and (5) are written as

$$\Delta V = \Sigma_e \frac{\mathrm{d}V_e}{\mathrm{d}h_e}\Delta h_e, \tag{8}$$

$$\Delta U_\epsilon = -\Sigma_e u_{\epsilon_e}\frac{\mathrm{d}V_e}{\mathrm{d}h_e}\Delta h_e, \tag{9}$$

and we can conclude that for the stiffest design the energy density along the shape u_{ϵ_s} to be designed must be constant,

$$u_{\epsilon_s} = \text{const.} \tag{10}$$

In the case of bone, the remodeling scenario described above, based on the phenomenological model, seems to realize the postulate of the constant value of the strain energy density. By balancing the SED value on the bone surface, the stiffness of the entire structure is ensured. In ideal conditions, when the bone structure is only rebuild, also the volume constraint

$$\Sigma_e \frac{\mathrm{d}V_e}{\mathrm{d}h_e} = 0. \tag{11}$$

In the real process, the bone volume undergoes continues changes. In this study the presented examples employ the scenario of bone remodeling, i.e. the employed scenario assumes changes of bone volume in the successive iterations.

So, the fixed volume constraint, resulting from the minimum compliance discussion is not a case in the bone remodeling. The optimization goal can be also formulated as a minimum volume problem with assumed fixed strain energy, what is described in [3]. The resulting condition concerning the strain energy density is the same like in the case of the minimum compliance, so the value of the strain energy density on the designed surface must be equal, when the volume is minimal by the assumed value of the strain energy in the structure.

In the model of bone remodeling, there is a special value of the strain energy density — the energy of homeostasis, when the balance between resorption and formation of the bone tissue is perfect. Figure 1 shows the computation scheme with strain energy density as a remodeling criterion.

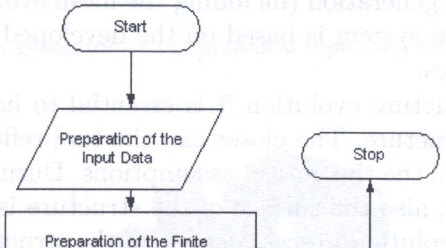

Fig. 1. Computation scheme with strain energy density (surface remodeling) as a remodeling criterion

3. THE STRUCTURE OF THE SYSTEM

In the presented system the algorithm of bone remodeling stimulated by mechanical loading is used [9]. The main structure of the presented simulation system is depicted in Fig. 2. The two attributes of trabecular bone remodeling phenomenon, surface adaptation and mechanosensitivity

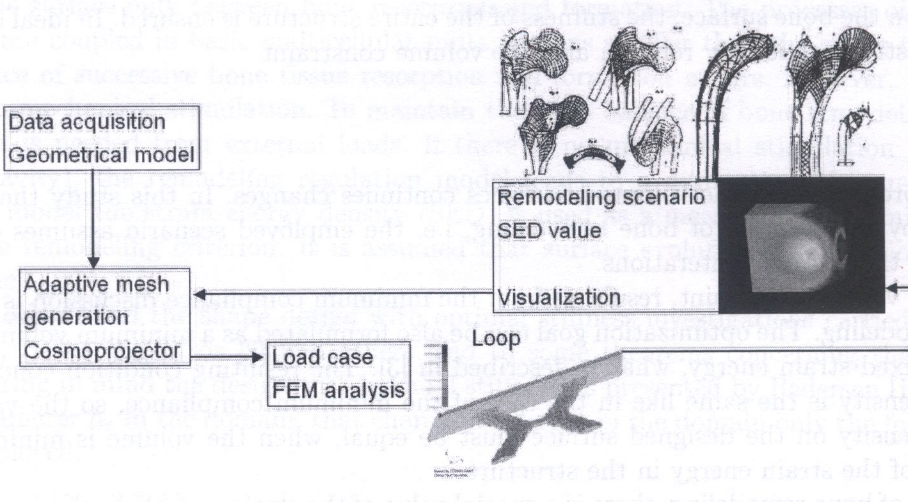

Fig. 2. Main structure of the presented simulation system

were the main frameworks for the optimization system. The simulation of the remodeling process needs both Finite Element mesh generation (including the mesh evolution) and structural stress and strain analysis. The optimization system is based on the developed for the trabecular bone surface adaptation simulation procedures.

