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Trefftz method for large deflection of plates with application of evolutionary algorithms

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The large deflection of thin plates by means of Berger equation is considered. An iterative solution of Berger equation by the method of fundamental solutions is proposed. In each iterative step the Berger equation can be considered as an inhomogeneous partial differential equation of the fourth order. The inhomogeneous term is interpolated by radial basis functions using thin plate splines. For the optimal choice of parameters of the fundamental solutions method an evolutionary algorithm is used. Numerical results for square plate with simply supported edges are presented to compare the obtained results with previous solutions.

1. Introduction

In the last decade, there has been considerable interest noticed in developing mesh-free methods for solution of the boundary value problems encountered in applied mechanics [15] and today, there are many versions of mesh-free methods. The family of these methods includes some versions of Trefftz methods as boundary methods. The Trefftz method can be understood as a method in which the differential equation is fulfilled exactly, whereas the boundary condition is fulfilled approximately. There are few possibilities in approximate fulfilment of the boundary condition in the frame of Trefftz method. One of them is the boundary collocation method (BCM) [14], where the boundary conditions are fulfilled in collocation manner.

Two different sets of functions, which satisfy exactly differential equations, are used in frame of BCM: T-complete Herrera functions [7] and fundamental solutions of governing equation [6]. In second case the method is known as the method of fundamental solutions (MFS). There are many application of BCM for linear elastic solutions of thin plates based on T-complete functions (e.g. [13, 16]) or fundamental solutions (e.g. [8, 9]). Then Trefftz method and the MFS as a special case, cannot be use in straight way for non-linear boundary value problems (BVPs) which appear at investigation of large deflection of plates. However, it doesn't mean that Trefftz method can not be used in any way for non-linear BVPs. First case when it can be used is BVPs with linear equation but with non-linear boundary conditions. Examples of such applications of this method are given in the papers [10, 17]. Second case known in literature is BVP with non-linear Poisson equation [1, 3, 5, 11, 12]. In the paper [5] the non-linear thermal explosions problem was solved by method of fundamental solutions. The radial basis functions were used for interpolation of right hand side on Picard iteration method which was used to tread non-linearity. In the paper [3], a method called "particular solution Trefftz method" was used. Another version of Trefftz method for solution of non-linear Poisson equation was presented in the paper [1]. For non-linear thermal conductivity problem by Kirchhoff transformation the nonlinearity exists only in boundary conditions. The non-linear algebraic equation was solved by stabilized continuation method. Kita *at al.* [11] considered steady state heat conduction problems for functionally gradient materials. In order to overcome the difficulty with non-linear Poisson equation the combination scheme of the Trefftz method with the computing point analysis method was presented. Also steady state heat conduction problem with temperature dependent conductivity was considered in the paper [12]. Combination of the method of fundamental solutions with Picard iteration was used for non-linear Poisson equation. Evolutionary algorithm was applied for the optimal determination of the method parameters. To the best knowledge of the authors there isn't a known application of MFS for large deflection of plates.

Linear plate theory is based on Kirchhoff model of plate. Kirchhoff theory assumes that normal to midplane remains straight prior to deformation and normal after deformation. Improved plate theories due to Reissner and Mindlin include the effect of transverse shear strains. Another plate theory appropriate for large deflection of thin plates was proposed by Berger [4]. Governing equation resulting from this theory is non-linear integro-differential equation and is applied with assumption that in-plane displacements are constrained at the boundary.

Solution of Berger equation by BEM was proposed in the papers [18, 19]. In the paper [18], a thin circular plate under a concentrated load was considered, while in the paper [19], the Berger equation was decomposed into two coupled partial differential equations of the second order. The local boundary integral equation method was applied to both equations. Such decomposition is possible for plate with straight edges of plate.

The purpose of the present paper is the application of the method of fundamental solutions for boundary value problem with Berger equation. The non-linearity is circumvented using algorithm of iteration proposed in the Sladeks' papers [18, 19]. In each iterative step the Berger equation can be considered as inhomogeneous partial differential equation of the fourth order. The inhomogeneous term is interpolated by radial basis functions. For the optimal choice of parameters of fundamental solutions method an evolutionary algorithm is used. Numerical results for square plate with simply supported edges are presented to compare the obtained results with previous solutions.

