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Application of discrete wavelet transformation in damage detection. Part I: Static and dynamic experiments

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The paper demonstrates the potential of Discrete Wavelet Transform (DWT) in damage detection. Efficiency of the method is demonstrated by the way of examples. In this study the numerically simulated static and dynamic experiments were used. One dimensional DWT was used to signal processing. Measurement errors were accounted for by introduction of white noise.

Keywords: damage detection, wavelet transformation

1. Introduction

The problem of localization and estimation of structural damage is one of the most important engineering problems. It is connected with the assessment of structure safety and serviceability. This issue belongs to a wider class of inverse problems, where unknown parameters of a system are determined using analysis of experimental data. Damage detection has focused much attention in the last two decades. Different methods have been proposed in the literature. Most of them were based on comparison of the response of the damaged and undamaged structures. It was a serious difficulty since identical boundary and loading conditions must be ensured for these two structures.

First papers concerning damage detection were grounded on the modal analysis of structural response. In papers [1] and [3] authors observed that damage leads to stiffness reduction, increase of damping and decrease of natural frequency of the structure. An extensive review of damage identification techniques based on information of changes in natural frequencies and shape modes of vibrations was presented in [8]. Unfortunately, global response of a structure is insensitive to localized damage. Hence, the uncertainty following from variation of the conditions in experiments can be larger than the precision of measuring gauges. Therefore, different ways of improvement of experiments were proposed in the literature. In [7] variable location of support or additional concentrated mass enhancing structural response was studied. Optimization of loads with the aim to better exhibit the discrepancy between responses of the damaged and undamaged structures was discussed in [19, 21, 22].

Lately, new methods have been applied to structural identification. They are called soft methods: fuzzy sets, genetic algorithms, artificial neural networks. The usage of genetic algorithms in damage identification was presented in [2]. Implementation of neural networks in structural identification was discussed in [27]. Fuzzy sets, genetic algorithms and neural networks were used simultaneously in [10]. The use of the wave propagation approach combined with the genetic algorithm and the gradient technique for damage detection in beam-like structures was presented in [18]. For the analysis of elastic wave propagation Spectral Element Method (SEM) based on the Fast Fourier Transform was used in [24].

Entirely new method rooted in mathematical theory of signal analysis emerged in the last years, namely wavelet transformation. It finds wide applications in compression and recognizing of images, signal denoising and also in solving boundary value problems. It allows, among others, effective analysis of non-stationary signals. Due to multiresolution decomposition of signals it makes possible to extract local small disturbances of a signal. It has become one of most promising tools in structural identification. Wavelet transformation can surprisingly well extract the desired detailed information from a numerous data representing the global response of a damaged structure. Moreover, when wavelet transformation is used to damage identification, usually experimental data referring only to damaged structure are processed. This is a major advantage, because usually the experimental data referring to undamaged structure are not accessible and computer assessed response of the undamaged structure can be imprecise because of inaccurate modelling.

The theory of wavelet transformation has been developed by Daubechies [5, 6], Mallat [20], Chui [4]. Newland showed in the paper [23] the potential of this tool in vibration signal analysis. One of the first applications of Wavelet Transform (WT) to damage detection was discussed in [26]. Effort of quantitative damage estimation basing on the analysis of signal transform coefficients was taken up in [9]. In [15] the method of damage localization and damage extent specification using Lipschitz exponent estimated by the WT was presented. The use of Walsh-wavelet packets in initial and boundary value problem and in structural identification was presented in [13, 14]. Effectiveness of WT in damage detection on examples of static and dynamic responses of beams was demonstrated in [11, 12, 17, 25]. In practical applications WT is used in discrete formulation. The application of 1-D Discrete Wavelet Transform (DWT) to the identification of damage in 1-D and 2-D structures was discussed in [16, 17]. The efficiency of DWT in damage detection in beams for various levels of noise in experimental data was studied in [11].

Some difficulty in application of DWT to damage identification can follow from the requirement of numerous measurements, which compose a structural response signal. Special techniques must be used to record simultaneously displacements, velocities or accelerations in 32 or 64 measurements points. Alternative approach was developed in [28], where the response was measured in one point but the excitation mechanical impulse was applied to several points of a mesh at the plate surface. Next, a Fast Fourier Transform was used to obtain the structural response in frequency domain.

The present paper continues discussion on the potential of DWT in structural damage identification by consideration of different types, forms and locations of damaged zones and by allowing for white noise in the response signal as a modelling of errors in experimental measurements.

2. General information on wavelet transformation

2.1. Background of wavelet transformation

To decompose an arbitrary signal $f(x)$ into an infinite sum of wavelets at different scales a wavelet transformation according to the expansion (1) is used,

$$f(x) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} c_{j,k} \, W(2^j x - k), \tag{1}$$

where $W(x)$ is a wavelet (mother) function. Integers j and k are dilation (scale) and translation (position) indices, respectively. The terms $c_{j,k}$ are numerical constants called wavelet coefficients. When j is negative, $W(2^j x - k)$ can always be expressed as a sum of terms $\phi(x-k)$, providing

$$f(x) = \sum_{k=-\infty}^{\infty} c_{\phi,k}\phi(x-k) + \sum_{j=0}^{\infty} \sum_{k=-\infty}^{\infty} c_{j,k} \, W(2^j x - k), \tag{2}$$

where $\phi(x)$ is a scaling (father) function and $c_{\phi,k}$ is a new set of coefficients.

Wavelet transformation is most often employed in discrete form due to the numerical effectiveness of procedures of DWT. In order to set up a DWT algorithm, the range of the independent variable x is limited to one unit interval. Hence, $f(x)$ is assumed to be defined for $0 \leq x \leq 1$, where x is a non-dimensional variable. Assuming that the range $< 0,1 >$ represents one period of a periodic signal $f(x)$, the wavelet expansion can be written in the form

$$f(x) = a_0\phi(x) + \sum_j \sum_k a_{2^j+k} W(2^j x - k). \tag{3}$$

The coefficients a_{2^j+k} represent the amplitudes of subsequent wavelets. The integer j describes different levels of wavelets, starting from $j = 0$. Integer k specifies the number of wavelets at each level, so that they cover the range $k = 0$ to $2^j - 1$.

The DWT used in this paper is an algorithm for computing coefficients a_{2^j+k} when $f(x)$ is sampled at equally spaced intervals over $0 \leq x \leq 1$. Since the number of sampled values is limited, every function $f(x)$ is approximated by $f_J(x)$ using $N = 2^J$ discrete values. The scale indicator is $j = 0, 1, \ldots, J-1$. Therefore, keeping in mind (3), the discrete signal decomposition can be written in the form

$$f_J = f_\phi + f_0 + f_1 + \cdots + f_j + \cdots + f_{J-2} + f_{J-1}, \tag{4}$$

or

$$f_J = S_M + D_M + D_{M-1} + \cdots + D_m + \cdots + D_2 + D_1, \tag{5}$$

where $m = J - j$. Each component in signal representation provides information at the scale level j. The last terms in (5) correspond with the most detailed signal representation (high frequency oscillations). The preceding representations offer the more and more rough information about the signal and correspond to the lower frequency oscillations. Therefore, the above method of signal representation is called a multi-resolution analysis (MRA).

The advantage of DWT is that in practical application it requires neither integration, nor explicit knowledge of scaling (father) function and wavelet (mother) function, generating transformation and inverse transformation.