To describe properly the structure evolution it is essential to have a tool which can be able to describe every change in the structure. The closer can the tool reflect the assumed conditions, the better can the simulation confirm the theoretical assumptions. During the simulation the volumetric FEM mesh is prepared. Besides, also the surface of the structure is controlled. It is important due to the fact that the structural evolution process occurs on the structure surface (like it is in the real process of trabecular bone adaptation).

To carry out the efficient simulation, the FEM mesh generator dedicated to this specific tasks must be used. Also the visualization tool should be able to provide the required graphic information about the state of computation. All these elements have been developed in the presented system and will be described below.

Since the visualization of the biological objects is based on digital images e.g. micro Computer Tomography (micro CT), the input to the system is based also on the collection of 2-dimensional images. After some graphical operations have been performed, the images of micro CT slices are directly used for building the 3-dimensional finite element mesh. The 2-dimensional image is first translated into a bitmap where '0' represents void and '1' the tissue (Fig. 3). On the bitmap the initial step of discretization is executed. The aim of this first step is to describe the areas filled with tissue. The user is able to set up the grid parameters and define the dimension of the discretization box. The discretizing procedure produces a 2-dimensional network of tetragonal elements according to the tissue image shape. After the two consecutive slices are discretized in this way, the second stage of mesh building is performed. The discretized 2-dimensional image is projected to the following one (Fig. 4). If there are areas containing tissue on both images, the boxes are created. Each box is in turn translated into 6 tetrahedral volume elements.

The user is able to describe the minimal volume of the tetrahedral element. Elements with lower volume than assumed will be not generated. The elements are also controlled with regard to the requirements of FEM. The information about nodes and elements is stored in a special data base and is as well translated into ABAQUS finite element system input file standard. In Fig. 5 the input information (collection of 2-dimensional pictures) and the resulting volumetric mesh are depicted. The system enables the surface control so from the whole structure the surface elements are distinguished. The information about these elements on the surface is stored separately.

The example whole structure and the 'surface' layer of the example projection between two slices are depicted in Fig. 6. The user is able to define the load and boundary conditions. This information

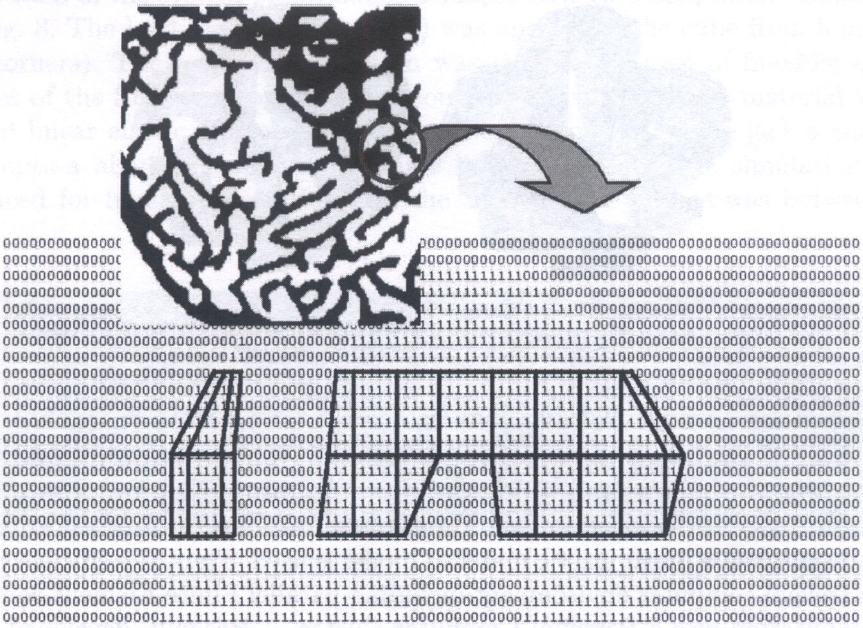

Fig. 3. Generic mesh generator — the graphical input and first step of discretization