2. Formulation of the boundary value problem

According the Berger hypothesis, the governing equation for large deformation of thin plates has the form

$$\left(\nabla^2 - \beta^2\right)\nabla^2 w(x,y) = \frac{q(x,y)}{D} \tag{1}$$

with the Berger constant expressed as

$$\beta^2 = \frac{6}{h^2 A}\int_S \left[\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \mathrm{d}S, \tag{2}$$

where w, q, D, A, and h denote the deflection, transversal load, plate stiffness, plate surface and plate thickness, respectively.

After introducing dimensionless variables

$$\tilde{X} = \frac{x}{a}, \quad \tilde{Y} = \frac{y}{a}, \quad b(\tilde{X},\tilde{Y}) = \frac{q(x,y)}{q_0}, \quad \tilde{W}(\tilde{X},\tilde{Y}) = \frac{w(x,y)\cdot D}{a^4 \cdot q_0}, \quad \tilde{H} = \frac{h}{a}, \quad \tilde{Q} = \frac{a^3 \cdot q_0}{D}, \tag{3}$$

one has a non-dimensional governing equation in the form

$$\nabla^4 \tilde{W}(\tilde{X},\tilde{Y}) - \tilde{B}^2\,\nabla^2 \tilde{W}(\tilde{X},\tilde{Y}) = b(\tilde{X},\tilde{Y}) \tag{4}$$

with the non-dimensional Berger constant

$$\tilde{B}^2 = \frac{6}{\tilde{H}^2}\tilde{Q}^2 \iint_{\tilde{X},\tilde{Y}} \left(\frac{\partial \tilde{W}(\tilde{X},\tilde{Y})}{\partial \tilde{X}}\right)^2 + \left(\frac{\partial \tilde{W}(\tilde{X},\tilde{Y})}{\partial \tilde{Y}}\right)^2 \mathrm{d}\tilde{X}\mathrm{d}\tilde{Y}, \tag{5}$$

where a is the characteristic dimension of the plate (width of a rectangular plate or radius of a circular plate), q_0 is the characteristic value of transversal load (intensity of uniform load or maximal value of space variable dependent load).

Equation (1) (or (4)) must be solved with appropriate boundary conditions resulting from the way of support of boundary of the plate. Thus, for each point on the boundary, two, of the following four, boundary conditions have to be satisfied,

$$w = \bar{w}\,, \tag{6}$$

$$\frac{\partial w}{\partial n} = \frac{\partial \bar{w}}{\partial n}\,, \tag{7}$$

$$M_n = \bar{M}_n\,, \tag{8}$$

$$Q_n + \frac{\partial M_{nt}}{\partial s} = \bar{V}_n\,, \tag{9}$$

where n is the normal to boundary contour, M_n is the bending moment per unit length of a section of plate perpendicular to the n direction, Q_n is the shearing force per unit length of the section of a plate and V_n is the effective shear force of the plate.

In the next example, a simply supported square plate is considered. The boundary condition in such a case are presented in Fig. 1

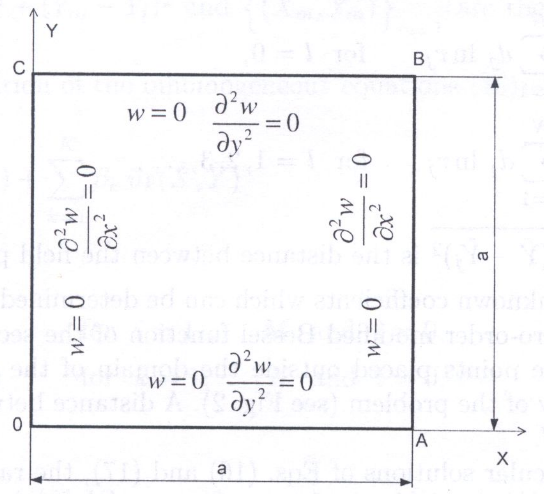

Fig. 1. Boundary conditions for simply supported square plate

3. Method of fundamental solutions

Equation (4) can be solved approximately using algorithm of iteration proposed in Sladeks' papers [18, 19] where some cases of BEM were used. According to their method, the Berger constant is calculated as