2.2. Characteristics of selected wavelets

In the following we will present selected wavelet families, their properties and main fields of application.

2.2.1. Orthogonal wavelets

Haar wavelet

Haar wavelet was discovered by Alfred Haar in 1910. From historical point of view, it is the first wavelet. It belongs to a wide group of orthogonal wavelets, that means to the group where the scaling function is orthogonal to itself with respect to its shifting, namely:

$$\int_{-\infty}^{\infty} \phi(x)\phi(x-m)\,dx = 0 \qquad \text{and} \qquad \int_{-\infty}^{\infty} \phi(x)\phi(x)\,dx = 1. \tag{6}$$

Haar wavelet is unique, symmetric, compactly supported and orthogonal. It is not continuous, therefore its applicability is limited.

Daubechies wavelet

Daubechies wavelets were invented by Ingrid Daubechies. These wavelets are asymmetrical, compactly supported, with sharp edges. They require a small number of coefficients and therefore they are widely used in image analysis. Wavelet transformation can be implemented extremely efficiently and the computer time of transformations increases only linearly with the dimension of the transformed vector.

2.2.1.1. Symmlets (Least Asymmetric Wavelets)

Symmlets wavelets were also constructed by Daubechies. Contrary to Daubechies wavelets, symmlets are as nearly symmetrical as possible. This symmetry guarantees continuity across replicas of the input and eliminates the large wavelet coefficients caused by boundary discontinuities. For these special features they are used in image coding applications.

2.2.1.2. Coiflets

Daubechies invented the coiflets under inspiration of Ronald Coifman. These wavelets are compactly supported and nearly symmetric. They are designed for a purpose of maintaining a close match between the trend values and the original signal values. They have simple sampling properties and therefore they are particularly useful in processing continuous time data.

2.2.2. Biorthogonal wavelets

Bspline wavelets

Bspline wavelets, as the name implies, have some orthogonality relationships between their filters. The main advantage in application the biorthogonal wavelets is that they permit to the use a much broader class of filters including symmetric filters.

3. Experiments in static and dynamic tests

3.1. Problem formulation

We use beam, frame or plate type models of damaged structures. The specific type of the structure does not make difference, provided that we can receive the response signal for any action (not necessarily defined). Our main task is to detect localization of damage in the structure, if such a damage exists. This localization will be determined basing on signal analysis of structural response to mechanical actions. The experiments are simulated numerically. The measurement errors are accounted for by introduction a white noise to computer simulated response signal. We consider numerical models of timber beams loaded by static forces (Fig. 1a, Fig. 1b). We use also dynamic forces for beams (Fig. 1c) and plates (Fig. 2a) made of steel, with Young modulus $E = 200$ GPa and density of the material $\varrho = 7850$ kg/m^3. In case of beams steady state harmonic vibrations without damping are considered using Bernoulli beam finite elements with diagonal mass matrix. In case of plates elastic longitudinal wave propagation with velocity $v = \sqrt{E/\varrho}$ was used employing 2D finite elements with four nodes, eight degrees of freedom. Damage in beams is modelled as local stiffness reduction (at a small prescribed region) or linear elastic hinge (with properly scaled stiffness parameter introduced at the point of damage). In the procedure of damage identification in beams various structural responses are analyzed, namely static vertical displacements and rotation angles or the amplitudes of dynamic displacement and acceleration. The structural response of this kind is a discrete signal, which is transformed using wavelet transform. In beams we used Haar and Daubechies wavelets.

In plates the rectangular, 4 nodes FEM shell elements are used and the number of elements is 80×40 in horizontal and vertical directions, respectively. Model of the specimen is presented in

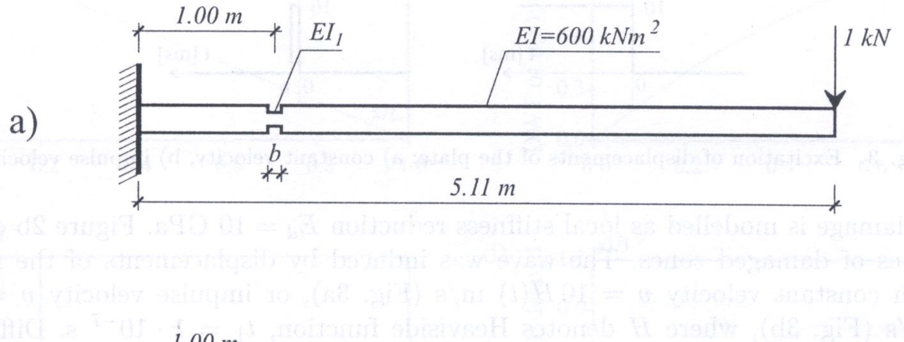

Fig. 1. Models of the beam structure: a, b) timber beams $E = 9000$ MPa, $A = 10\times20 = 200$ cm^2, c) steel beam $E = 200$ GPa, $A = 3\times6 = 18$ cm^2

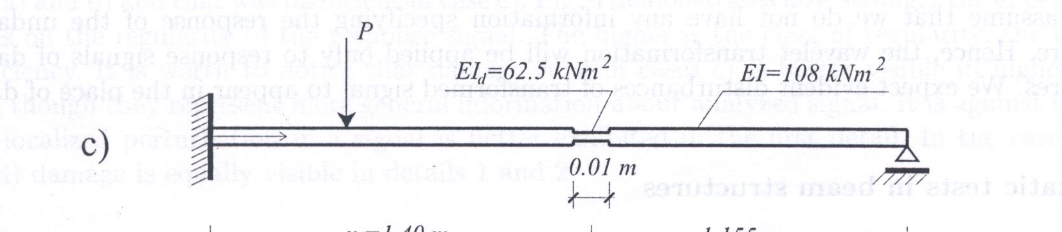

Fig. 2. Model of the plate structure with the damaged areas: a) regular shape of specimen, b) types of damage

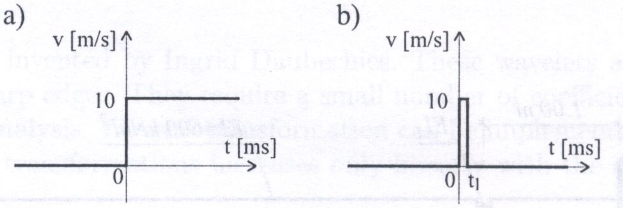

Fig. 3. Excitation of displacements of the plate: a) constant velocity, b) impulse velocity

Fig. 2a. The damage is modelled as local stiffness reduction $E_d = 10$ GPa. Figure 2b demonstrates different shapes of damaged zones. The wave was induced by displacements of the right edge of the plate with constant velocity $v = 10\,H(t)$ m/s (Fig. 3a), or impulse velocity $v = 10\,[H(t) - H(t - t_1)]$ m/s (Fig. 3b), where H denotes Heaviside function, $t_1 = 1 \cdot 10^{-7}$ s. Different signals of the structural dynamic response are analyzed, namely horizontal and vertical components of displacements (**u**), velocities (**v**) and accelerations (**a**). All values were measured in discrete points along vertical or horizontal line in the plate. These signals are processed using Discrete Wavelet Transformation (DWT). In plates wavelet called Daubechies (daublet4) is used.

In the paper we used two types of wavelets namely Haar and Daubechies. The first one is the simplest wavelet, whereas the latter one requires small number of coefficients. Both types of wavelets are orthogonal, therefore they are extremely efficient in numerical analysis. It follows from our experience that the users have to try different details and find which detail detects in the best way defects.