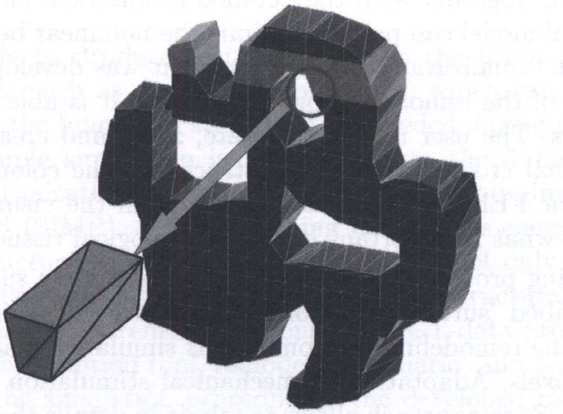

Fig. 4. Generic mesh generator — projection between layers and generation of tetrahedral mesh

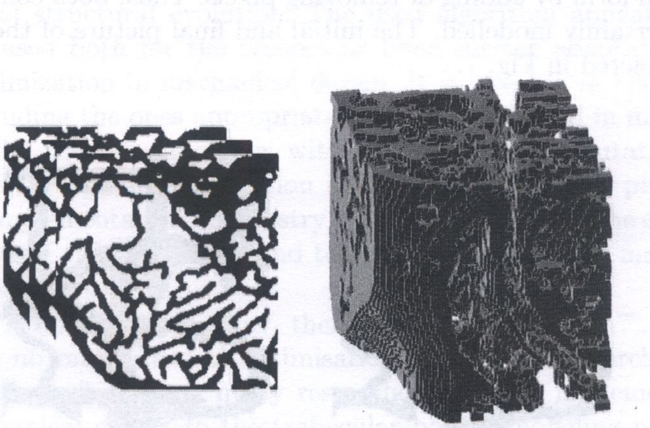

Fig. 5. Generic mesh generator — the graphical input and tetrahedral mesh

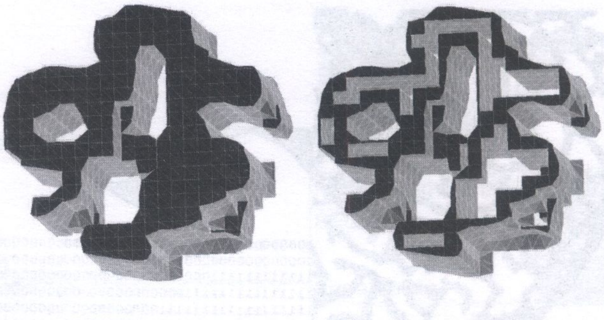

Fig. 6. Generic mesh generator — the surface layer control

is also translated into the specific database and attached to the ABAQUS input standard file. The remodeling surface is defined as a result of the difference between the whole surface and the surface with the defined loads and boundary conditions.

Biological objects are very complicated and need many elements for proper shape modeling. It is especially important when a linear elastic material model is used. Thus, the efficient numerical simulation demands special computer environment including parallel Finite Element computations. The base for the computation was a Personal Computer cluster — Beowulf, developed in the Division of Machine Design Methods at Poznań University of Technology. The parallel computations were performed with the use of Message Passing Interface (MPI) — the program MPICH, the Linux implementation of MPI standard. For the decomposition of the computation domain a decomposition tool — METIS was used. Together with the detailed geometrical model of the analyzed structure, the linear elastic material model can precisely mimic the nonlinear behavior of the structure and its evolution. For the result visualization, own environment was developed. The generic code enables the robust presentation of the remodeling process results. It is able to collect information coming from different processors. The user is able to rotate, zoom and create a movie. It is also possible to display a 2-dimensional cross-section of the structure. The color map can illustrate the value of required variables. The FEM environment together with the visualization tool is useful also for million-element meshes, what is important in case of biological tissue problems.

Because the remodeling process occurs on the trabecular bone surface only, for structure adaptation merely the described 'surface' layer of the structure is taken into consideration. Contrary to other voxel models, the remodeling phenomenon is simulated exactly on the analyzed structure without recalling the voxels. Adaptation to mechanical stimulation results in altering the surface position in virtual space. Such approach allows to mimic in details the real biological process of bone formation and resorption. The surface position adaptation is carried out on the 2-dimensional input images in the graphical form by adding or removing pixels. Thus, both consolidation and separation of the tissue can be certainly modelled. The initial and final picture of the graphical operations on the single slice are depicted in Fig. 7.