$$\tilde{B}_l^2 = \frac{6}{\tilde{H}^2} Q^2 \iint_{\tilde{X},\tilde{Y}} \left(\frac{\partial \tilde{W}^{(l)}(\tilde{X},\tilde{Y})}{\partial \tilde{X}} \right)^2 + \left(\frac{\partial \tilde{W}^{(l)}(\tilde{X},\tilde{Y})}{\partial \tilde{Y}} \right)^2 d\tilde{X}\, d\tilde{Y}\,, \tag{10}$$

where $\tilde{W}^{(l)}$ is the l-th iterative solution of a linear equation in the form

$$\nabla^4 \tilde{W}(\tilde{X},\tilde{Y}) - \tilde{k}^2\, \nabla^2 \tilde{W}(\tilde{X},\tilde{Y}) = b(\tilde{X},\tilde{Y}), \tag{11}$$

with

$$\tilde{k}^2 = \tilde{B}_{l-1}^2 \qquad \text{and} \qquad \tilde{B}_{-1}^2 = 0. \tag{12}$$

In our proposition, at each iteration step, the method of fundamental solutions with radial basis function for interpolation right hand side is used for solution of Eq. (11). As yet, this method was used for non-linear Poisson equation [3, 5]. Here an attempt is presented to use this for the Berger equation. In that case, the solution of Eq. (11) is assumed in the form

$$\tilde{W} = \tilde{W}_h + \tilde{W}_n \tag{13}$$

where $\tilde{W}_h$ is a general solution of the homogeneous solution, i.e.

$$\nabla^4 \tilde{W}_h(\tilde{X}, \tilde{Y}) = 0 \qquad \text{for } l = 0, \tag{14}$$

and

$$\nabla^4 \tilde{W}_h(\tilde{X}, \tilde{Y}) - \tilde{k}^2\, \nabla^2 \tilde{W}_h(\tilde{X}, \tilde{Y}) = 0 \qquad \text{for } l = 1, 2, 3, \ldots, \tag{15}$$

whereas $\tilde{W}_n$ is the particular solution of the inhomogeneous Eq. (11), i.e.

$$\nabla^4 \tilde{W}_n(\tilde{X}, \tilde{Y}) = b(\tilde{X}, \tilde{Y}) \qquad \text{for } l = 0, \tag{16}$$

and

$$\nabla^4 \tilde{W}_n(\tilde{X}, \tilde{Y}) - \tilde{k}^2\, \nabla^2 \tilde{W}_n(\tilde{X}, \tilde{Y}) = b(\tilde{X}, \tilde{Y}) \qquad \text{for } l = 1, 2, 3, \ldots \tag{17}$$

Using the method of fundamental solutions, the general solution of homogeneous equation is taken in the form

$$\tilde{W}_h = \sum_{k=1}^{N} c_j r_j^2 \ln(r_j) + \sum_{k=1}^{N} d_j \ln r_j \qquad \text{for } l = 0, \tag{18}$$

$$\tilde{W}_h = \sum_{k=1}^{N} c_j\, K_0(\tilde{k} r_j) + \sum_{k=1}^{N} d_j \ln r_j \qquad \text{for } l = 1, 2, 3, \ldots \tag{19}$$

where $r_j = \sqrt{(\tilde{X} - \tilde{X}_j)^2 + (\tilde{Y} - \tilde{Y}_j)^2}$ is the distance between the field point $(\tilde{X}, \tilde{Y})$ and the source points $(\tilde{X}_j, \tilde{Y}_j)$, c_j, d_j are unknown coefficients which can be determined from satisfaction of boundary conditions, K_0 is the zero-order modified Bessel function of the second kind, N is the number of source points. The source points placed outside the domain of the problem on source contour which is similar to boundary of the problem (see Fig. 2). A distance between boundary contour and source contour is equal to S.