We assume that we do not have any information specifying the response of the undamaged structure. Hence, the wavelet transformation will be applied only to response signals of damaged structures. We expect evident disturbances of transformed signal to appear in the place of damage.

3.2. Static tests in beam structures

3.2.1. Different kinds of signals and damage models

The efficiency of damage detection depending on the kinds of measured structural response and damage will be studied. Two kinds of damage will be considered: smeared over a small region and concentrated damage, resulting in displacement functions C^1 and C^0, respectively. In this example we consider static vertical displacements and rotation angles of a cantilever beam presented in Fig. 1. The beam with stiffness $EI = 600$ kNm^2 has damage localized 1.00 m measured from the support. The damage is modelled as stiffness reduction EI_1 on the length b, or as elastic joint with stiffness k. Stiffness k of the elastic joint is defined so that the rotation in this joint is equal to the slope increment on length b. Displacements u and slopes u' were derived analytically, then the values were discretized for 512 points in the beam span (the step was 0.01 m). All signals were transformed using Haar wavelets. Graphs in Figs. 4a, b, c and d illustrate the DWT of the following discretized response functions:

a) slope function for the damage modelled as stiffness reduction ($b = 0.01$ m, $EI_1 = 205.8$ kNm^2), signal C^0,

b) slope function for the damage modelled as elastic joint ($b = 0.01$ m, $k = 51442.5$ kNm/rad), signal discontinuous,

c) vertical displacement for the damage modelled as stiffness reduction ($b = 0.02$ m, $EI_1 = 205.8$ kNm^2), signal C^1,

d) vertical displacement for the damage modelled as elastic joint ($b = 0.02$ m, $k = 10290$ kNm/rad), signal C^0.

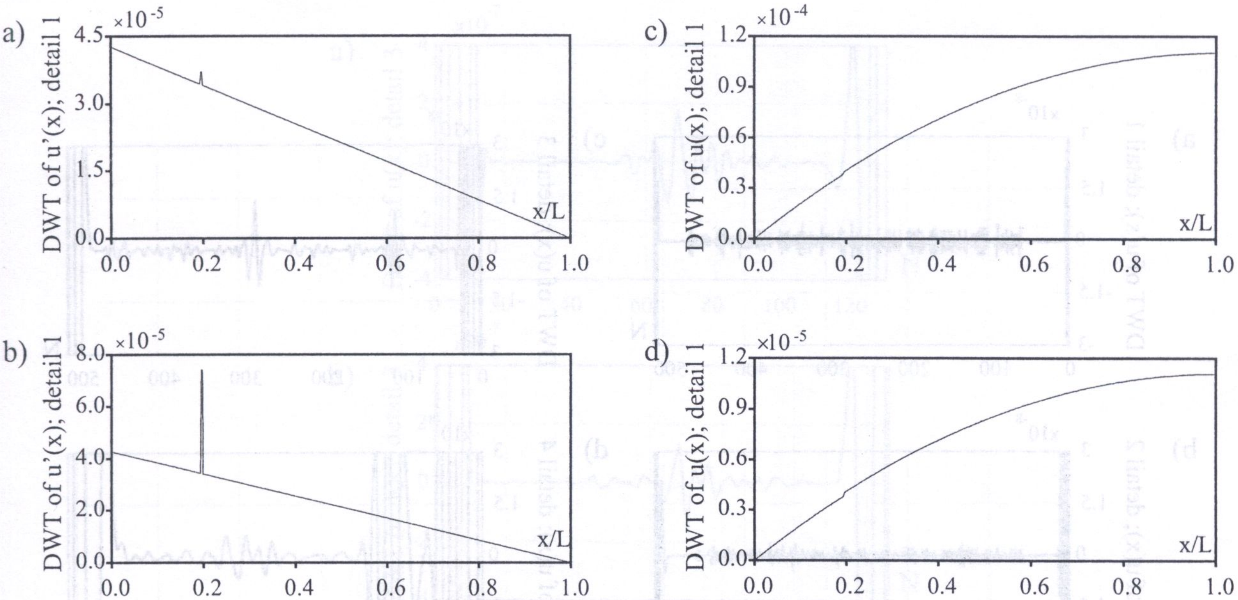

Fig. 4. Wavelet transforms (Haar wavelet, detail 1) in cases a), b), c), d)

The above graphs show that wavelet transformation properly detects damage in case of response signals a) and b) and that was inefficient in case c). Fig. 4 demonstrates how strongly the effectiveness depends on the regularity of the response signal. The higher is the class of regularity, the lower is the efficiency. It is worth to notice that damage zone in cases c) is better visible in higher order details, though they represent more general information about analyzed signal. It is against the rule that a localized perturbation in a signal is better exhibited in the first detail. In the case of the signal d) damage is equally visible in details 1 and 2.

3.2.2. Force localization and the scale level of signal decomposition

Static vertical displacements were analyzed as structural response signals. A propped cantilever beam model, presented in Fig. 1c, was loaded with a concentrated force $P = 1$ kN. The beam with bending stiffness $EI = 108$ kNm^2 has damage localized 1.40 m from the clamped end. The damage is modelled as stiffness reduction $EI_d = 62.5$ kNm^2 on the portion $b = 0.01$ m. The response signal was computed in 512 points, uniformly distributed at the longitudinal axis of the beam. Wavelet named daublet8 was used in this transformation.

In Figs. 5 and 6 details D_1, D_2, D_3, D_4 of the wavelet transformation of static response for the force localization at the points $x = 0.5$ m and $x = 1.5$ m are presented. Note in these figures, that the damage (and also the force) localization is demonstrated in the utmost evident way by the detail D_3. Defect is visible for both force positions, though greater effect was obtained for the case when the force was localized near the damage. It is worth to notice that in all graphs high disturbances at the ends of interval appeared and they increased in successive higher order signal representations.

3.2.3. Number of measurement points

One of the key problems in the damage identification is the prediction of the minimal number of measurements, required for proper data processing. Figure 7 provides the first insight into this problem. On the successive graphs 7a,b,c, the wavelet transformations of the static response represented by 128, 64 and 32 uniformly distributed measurement points are depicted.