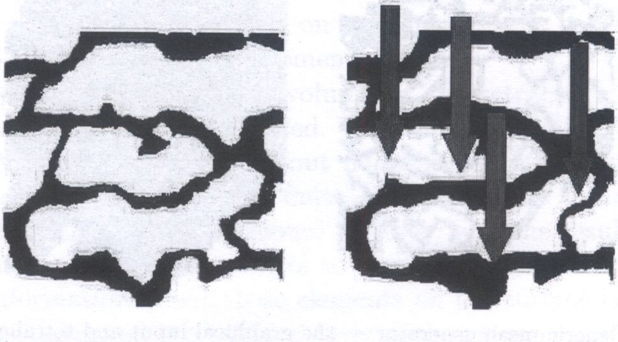

Fig. 7. 2D image modification — evolution of the structure

As an illustration of the system performance a simple structure adaptation under compression is presented in Fig. 8. The load (compression force) was applied to the cube from four different areas (four bottom corners). The boundary condition was the fixed degree of freedom on the opposite side. Two values of the SED forming the "lazy zone" were assumed. The material was assumed to be isotropic and linear elastic. The result remains the well-known garage jack stand. The scenario made no assumption about the material volume limit. Therefore, the simulation stopped when there was no need for further adaptation and the SED on the surface was between the assumed limits.

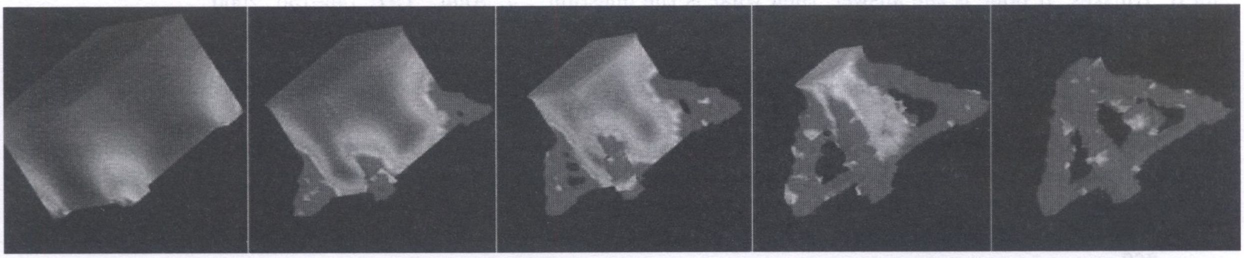

Fig. 8. Simulation of mechanical structure evolution — the adaptation of the simple test structure under compression

4. CONCLUDING REMARKS

The presented method is able to produce results similar to the topology optimization method and has some special features, such as domain independence, which can be useful in mechanical design especially in case when the functional structures are needed during the optimization process. The result of presented example simulation is very similar to the 3-dimensional solution given in the Nastran Topology Optimization module presentation [12] obtained with use of standard topology optimization method (SIMP). It is astonishing how simple energetic assumption applied only on the surface of the structure, leads to the optimal design not only as a shape optimization but also as a topology optimization. The presented system of structural optimization based on biological principle of trabecular bone remodeling consists of CT-data processing interface, robust 3D mesh generation tool, mesh evolution tool, remodeling scenario, and parallel FEM software and environment. The crucial in the simulation procedure is the developed method of Finite Element mesh generation and evolution, including the simultaneous control of the structural surface. The mesh control method enable the separation and consolidation of the structural elements, what is very important in case of structural evolution. The used algorithm appeared to be very flexible and the system can be used both for the trabecular bone surface adaptation simulation as well as for the structural optimization in mechanical design. It is possible to test different scenarios of the bone remodeling including the ones appropriate for materials found in mechanical applications. Well known Wasiutyński's theorem, together with this efficient computational environment are crucial enablers of a new structural optimization approach and are very promising for the future investigations as well as implementation in industry oriented optimizers. The optimization procedure comprise the optimization of the size, shape and topology, related to the material properties what is natural to biological structures.

In [5] Huiskes asked "If bone is the answer, then what is the question?". He stated in the conclusion, that "There are no mathematical optimisation rules for bone architecture; there is just a biological regulatory process". In fact, many researchers tried to implement optimization techniques known from mechanical design to the trabecular bone remodeling problem. In contrast to such approach, authors promote the opposite direction — how to utilize the observed phenomenon in mechanical design, bearing in mind the theorems of mechanics.

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