For construction of particular solutions of Eqs. (16) and (17), the radial basis functions (RBFs) is used. Let $\left\{P_i = (\tilde{X}_i, \tilde{Y}_i)\right\}_{i=1}^{M}$ denote the set of M collocation points in Ω, of which $\left\{(\tilde{X}_i, \tilde{Y}_i)\right\}_{i=1}^{M_1}$ are interior points and $\left\{(\tilde{X}_i, \tilde{Y}_i)\right\}_{i=1}^{M_2}$ are boundary points ($M = M_1 + M_2$). The right-hand side function in Eq. (16) or (17) is approximated by RBFs as

$$b(\tilde{X}, \tilde{Y}) = \sum_{i=1}^{M} \alpha_i\, \hat{\varphi}_i(r_i) + \sum_{k=1}^{K} \beta_k\, \tilde{\varphi}_k(\tilde{X}, \tilde{Y}) \tag{20}$$

where $\hat{\varphi}_i(r_i) = \hat{\varphi}_i\left(\sqrt{(\tilde{X} - \tilde{X}_i)^2 + (\tilde{Y} - \tilde{Y}_i)^2}\right)$ are the RBFs and $\left\{\tilde{\varphi}_k(\tilde{X}, \tilde{Y})\right\}_{k=1}^{K}$ is the complete basis for d-variate polynomials of the degree $\leq m - 1$. The coefficients α_i and β_i can be found by solving the system of linear equations

$$\sum_{i=1}^{M} \alpha_i\, \hat{\varphi}_i(r_{im}) + \sum_{k=1}^{K} \beta_k\, \tilde{\varphi}_k(\tilde{X}_m, \tilde{Y}_m) = b(\tilde{X}_m, \tilde{Y}_m), \qquad 1 \leq m \leq M, \tag{21}$$

$$\sum_{i=1}^{M} \alpha_i\, \tilde{\varphi}_k(\tilde{X}_i, \tilde{Y}_i) = 0, \qquad 1 \leq k \leq K, \tag{22}$$

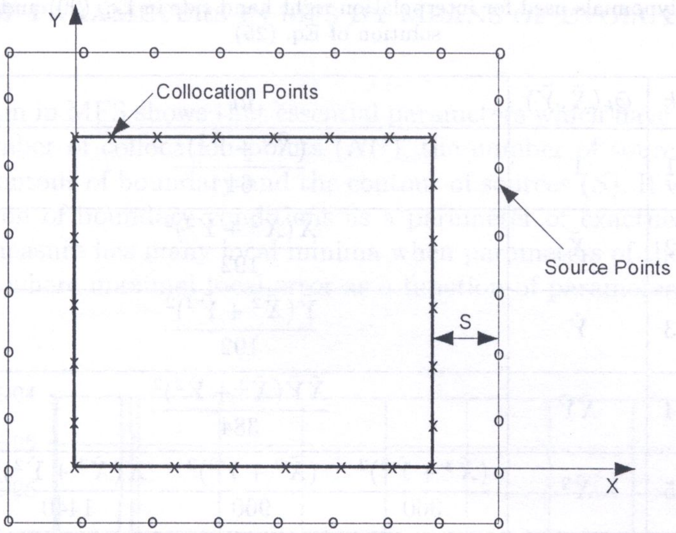

Fig. 2. Distribution of collocation and source points

where $r_{im} = \sqrt{(\tilde{X}_m - \tilde{X}_i)^2 + (\tilde{Y}_m - \tilde{Y}_i)^2}$ and $\left\{(\tilde{X}_m, \tilde{Y}_m)\right\}_{m=1}^{M}$ are the interpolation points placed in $\Omega \cup \partial\Omega$.

Now, the particular solution of the inhomogeneous equations (16) and (17) is taken in the form

$$\tilde{W}_n(\tilde{X},\tilde{Y}) = \sum_{i=1}^{M} \alpha_i\, \hat{u}_i(r_i) + \sum_{k=1}^{K} \beta_k\, \tilde{u}_k(\tilde{X},\tilde{Y}) \tag{23}$$

where

$$\nabla^4 \hat{u}_i(\tilde{X},\tilde{Y}) = \hat{\varphi}_i(\tilde{X},\tilde{Y}) \qquad \text{for } i = 1,\ldots,M \text{ and } l = 0, \tag{24}$$