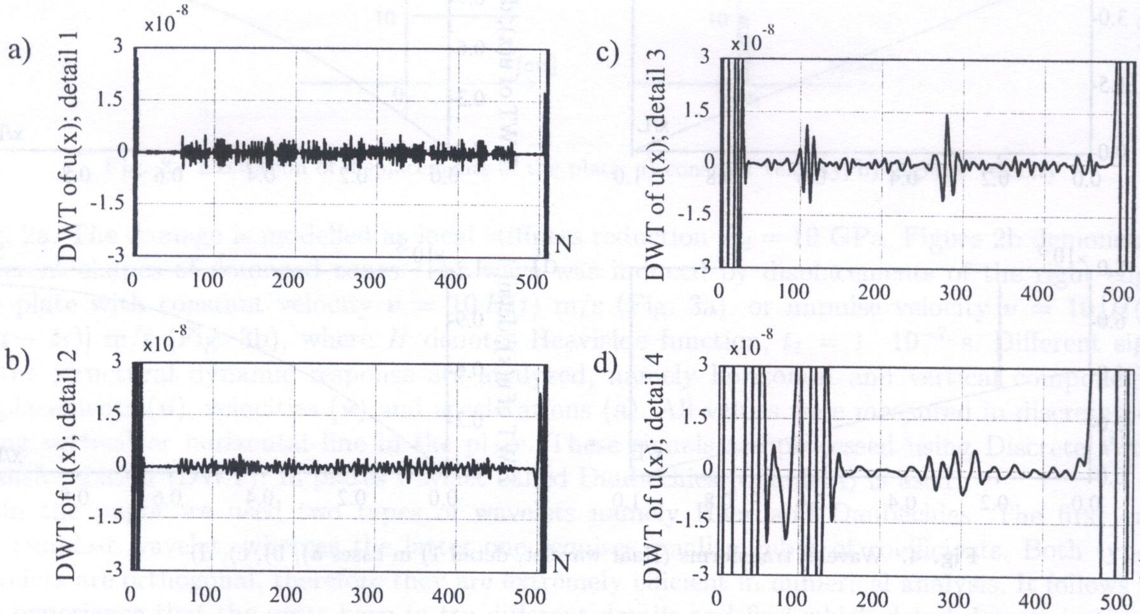

Fig. 5. Wavelet transforms using wavelet daublet8 of static displacement signal for the force localization $x = 0.5$ m, number of nodes $N = 512$: a) detail D_1, b) D_2, c) D_3, d) D_4

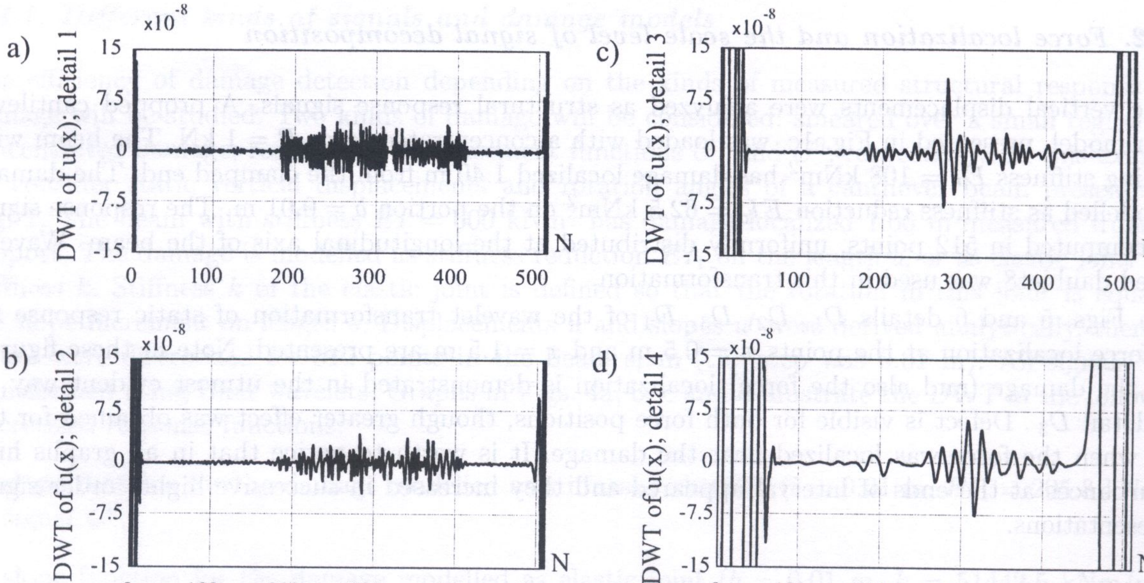

Fig. 6. Wavelet transforms using wavelet daublet8 of static displacement signal for the force localization $x = 1.5$ m, $N = 512$: a) detail D_1, b) D_2, c) D_3, d) D_4

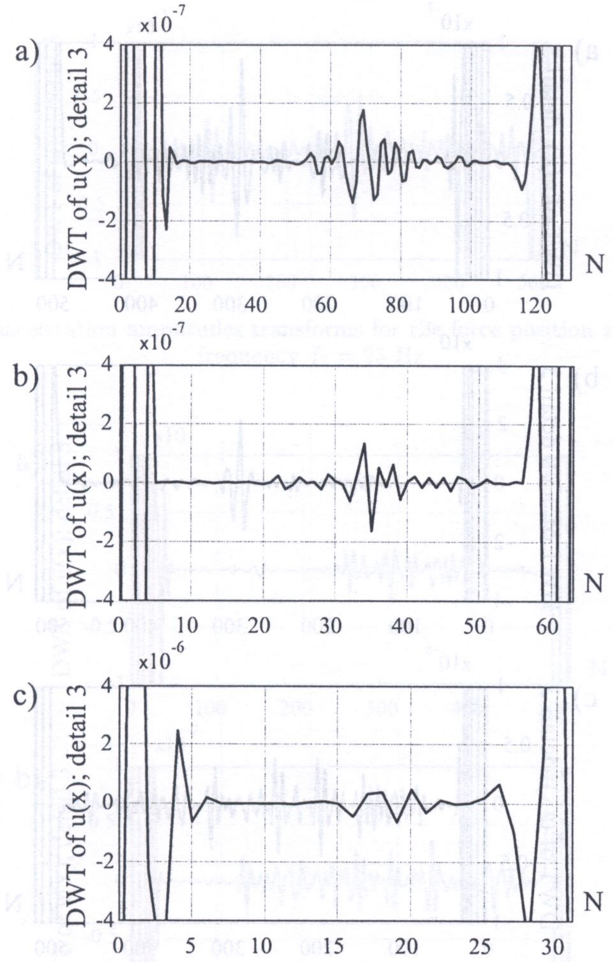

Fig. 7. Wavelet transforms of static displacement signal for the damage position $x_d = 1.4$ m and force localization $x = 1.5$ m; number of nodes: a) $N = 128$, b) $N = 64$, c) $N = 32$

Basing on Fig. 7 we can assume that in our case the minimal number of measurements was 64. However, it is difficult to provide a general rule for the number of experimental data, which is indispensable for damage identification. This number depends on many factors, for example damage quantity with reference to the rest, analyzed part of the structure. Future research will tend towards minimization of the number of measurements and improvement of tools of signal processing.

3.3. Dynamic tests of beams and plates

3.3.1. Different frequencies of periodic loading of beams

To examine the efficiency of damage identification for different signals representing dynamic structural responses, a propped cantilever beam model presented in Fig. 1c was used.

The beam with bending stiffness $EI = 108$ kNm^2 has damage localized 1.40 m from the clamped end. The damage is modelled as stiffness reduction $EI_d = 62.5$ kNm^2 on the portion $b = 0.01$ m. All dynamic responses were computed using FEM (ABAQUS program). Steady-state vibrations excited by a dynamic force $P(t) = 1\cos(2\pi ft)$ were examined. Three frequencies of dynamic excitation were assumed, namely: $f_1 = 10$ Hz, $f_2 = 25$ Hz and $f_3 = 80$ Hz. The frequency f_2 was close to the first eigenfrequency of the system (32.81 Hz), whereas the frequency f_3 was near to the second one (106.50 Hz). Concentrated force was localized in various points: $x = 0.5$ m, $x = 1.0$ m, $x = 1.5$ m and $x = 2.0$ m. As a structural response signal the amplitudes of vertical displacements and accelerations