$$\nabla^4 \tilde{u}_k(\tilde{X},\tilde{Y}) = \tilde{\varphi}_k(\tilde{X},\tilde{Y}) \qquad \text{for } k = 1,\ldots,K \text{ and } l = 0, \tag{25}$$

and

$$\nabla^4 \hat{u}_i(\tilde{X},\tilde{Y}) - k^2 \nabla^2 \hat{u}_i = \hat{\varphi}_i(\tilde{X},\tilde{Y}) \qquad \text{for } i = 1,\ldots,M \text{ and } l = 1,2,3,\ldots, \tag{26}$$

$$\nabla^4 \tilde{u}_k(\tilde{X},\tilde{Y}) - \tilde{k}^2 \nabla^2 \tilde{u}_k = \tilde{\varphi}_k(\tilde{X},\tilde{Y}) \qquad \text{for } k = 1,\ldots,K \text{ and } l = 1,2,3,\ldots \tag{27}$$

Nowadays, one can find in the literature many propositions of RBFs. Some numerical experiments related with application of five types of these function for solution of the Poisson equation are given in the paper [20]. Here, the RBF known as thin plate spline is used. This function has the form

$$\hat{\varphi}_i(r_i) = r_i^2 \ln r_i\,. \tag{28}$$

Solution of the differential equations (24) and (26) has the forms

$$\hat{u}_i = \frac{r_i^6\,(6\ln(r_i) - 5)}{3456} + \frac{r_i^2}{64} \qquad \text{for } l = 0, \tag{29}$$

and

$$\hat{u}_i = -\frac{r_i^2 \ln r_i}{\tilde{k}^4} - \frac{r_i^4 \ln r_i}{16\tilde{k}^2} + \frac{r_i^4}{32\tilde{k}^2} \qquad \text{for } l = 1,2,3,\ldots, \tag{30}$$

respectively.

Table 1. Forms of polynomials used for interpolation right hand side in Eq. (20) and appropriate particular solution of Eq. (25)

k	$\tilde{\varphi}_k(\tilde{X},\tilde{Y})$	$\tilde{u}_k$
1	1	$\dfrac{(\tilde{X}^2+\tilde{Y}^2)^2}{64}$
2	$\tilde{X}$	$\dfrac{\tilde{X}(\tilde{X}^2+\tilde{Y}^2)^2}{192}$
3	$\tilde{Y}$	$\dfrac{\tilde{Y}(\tilde{X}^2+\tilde{Y}^2)^2}{192}$
4	$\tilde{X}\tilde{Y}$	$\dfrac{\tilde{X}\tilde{Y}(\tilde{X}^2+\tilde{Y}^2)^2}{384}$
5	$\tilde{X}^2$	$\dfrac{(\tilde{X}^3+\tilde{Y}^2)^2}{360}-\dfrac{(\tilde{X}^2+\tilde{Y}^2)^2}{960}-\dfrac{\tilde{X}(\tilde{X}^2+\tilde{Y}^2)^2}{1440}$
6	$\tilde{Y}^2$	$\dfrac{(\tilde{X}^2+\tilde{Y}^3)^2}{360}-\dfrac{(\tilde{X}^2+\tilde{Y}^2)^2}{960}-\dfrac{\tilde{Y}(\tilde{X}^2+\tilde{Y}^2)^2}{1440}$

Table 2. Forms of polynomials used for interpolation right hand side in Eq. (20) and appropriate particular solution of Eq. (27)

k	$\tilde{\varphi}_k(\tilde{X},\tilde{Y})$	$\tilde{u}_k(\tilde{X},\tilde{Y})$
1	1	$\tilde{u}_1=-\dfrac{1}{4\tilde{k}^2}(\tilde{X}^2+\tilde{Y}^2)-\dfrac{1}{\tilde{k}^4}$
2	$\tilde{X}$	$\tilde{u}_2=-\dfrac{1}{8\tilde{k}^2}\tilde{X}(\tilde{X}^2+\tilde{Y}^2)-\dfrac{1}{\tilde{k}^4}\tilde{X}$
3	$\tilde{Y}$	$\tilde{u}_3=-\dfrac{1}{4\tilde{k}^2}\tilde{Y}(\tilde{X}^2+\tilde{Y}^2)-\dfrac{1}{\tilde{k}^4}\tilde{Y}$
4	$\tilde{X}\tilde{Y}$	$\tilde{u}_4=-\dfrac{1}{12\tilde{k}^2}\tilde{X}\tilde{Y}(\tilde{X}^2+\tilde{Y}^2)-\dfrac{1}{3\tilde{k}^4}\tilde{X}\tilde{Y}$
5	$\tilde{X}^2$	$\tilde{u}_5=-\dfrac{1}{14\tilde{k}^2}\tilde{X}^2(\tilde{X}^2+\tilde{Y}^2)-\dfrac{1}{\tilde{k}^2}\left(-\dfrac{1}{14\tilde{k}^2}\tilde{X}^2-\dfrac{1}{84}\tilde{Y}\right)$
6	$\tilde{Y}^2$	$\tilde{u}_6=-\dfrac{1}{14\tilde{k}^2}\tilde{Y}^2(\tilde{X}^2+\tilde{Y}^2)-\dfrac{1}{\tilde{k}^2}\left(-\dfrac{1}{14\tilde{k}^2}\tilde{Y}^2-\dfrac{1}{84}\tilde{X}\right)$

Forms of polynomials used for interpolation of the right hand side with appropriate particular solutions are given in Tables 1–2.

After determination of the interpolation coefficients α_i and β_k, the coefficients c_j and d_j can be determined from satisfaction of appropriate boundary condition in a collocation manner. For better satisfaction of boundary conditions, the number of collocation points NC is greater than the number of source points N. An overdetermined system of linear equations resulting from satisfaction of boundary conditions is solved in the least squares sense.

Iteration procedure is done until convergence condition in the form

$$\left\|\tilde{W}_{l+1}-\tilde{W}_l\right\| \le TOL \tag{31}$$

is achieved, where TOL is assumed the tolerance of the solution error.

4. Optimization of parameters in MFS by means of evolutionary algorithm

Analysis of the algorithm in MFS shows that essential parameters which have influence on exactness of method are: the number of collocation points (NC), the number of source points (N), and the distance between the contour of boundary and the contour of sources (S). If we choose the maximal local error of satisfaction of boundary conditions as a parameter of exactness of the method, one can observe that this measure has many local minima when parameters of the method are changed. For example see Fig. 3 where maximal local error as a function of parameters is shown.

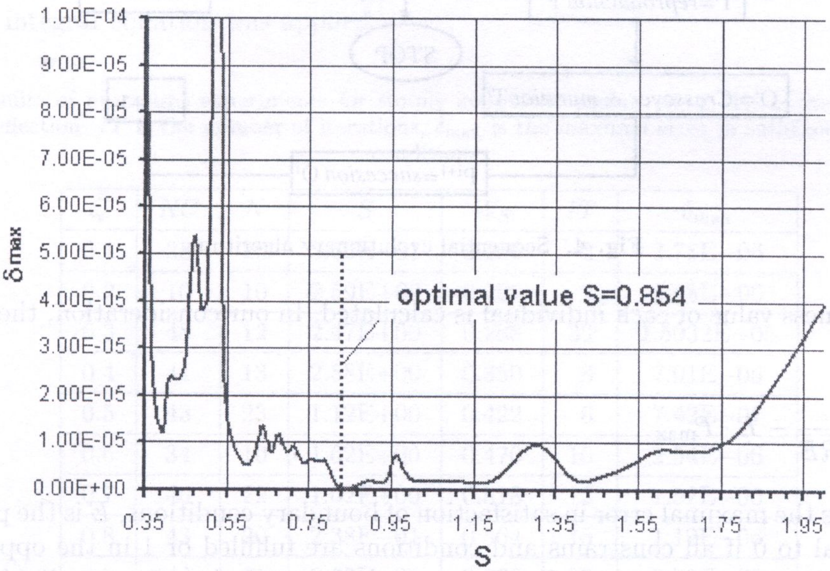

Fig. 3. Maximal local error of satisfaction of boundary conditions $\delta_{\max}$ as a function of distance between the boundary contour and the source contour S

We note that once the problem has been formulated as an optimisation problem then various optimisation algorithms may be used in order to locate the optimum of the objective function. The efficiency of a particular optimisation method clearly depends on the form of the objective function. In the problem considered in this paper, the objective function has a complex non-linear structure. Moreover, an analytical expression for the objective function for every possible solution of the problem is not known. Therefore evolutionary algorithms appear to be very suitable for optimizing the objective function of the problem considered since they do not require knowledge of the gradient of the objective functions, which makes them particularly suited to optimization problems for which an analytical expression for the fitness function is unknown.