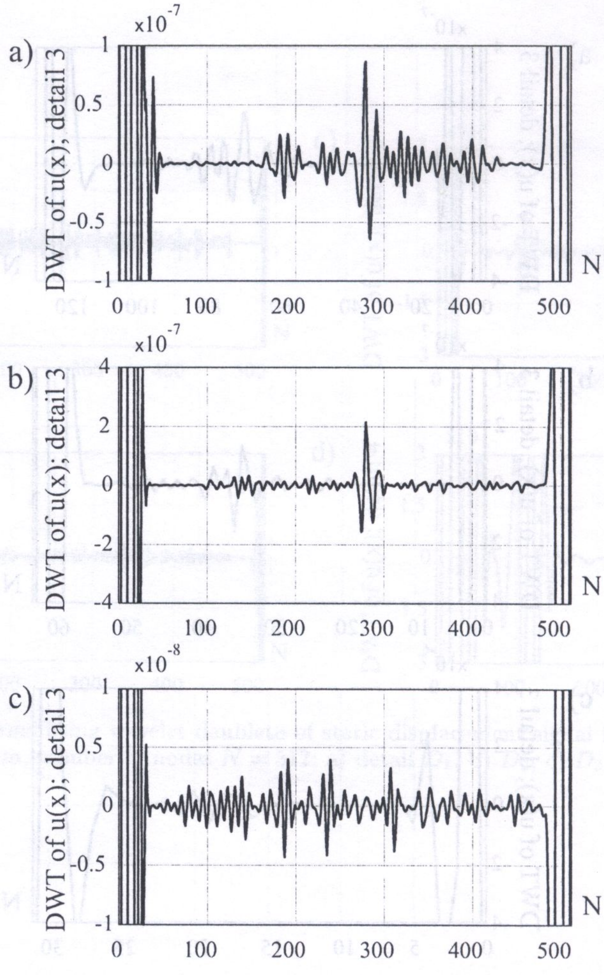

Fig. 8. Detail 3 of displacement amplitudes transforms for the harmonic force localization $x = 1.5$ m, $N = 512$ and frequencies: a) $f_1 = 10$ Hz, b) $f_2 = 25$ Hz, c) $f_3 = 80$ Hz

were analyzed. In each case the response signal was computed in 512 points, uniformly distributed at the longitudinal axis of the beam. These data were treated as respective discrete signals in the space domain and were processed using DWT. Usually, wavelets named daublet8 were used in this transformation. As a result of the transformation we obtained evident disturbances in the place of damage in the most detailed signal representations, namely D_1, D_2 or D_3.

To show the efficiency of dynamic response in damage identification, the transform of this response is presented in Fig. 8. The force P was localized at $x = 1.5$ m and vertical displacements were analyzed. On the graphs 8a, b and c, the detail 3 is presented for frequencies of excitation $f_1 = 10$ Hz, $f_2 = 25$ Hz, $f_3 = 80$ Hz, respectively. Figure 8 demonstrates that in this case damage was properly identified for the frequencies f_1 and f_2. The frequency f_3 has not been useful. Of course, it is connected with the specified damage position which was near the inflexion point of the displacement line. Next, we studied the usefulness of the response signal in the form of accelerations. Note that it is often easier to measure in situ the accelerations than the displacements.

Figure 9 illustrates the wavelet transform of the accelerations for the case f_2 identical to the case shown in Fig. 8b. Unfortunately, identification based on transform of acceleration amplitudes has not brought expected results. It needs next studies. Let us come back to the problem of a choice of excitation frequency. If we change our model so that the dynamic force is acting at the point $x = 2.0$ m and the damage will be defined at the point $x_d = 0.75$ m, proper damage identification will occur only for the frequency $f_3 = 80$ Hz. This phenomena was shown in Fig. 10. We observe that the dynamic structural response signals can be efficiently used in damage detection. The application of dynamic excitation provides more possibilities in planning the experiment.

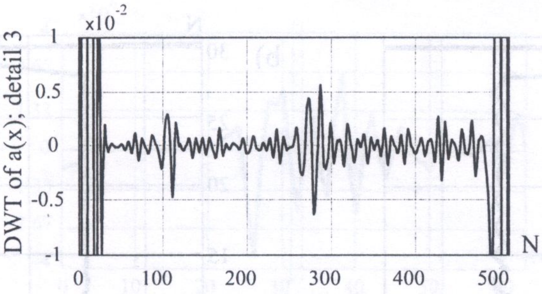

Fig. 9. Detail 3 of acceleration amplitudes transforms for the force position $x = 1.5$ m, N=512 and frequency $f_2 = 25$ Hz

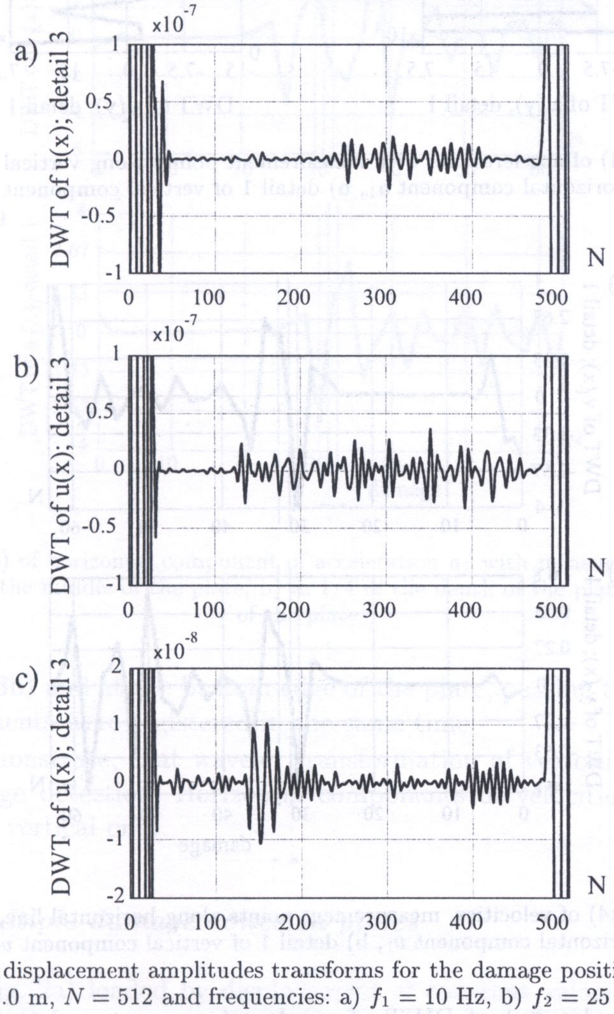

Fig. 10. Detail 3 of displacement amplitudes transforms for the damage position $x_d = 0.75$ m, force localization $x = 2.0$ m, $N = 512$ and frequencies: a) $f_1 = 10$ Hz, b) $f_2 = 25$ Hz, c) $f_3 = 80$ Hz

3.3.2. Location of the damage in plates

Henceforth, numerically simulated experiments of wave propagation will be used to damage detection. We assume that the plate is built-in rigidly at the left boundary and that the waves are induced by prescribed displacement at the right boundary. Consider a steel plate (Fig. 2a) loaded by boundary displacements of constant velocity (Fig. 3a). The damage is modelled within 3 FEM elements at the bottom edge of the plate (Fig. 2b, case 4). The measurements in all points along vertical or horizontal line were registered in the same point in time domain, when the front of the wave had just passed the damaged area. The prior numerical experiments revealed that DWT analyses at this moment were most efficient.

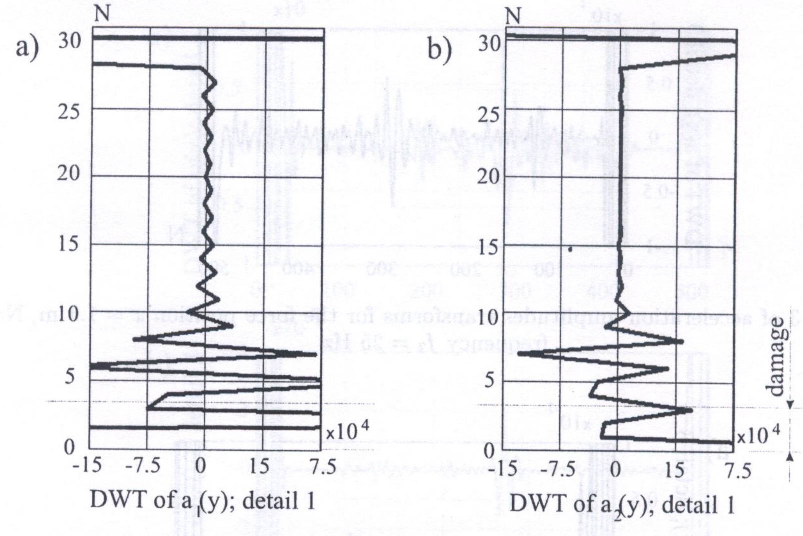

Fig. 11. DWT (daublet4) of accelerations with measurement points along vertical line, $N = 32$: a) detail 1 of horizontal component a_1, b) detail 1 of vertical component a_2

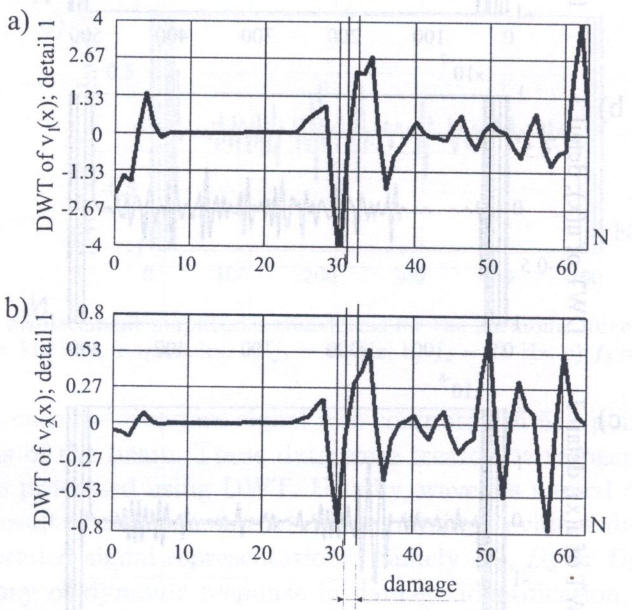

Fig. 12. DWT (daublet4) of velocities, measurement points along horizontal line, $N = 64$: a) detail 1 of horizontal component v_1, b) detail 1 of vertical component v_2

Figure 11 presents the detail 1 of DWT of accelerations measured in 32 points along vertical cross-section. We observe disturbances within the area of 10 elements, although the damaged zone was limited to 3 elements between measurement points 0 and 3, indicated at the vertical axis. This can be the effect of reflection and diffraction of the wave.

Figure 12 presents the detail 1 of DWT of velocities measured in 64 points along horizontal line at the bottom edge of the plate. It passes through the damaged area, which is between measurement points 31 and 32, indicated at the horizontal axis.

It was found that the effectiveness of damage detection strongly depends on the location of measurement points. The further they are from the damaged area the worse is the effectiveness (Fig. 13). It presents the DWT of horizontal component of acceleration a_1, when the measurement points are along the horizontal lines: in the center line (Fig. 13a), along the line at 1/4 of the

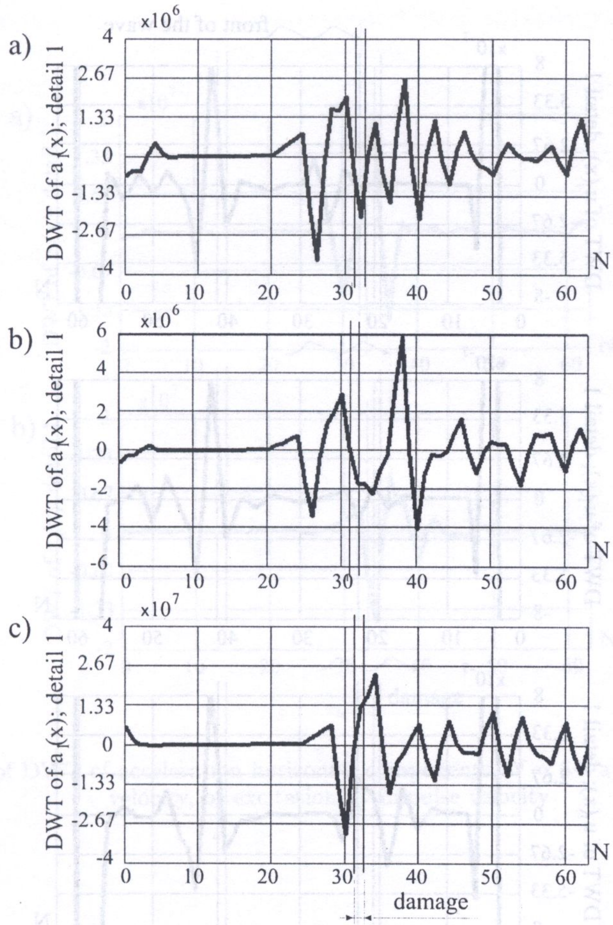

Fig. 13. DWT (daublet4) of horizontal component of acceleration a_1 with measurement points along horizontal line, $N = 64$: a) in the middle of the plate, b) in 1/4 of the depth of the plate, c) at the bottom edge of the plate

depth of the plate (Fig. 13b) and at the bottom edge of the plate, passing through the damage area (Fig. 13c). The measurements were registered in the same time.

Figures 11 and 12 demonstrate, that wavelet transformation of velocities and accelerations are equally efficient in damage detection. Horizontal components of velocities and accelerations are evidently better than the vertical ones.

3.3.3. Detection of multiple damage zones in plates

Consider a steel plate (Fig. 2a) loaded by displacement of constant velocity (Fig. 3a), induced at the right boundary of the plate. Assume that there are two damage zones in the interior of the plate (Fig. 2b, cases 1 and 2). Damage zones are located between points 19–22 and 40–41 indicated at the horizontal axis. Let the horizontal component of dynamic displacements u_1 be measured in 64 points along the horizontal line passing through the damage zones. Let these displacements be registered, in three points of time, (a) when the front of wave has passed the first damage zone and has not reached the second one, (b) and (c) when the front has passed both zones. Figure 14 presents the detail 1 of the DWT of these signals. It demonstrates that the effectiveness of damage detection strongly depends on the proper time point of measurement. In the place of the front of wave similar intensity of disturbance can be observed as in the place of damage. Therefore, the DWT of several response signals, referring to different points of time and position in space, must be used to detect multiple damage zones.