In the proposed consideration, the evolutionary algorithm is used for optimal choice of parameters of MFS. Evolutionary algorithm (EA) is a stochastic searching method based on the principles of population genetics and biological evolution. The EA can be considered as the modified genetic algorithm in which population are coded by the floating point representation [2]. The real code is used in algorithm which is presented on Fig. 4.

The problem solution using evolutionary algorithms is carried out by the following procedure:

Step 1. A population pool of individuals with various chromosomes is installed (the first generation). In our case, the individuals are solutions of BVPs and the chromosomes contain the information about the method parameters. The chromosomes of the initial individuals are set randomly.

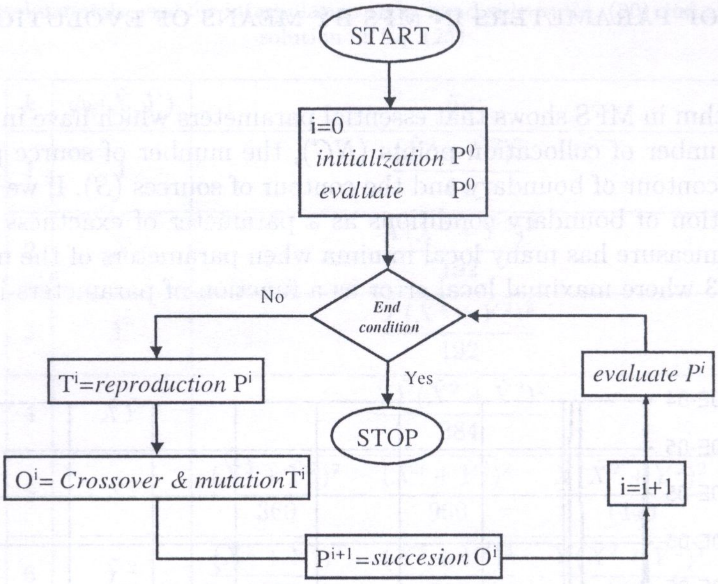

Fig. 4. Sequential evolutionary algorithm

Step 2. The fitness value of each individual is calculated. In our consideration, the fitness function has the form

$$P = \frac{1}{\sqrt{MRE}} - E \cdot P_{\max} \tag{32}$$

where MRS is the maximal error in satisfaction of boundary conditions, E is the penalty function, which is equal to 0 if all constrains and conditions are fulfilled or 1 in the opposite case, $P_{\max}$ is the value of the fitness function for the currently best solution. Some individuals (solution of BVPs) with low fitness are deleted from the population. The selection is carried out by a set of rules based on the fitness values.

Step 3. The crossover is carried out among the chromosomes of individuals. These individuals, general called "parents", are the survivors of the selection that occurs in **Step 2**. "Crossover points" are placed randomly in the chromosomes. The parts of parents' chromosomes between the crossover points are exchanged with each other to produce the offspring.

Step 4. Mutations occur at small probability in the chromosome of each of the offspring individually to produce "children". Genes of the chromosomes are changed randomly with some established probability. This mutation procedure produces children, which, because solely the crossover does not produce them, keeps the population pool diverse. The population pool of the individuals obtained in this procedure is called the "next generation".

Step 5. **Steps 2–4** are repeated until the terminating condition is satisfied. When the condition is satisfied, the individual with the maximum fitness value become the quasi-optimal solution of BVP.

The other parameters appearing in the method of fundamental solutions are not searched by evolutionary algorithm. The following values of these parameters were assumed:

– number of inner interpolation points $M = 10 \cdot 10$ points arranged uniformly in the considered region,

– number of polynomial terms $\tilde{\varphi}_k(r_m)$ — experiments were done for $K = 6$.