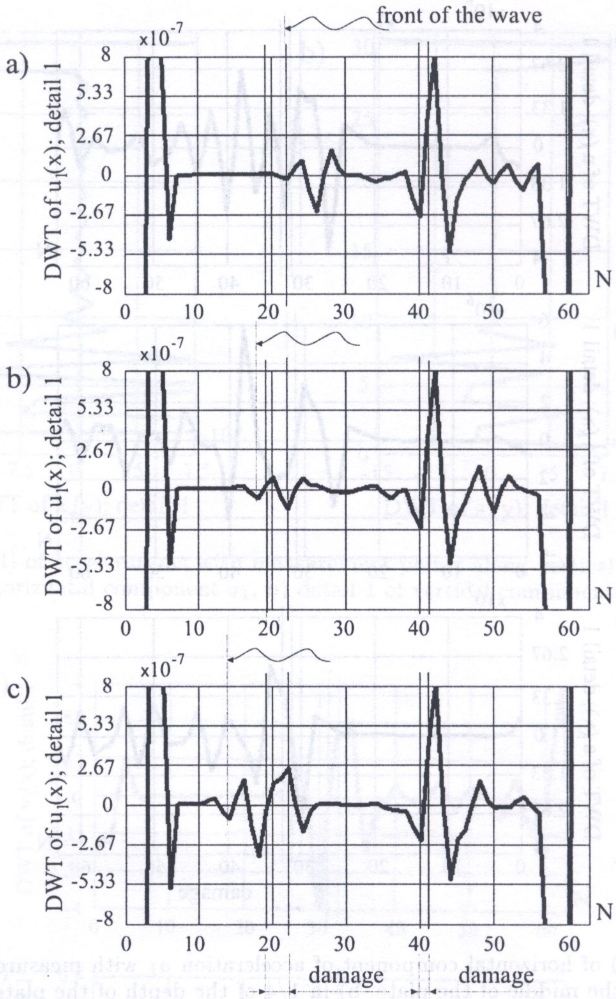

Fig. 14. Detail 1 of DWT (daublet4) of horizontal component of displacements u_1 with measurement points along horizontal line passing through damage zones, $N = 64$: a) the front of the wave has passed the first damage zone and approached the second one, b) the front of the wave has just passed the second damage zone, c) the front has moved further

3.3.4. Type of dynamic excitation

In this section we will compare the effectiveness of damage detection for different types of excitation of elastic wave, namely constant velocity (Fig. 3a) and impulse velocity (Fig. 3b).

Consider a plate shown in Fig. 2a and assume the damaged zone within 8 FEM elements, illustrated in Fig. 2b, case 3. Let the dynamic response signals be measured in the same point of time in both cases of excitation.

Figures 15 and 16 refer to the case when the response signal was measured in 64 points along horizontal line. The measurement lines passed through the damage zone. The details 1 of horizontal components of the acceleration and velocity were used, because the vertical components were less efficient.

Figures 15 and 16 prove that both types of excitation are equally useful in damage detection. It is important because execution of impulse excitation is easier in practical experiments than constant velocity excitation.

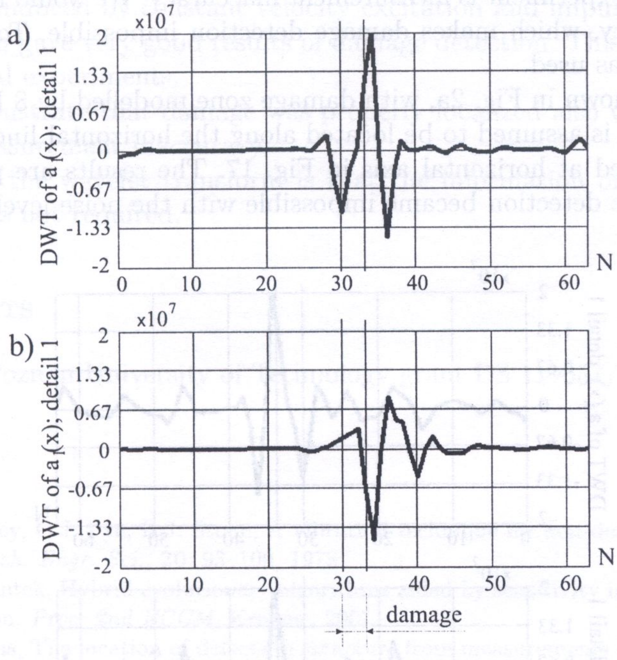

Fig. 15. Detail 1 of DWT of acceleration horizontal components, $N = 64$: a) excitation by constant velocity, b) excitation by impulse velocity

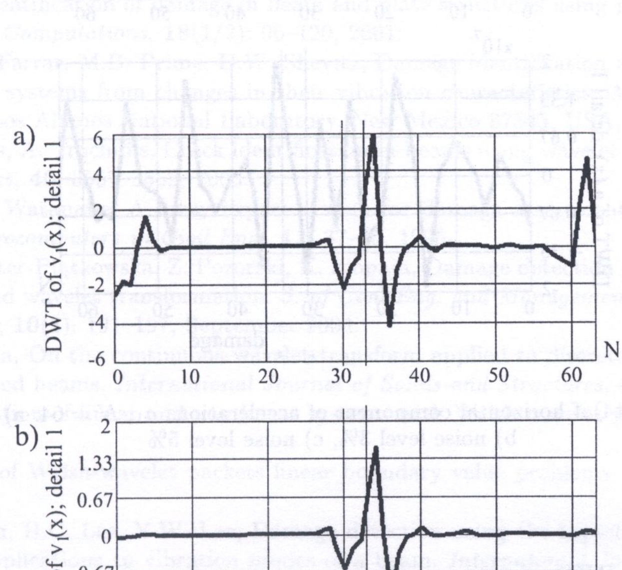

Fig. 16. Detail 1 of DWT of velocity horizontal components, $N = 64$: a) excitation by constant velocity, b) excitation by impulse velocity

3.3.5. Noise in measured data

Inevitable element of any experiment is measurement inaccuracy. We would like to estimate the level of measurement inaccuracy, which makes damage detection impossible. To model the inaccuracy, a white noise generator was used.

Consider a structure shown in Fig. 2a, with damage zone modelled by 8 FEM elements (Fig. 2b, case 3). The damage zone is assumed to be located along the horizontal line between measurement points 30 and 33, indicated at horizontal axis in Fig. 17. The results are presented in Fig. 17. It shows that proper damage detection became impossible with the noise level 5%.

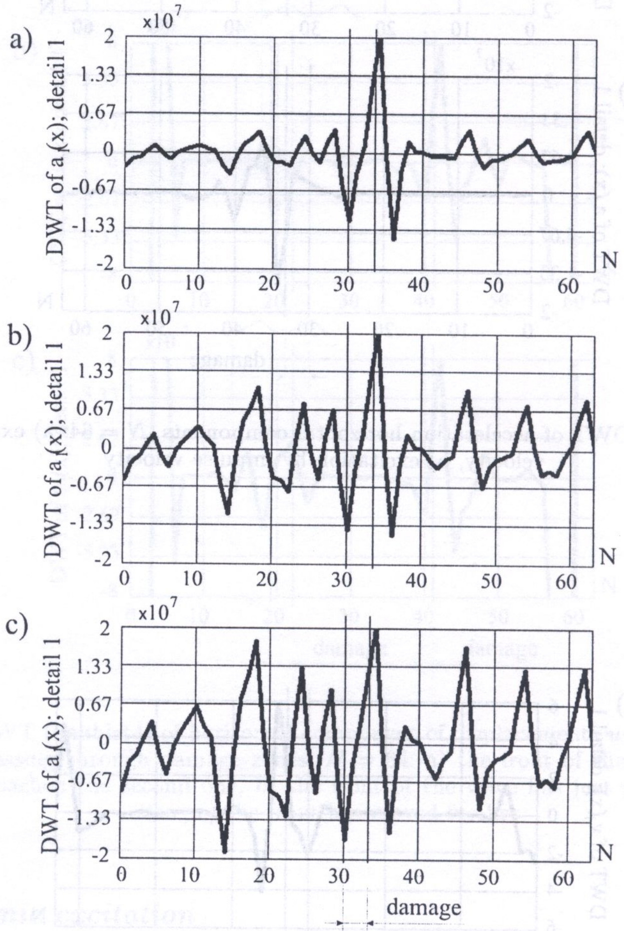

Fig. 17. DWT (daublet4)of horizontal component of accelerations a_1, $N = 64$: a) with noise level 1%, b) noise level 3%, c) noise level 5%