5. Numerical example

Numerical results are presented for simply supported square plate with a uniform load q_0. For comparison purpose the dimensions and other parameters of the plate are the same as in Sladeks' paper [19]. The side length and plate thickness are chosen as 8 and 0.03 m, respectively. The material parameters are following: Young modulus $E = 10^{12}$ N/m^2 and Poisson ratio $\nu = 0.3$. In our numerical experiments the loading parameter $\tilde{Q} = \frac{a^3 q_0}{D}$ was changed from 0.1 to 1.0. Results of numerical experiments are given in Table 3 and in Fig. 5. In Table 3, the values NC – the number of collocation points, N – the number of source points, and S – the distance between the source and the boundary contours are obtained by means of the evolutionary algorithm.

The obtained results for maximal deflection are practically the same as in the paper [19] where local boundary integral equation was applied.

Table 3. Results of numerical experiments for simply supported square plate; W_S is the maximum non-dimensional deflection, IT is the number of iterations, $\delta_{\max}$ is the maximal error in satisfaction of boundary conditions

$\tilde{Q}$	NC	N	S	$\tilde{W}_S$	IT	$\delta_{\max}$
0.1	28	20	5.69E−01	0.106	2	3.78E−06
0.2	10	10	2.59E+00	0.196	2	2.68E−06
0.3	48	12	2.47E+00	0.288	39	1.5032E−05
0.4	41	13	2.58E+00	0.350	8	7.01E−06
0.5	43	23	1.12E+00	0.422	6	7.42E−07
0.6	34	10	1.62E+00	0.470	10	2.54E−06
0.7	45	22	1.62E+00	0.518	12	1.52E−06
0.8	43	20	2.38E−01	0.564	15	1.12E−06
0.9	40	11	8.69E−01	0.606	29	2.73E−05
1.0	28	10	8.54E−01	0.640	29	4.20E−07

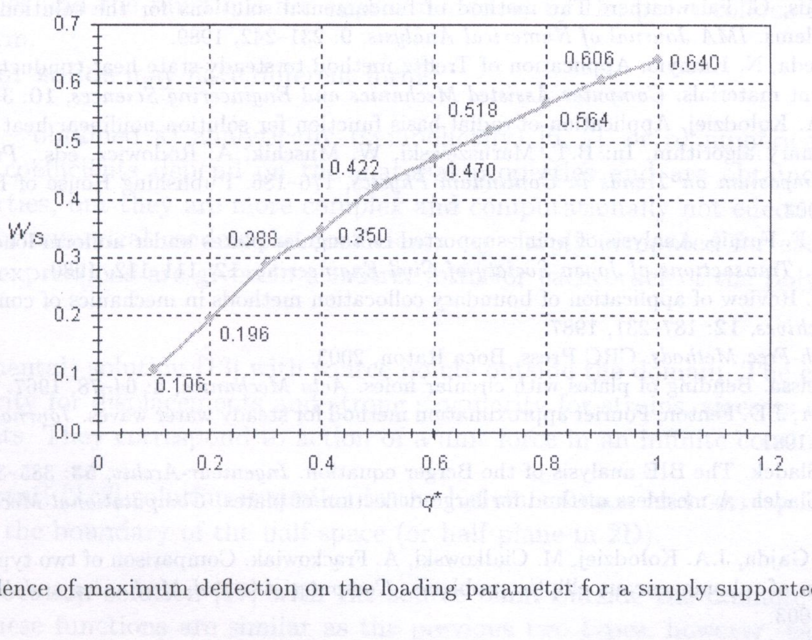

Fig. 5. Dependence of maximum deflection on the loading parameter for a simply supported square plate

6. Conclusion

In the paper, the method of fundamental solutions as a kind of Trefftz method was successfully applied to solve problems of bending of thin elastic plates with large deflection described by the Berger equation. This equation was solved approximately using algorithm of iteration proposed in the Sladeks' papers [18, 19] where they used some kinds of BEM. In our method, an inhomogeneous term is interpolated by radial basis functions. For the optimal choice of parameters of the fundamental solutions method, the evolutionary algorithm was used. Numerical results for a square plate with simply supported edges are presented. The paper presents a new application of the method of fundamental solutions to non-linear BVP with a four-order differential equation.

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