4. Concluding remarks

The implementation of the Discrete Wavelet Transform (DWT) to the damage detection basing on static and dynamic experiments was presented. The experiments were numerically simulated by FEM using ABAQUS system. The effectiveness of the method was studied by the way of numerous examples. The examples demonstrated that DWT provides good information on the existence and position of damage. In this paper the one dimensional DWT was used.

The examples confirmed that the effectiveness of damage localization strongly depends on the location of measurement points and on the point of time of measurement. Therefore detection of single or multiple damage zones requires the DWT processing of several structural response signals measured in various points in space and time domains. The damage was properly detected regardless of its location, in the middle or at the edge of the plate.

Elastic waves were induced by constant velocity excitation and impulse velocity excitation. The latter type of excitation gave very good results of damage detection. This type of excitation is much easier to execute in real experiments.

The examples demonstrate that damage was properly localized also with some specified level of noise, representing measurement inaccuracy.

Great advantage of the wavelet transform is that the information on the response data of the undamaged structure is not required.

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REFERENCES

[1] R.D. Adams, P. Cawley, C.J. Pye, B.J. Stone, A vibration technique for non-destructive assessing the integrity of structures. *J.of Mech. Engr. Sci.*, **20**: 93–100, 1978.

[2] T. Burczyński, P. Orantek, Hybrid evolutionary algorithms aided by sensitivity information in identification and structural optimization. *Proc. 2nd ECCM*, Kraków, 2001.

[3] P. Cawley, R.D. Adams, The location of defects in structure from measurements of natural frequencies. *J. Strain Anal.*, **14**: 49–57, 1979.

[4] C.K. Chui, An introduction to Wavelets. *Academic Press. Inc.*, San Diego, 1992.

[5] I. Daubechies, The wavelet transform, time-frequency localization and signal analysis. *IEEE Trans. on Information Theory*, **36**: 961–1005, 1990.

[6] I. Daubechies, Ten lectures on wavelets. *PA: SIAM*, Philadelphia, 1992.

[7] K. Dems, Z. Mróz, Identification of damage in beam and plate structures using parameter dependent frequency changes. *Engineering Computations*, **18**(1/2): 96–120, 2001.

[8] S.W. Doebling, C.R. Farrar, M.B. Prime, D.W. Shevitz, Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review. *Report LA-13070-MS, UC-900*, Los Alamos National Laboratory, New Mexico 87545, USA, May, 1996.

[9] E. Douka, S. Loutridis, A. Trochidis, Crack identification in beams using wavelet analysis. *International Journal of Solid and Structures*, **40**: 3557–3569, 2003.

[10] H. Furuta, J. He, E. Watanabe, A fuzzy expert system for damage assessment using genetic algorithms and neural networks. *Microcomputers in Civil Eng.*, **11**: 37–45, 1996.

[11] A. Garstecki, A. Knitter-Piątkowska, Z. Pozorski, K. Ziopaja, Damage detection using parameter dependent dynamic experiments and wavelet transformation. *J. of Civil Eng. and Management, Vilnius Gediminas Technical University, Lithuania*, **10**(3): 191–197, September 2004.

[12] A. Gentile, A. Messina, On the continuous wavelet transform applied to discrete vibrational data for detecting open cracks in damaged beams. *International Journal of Solids and Structures*, **40**: 295–315, 2003.

[13] W. Glabisz, Identification of linear and non-linear systems with Walsh wavelet packets. *Arch. Civ. Mech. Eng.*, **1**: 47–62, 2001.

[14] W. Glabisz, The use of Walsh-wavelet packets linear boundary value problems. *Comput. Struct.*, **82**: 131–141, 2004.

[15] J.-C. Hong, Y.Y. Kim, H.C. Lee, Y.W. Lee, Damage detection using the Lipschitz exponent estimated by the wavelet transform: applications to vibration modes of a beam. *International Journal of Solids and Structures*, **39**: 1803–1816, 2002.

[16] A. Knitter-Piątkowska, A. Garstecki, Wavelet transformation in damage identification by dynamic tests. *Proc. 75th Annual Scientific Conference – GAMM*, Dresden, Germany, March 21-27, 2004.

[17] A. Knitter-Piątkowska, Z. Pozorski, K. Ziopaja, A. Garstecki, Damage detection using dynamic approach and sensitivity analysis. *Proc. CMM-2003*, Gliwice/Wisła, Poland, June 3-6, 2003.

[18] M. Krawczuk, Application of spectral beam finite element with a crack and iterative search technique for damage detection. *Finite Elements in Analysis and Design*, **38**: 537–548, 2002.

[19] J. Lee, R.T. Haftka, O.H.Jr Griffin, L.T. Watson, M.D. Sensmeier, Detecting delaminations in a composite beam using anti-optimization. *Struct. Optim.*, **8**: 93–100, 1994.

[20] S. Mallat, A theory for multiresolution signal decomposition: the wavelet representation. *IEEE Trans. Pattern Anal. and Machine Intell.*, **11**: 674–693, 1989.

[21] Z. Mróz, A. Garstecki, Optimal loading conditions in design and identification of structures. Part 1: Discrete formulation. *Int. J. Struct. and Multidisciplinary Optim.*, **29**: 11–18, 2005.

[22] Z. Mróz, A. Garstecki, Optimal loading in identification and design of structures. *Proc. 2nd WCSMO*, (W. Gutkowski, Z. Mróz, Eds.), Zakopane, IFTR, 279–284, 1997.

[23] D.E. Newland, *Random vibrations, spectral and wavelet analysis.* 3rd ed., Longman, Harlow and John Wiley, New York 1993.

[24] M. Palacz, M. Krawczuk, W. Ostachowicz, The spectral finite element model for analysis of flexural-shear coupled wave propagation. Part 1: Laminated multilayer composite. *Composite Structures*, **68**: 37–44, 2005.

[25] Q. Wang, X. Deng, Damage detection with spatial wavelets. *J. Solid and Struct.*, **36**: 3443–3468, 1999.

[26] W.J. Wang, P.D. McFadden, Application of wavelets to gearbox vibration signals for fault detection. *Journal of Sound and Vibration*, **192**: 927–939, 1996.

[27] Z. Waszczyszyn, L. Ziemiański, Neural networks in mechanics of structures and materials - new results and prospects of applications. *Computers and Structures*, **79**(22–25): 2261–2276, September, 2001.

[28] K. Wilde, M. Rucka, Damage detection in rectangular plates by continuous two-dimensional wavelet transformation. *Eurodyn 2005 Conference*, Paris, France, September 4-7, 2005.